# Compressibility of the electron gas: Analytical results for width effects within the Hartree-Fock approximation

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(Received 8 September 1995)

We present analytical results for the compressibility of the interacting electron gas within the Hartree-Fock approximation. For the three-dimensional and the ideally two-dimensional electron gas, the well-known results from the literature are found. For the finite width effects in the two-dimensional (quantum well) and the one-dimensional (quantum wire) electron gas, analytical results are described. The explicit form of the interaction potential between the charged carriers enters into the expression for the compressibility: finite-width effects reduce the corrections induced by interaction effects. Our predictions could be tested by capacitance measurements.

## I. INTRODUCTION

For the interacting three-dimensional electron gas, it is well known that analytical results for the ground-state energy can be obtained within the Hartree-Fock approximation (HFA).[1,2] The same is true for the ideally two-dimensional electron gas, where extension effects are neglected.[3] The compressibility also can be calculated analytically. For heterostructures and quantum wells[4–6] and for quasi-one-dimensional systems,[7] analytical results for the exchange energy are not available. In general, one calculates the exchange energy for such systems, and the compressibility is obtained by the first and second (numerical) derivative of the exchange energy with respect to the carrier density.[5,6] The HFA is, as the lowest-level approximation for many-body effects, of considerable importance, and research on this topic is still going on.[8]

The influence of many-body effects on the compressibility has recently been measured in GaAs quantum wells,[6] and within these measurements the importance of exchange effects has been demonstrated. Finite-width effects for the compressibility concerning these experiments have been discussed in Ref. 5. However, the compressibility was calculated by the method described above.

In this paper we present a direct method to calculate the compressibility, which uses the fact that the exchange energy is given in terms of the static structure factor, and that the static structure factor depends on the electron density. We find that in one-dimensional systems the static structure factor is such a simple function that we obtain an analytical result for the compressibility. Moreover, for two-dimensional systems we derive an equation which can be used very easily by experimenters, and which allows them to calculate finite-width effects for the compressibility by evaluating an integral. In addition, an approximate expression is derived which is in good agreement with the exact result.

The paper is organized as follows. The model and theory are described in Sec. II. In Sec. III we present the analytical and numerical results. We discuss our theory in comparison to experiments in Sec. IV, and we conclude in Sec. V. In the Appendix we give some additional results for systems where the confinement depends on the electron density.

## II. MODEL AND THEORY

For an interacting electron gas in $d$ dimensions, the density parameter $r_s$ is given by the carrier density $N_d$ as $r_s=[3/4\pi N_3 a^{*3}]^{1/3}$, $r_s=[1/\pi N_2 a^{*2}]^{1/2}$, and $r_s=1/2N_1 a^*$, respectively. $r_s$ is the mean particle distance in units of the effective Bohr radius. $a^*=\varepsilon_L/m^*e^2$ is the effective Bohr radius defined with the effective electron mass $m^*$, background dielectric constant $\varepsilon_L$, and electron charge $e$. For the Planck constant we use $h/2\pi=1$. The energy scale is the effective Rydberg, defined by $\mathrm{Ry}^*=1/2m^*a^{*2}$. The electron densities $N_d$ define the Fermi wave number $k_F$ via $N_3=g_v k_F^3/3\pi^2$, $N_2=g_v k_F^2/2\pi$, and $N_1=2g_v k_F/\pi$. $g_v$ is the valley degeneracy. The Fourier transform of the interaction potential between the carriers is written as $V(q)$. Explicitly, we use $V(q)=4\pi e^2/\varepsilon_L q^2$ for $d=3$, $V(q)=2\pi e^2 F(qb)/\varepsilon_L q$ for $d=2$, and $V(q)=e^2 f(qb)/2\varepsilon_L$ for $d=1$. $F(qb)$ is the form factor for width effects and for the ideally two-dimensional electron gas, where width effects are neglected $F(qb)=1$. $b$ is the width parameter of the quantum well. For quasi-one-dimensional systems, the width effects are always important, and are described by $f(qb)$. $b$ is the width parameter of the wire. We assume that the width parameter does not depend on the electron density. In systems where $b$ depends on the electron density, our results become quite complicated, and are given in the Appendix.

Within the HFA the ground-state energy $\varepsilon_{\mathrm{HFA}}$ per particle can be expressed as[1,2]

$$\varepsilon_{\mathrm{HFA}}(r_s)=\varepsilon_{\mathrm{kin}}(r_s)+\varepsilon_{\mathrm{ex}}(r_s). \tag{1}$$

The kinetic energy per particle of a $d$-dimensional electron gas is given as $\varepsilon_{\mathrm{kin}}(r_s)/\mathrm{Ry}^*=C(d)/r_s^2$, with $C(3)=2.2099/g_v^{2/3}$, $C(2)=1/g_v$, and $C(1)=0.2056/g_v^2$. The exchange energy is calculated by taking into account the Coulomb interaction between electrons and the exchange hole due to the Fermi statistic. Our calculation holds for dimensions $d=1$, 2, and 3. In order to obtain analytical results for the compressibility, we do not take into account zero momentum transfer processes resulting from the local non-neutrality in systems with finite width. Our exchange energy does not include the Hartree energy and higher-order terms (with band filling, the electron wave functions in the confinement directions are slightly modified). Within these approximations the exchange energy per particle is given by

$$\varepsilon_{\rm ex}(r_s)=-\frac{1}{2L^d}\sum_{q\neq 0} V(q)[1-S_0(q)], \qquad (2a)$$

where $S_0(q)$ is the static form factor[1] or the static structure factor[2] of the free-electron gas. $L$ is the length of the system. $S_0(q)$ depends only on the variable $y=q/2k_F$, and can be expressed as

$$S_0(q\neq 0)=1-\sum_{\mathbf{k}} n_{\mathbf{q}} n_{\mathbf{k}+\mathbf{q}}/N. \qquad (2b)$$

$n_{\mathbf{q}}$ is the Fermi distribution function at temperature zero defined as $n_{\mathbf{q}}=2g_v$ if $|\mathbf{q}|\leqslant k_F$ and $n_{\mathbf{q}}=0$ if $|\mathbf{q}|>k_F$ and $N$ is the number of particles.

The compressibility $\kappa_d$ is given as[2]

$$\frac{1}{\kappa_d}=\frac{N_d r_s}{d^2}\left\{(1-d)\frac{d\varepsilon}{dr_s}+r_s\frac{d^2\varepsilon}{dr_s^2}\right\}. \qquad (3a)$$

According to Eq. (1) the compressibility $\kappa_{{\rm HFA}d}$ in the HFA is given as

$$\frac{1}{\kappa_{{\rm HFA}d}}=\frac{1}{\kappa_{0d}}-\frac{N_d r_s}{d^2}\left\{(d-1)\frac{d\varepsilon_{\rm ex}}{dr_s}-r_s\frac{d^2\varepsilon_{\rm ex}}{dr_s^2}\right\}, \qquad (3b)$$

with $\kappa_{0d}=d^2/[2(2+d)N_d\varepsilon_{\rm kin}(r_s)]=m^*d^2r_s^2a^{*2}/[(2+d)N_dC(d)]$ as the compressibility of the free-electron gas, and with $m^*=1/2{\rm Ry}^*a^{*2}$. One obtains $\kappa_{03}=4\pi(4/9\pi)^{2/3}g_v^{2/3}r_s^5a^{*5}m^*$ for $d=3$, $\kappa_{02}=\pi g_v r_s^4a^{*4}m^*$ for $d=2$, and $\kappa_{01}=32g_v^2r_s^3a^{*3}m^*/\pi^2$ for $d=1$.

We note that $d\varepsilon_{\rm ex}/dr_s\propto d\varepsilon_{\rm ex}/dk_F\propto dS_0/dk_F\propto dS_0/dy$, and that an additional $r_s$ dependence results from the fact that $1-S_0(q)=0$ for $q>2k_F$. For $d=1$, 2, and 3, Eq. (3b) can be written as

$$\frac{\kappa_{0d}}{\kappa_{{\rm HFA}d}}=1-\frac{2^{d-2}k_F^d}{(2\pi)^d(2+d)\varepsilon_{\rm kin}(r_s)} \times\int d^dy V(2k_Fy)y\left\{(d-1)\frac{dS_0(y)}{dy}-y\frac{d^2S_0(y)}{d^2y}\right\} \qquad (4)$$

and $y=q/2k_F$. Explicitly we find

$$\frac{\kappa_{03}}{\kappa_{\rm HFA3}}=1-\frac{2}{3\pi}\left(\frac{4g_v}{9\pi}\right)^{1/3} r_s\int_0^1 dy\; y\{2S_0'(y)-yS_0''(y)\} \qquad (5a)$$

for three dimensions,

$$\frac{\kappa_{02}}{\kappa_{\rm HFA2}}=1-\frac{(2g_v)^{1/2}}{4} r_s\int_0^1 dy\; yF(2k_Fby)\{S_0'(y)-yS_0''(y)\} \qquad (5b)$$

for two dimensions, and

$$\frac{\kappa_{01}}{\kappa_{\rm HFA1}}=1+\frac{2g_v}{\pi^2} r_s\int_0^1 dy\; y^2 f(2k_Fby)S_0''(y) \qquad (5c)$$

for quasi-one-dimensional systems.

## III. RESULTS AND DISCUSSION

### A. Analytical results

We note that $S_0(y\neq 0)=3y/2-y^3/2$ for $y\leqslant 1$ and $S_0(y)=1$ for $y>1$ in three dimensions, $S_0(y\neq 0)=2\{\arcsin(y)+y(1-y^2)^{1/2}\}/\pi$ for $y\leqslant 1$ and $S_0(y)=1$ for $y>1$ in two dimensions, and $S_0(y\neq 0)=|y|$ for $|y|\leqslant 1$ and $S_0(y)=1$ for $|y|>1$ in one dimension. Accordingly, for $y\leqslant 1$, we get for $(d-1)dS_0(y)/dy-yd^2S_0(y)/d^2y$ the simple expressions 3 for $d=3$, $4/[\pi(1-y^2)^{1/2}]$ for $d=2$, and $y\delta(y-1)$ for $d=1$. For $y>1$ we find 0 in all dimensions. It is the simple form $S_0''(y)=-\delta(y-1)$ for one dimension, which allows us to derive analytical results for quasi-one-dimensional systems; see Eq. (5c).

With our analytical results we can calculate $\kappa_{0d}/\kappa_{{\rm HFA}d}$ in an explicit form. We find[2]

$$\frac{\kappa_{03}}{\kappa_{\rm HFA3}}=1-\frac{1}{\pi}\left(\frac{4g_v}{9\pi}\right)^{1/3} r_s \qquad (6)$$

for three dimensions,

$$\frac{\kappa_{02}}{\kappa_{\rm HFA2}}=1-\frac{(2g_v)^{1/2}}{\pi} r_s\int_0^1 dy\; F(2k_Fby)\frac{y}{(1-y^2)^{1/2}} \qquad (7)$$

for two dimensions, and

$$\frac{\kappa_{01}}{\kappa_{\rm HFA1}}=1-\frac{2g_v}{\pi^2} r_s f(2k_Fb) \qquad (8)$$

for quasi-one-dimensional systems. Equation (7) for two-dimensional systems and Eq. (8) for one-dimensional systems are the fundamental results of this paper. Note that a negative compressibility for large $r_s$ does not imply an instability: the positive background charge in the jellium model stabilizes the system. A negative compressibility of the electronic systems was seen in the experiment.[6]

For ideally two-dimensional systems, where $F(qb)=1$, with $\int_0^1 dy\; y/(1-y^2)^{1/2}=1$ we obtain the well-known result

$$\frac{\kappa_{02}}{\kappa_{\rm HFA2}}=1-\frac{(2g_v)^{1/2}}{\pi} r_s. \qquad (9)$$

In Eq. (7) the term $y/(1-y^2)^{1/2}$ becomes singular for $y=1$. Therefore, we replace $F(2k_Fby)$ in Eq. (7) by $F(2k_Fb)$, and obtain the approximate result

$$\frac{\kappa_{02}}{\kappa_{\rm HFA2}}=1-\frac{(2g_v)^{1/2}}{\pi} r_sF(2k_Fb), \qquad (10)$$

which gives experimenters a good estimate about the importance of width effects. In the following we present some numerical results for quantum wells and quantum wires.

### B. Numerical results: Two-dimensional systems

In quantum wells (QW's) of width $w$, the form factor with $x=qw$ for the width effects can be written as[9]

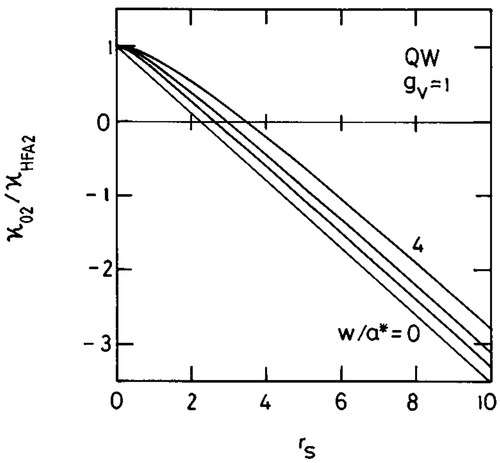


FIG. 1. Inverse compressibility $1/\kappa_{HFA2}$ (in units of the inverse compressibility of the free-electron gas $1/\kappa_{02}$) vs random phase approximation (RPA) parameter $r_s$ of a quantum well of width $w$ for $w=4a^*$, $w=2a^*$, $w=a^*$, and $w=0$.

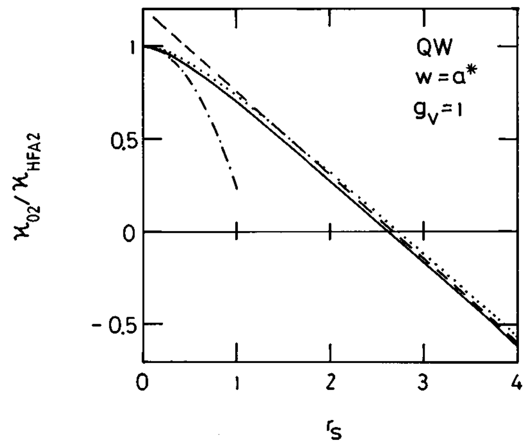


FIG. 2. Inverse compressibility $1/\kappa_{HFA2}$ (in units of the inverse compressibility of the free-electron gas $1/\kappa_{02}$) vs RPA parameter $r_s$ of a quantum well of width $w=a^*$. The solid line corresponds to Eq. (7). The dotted line corresponds to the approximate expression according to Eq. (10). The dashed and dashed-dotted lines correspond to Eqs. (12) and (13).

$$F(x)=\frac{1}{4\pi^2+x^2}\left\{3x+\frac{8\pi^2}{x}-\frac{32\pi^4}{x^2}\frac{1-\exp(-x)}{4\pi^2+x^2}\right\}. \tag{11}$$

Numerical results for the inverse compressibility of quantum wells according to Eq. (7) are shown in Fig. 1 for different well widths. $1/\kappa_{HFA2}$ increases with increasing well width. This means that many-body effects are weaker in quantum wells with larger widths.

Equation (7) can be used to obtain asymptotical results for large and small densities. With $F(x\to 0)=1-(1/3-5/4\pi^2)x+O(x^2)$,[9] for $2k_Fw\ll 1$ $[r_s\gg r_s^*=(8/g_v)^{1/2}w/a^*]$ we obtain

$$\frac{\kappa_{02}}{\kappa_{HFA2}}=1-\frac{(2g_v)^{1/2}}{\pi}r_s\left\{1+(2/g_v)^{1/2}\left(\frac{5}{8\pi}-\frac{\pi}{6}\right)\frac{w}{r_sa^*}\right\}. \tag{12}$$

With $F(x\to\infty)=3/x$, for $2k_Fw\gg 1$ $[r_s\ll r_s^*]$ we find

$$\frac{\kappa_{02}}{\kappa_{HFA2}}=1-\frac{3}{4}g_v\frac{a^*}{w}r_s^2, \tag{13}$$

which is the expression for large $w$ and (or) large density. Equations (12) and (13) are valid for a two-dimensional electron gas with only the lowest subband occupied: the intersubband energy $E_{21}\approx 3\mathrm{Ry}^*a^{*2}\pi^2/w^2$ decreases with increasing $w$, and $E_{21}$ must be larger than the Fermi energy $\varepsilon_F$ in order that the one-subband calculation is still valid. This condition leads to $k_Fw<3^{1/2}\pi\approx 5.4$. This condition works against the validity range of Eq. (13): $k_Fw\gg 1/2$. We conclude that the validity range of Eq. (13) becomes $0.5\ll k_Fw<5.4$ $(0.26\leq r_sa^*/w\ll 2.83)$. For $k_Fw>5.4$ the one-subband approximation is not justified, and all subbands have to be taken into account. Equation (13) has a very small range of validity; see Fig. 2 for $w=a^*$. Nevertheless, we think Eq. (13) shows that finite-width effects cannot be neglected at high density: compare Eqs. (13) and (9).

In Fig. 2 we compare the various approximate expressions for the inverse compressibility of a quantum well of width $w=a^*$. We mention that the analytical result [Eq. (10)] is in very good agreement with the exact result [Eq. (7)]; see Fig. 2. Equation (12) is also in very good agreement with the exact result for $r_s>1$. In Fig. 2 the relevant parameter is $r_s^*=2.8$.

Some analytical and numerical results for heterostructures, where the confinement parameter depends on the electron density, are given in the Appendix.

### C. Numerical results: Quasi-one-dimensional systems

For quasi-one-dimensional systems we study two models. First, we study cylindrical wires (CW's) where the confinement potential is zero for $|\mathbf{r}|<R_0$ and infinite for $|\mathbf{r}|>R_0$. The width parameter $b$ corresponds to $R_0$. The form factor with $x=qR_0$ is written as[10]

$$f(x)=\frac{144}{x^2}\left[\frac{1}{10}-\frac{2}{3x^2}+\frac{32}{3x^4}-64\frac{I_3(|x|)K_3(|x|)}{x^4}\right]. \tag{14}$$

$I_3(x)$ and $K_3(x)$ are the modified Bessel functions of order 3.[11] Numerical results for the inverse compressibility versus $r_s$ are shown in Fig. 3 for different wire radii. The solid dots in Fig. 3 represent the inverse compressibility for $R_0=a^*$, when correlation effects are taken into account.[12] Note that the results within the HFA are in reasonable agreement with the solid dots in order to argue that the HFA can be used to estimate many-body effects for the compressibility. The condition that only the lowest subband is occupied is expressed as $r_s>0.25R_0/g_va^*$.

Second, we study quantum wires where the confinement potential is described by an oscillator potential (OW) with width parameter $c$. The form factor with $x=qc$ is written as[13]

$$f(x)=2E_1(x^2)\exp(x^2), \tag{15}$$

where $E_1(x)$ is related to the exponential-integral function.[11] Numerical results for the inverse compressibility for the oscillator confinement are given in Fig. 4. Note that many-body

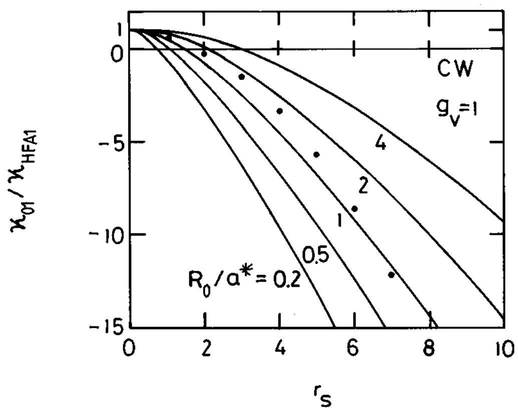


FIG. 3. Inverse compressibility $1/\kappa_{\mathrm{HFA1}}$ (in units of the inverse compressibility of the free-electron gas $1/\kappa_{01}$) vs RPA parameter $r_s$ of a cylindrical quantum wire of width $R_0$ for $R_0=4a^*$, $R_0=2a^*$, $R_0=a^*$, $R_0=a^*/2$, and $R_0=a^*/5$. The solid dots represent the inverse compressibility for $R_0=a^*$ with exchange and correlation taken into account (Ref. 12).

effects are somewhat smaller in wires with oscillator confinement than in wires with infinite confinement for $|\mathbf{r}|>R_0$; compare Fig. 3 with Fig. 4 for $c=R_0$. For the oscillator model the condition that only the lowest subband is occupied can be written as $r_s>\pi c/4g_v a^*$.

## IV. COMPARISON WITH EXPERIMENTS

Within the HFA the correlation effects are neglected, and this approximation is only valid for $r_s<1$. In general, however, it is fair to say that correlation effects, important for $r_s>1$, do not dramatically modify $\kappa_{0d}/\kappa_d$; see Refs. 2, 5, and 12, and our Fig. 3. The analytical results presented in this paper can be used to estimate the importance of many-body effects, and should be helpful to experimenters. Of course, the main motivation of this paper was to calculate the effects of a finite width in low-dimensional systems. Our results are for zero temperature; finite-temperature effects can be calculated following the lines given in Ref. 8.

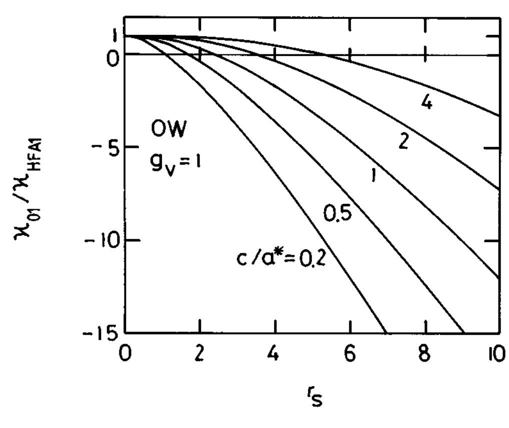


FIG. 4. Inverse compressibility $1/\kappa_{\mathrm{HFA1}}$ (in units of the inverse compressibility of the free-electron gas $1/\kappa_{01}$) vs the RPA parameter $r_s$ of a quantum wire with an oscillator confinement of width parameter $c$ for $c=4a^*$, $c=2a^*$, $c=a^*$, $c=a^*/2$, and $c=a^*/5$.

In confined systems the electron density can be large, and can be varied within one sample. In GaAs quantum wells with $a^*=100$ Å, the value $r_s=1$ corresponds to a density $N=3.2\times10^{11}$ cm$^{-2}$. Experiments are made in the density range $N=(0.5–7)\times10^{11}$ cm$^{-2}$ (corresponding to $0.7<r_s<2.5$) with $w\sim(1–3)a^*$. These facts open up a systematic study of many-body effects. In general one finds that $1/\kappa<1/\kappa_{\mathrm{HFA}}$ for $d=3$ due to correlation effects. However, in confined systems the correlation energy[4,5] is no longer a monotonic function of the carrier density, and in systems with finite width the relation $1/\kappa<1/\kappa_{\mathrm{HFA}}$ is not always valid. For details of quantum wire systems, where correlation effects are taken into account, see Ref. 12.

Information about the compressibility is available via capacitance measurements.[3] The capacitance $C_d$ per volume $L^d$ of the interacting electron gas is a function of the chemical potential $\mu$: $L^d/C_d=(1/e^2)d\mu/dN_d$.[14] With $1/\kappa_d=N_d^2 d\mu/dN_d$,[2] one finds

$$\frac{L^d}{C_d}=\frac{1}{e^2\rho_{0d}(\varepsilon_F)}\frac{\kappa_{0d}}{\kappa_d}. \tag{16}$$

$\varepsilon_F$ is the Fermi energy, and $\rho_{0d}(\varepsilon_F)$ the density of states of the free-electron gas at the Fermi energy $\varepsilon_F$: $\rho_{0d}(\varepsilon_F)=N_d d/2\varepsilon_F$. Neglecting many-body effects ($\kappa_{0d}/\kappa_d=1$), one obtains $C_d\propto\rho_{0d}(\varepsilon_F)$ with $\rho_{02}(\varepsilon_F)=$const, and $\rho_{01}(\varepsilon_F)\propto1/\varepsilon_F^{1/2}$. Note that if $\kappa_{0d}/\kappa_d<0$, the capacitance also becomes negative.

Capacitance measurements of two-dimensional electron systems[6,14] indicate that information about many-body effects can be obtained by such measurements. Capacitance measurements of quantum wires have been used to obtain information about the subband structure (the density of states) and modifications of this subband structure with increasing magnetic field.[15,16] $\kappa_{01}/\kappa_1=1$ is used for the analysis of these experiments. This is justified for large wire radius and large density. We suggest that capacitance measurements of quantum wires with only one subband occupied could be used to study many-body effects (the factor $\kappa_{01}/\kappa_1$) in one-dimensional systems. We note, however, that the Hartree contribution, which depends on the distribution of the dopant ions, have been neglected in our calculation. We also mention that $\rho_{01}(\varepsilon_F)$ is not constant if the density varies; however, this factor is independent of the confinement, while $\kappa_{01}/\kappa_1$ depends on the confinement and the carrier density. This might help to interpret experimental results using Eq. (16).

## V. CONCLUSION

We have derived quasianalytical results for the compressibility of two-dimensional interacting electron gases with finite width, and analytical results for the compressibility of quasi-one-dimensional interacting electron gases. Our results should be useful for experimenters, and can be tested in experiments via capacitance measurements.

## ACKNOWLEDGMENTS

The ''Laboratoire de Physique des Solides (ERS 111)'' is ''Laboratoire associé au Centre National de la Recherche Scientifique (CNRS).''

## APPENDIX

### 1. Two-dimensional systems

In order to derive Eq. (5b) for two-dimensional systems, we assumed that the width parameter is independent of the electron density. If the width parameter is density dependent, we derive

$$\frac{\kappa_{02}}{\kappa_{\mathrm{HFA2}}}=1-\frac{g_v^{1/2}r_s}{2^{3/2}}\int_0^1 dy\ F(2k_Fby)K(r_s,y), \quad \text{(A1a)}$$

with

$$K(r_s,y)=\{\alpha(r_s)yS_0'(y)+\beta(r_s)y^2S_0''(y)+\gamma(r_s)[1-S_0(y)]\}. \quad \text{(A1b)}$$

The coefficients $\alpha$, $\beta$, and $\gamma$ are expressed as

$$\alpha(r_s)=1+\frac{3r_sb'}{b}-\frac{4r_s^2b'^2}{b^2}+\frac{r_s^2b''}{b}, \quad \text{(A2a)}$$

$$\beta(r_s)=-1+\frac{2r_sb'}{b}-\frac{r_s^2b'^2}{b^2}, \quad \text{(A2b)}$$

and

$$\gamma(r_s)=\frac{r_sb'}{b}+\frac{2r_s^2b'^2}{b^2}-\frac{r_s^2b''}{b}, \quad \text{(A2c)}$$

with $b'=db/dr_s$ and $b''=d^2b/dr_s^2$. When $b$ is independent of $r_s$, one obtains $\alpha(r_s)=1$, $\beta(r_s)=-1$, and $\gamma(r_s)=0$, and Eq. (A1a) becomes equal to Eq. (5b).

For GaAs heterostructures (HS) with vanishing depletion density $N_D$, where $b\propto r_s^{2/3}$, the form factor is given by

$$F(x)=\frac{1}{(1+x)^3}\left[1+\frac{9}{8}x+\frac{3}{8}x^2\right]. \quad \text{(A3)}$$

The width parameter is written as[3] $(b/a^*)^3=2r_s^2/[33(1+32N_D/11N_2)]$. For the inverse compressibility, we obtain

$$\frac{\kappa_{02}}{\kappa_{\mathrm{HFA2}}}=1-\frac{(2g_v)^{1/2}r_s}{9\pi}\int_0^1 dy\ F(2k_Fby)\times\left[4\pi-8\arcsin(y)+\frac{y}{(1-y^2)^{1/2}}\right]. \quad \text{(A4)}$$

Within the approximation $F(2k_Fby)=F(2k_Fb)$, the integral in Eq. (A4) can be calculated. From Eq. (A4) we derive the approximate result

$$\frac{\kappa_{02}}{\kappa_{\mathrm{HFA2}}}=1-\frac{(2g_v)^{1/2}}{\pi}r_sF(2k_Fb), \quad \text{(A5)}$$

which is identical to the approximate result given in Eq. (10), where density effects of the width have been neglected.

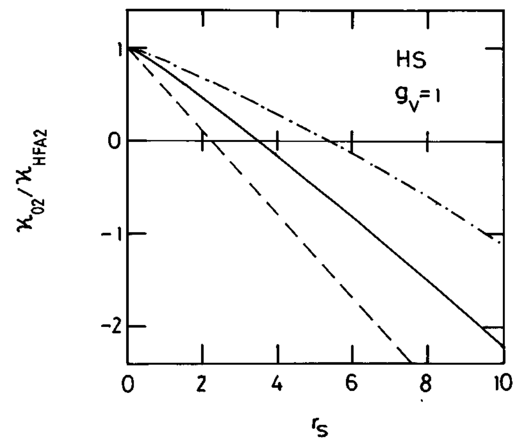


FIG. 5. Inverse compressibility $1/\kappa_{\mathrm{HFA2}}$ (in units of the inverse compressibility of the free-electron gas $1/\kappa_{02}$) vs the RPA parameter $r_s$ of a heterostructure with a width parameter $b$ (solid line). The dashed-dotted line represents the approximate result according to Eq. (A5). The dashed line represents the ideally two-dimensional electron gas with a zero width according to Eq. (9).

Numerical results for the inverse compressibility of HS with $N_D=0$ according to Eqs. (A4) and (A5) are shown in Fig. 5. The difference between the curves corresponding to Eqs. (A4) and (A5) is due to the density dependence of the width. We note that, for heterostructures, the density dependence of the width is important.

### 2. One-dimensional systems

If the width parameter depends on the electron density, the compressibility of one-dimensional systems, as given in Eq. (8), has to be generalized. We find

$$\frac{\kappa_{01}}{\kappa_{\mathrm{HFA1}}}=1-\frac{2g_vr_s}{\pi^2}\left\{f(2k_Fb)\alpha(r_s)+\int_0^1 dy\ f(2k_Fby)[\beta(r_s)+y\gamma(r_s)]\right\}, \quad \text{(A6)}$$

with

$$\alpha(r_s)=1-\frac{2r_sb'}{b}+\frac{r_s^2b'^2}{b^2}, \quad \text{(A7a)}$$

$$\beta(r_s)=\frac{2r_s^2b'^2}{b^2}-\frac{r_s^2b''}{b}, \quad \text{(A7b)}$$

and

$$\gamma(r_s)=\frac{2r_s^2b''}{b}-\frac{6r_s^2b'^2}{b^2}+\frac{4r_sb'}{b}. \quad \text{(A7c)}$$

When $b$ is independent of $r_s$, one obtains $\alpha(r_s)=1$, $\beta(r_s)=0$, and $\gamma(r_s)=0$, and Eq. (8) is found.

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