Shot noise in the presence of phonon-assisted transport through quasiballistic nanowires

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We work out a theory of the voltage-dependent noise in semiconductor nanowires under the condition of non-Ohmic phonon-assisted quasiballistic transport. We assume that weak scattering of the electrons confined within the nanowire is due to their interaction with the bulk acoustic and/or optical phonons. A general expression for the noise is derived. It is used to consider particular cases of interest. The dependence of noise intensity on the applied voltage is investigated. For low temperatures, a remarkable threshold effect is predicted.

I. INTRODUCTION

The purpose of the present paper is to work out a theory of noise in nanowires under the condition of the non-Ohmic phonon-assisted transport. We will consider the so-called quasiballistic regime where the conduction electrons within the nanowire either move without collisions or suffer a collision with a phonon. The impurity scattering is always assumed weak enough to make conductance steps clearly observable at low temperatures. We will be interested in the contribution to the noise that is due to the discreteness of the electron charges, depends on the applied voltage V, and vanishes when $V \to 0$. To establish a continuity with the existing terminology we will sometimes call this "shot noise." Such noise can persist down to zero temperatures. In other words, we consider the noise caused by the interaction of the conduction electrons with phonons.

The non-Ohmic phonon-assisted transport in nanowires has been recently considered both for the acoustic- and optical-phonon scattering (see Refs. 1,2, cf. also with Ref. 3). There the hot-electron transport regime was investigated and the variation of the total ballistic current ΔJ in a spatially uniform conductor that is due to the electron-phonon scattering was calculated. This variation is assumed to be sufficiently small so that most of the current is still transported ballistically. The non-Ohmic behavior is due to the fact that the rate of electron-phonon collisions is sensitive to the form of the electron distribution, which in a ballistic conductor strongly depends on the applied voltage. In fact, it was implied in Refs. 1 and 2 that the potential is almost constant along the wire and its main drops are within the contacts (one can always choose the form of gate electrodes to satisfy this condition).

It turns out that the backscattering of electrons determines the phonon-controlled portion of the current ΔJ. The energies exchanged with the phonon system can be much bigger than $k_B T$ and are in such a case determined by the potential difference V. Only those phonons contribute to the phonon-controlled part of the current whose quasimomenta (or rather their projections on the direction x of the electron propagation) are large enough to reverse the quasimomenta of the scattered electrons. At low temperature the number of such equilibrium phonons can be exponentially small. However, if the applied voltage is sufficiently large such phonons *can be emitted*, thus greatly enhancing the phonon-controlled part of the current. This can lead to a remarkable threshold effect for the generation of phonons with a given energy $\hbar\omega$ provided that eV exceeds $\hbar\omega$. Our purpose in particular is to discuss voltage-dependent noise under these conditions.

Recently a general relation between the shot noise spectral density P and the transmission properties of a mesoscopic conductor has been derived for the elastic electron scattering.[4–8] If the incident channel states do not mix with each other,[4–6] this relation takes the following form:

$$P = 2e|V|\frac{e^2}{h}\sum_n T_n(1 - T_n), \tag{1}$$

where T_n are the channel transmission coefficients. For the case of an arbitrary mesoscopic conductor with mixing channels this relation was generalized by Büttiker, Ref. 7 (see also a paper by Martin and Landauer, Ref. 8). It was shown there that when the voltage V is applied to an arbitrary two-terminal mesoscopic conductor, the zero-frequency, zero-temperature shot noise power is given by

$$P = 2e|V|\frac{e^2}{h}\mathrm{Tr}[\hat{t}\hat{t}^\dagger(1 - \hat{t}\hat{t}^\dagger)], \tag{2}$$

where $\hat{t}$ is the transmission matrix of the conductor evaluated at the Fermi energy.

These equations solve the problem in general. In order to find the explicit expression for P for any particular mesoscopic system one should analyze the structure of the transmission matrix. For a few-channel finite-length mesoscopic conductor even in the presence of some degree of disorder this problem is, in principle, not difficult provided that the transmission matrix can be found either analytically or numerically. In contrast, for a many-channel conductor with complicated internal scattering structure the explicit evaluation of $\hat{t}$ is a formidable task. However, as was shown by Beenakker and Büttiker,[9] in order to find shot noise power it is enough sometimes to know only *the statistical properties* of $\hat{t}\hat{t}^\dagger$ and $(\hat{t}\hat{t}^\dagger)^2$ eigenvalues. For the particular case of a diffusive quasi-one-dimensional metallic conductor they found that shot noise is partially suppressed as compared to

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the "full shot noise" level. The latter corresponds to the noise of a classical tunnel junction with the same conductance. Namely, they found

$$P=\gamma\frac{2e^2}{h}e|V|G, \tag{3}$$

where the "reduction factor" $\gamma=1/3$, and G is the conductance of the sample. The same result for the shot noise power of the diffusive metallic mesoscopic conductor was obtained independently by Nagaev[10] by making use of the the Boltzmann equation approach.

In general, the shot noise has been widely investigated for mesoscopic systems with different types of electron transport. However, the analysis given in most of the papers is restricted to the situation of purely *elastic* scattering[5–8,11] when the ideal quantum coherence persists within a conductor. Moreover, it is generally believed [see, e.g., Refs. 9, 12, and 13 and the comprehensive review by Landauer (Ref. 14)] that *inelastic* scattering destroys coherence and leads to a suppression of the shot noise. This, however, is not always the case. For instance, recently it was shown by Kozub and one of the authors of the present paper[15] that at least an inelastic electron-electron scattering, even for the case where it is so strong that it controls the form of the electron distribution function, does not essentially suppress shot noise of a diffusive conductor. In the electron-temperature approximation the zero-frequency, zero-lattice-temperature power of shot noise was shown to be

$$P=\frac{\sqrt{3}}{2}\frac{e^2}{h}e|V|G, \tag{4}$$

so that the reduction factor in this case is $\gamma=\sqrt{3}/4$. Although it differs from the result of Beenakker, Büttiker, and Nagaev (BBN), it does not produce a strong suppression of noise. The obtained prefactor is universal in the same sense as the BBN prefactor: it holds for any quasi-one-dimensional geometry without dependence on the degree of disorder.

Both Eq. (3) and Eq. (4) are valid for many-channel conductors with a rather strong, namely, diffusive, elastic scattering. Let us now turn to, in some sense, the opposite limit, namely, that of a one-channel conductor. According to Eq. (1), provided that only elastic processes are present the shot noise power of such a system is $P=2e|V|(e^2/h)T_0(1-T_0)$, where T_0 is the channel transmission. Shot noise is finite if and only if the transmission coefficient is neither 0 nor 1. An ideal one-channel conductor, where the elastic scattering is absent and therefore $T_0=1$, does not produce any noise. We will show that the inelastic scattering by phonons alters this situation. At nonzero temperature or voltage, electrons may be scattered in the course of phonon emission or absorption, which brings about shot noise as well.

An appearance of shot noise in an ideal conductor due to the inelastic electron-phonon scattering has been pointed out for the first time, to our knowledge, by Kulik and Omel'yanchuk[16] for a classical three-dimensional (3D) ballistic point contact. The purpose of the present paper is to study the effects of the electron-phonon scattering on shot noise of quasiballistic nanowires with a strongly quantized resistance. We will derive a formula for noise, which takes into account the inelastic scattering by phonons. Then we will make use of this formula to study particular cases of interest.

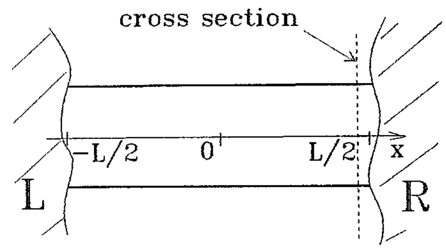

FIG. 1. Two-terminal Landauer resistor.

II. GENERAL RESULTS

Let us consider a nanowire of a length L along the x axis. In the spirit of the Landauer-Büttiker-Imry[17] approach we assume the wire to be connected with the reservoirs which we call "left" (L) and "right" (R), each of these being in equilibrium with itself. If the wire is long enough the electron motion along the x axis (see Fig. 1) may be, as already mentioned, treated classically, and one can use a semiclassical kinetic theory to treat the electron transport in this direction. This means that for our purpose it is sufficient to introduce and calculate a classical one-dimensional distribution function $F_n(x,p,t)$ for the electrons (where the channel index n is considered as a parameter) while the collisions should be treated quantum mechanically.

The introduction of the phonons into this picture can be done along the lines worked out in Refs. 1 and 3 [see also Ref. 11, where a semiclassical theory was used to derive a semiclassical analog for the shot noise formula, Eq. (2)]. We may also add that in Ref. 2 two approaches are used to treat the same transport problem concerning weak phonon scattering in nanowires. One of them is based on a time-dependent quantum theory exploiting the diagrammatic techniques; another one is semiclassical. The results obtained by the two methods are the same.

The semiclassical transport theory allows one not only to find the time-averaged electron distribution function $\bar{F}$ in the wire and, correspondingly, the mean current through a conductor, but the temporal fluctuations of both of these quantities as well. Using the approach described in Refs. 19 and 20 (see also Ref. 21), we will study the time evolution equations for $\bar{F}$ and $\langle \delta F \delta F\rangle$ (where we define δF as $\delta F\equiv F-\bar{F}$), which both have the form of quasiclassical Boltzmann equation with an electron-phonon collision term.

To avoid the excessive proliferation of indexes and at the same time to demonstrate the general scheme we will drop the indices of transverse quantization of electrons, restoring them only in *some* equations. We start with the equation for the average distribution function $\bar{F}(x,p,t)$:

$$\frac{\partial \bar{F}(x,p,t)}{\partial t}+v\frac{\partial \bar{F}(x,p,t)}{\partial x}=I\{\bar{F}(x,p,t)\}, \tag{5}$$

where p and v are the x components of the electron quasimomentum and velocity. $I\{\bar{F}(x,p,t)\}$ is the electron-phonon collision integral which, as usual, is a difference of "in" and "out" terms:

$$I\{\bar{F}(p,x,t)\}=I^{(\mathrm{in})}\{\bar{F}(p,x,t)\}-I^{(\mathrm{out})}\{\bar{F}(p,x,t)\},$$

$$I^{(\mathrm{in})}\{\bar{F}(p,x,t)\}=\frac{2\pi}{\hbar}\sum_{p'\mathbf{q}}|V_{p\mathbf{q}}|^2\bar{F}(p',x,t)[1-\bar{F}(p,x,t)]\times[\delta_{p'-\hbar q_x,p}\delta(\varepsilon_{p'}-\hbar\omega_q-\varepsilon_p)(N_q+1)+\delta_{p'+\hbar q_x,p}\delta(\varepsilon_{p'}+\hbar\omega_q-\varepsilon_p)N_q], \quad (6)$$

$$I^{(\mathrm{out})}\{\bar{F}(p,x,t)\}=\frac{2\pi}{\hbar}\sum_{p'\mathbf{q}}|V_{p\mathbf{q}}|^2\bar{F}(p,x,t)[1-\bar{F}(p',x,t)]\times[\delta_{p-\hbar q_x,p'}\delta(\varepsilon_p-\hbar\omega_q-\varepsilon_{p'})(N_q+1)+\delta_{p+\hbar q_x,p'}\delta(\varepsilon_p+\hbar\omega_q-\varepsilon_{p'})N_q]. \quad (7)$$

Here $V_{p\mathbf{q}}$ is the matrix element of electron-phonon interaction. Here we consider interactions of electrons with the three-dimensional bulk phonons (although the phonons confined within the nanostructure could have been easily included into the scheme). The summation over the phonon branches is implied. We integrate in Eqs. (6) and (7) over the three components of the phonon wave vector. $\mathbf{q}_\perp$ indicates the two transverse wave-vector components. The third component is given by

$$q_x=\pm(p-p')/\hbar.$$

Therefore the third integration is equivalent to the integration over the electron quasimomentum p' because of the conservation of quasimomentum.

As a result, these equations can be transformed into the following form (we write all the indexes explicitly here):

$$I^{(\mathrm{out})}\{F_n(p,x)\}=F_n(p,x)\sum_{n'}\int_{-\infty}^{\infty}\frac{dp'}{2\pi\hbar}[1-F_{n'}(p',x)]\int\frac{d^2\mathbf{q}_\perp}{(2\pi)^2}|\langle n'|\exp(i\mathbf{q}_\perp\cdot\mathbf{r}_\perp)|n\rangle|^2\times W(\mathbf{q})[N_\mathbf{q}\delta(\epsilon'-\epsilon-\hbar\omega_\mathbf{q})+(N_\mathbf{q}+1)\delta(\epsilon'-\epsilon+\hbar\omega_\mathbf{q})], \quad (8)$$

$$I^{(\mathrm{in})}\{F_n(p,x)\}=[1-F_n(p,x)]\sum_{n'}\int_{-\infty}^{\infty}\frac{dp'}{2\pi\hbar}F_{n'}(p',x)\int\frac{d^2\mathbf{q}_\perp}{(2\pi)^2}|\langle n'|\exp(i\mathbf{q}_\perp\cdot\mathbf{r}_\perp)|n\rangle|^2W(\mathbf{q})[(N_\mathbf{q}+1)\delta(\epsilon'-\epsilon-\hbar\omega_\mathbf{q})+N_\mathbf{q}\delta(\epsilon'-\epsilon+\hbar\omega_\mathbf{q})], \quad (9)$$

where $\phi_n(r_\perp)$ are the wave functions of transverse quantization, n is the channel index, $\epsilon\equiv\epsilon_n(p)$, $\epsilon'\equiv\epsilon_{n'}(p')$, and

$$W(\mathbf{q})|\langle n'|\exp(i\mathbf{q}_\perp\cdot\mathbf{r}_\perp)|n\rangle|^2=\frac{2\pi}{\hbar}|V_{p\mathbf{q}}|^2\mathscr{V}, \quad (10)$$

where $\mathscr{V}$ is the volume where phonons propagate.

In the isotropic[22] approximation for the scattering by acoustic phonons,[23]

$$W(\mathbf{q})=\frac{\pi\Lambda^2q^2}{\rho\omega_\mathbf{q}}, \quad (11)$$

where Λ is the deformation potential constant for the longitudinal phonons, and ρ is the mass density. For the scattering by optical phonons,[24]

$$W(\mathbf{q})=(2\pi)^2\omega_0e^2\left(\frac{1}{\epsilon_\infty}-\frac{1}{\epsilon_0}\right)\frac{1}{q^2}, \quad (12)$$

where ω_0 is the longitudinal optical-phonon frequency at $\mathbf{q}=0$, and ϵ_∞ and ϵ_0 are the high-frequency and the static dielectric susceptibilities, respectively.

Here we take into account only the interaction between electrons within a nanowire and bulk phonons and neglect the same interaction within the contacts (as in Refs. 1–3). This can be justified, for instance, for the following typical physical situation. We assume that contacts taper into a wire adiabatically (see Ref. 25). As, however, the width of the contacts is much larger than the width of a wire the number of channels within the contacts is also much larger. Most of these channels are not current carrying, as the electrons belonging to them are reflected from the wire region back into the corresponding contact. As for the current-carrying channels, one can easily check that the rate of electron-electron (e-e) collisions is strongly suppressed for the one-dimensional geometry. To do this it is sufficient to analyze the energy and quasimomentum conservation law for the e-e collisions (and take into consideration that the initial electron states should be filled while the final states should be empty). We have

$$\epsilon_n(p)+\epsilon_{n'}(p')=\epsilon_{n''}(p+\hbar q_x)+\epsilon_{n'''}(p'-\hbar q_x).$$

For instance, for a one-channel situation ($n=n'=n''=n'''$), particularly interesting for us in the present paper, these collisions are strictly forbidden.

As the contact region widens the number of channels is enhanced and the e-e collisions rapidly become more and more effective. As such collisions destroy the phase coherence of the electron wave functions this means that the principal variation of the (quasi)ballistic conductance is due to the electron-phonon scattering events where the electrons within the wire (rather than those within the contacts) take part.

In the spirit of the Landauer-Büttiker-Imry approach,[17,18] we take the boundary conditions for the transport equation, Eq. (5), to be in the form

$$\bar{F}(p>0,x=-L/2)=f_L(p)\equiv\frac{1}{\exp[(\varepsilon_p-\mu_L)/k_BT]+1},$$

$$\bar{F}(p<0,x=+L/2)=f_R(p)\equiv\frac{1}{\exp[(\varepsilon_p-\mu_R)/k_BT]+1}. \tag{13}$$

Here μ_L and μ_R are the chemical potentials of the reservoirs, and T is the temperature. The difference between the chemical potentials is the voltage bias across a conductor: $eV=\mu_L-\mu_R$. It is always assumed much smaller than the chemical potentials of the reservoirs themselves.

In the absence of collisions with phonons and under the stationary conditions, the solution of Eq. (5) is

$$\bar{F}^{(0)}(x,p)=\theta(p)f_L(p)+\theta(-p)f_R(p), \tag{14}$$

where $\theta(p)$ is the step function. This solution describes a ballistic motion in the absence of scattering.

Adding a weak electron-phonon interaction results in

$$\bar{F}=\bar{F}^{(0)}+\overline{\Delta F}$$

with $\overline{\Delta F}$ satisfying the first iteration of the Boltzmann equation

$$v\,\partial\,\overline{\Delta F}/\partial x=I\{\bar{F}^{(0)}\}.$$

Taking into account the boundary conditions, Eq. (13), and assuming that zero of the coordinate system is at the midpoint of the wire (see Fig. 1) we arrive at a solution of this equation in the form:[3]

$$\overline{\Delta F}(x,p)=\frac{1}{v}\left[x+\frac{L}{2}\mathrm{sign}p\right]I\{\bar{F}^{(0)}(p)\}. \tag{15}$$

We assume that the phonons are in equilibrium and hence $N_{\mathbf{q}}$ is the Bose function. The detailed balance guarantees a vanishing collision term for the equilibrium distribution function at constant temperature and chemical potential. This means that the distribution function (14) gives finite contribution to the collision term if and only if p and p' are of opposite sign, so that their chemical potentials are different. In other words, only those phonons contribute that can *backscatter* the electrons — see Ref. 3.

These considerations give the *averaged* electron distribution function along the wire in the presence of a weak inelastic scattering. As for the *fluctuating part* of the distribution function δF according to Refs. 19 and 20 the correlation function $\langle\delta F(x',p',t')\delta F(x,p,t)\rangle$ satisfies for $t'>t$ the Boltzmann equation, Eq. (5), in the first set of variables with the following one-time correlation function as the initial condition

$$\langle\delta F(x',p',t)\delta F(x,p,t)\rangle=h\,\delta(x-x')\delta(p-p')\times\bar{F}(x,p,t)[1-\bar{F}(x,p,t)]. \tag{16}$$

Let us now turn to the two-terminal scattering geometry shown in Fig. 1. The current through the cross section of a wire, which we choose to be near the right reservoir, is

$$J(t)=\frac{2e}{h}\int_{-\infty}^{\infty}dp\;vF(x=L/2,p,t), \tag{17}$$

where factor 2 is due to the spin degeneracy. In complete analogy with the distribution function, the current has the average value $\bar{J}$ and the fluctuating part, $\delta J(t)=J(t)-\bar{J}$. As before, we assume time independence of both $\bar{F}$ and $\bar{J}$. The current noise spectral density in the limit of zero frequency is given by the Fourier transform of the current-current correlation function:

$$P=4\int_0^{\infty}dt\langle\delta J(t)\delta J(0)\rangle=4\left(\frac{e}{h}\right)^2\int_0^{\infty}dt\int_{-\infty}^{\infty}dp\int_{-\infty}^{\infty}dp'vv'\chi, \tag{18}$$

where

$$\chi=\langle\delta F(L/2,p',t)\delta F(L/2,p,0)\rangle.$$

Here we have made use of the conservation of the current, which permits one to choose any cross section to calculate the current fluctuations for $\omega\to 0$. We make the same choice as above, namely $x=L/2$. We will consider χ separately for the positive and negative values of p.

Electrons with $p>0$ reach the right reservoir without further scattering. Therefore, χ contains only the term proportional to $\delta(t)$:

$$\chi(p>0)=\frac{h}{|v'|}\delta(t)\delta(p'-p)\bar{F}(L/2,p)[1-\bar{F}(L/2,p)], \tag{19}$$

where $\bar{F}(L/2,p)$ is given by the sum of Eqs. (14) and (15), taken at $x=L/2$ and $p>0$:

$$\bar{F}(L/2,p>0)=f_L(p)+\frac{L}{|v|}I^{(\mathrm{in})}\{f_L(p)\}-\frac{L}{|v|}I^{(\mathrm{out})}\{f_L(p)\}. \tag{20}$$

Electrons with $p<0$ can be, in fact, backscattered within the wire and cross the cross section $x=L/2$ again. In order to take into account all backscattering trajectories we, at first, divide the wire into small pieces $[x_1,x_2]$, $[x_2,x_3]$, . . . ,$[x_i,x_{i+1}]$, . . . (see Fig. 1). Then we introduce the probability for an electron from the right reservoir to be backscattered per unit of time, $\mathscr{U}\{p<0\}\to\{p_k>0\}$ and sum over the contributions from all the pieces. As a result, we obtain

$$\chi(p<0)=\frac{h}{|v'|}\left[\delta(t)\,\delta(p'-p)+\sum_{ik}h\frac{\Delta x_i}{|v|}\mathscr{U}\{p<0\}\to\{p_k>0\}\,\delta(t-t_i)\,\delta(p'-p_k)\right]\bar{F}(L/2,p)[1-\bar{F}(L/2,p)]. \quad (21)$$

Here p_k is a final electron state after backscattering, t_i is the time spent in the wire, and $\Delta x_i \equiv x_{i+1}-x_i$. The distribution function $\bar{F}(L/2,p)$ for electrons with $p<0$ is exactly $f_R(p)$, as is clearly seen from Eqs. (14) and (15).

Substitution of Eqs. (19) and (21) into the expression for the shot noise spectral density, Eq. (18), and the integration over t and p' finally gives

$$P=4\left(\frac{e}{h}\right)^2\int_{-\infty}^{\infty}dp|v|\left\{\theta(p)\frac{1}{2}\left(f_L+\frac{L}{|v|}I^{(\mathrm{in})}\{f_L\}-\frac{L}{|v|}I^{(\mathrm{out})}\{f_L\}\right)\left(1-f_L-\frac{L}{|v|}I^{(\mathrm{in})}\{f_L\}+\frac{L}{|v|}I^{(\mathrm{out})}\{f_L\}\right)\right.$$
$$\left.+\theta(-p)\left[\frac{1}{2}-\frac{L}{|v|}\frac{\delta I^{(\mathrm{out})}\{f_R\}}{\delta f_R}\right]f_R[1-f_R]\right\}. \quad (22)$$

Here

$$\frac{\delta I^{(\mathrm{out})}\{f_R(p)\}}{\delta f_R(p)}=\sum_{p'}\mathscr{U}\{p<0\}\to\{p'>0\}$$

is the variational derivative of the collisional integral, and $f_{L,R}\equiv f_{L,R}(p)$.

Equation (22) should be understood in the following way. It contains terms of the zeroth, first, and second order in the small parameter

$$\frac{L}{|v|}I^{(\mathrm{in,out})}\{f_{L,R}(p)\}.$$

In general, it is not always permissible to retain the terms of the second order. However, we have not discarded them because they are meaningful in some cases (see below — Sec. III A).

The obtained Eq. (22) is what we are looking for: it gives the expression for the shot noise power in one-dimensional nanowires with weak inelastic scattering by phonons taken into account. This general formula allows one to study various cases of interest. In particular, we will be interested in three regimes, namely, $eV \gg k_BT$, $eV \ll k_BT$, and, finally, a quasielastic scattering regime.

Before starting to discuss these cases, we would like to give a detailed many-channel expression for the shot noise spectral density in a quantum quasiballistic resistor:

$$P=4\left(\frac{e}{h}\right)^2\sum_n\int_{-\infty}^{\infty}dp|v|\left\{\theta(p)\frac{1}{2}\left(f_L^{(n)}+\frac{L}{|v|}I^{(\mathrm{in})}\{f_L^{(n)}\}-\frac{L}{|v|}I^{(\mathrm{out})}\{f_L^{(n)}\}\right)\left(1-f_L^{(n)}-\frac{L}{|v|}I^{(\mathrm{in})}\{f_L^{(n)}\}+\frac{L}{|v|}I^{(\mathrm{out})}\{f_L^{(n)}\}\right)\right.$$
$$\left.+\theta(-p)\left[\frac{1}{2}-\frac{L}{|v|}\frac{\delta I^{(\mathrm{out})}\{f_R^{(n)}\}}{\delta f_R^{(n)}}\right]f_R^{(n)}[1-f_R^{(n)}]\right\}. \quad (23)$$

Here

$$f_{L,R}^{(n)}\equiv f_{L,R}[\varepsilon_n(0)+p^2/2m],$$

where $\varepsilon_n(0)$ is the position of the subband bottom, and m is the electron's effective mass.

III. APPLICATIONS OF GENERAL RESULTS

In this section we restrict ourselves to the one-channel case.

A. Large-voltage, zero-temperature case

Let us consider now the case, when the voltage bias $eV=\mu_L-\mu_R$ applied across the conductor is finite while the temperature is assumed to be zero. In this case $N_q=0$ and only processes of phonon emission are possible. The distribution functions for electrons in the leads are the step functions:

$$f_{L,R}(E)=\theta(\mu_{L,R}-E). \quad (24)$$

This leads to a substantial simplification of the problem. In particular, $I^{(\mathrm{in})}\{f_L(p)\}$ and $\delta I^{(\mathrm{out})}\{f_R(p)\}/\delta f_R(p)$ become zero, while $(L/|v|)I^{(\mathrm{out})}\{f_L(p>0)\}=f_L(p)\mathscr{R}(\varepsilon_p)$, where

$$\mathscr{R}(\varepsilon_p)=\frac{L}{|v|}\int_{-\infty}^{0}\frac{dp'}{2\pi\hbar}\int\frac{d^2\mathbf{q}_\perp}{(2\pi)^2}\,\theta([\varepsilon_p-\mu_R]-\hbar\omega_q)$$
$$\times|\langle 0|\exp(i\mathbf{q}_\perp\mathbf{r}_\perp)|0\rangle|^2W(q)\,\delta(\varepsilon_p-\hbar\omega_q-\varepsilon_{p'}) \quad (25)$$

Here we introduced, in particular, the effective "inelastic reflection" coefficient $\mathscr{R}(\varepsilon_p)$, which describes efficiency of electron backscattering in the course of phonon emission.

For the shot noise spectral density we now have, making use of Eq. (22):

$$P=\frac{2e^2}{h}\int_{\mu_R}^{\mu_R+e|V|}dE\ \mathscr{R}(E)[1-\mathscr{R}(E)]. \qquad (26)$$

Therefore, the shot noise power is determined by the energy dependence of the "inelastic reflection" coefficient $\mathscr{R}(E)\ll 1$.

We wish to emphasize that here we retain the terms quadratic in the reflection coefficient (along with terms linear in $\mathscr{R}$). At the same time we do not take into account the quadratic terms while calculating the corrections to the distribution function itself due to the electron scattering [see Eq. (15)]. This is permissible, because in the case under discussion ($eV\gg k_BT$) the latter are of the *third* order in $\mathscr{R}$. Indeed, the second-order corrections to ΔF could appear after taking into account the second scattering event for an electron that has already experienced a scattering. However, the probability of this event is proportional to the number of empty states $1-F(p)$, which in its turn is proportional to $\mathscr{R}$.

We start by considering the 3D extended acoustic phonons for which the matrix elements of the electron-phonon interaction are given by Eq. (11). We are interested in the electron backscattering, therefore there should be some minimal wave vector for a phonon to be emitted. Namely, $q_{\min}=2p_F/\hbar$, where p_F is the Fermi momentum. This leads to a *threshold* in the inelastic reflection coefficient energy dependence, and, furthermore, in $P(V)$.

Indeed, $\mathscr{R}(E)=0$ for $E<2p_Fs\equiv E_{\rm th}$, where we introduce the notation $E=\varepsilon_p-\mu_R$. One can show that for energies near the threshold when $(E-E_{\rm th})/E_{\rm th}\ll 1$, in the first approximation in this small parameter, the coefficient of the inelastic reflection is

$$\mathscr{R}(E)=\frac{E-E_{\rm th}}{E_{\rm th}}R_0, \qquad (27)$$

where

$$R_0=\frac{Lm^2W(q_{\min})}{\hbar^4\pi^2}. \qquad (28)$$

Here we have taken into account that in real structures $E_{\rm th}=2p_Fs$ is of the same order as $\hbar s/d$. The shot noise power in this limit is

$$P(V)=\frac{e^2}{h}R_0\frac{(e|V|-E_{\rm th})^2}{E_{\rm th}}. \qquad (29)$$

Well above the threshold, where $E/E_{\rm th}\gg 1$, the matrix element

$$|\langle 0|\exp(i\mathbf{q}_\perp\mathbf{r}_\perp)|0\rangle|\propto(q_\perp d)^{-2},$$

d being the thickness of the nanowire. Then $R(E)=R_0$, while the shot noise power becomes proportional to the applied voltage

$$P(V)=\frac{2e^2}{h}e|V|R_0. \qquad (30)$$

The overall dependence of the shot noise power $P(V)$ of the two-terminal mesoscopic nanowire in the phonon-assisted quasiballistic resistance regime is shown in Fig. 2.

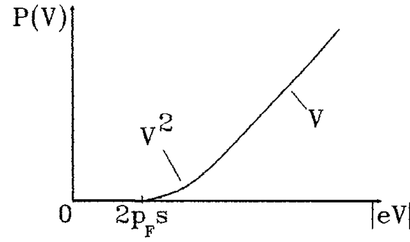

FIG. 2. The overall dependence of the shot noise power $P(V)$ of two-terminal mesoscopic nanowire in the regime of phonon-assisted quasiballistic resistance.

One sees that the nonequilibrium noise voltage dependence in this case differs from a simple linear law which is due to the inelastic type of the electron backscattering within the wire.

Let us now consider the inelastic reflection due to the 3D polar optical-phonon emission. Optical phonon is the simplest example of a phonon mode with a nonvanishing minimal frequency. The generalization to the other modes of such type, like the phonons confined within a nanostructure, or the confined optical phonons, is straightforward.

We denote the optical-phonon energy by $\hbar\omega_0$ and do not take into account the phonon dispersion. The matrix elements of the electron-phonon interaction are given by Eq. (12) in this case. Under these conditions the contribution of the optical-phonon emission processes to $\mathscr{R}$ vanishes for the electron energies smaller than $\hbar\omega_0$ while in the vicinity of $\hbar\omega_0$ the reflection coefficient exhibits a jump,

$$\Delta\mathscr{R}_{\hbar\omega_0}=\mathscr{R}_{\hbar\omega_0+0}-\mathscr{R}_{\hbar\omega_0-0}\approx R_0, \qquad (31)$$

where $W(q)$ in R_0 is given by Eq. (12). The shot noise power $P(V)$ has a bend when V crosses $\hbar\omega_0/e$.

B. Linear response regime

Let us consider the case of the small applied voltage $eV=\mu_L-\mu_R\ll k_BT$. In order to specify the notations we take f_R as an "equilibrium" distribution function and denote it f_0, while f_L is "shifted" (with $\mu_L=\mu_R+eV$). As $eV\ll k_BT$ we expand f_L up to the second order in eV:

$$f_L(\varepsilon_p,\mu_L,T)=f_0+eV\frac{\partial f_0}{\partial\varepsilon_p}+\frac{(eV)^2}{2}\frac{\partial^2 f_0}{\partial\varepsilon_p^2}. \qquad (32)$$

Now we should substitute this expansion into Eq. (22). The obtained expression contains the voltage-independent part responsible for the thermal noise, and the terms proportional to the voltage squared, i.e., a nonequilibrium contribution to the noise. Before writing them in the explicit form it is convenient to introduce the coefficients of the "inelastic reflection" $\mathscr{R}_E$ "inelastic 'in'-term" $\mathscr{P}_E$ that make the expressions more concise. Making use of Eqs. (6), (7), and (32) we have

$$\frac{L}{|v|}I^{(\mathrm{out})}\{f_L(\varepsilon_p)\}=f_L(\varepsilon_p)\frac{L}{|v|}\sum_{p'\mathbf{q}}\frac{2\pi}{\hbar}|V_{p\mathbf{q}}|^2 \times\theta(-p')[1-f_0(\varepsilon_{p'})][\cdots] \equiv\left[f_0+eV\frac{\partial f_0}{\partial\varepsilon_p}+\frac{(eV)^2}{2}\frac{\partial^2 f_0}{\partial\varepsilon_p^2}\right]\mathscr{R}_{\varepsilon_p},$$

$$\frac{L}{|v|}I^{(\mathrm{in})}\{f_L(\varepsilon_p)\}=[1-f_L(\varepsilon_p)]\frac{L}{|v|}\sum_{p'\mathbf{q}}\frac{2\pi}{\hbar}|V_{p\mathbf{q}}|^2 \times\theta(-p')f_0(\varepsilon_{p'})[\cdots] \equiv\left[1-\left(f_0+eV\frac{\partial f_0}{\partial\varepsilon_p}+\frac{(eV)^2}{2}\frac{\partial^2 f_0}{\partial\varepsilon_p^2}\right)\right]\mathscr{P}_{\varepsilon_p}, \tag{33}$$

where $[\cdots]$ stands for the terms in Eqs. (6) and (7), which contain δ functions.

There is an important relation between $\mathscr{R}_E$ and $\mathscr{P}_E$ that is due to the fact that the collisional integral vanishes after a substitution of the equilibrium distribution function, $I\{f_0\}=0$,

$$[1-f_0(\varepsilon_p)]\mathscr{P}_{\varepsilon_p}-f_0(\varepsilon_p)\mathscr{R}_{\varepsilon_p}=0. \tag{34}$$

Exploiting this relation and Eq. (33) we have, from Eq. (22),

$$P=P_{\mathrm{th}}+P_1(V), \tag{35}$$

$$P_{\mathrm{th}}=4\frac{e^2}{h}\int dE\; f_0(1-f_0)(1-\mathscr{R}_E), \tag{36}$$

$$P_1(V)=(eV)^2\frac{2e^2}{h}\int dE\left\{\left(\frac{\partial f_0}{\partial E}\right)^2[2(\mathscr{R}_E+\mathscr{P}_E)-1] +\frac{1}{2}\frac{\partial^2 f_0}{\partial E^2}[1-2f_0+\mathscr{P}_E-\mathscr{R}_E]\right\}. \tag{37}$$

Here P_{th} is the thermal noise power, while $P_1(V)$ is the nonequilibrium contribution to noise. One can see that

$$(1/2)\int dE(\partial^2 f_0/\partial E^2)(1-2f_0)=\int dE(\partial f_0/\partial E)^2,$$

and, therefore,

$$P_1(V)=(eV)^2\frac{2e^2}{h}\int dE\left[\left(\frac{\partial f_0}{\partial E}\right)^2 2(\mathscr{R}_E+\mathscr{P}_E) +\frac{1}{2}\frac{\partial^2 f_0}{\partial E^2}(\mathscr{P}_E-\mathscr{R}_E)\right]. \tag{38}$$

After substituting the explicit form of $f_0=[\exp(E/k_BT)+1]^{-1}$ (we refer here all energies to the chemical potential μ_R), and taking into account Eq. (34), which gives

$$\mathscr{R}_E-\mathscr{P}_E=\tanh(E/2k_BT)(\mathscr{P}_E+\mathscr{R}_E),$$

we arrive at the following result:

$$P_1(V)=(\mathrm{eV})^2\frac{2e^2}{h(k_BT)^2}\int_{-\infty}^{+\infty}dE\,\frac{1-\sinh^2(E/2k_BT)}{8\cosh^4(E/2k_BT)} \times[\mathscr{R}_E+\mathscr{P}_E]=\frac{1}{3}\frac{(eV)^2}{k_BT}\frac{e^2}{h}\mathscr{R}_{E\approx 0}, \tag{39}$$

(40)

where $\mathscr{R}_{E\approx 0}=R_0$, see Eq. (28).

C. Quasielastic scattering

It is instructive to show how one can obtain the expression for shot noise assuming the electron-phonon scattering to be quasielastic. We compare our result with that obtained by Beenakker and van Houten in Ref. 11.

If the scattering is quasielastic the terms in the collisional integral, due to the Pauli principle, such as $[1-f_\alpha(\varepsilon_p)]$, cancel. Then the conservation of the number of particles *for each energy value* gives

$$|I^{(\mathrm{out})}\{f_{L(R)}(E)\}|=|I^{(\mathrm{in})}\{f_{R(L)}(E)\}|\equiv\frac{|v|}{L}f_{L(R)}\mathscr{R}_E. \tag{41}$$

Substituting this into Eq. (22) and rearranging the terms one gets the expression for shot noise induced by weak *quasielastic* electron-phonon scattering:

$$P=P_1+P_2, \tag{42}$$

$$P_1=2\frac{e^2}{h}\int dE(1-\mathscr{R}_E)[f_L(1-f_L)+f_R(1-f_R)], \tag{43}$$

$$P_2=2\frac{e^2}{h}\int dE\;\mathscr{R}_E(1-\mathscr{R}_E)(f_L-f_R)^2. \tag{44}$$

This expression is analogous to the result obtained by Beenakker and van Houten in Ref. 11 for shot noise, which is a consequence of the purely elastic scattering. Here, however, it is permissible to retain the terms proportional to $\mathscr{R}^2$ only, provided that $eV\gg k_BT$.

IV. CONCLUSION

We have several conclusive remarks, the first of which concerns this important and interesting question: How can one separate experimentally the phonon-induced contribution to the shot noise found in the present paper from other contributions such as a weak impurity-scattering-induced noise, a noise caused by the two-level systems, etc? The answer is quite simple and promising. The contribution we found depends in a specific way on the Fermi quasimomentum [see Eq. (29)]. The Fermi quasimomentum can be easily tuned in practice, so that one can separate the contribution to the shot noise under discussion.

We assume the transport to be quasiballistic, which means that the electron-phonon collisional events must be rare. To satisfy this requirement, the electron mean free path should be somewhat bigger than the length of the quantum wire L.

We have considered here the case of the simplest geometry, i.e., a quantum wire of a constant cross section. However, the results of our consideration can be easily generalized for the case of a so-called adiabatic transport (see Ref. 25), where the variation of the potential profile is smooth on the scale of the de Broglie wavelength.

The phonons throughout the paper are assumed to be in equilibrium. An important generalization would be to consider the transport and noise under the nonequilibrium phonon conditions.

A generalization taking into consideration phonons confined within a nanostructure (see Ref. 26) is rather straightforward. To take into account phonons localized near a nanostructure due to the dynamical screening (cf. with Ref. 27) may be more involved but still very important.

To summarize, we have developed a theory of the shot noise in quantum quasiballistic channels under the condition of a non-Ohmic phonon-assisted transport. A general formula for the shot noise caused by the weak electron-phonon scattering in an otherwise purely ballistic quantum channels is derived. The general results are used to work out expressions for some particular cases of interest. We have studied the cases of small ($k_B T \ll eV$) and large ($k_B T \gg eV$) temperatures, as well as that of the quasielastic scattering. For small temperatures a remarkable threshold effect for the shot noise is predicted. Such an effect can be used to analyze the spectrum of the phonons interacting with the electrons of a nanowire.

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