Influence of typical environments on quantum processes

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We present the results of studying the influence of different environmental states on the coherence of quantum processes. We choose to discuss a simple model that describes two electronic reservoirs connected through tunneling via a resonant state. The model could, e.g., serve as an idealization of inelastic resonant tunneling through a double-barrier structure. We develop Schwinger's closed time path formulation of nonequilibrium quantum statistical mechanics, and show that the influence of the environment on a coherent quantum process can be described by the value of a generating functional at a specific force value, thereby allowing for a unified discussion of destruction of phase coherence by various environmental states: thermal state, classical noise, time-dependent classical field, and a coherent state. The model allows an extensive discussion of the influence of dissipation on the coherent quantum process, and expressions for the transmission coefficient are obtained in the possible limits.

I. INTRODUCTION

The progress in fabrication of submicron structures has led to a wealth of new structures whose transport properties are dominated by the feature that electronic transport through the structure takes place coherently. However, additional degrees of freedom in addition to those of single electrons are present and it is of importance to account for their influence on coherent processes. In the following we shall investigate how different environmental states influence the quantum process of main interest. Although we shall choose a model that is directly relevant to transport properties of a double-barrier structure or transport through a quantum dot, the technique we develop is of general interest.

The paper is organized as follows: In Sec. II we model the system whose transport properties will be studied, and in Sec. III we show how to describe the transmission probability of a structure in terms of Green's functions. In Sec. IV we discuss the main approximation, which will allow an analytical description of the influence of dissipation on the transmission properties. In Sec. V we introduce the closed time path formulation and develop the description of the destruction of phase coherence in terms of a generating functional technique, and in Sec. VI we discuss the effects of different environmental states: thermal and coherent states, the external field case, and the fluctuating level model. We shall for all these cases derive closed expressions for the transmission probabilities, and compare the results in order to notice similarities and differences in the influence of different environments. In Sec. VII we discuss destruction of phase coherence using the generic Aharonov-Bohm situation. Finally in Sec. VIII we summarize and conclude.

II. THE MODEL FOR STUDYING DESTRUCTION OF PHASE COHERENCE

In this section we shall set up a minimal model describing the dissipative feature of the general quantum transport problem. The simplifications are introduced in order to be able to treat analytically the influence of environments on the transport process. We shall study the general problem of quantum transport where electronic current reservoirs provide electrons to an active region where interaction with an environment can take place. The reservoirs correspond in reality to large electrodes, and we can describe the Hamiltonians for the left and right electrodes in terms of their electron energy levels:

$$H_\sigma = \sum_{\mathbf{p}} \epsilon_{\mathbf{p},\sigma} a^\dagger_{\mathbf{p},\sigma} a_{\mathbf{p},\sigma}. \tag{1}$$

The quantum number $\mathbf{p}$ labels the momentum of the electron states, $\sigma = l,r$ refers to the left and right electrode, respectively, and $a^\dagger_{\mathbf{p},\sigma}$ creates particles in states with these quantum numbers with the corresponding energy $\epsilon_{\mathbf{p},\sigma}$.

The central active (sample) region can in the absence of coupling to the reservoirs be described by a Hamiltonian or equivalently its energy levels. The single-particle-energy at the central site, labeled c, is ϵ_c corresponding to the term in the Hamiltonian

$$H_s = \epsilon_c a^\dagger_c a_c. \tag{2}$$

In the following we shall consider a model that allows extensive analytical calculations, and therefore restrict the number of levels relevant in the central region to one. Eventually we shall consider also the case of two levels. In the event that the reservoirs are connected through the central site, electrons are transmitted between them. Such a situation can quite generally be modeled by transfer matrix elements, $V_{\mathbf{p},\sigma}$, between the reservoirs and the central region. The coupling of the electrodes to the central site is therefore described by the tunneling Hamiltonian[1]

$$H_t = \sum_{\mathbf{p}\sigma} \{V_{\mathbf{p}\sigma} a^\dagger_c a_{\mathbf{p}\sigma} + \text{H.c.}\}. \tag{3}$$

The transfer matrix elements are here considered to be phenomenological parameters, but can of course for any chosen microscopic model of say a double barrier be expressed in

terms of the potential profile and the carrier mass. The Hamiltonian H_e for the electronic system of interest is therefore given by

$$H_e = H_l + H_r + H_s + H_t . \tag{4}$$

Within the sample region we allow for interaction with an environment. For our purpose we can quite generally assume a bosonic environment with a corresponding Hamiltonian H_b that has the standard normal mode form

$$H_b = \sum_\alpha \hbar\omega_\alpha \{ b_\alpha^\dagger b_\alpha + \tfrac{1}{2} \} \tag{5}$$

in terms of the bosonic creation and annihilation operators, $b_\alpha^\dagger$ and b_α.

For the coupling of an electron at the central site we take a linear coupling to the normal modes

$$H_i = a_c^\dagger a_c X, \tag{6}$$

where X is the collective environment displacement operator

$$X = \sum_\alpha \lambda_\alpha \{ b_\alpha^\dagger + b_\alpha \}, \tag{7}$$

and λ_α the coupling constant to mode α.

The resulting Hamiltonian for system and environment,

$$H = H_e + H_b + H_i , \tag{8}$$

has been discussed in a variety of contexts,[2] most recently in the context of inelastic resonant tunneling,[3–5] where the model has served as a simplified description of the influence of interaction with phonons on the transport properties of a double-barrier structure. In the present account we shall not only discuss a thermal environment, but a variety of environmental states and their influence. A purpose of the paper is to present a calculational scheme that allows a unified description of arbitrary environments, and in a direct way exhibits the physical content of a model so that calculations within more realistic models can be made tractable. We now proceed to describe in detail the transmission properties of the system under consideration.

III. TRANSMISSION PROPERTIES

In mesoscopic physics where the main feature of electronic transport is its coherence, the transport description can be expressed in terms of the scattering properties of the mesoscopic structure. The choice of model has been dictated by this feature, and in the following we shall study the quantum-mechanical problem of transmission of an electron through a region where it can interact with additional degrees of freedom. We wish, therefore, to calculate the transmission coefficient for a particle emitted, say, from the left reservoir under the assumption that it propagates via the central site, where it is allowed to interact with an environment, to the right reservoir. In accordance with the initial condition of an electron impinging from the left reservoir, we can assume that at some initial time t_i the two subsystems, particle and environment, are decoupled, so that the initial state is described by a separable statistical operator

$$\rho_i = P_{\mathbf{p}',l} * \rho_b , \tag{9}$$

where $P_{\mathbf{p}',l}$ is the projection operator describing an electron in the left electrode in momentum state $\mathbf{p}'$

$$P_{\mathbf{p}',l} = |\mathbf{p}',l\rangle\langle\mathbf{p}',l| . \tag{10}$$

The state of the environment is presently arbitrary and described by the statistical operator ρ_b. As regards the transmission problem, given the above initial state, all information can be extracted from the probability, $P_{\mathbf{p}',l\to\mathbf{p},r}(t)$, to find the electron in the right electrode at time t with momentum $\mathbf{p}$. This conditional probability is given by the expression

$$P_{\mathbf{p}',l\to\mathbf{p},r}(t) = \mathrm{Tr}(\rho_i U^\dagger(t,t_i) P_{\mathbf{p},r} U(t,t_i)), \tag{11}$$

where $P_{\mathbf{p},r}$ is the projection operator

$$P_{\mathbf{p},r} = |\mathbf{p},r\rangle\langle\mathbf{p},r| \tag{12}$$

corresponding to the assumed final outgoing particle state with momentum $\mathbf{p}$ in the right electrode, and

$$U(t,t_i) = e^{-(i/\hbar)H(t-t_i)} \tag{13}$$

is the evolution operator corresponding to the total Hamiltonian. The trace with respect to all the degrees of freedom is denoted by Tr. The absence of any environment operator discriminating the final states of the environment is in accordance with the typical experimental condition pertaining to the electronic conduction process, namely, that the environmental degrees of freedom are not observed.

For an electron to propagate between the reservoirs it first has to enter the sample region and, last, exit it. In the event we only explicitly consider interaction with the environment in the sample region, we can exploit this feature and introduce a discussion in terms of the effective coupling between reservoirs and the sample region. The chosen model has this feature to the extreme. In the assumed model the electron only couples to the environment at the central site. We can therefore express the transition probability in a form that explicitly only involves the dynamics of the electron at the central site and the environment by simply noting that in order to calculate the amplitude for a transition from the left to the right reservoir, the first propagation has to be from the left reservoir to the central site, and the last propagation from the central site to the right reservoir. The transition probability can therefore be rewritten as

$$\begin{aligned} P_{\mathbf{p}',l\to\mathbf{p},r}(t) = \frac{1}{\hbar^4} |V_{\mathbf{p}r}|^2 |V_{\mathbf{p}'l}|^2 \int_0^t dt_1 \int_0^t dt_2 \int_0^t dt_3 \int_0^t dt_4 \\ \times \exp\left\{ \frac{i}{\hbar} \epsilon_{\mathbf{p},r}(t_2 - t_3) + \frac{i}{\hbar} \epsilon_{\mathbf{p}',l}(t_4 - t_1) \right\} \\ \times \langle \hat{G}_c^A(t_4,t_3) \hat{G}_c^R(t_2,t_1) \rangle , \end{aligned} \tag{14}$$

where $\langle\ \rangle = \mathrm{tr}(\rho_b \ldots)$ is shorthand for the trace with respect to the environmental degrees of freedom, weighted with respect to the initial environment state, and

$$\hat{G}_c^R(t,t') = -i\theta(t-t')\langle 0|[a_c(t), a_c^\dagger(t')]|0\rangle \tag{15}$$

is the retarded Green's operator (operator with respect to the environmental degrees of freedom) for the central site dynamics as

$$a_c(t)=e^{(i/\hbar)Ht}a_c e^{-(i/\hbar)Ht}, \tag{16}$$

and $|0\rangle$ denotes the particle vacuum state. The advanced Green's operator is given by Hermitian conjugation,

$$\hat{G}_c^A(t,t')=[\hat{G}_c^R(t',t)]^\dagger, \tag{17}$$

where † denotes Hermitian conjugation. In Eq. (14), the arbitrary initial time has been chosen at time zero, $t_i=0$.

In many cases of interest we do not need the full information on the transition probability as a function of time. For instance, if we are only interested in the average particle flow in a steady state we are only interested in the transition probability per unit time,

$$w_{\mathbf{p}',l\to\mathbf{p},r}=\lim_{t\to\infty}\frac{P_{\mathbf{p}',l\to\mathbf{p},r}(t)}{t}, \tag{18}$$

or the transmission coefficient for making a transition between a state of energy ϵ' in the left reservoir and a state of energy ϵ in the right reservoir:

$$T(\epsilon,\epsilon')=h\sum_{\mathbf{p}\mathbf{p}'} w_{\mathbf{p}',l\to\mathbf{p},r}\delta(\epsilon-\epsilon_{\mathbf{p},r})\,\delta(\epsilon'-\epsilon_{\mathbf{p}'l}). \tag{19}$$

Invoking the scattering approach for the description of transport properties for coherent quantum processes[6] we have at zero temperature the contribution to the conductance at energy ϵ',

$$G(\epsilon')=\frac{2e^2}{h}T(\epsilon'), \tag{20}$$

where

$$T(\epsilon')=\int_0^\infty d\epsilon\; T(\epsilon,\epsilon') \tag{21}$$

is the total transmission coefficient, the probability to reach the right reservoir for a state of energy ϵ' in the left electrode. The factor of two is the spin degeneracy factor of the electron.

IV. THE WIDE-BAND APPROXIMATION

We have in the previous section reduced the expression for the transition probability to an expression that only involves the dynamics at the central site. However, since both tunneling and interaction with the environment are present the dynamics is complicated and no closed expression for the transmission probability can be found; that is, without any further assumption no simplification is possible as the phonon average cannot be done explicitly. If we, however, assume that the widths W of the electronic energy bands of the electrodes are the largest energy in the problem, $W>\hbar\omega_\alpha,\Gamma$, it is possible to obtain closed expressions for the transition probability for various cases of environmental states. In this wide-band limit an electron at the central site decays into a continuum of states in the electrodes, and we expect to have a quasistationary state that decays exponentially in time. To utilize this property of the electrodes we first discuss the lifetime for occupation of the central site in the absence of an environment.

In the absence of coupling to the environment the retarded Green's operator for the central site, $\hat{G}_c^R$, reduces to the c-number Green's function, equaling the amplitude G_c^R for the particle to remain at the central site. The Fourier transformed amplitude satisfies the Dyson equation,

$$G_c^R(\epsilon)=g_c^R(\epsilon)+g_c^R(\epsilon)\Sigma_c^R(\epsilon)G_c^R(\epsilon), \tag{22}$$

where the central site self-energy is given by

$$\Sigma_c^R(\epsilon)=\sum_{\mathbf{p}\sigma}|V_{\mathbf{p}\sigma}|^2 g_{\mathbf{p}\sigma}^R(\epsilon), \tag{23}$$

and describes the coupling of the central site to the reservoirs, as indicated by the appearance of the electrode propagators. A lower capital is used to designate a subsystem Green's function, implying that the electron is not allowed to propagate in or out of the electrodes, i.e., its dynamics is determined by the Hamiltonian with all connecting elements, $V_{\mathbf{p},\sigma}$, set equal to zero.

In the absence of tunneling between the electrodes and the central site we have for the isolated electrode Green's functions

$$g_{\mathbf{p}\sigma}^R(\epsilon)=\frac{1}{\epsilon-\epsilon_{\mathbf{p},\sigma}+i0}, \tag{24}$$

and for the central site Green's function

$$g_c^R(\epsilon)=\frac{1}{\epsilon-\epsilon_c+i0}. \tag{25}$$

In the presence of coupling between electrodes and the central site the Dyson equation yields the central site Green's function in terms of the self-energy,

$$G_c^R(\epsilon)=\frac{1}{\epsilon-\epsilon_c-\Sigma_c^R(\epsilon)}. \tag{26}$$

The escape rate from the central site is given by the imaginary part of the self energy

$$\Gamma(\epsilon)=-\mathrm{Im}\Sigma_c^R(\epsilon)=\pi\sum_{\mathbf{p}\sigma}|V_{\mathbf{p}\sigma}|^2\delta(\epsilon-\epsilon_{\mathbf{p}\sigma}). \tag{27}$$

If the hopping matrix elements vary slowly with energy in the resonance region, $\epsilon\sim\epsilon_c$, and the energy bands in the reservoirs are wide so that also the electrode density of states, $N_\sigma(\epsilon)=\Sigma_{\mathbf{p}}\delta(\epsilon-\epsilon_{\mathbf{p}\sigma})$, has a weak energy dependence, we can neglect the energy dependence of the escape rate, and by analyticity we have

$$\Sigma_c^R(\epsilon)=\int_{-\infty}^{\infty}\frac{d\epsilon'}{\pi}\frac{\Gamma(\epsilon')}{\epsilon-\epsilon'+i0}, \tag{28}$$

which implies that the real part of the self-energy vanishes in the wide-band limit.

The wide-band approximation simplifies the problem considerably as the time-dependent Green's function becomes a decaying exponential,

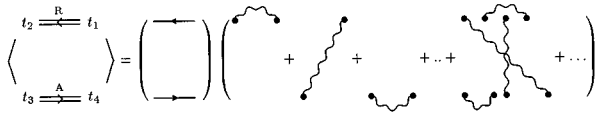

FIG. 1. The Feynman diagrammatic representation of the decoupling of particle and environment dynamics for the amplitude correlator $\langle \hat{G}_c^A(t_4,t_3)\hat{G}_c^R(t_2,t_1)\rangle$ in the wide-band approximation. The double lines denote the full central site Green's functions while the single lines denote the central site Green's functions in the absence of coupling to the environment. The curly lines denote the correlator of the environment operator.

$$G_c^R(t,t') = -i\,\theta(t-t')e^{-(i/\hbar)(\epsilon_c - i\Gamma)(t-t')}, \tag{29}$$

and the Green's function satisfies, for times $t>t''>t'$, the relation

$$G_c^R(t,t'')G_c^R(t'',t') = G_c^R(t,t'). \tag{30}$$

This group property leads to a tremendous simplification of the interacting problem as the tunneling dynamics at the central site and the environment dynamics decouple. If the group property, Eq. (30), is valid we have for the central site amplitude correlations

$$\langle \hat{G}_c^A(t_4,t_3)\hat{G}_c^R(t_2,t_1)\rangle = G_c^A(t_4,t_3)G_c^R(t_2,t_1)Z(t_4,t_3,t_2,t_1), \tag{31}$$

where we have introduced the influence function describing the effect of the environment

$$Z(t_4,t_3,t_2,t_1) = \langle (\tilde{T}e^{(i/\hbar)\int_{t_4}^{t_3} dtX(t)})(Te^{-(i/\hbar)\int_{t_1}^{t_2} dtX(t)})\rangle \tag{32}$$

with the environment variable in the interaction picture

$$X(t) = e^{(i/\hbar)H_b t}Xe^{-(i/\hbar)H_b t}, \tag{33}$$

and T and $\tilde{T}$ denotes the time and antitime ordering operators, respectively. The decoupling of the particle and environment degrees of freedom is visualized using Feynman diagrams in Fig. 1.

Because of the group property, Eq. (30), the central site dynamics has a special behavior: the lifetime of the central site state is independent of the coupling to the environment, as the probability $P_c(t)$ for the particle to remain at the central site after a time span t, if initially at the central site, is

$$P_c(t) = \langle \hat{G}_c^A(0,t)\hat{G}_c^R(t,0)\rangle, \tag{34}$$

and noting that $Te^{-(i/\hbar)\int_0^t d\bar{t}X(\bar{t})}$ is the interaction picture time evolution operator for the bath, and consequently unitary, we have $Z(0,t,t,0)=1$. Therefore by Eq. (31) we have for the staying probability

$$P_c(t) = \langle G_c^A(0,t)G_c^R(t,0)\rangle = e^{-(2\Gamma/\hbar)t}, \tag{35}$$

which is independent of the coupling to the environment.[4]

Before we go on to calculate the influence function Z, which contains all information about the influence of the environment for various environmental states, we briefly discuss the transmission problem in the absence of coupling to the environment. We shall explicitly assume the exponential form Eq. (29) for the decay amplitude.

If we for the moment neglect the coupling to the environment we have for the transition probability

$$P_{\mathbf{p}',l\to\mathbf{p},r}(t) = \frac{1}{\hbar^4}|V_{\mathbf{p},r}|^2|V_{\mathbf{p}',l}|^2\int_0^t dt_1\int_{t_1}^t dt_2\int_0^t dt_3\int_0^{t_3} dt_4 \times e^{(i/\hbar)\epsilon_{\mathbf{p},r}(t_2-t_3)+(i/\hbar)\epsilon_{\mathbf{p},l}(t_4-t_1)} \times e^{-(i/\hbar)(\epsilon_c - i\Gamma)(t_2-t_1)+(i/\hbar)(\epsilon_c+i\Gamma)(t_3-t_4)}. \tag{36}$$

The integrations are readily done and we obtain for the transition probability per unit time

$$w_{\mathbf{p}',l\to\mathbf{p},r} = \lim_{t\to\infty}\frac{P_{\mathbf{p}',l\to\mathbf{p},r}(t)}{t} = \frac{2\pi}{\hbar}\frac{|V_{\mathbf{p},r}|^2|V_{\mathbf{p}',l}|^2}{(\epsilon_{\mathbf{p}',l}-\epsilon_c)^2+\Gamma^2}\,\delta(\epsilon_{\mathbf{p},r}-\epsilon_{\mathbf{p}',l}), \tag{37}$$

or equivalently for the transmission coefficient

$$T(\epsilon,\epsilon') = \frac{4\Gamma_l\Gamma_r}{(\epsilon'-\epsilon_c)^2+\Gamma^2}\,\delta(\epsilon-\epsilon') = \frac{2\Gamma_l\Gamma_r}{\Gamma}A_c(\epsilon')\,\delta(\epsilon-\epsilon'), \tag{38}$$

where we have introduced the central site spectral weight function

$$A_c(\epsilon') = i(G_c^R(\epsilon')-G_c^A(\epsilon')) = \frac{2\Gamma}{(\epsilon'-\epsilon_c)^2+\Gamma^2} \tag{39}$$

and the left and right escape rates

$$\Gamma_\sigma = \pi\sum_{\mathbf{p}}|V_{\mathbf{p}\sigma}|^2\delta(\epsilon-\epsilon_{\mathbf{p}\sigma}). \tag{40}$$

We observe that the transmission coefficient has the expected resonant character of the Breit-Wigner formula.[7]

In the absence of coupling to the environment we therefore have the contribution to the conductance at energy ϵ',

$$G(\epsilon') = \frac{4e^2}{\pi\hbar}\frac{\Gamma_l\Gamma_r}{(\epsilon'-\epsilon_c)^2+\Gamma^2}. \tag{41}$$

Having discussed briefly the uncoupled case, we now turn to calculate the influence function Z for various environmental states. We shall invoke the assumption of a wide-band width allowing for the decoupling expressed by Eq. (31), and obtain in this approximation the following expression for the transmission coefficient:

$$T(\epsilon,\epsilon') = \lim_{t\to\infty}\frac{1}{t}\frac{2\Gamma_l\Gamma_r}{\pi\hbar^3}\int_0^t dt_1\int_{t_1}^t dt_2\int_0^t dt_3\int_0^{t_3} dt_4 \times Z(t_4,t_3,t_2,t_1)e^{(i/\hbar)\epsilon(t_2-t_3)+(i/\hbar)\epsilon'(t_4-t_1)} \times e^{-(i/\hbar)(\epsilon_c-i\Gamma)(t_2-t_1)+(i/\hbar)(\epsilon_c+i\Gamma)(t_3-t_4)}. \tag{42}$$

In the wide-band limit we notice the integral of the transmission coefficient is related to the staying probability at the central site:

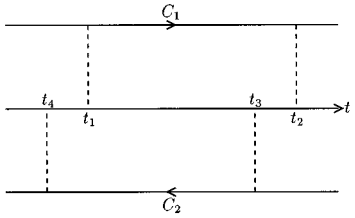

FIG. 2. Closed time path C extending from $-\infty$ to $+\infty$ along C_1 and back again along C_2.

$$\int_{-\infty}^{\infty} d\epsilon' T(\epsilon') = \frac{8\pi\Gamma_l\Gamma_r}{\hbar}\int_0^{\infty} dt\ P_c(t), \tag{43}$$

and we can therefore conclude that the integral of $T(\epsilon')$ is unaffected by the coupling to the environment since the staying probability $P_c(t)$ is unaffected.

V. CLOSED TIME PATH FORMULATION FOR THE INFLUENCE FUNCTION

In order to calculate the influence of the environment on the otherwise coherent quantum process, the task is to calculate the influence function given in Eq. (32). The first step in achieving such a goal is to transform the operator expression into an analytical one. A general and convenient method for performing this transformation has been devised by Schwinger.[8] The key point to note for employing this method for the present purpose is to note that the influence function Z quite generally can be expressed in terms of a closed time path generating functional provided the ''force'' is chosen properly as

$$Z(t_4,t_3,t_2,t_1) = \langle T_C e^{-(i/\hbar)\int_c d\tau f(\tau)X(\tau)}\rangle\big|_{f(\tau)=f^0(\tau)}, \tag{44}$$

where C is the closed time path extending from $-\infty$ to $+\infty$ along C_1 and back again along C_2, as illustrated in Fig. 2, and T_C is the contour ordering symbol, ordering the environment operators $X(\tau)$ according to their contour label position on the contour C (earliest to the right) and $f^0(\tau)$ is the function on the upper and lower branches of the contour specified by

$$f^0(\tau) = \begin{cases} f_1^0(t) &= \theta(t_2-t)-\theta(t_1-t), \quad t=\tau\in C_1 \\ f_2^0(t) &= \theta(t_3-t)-\theta(t_4-t), \quad t=\tau\in C_2. \end{cases} \tag{45}$$

The closed time path generating functional

$$Z[f] \equiv \langle T_C e^{-(i/\hbar)\int_c d\tau f(\tau)X(\tau)}\rangle \tag{46}$$

is therefore the quantity of interest.

The influence of the environment only appears through the state of the environment in the generating functional, and different environments can now be handled on an equal footing corresponding to just substituting the proper environmental state, i.e., the corresponding statistical operator.

In the present formulation we note that corresponding to the influence of an environment on a given physical quantity there corresponds a function, here denoted the proper ''force,'' which when inserted into the generating functional completely specifies the influence of the environment.

In the following section we calculate the generating functional and the influence function for various environmental states.

VI. THE INFLUENCE FUNCTION FOR TYPICAL ENVIRONMENTAL STATES

In the preceding section we showed that in the presented model the influence functional refers explicitly only to the environmental degrees of freedom and is characterized by a single ''force.'' This simplifying feature will allow us to obtain closed expressions for the influence function for typical environmental states.

A. Thermal environment

We first discuss the quite common physical situation where the environment is acting as a heat bath. This could, for example, be the case in question where the Hamiltonian is thought to represent a resonant tunneling structure, the lattice degrees of freedom of the crystal acting as the heat bath.

A heat bath is characterized by a single macroscopic parameter, its temperature T, and the environmental state is in this case described by the equilibrium statistical operator

$$\rho_b = \rho_T = \frac{e^{-H_b/k_BT}}{\mathrm{tr}e^{-H_b/k_BT}}. \tag{47}$$

The average of the generating functional is then Gaussian, yielding the quadratic form

$$Z[f] = \exp\left\{-\frac{i}{2\hbar^2}\int_c d\tau\int_c d\tau' f(\tau)D(\tau,\tau')f(\tau')\right\}, \tag{48}$$

where

$$D(\tau,\tau') = -i\langle T_C[X(\tau)X(\tau')]\rangle \tag{49}$$

is the contour ordered bath Green's function.

It is convenient for the physical interpretation to split the exponent appearing in the generating functional into real and imaginary parts[9]

$$Z[f] = \exp\left\{-\frac{i}{\hbar^2}\int_{-\infty}^{\infty} dt\int_{-\infty}^{\infty} dt'[2f_-(t)D^R(t,t')f_+(t') + f_-(t)D^K(t,t')f_-(t')]\right\}, \tag{50}$$

where

$$f_\pm(t) = \tfrac{1}{2}[f_1(t)\pm f_2(t)] \tag{51}$$

and

$$D^R(t,t') = -i\,\theta(t-t')\langle[X(t),X(t')]\rangle \tag{52}$$

is the retarded bath propagator, and

$$D^K(t,t') = -i\langle\{X(t),X(t')\}\rangle \tag{53}$$

is the correlation or Keldysh bath propagator.[10]

For the present thermal case we have for the retarded Green's function

$$D^R(t,t')=-\frac{\hbar^2}{2}\theta(t-t')\int_0^\infty d\omega\ J(\omega)\sin\omega(t-t'), \tag{54}$$

where

$$J(\omega)=\frac{4}{\hbar^2}\sum_\alpha \lambda_\alpha^2\delta(\omega-\omega_\alpha)=-\frac{4}{\pi\hbar^2}\mathrm{Im}D^R(\omega) \tag{55}$$

is the spectral function completely characterizing the influence of the microscopic degrees of freedom of the bath. For the correlation function we similarly have

$$D^K(t,t')=-i\frac{\hbar^2}{2}\int_0^\infty d\omega\ J(\omega)\coth\frac{\hbar\omega}{2k_BT}\cos\omega(t-t'). \tag{56}$$

The Fourier transforms of the retarded and the correlation propagator is connected according to the fluctuation-dissipation theorem

$$D^K(\omega)=2i\ \mathrm{Im}D^R(\omega)\coth\frac{\hbar\omega}{2k_BT}. \tag{57}$$

The calculation of the influence of the thermal reservoir on the transition probability has now been reduced to the performance of simple integrals that are readily done and we obtain for the influence function

$$Z(t_4,t_3,t_2,t_1)=Z^R(t_4,t_3,t_2,t_1)Z^K(t_4,t_3,t_2,t_1), \tag{58}$$

where the contribution from the retarded bath propagator is given by

$$Z^R(t_4,t_3,t_2,t_1)=e^{(i/4)\lambda(t_2-t_1+t_4-t_3)}\tilde{Z}^R(t_4,t_3,t_2,t_1), \tag{59}$$

with

$$\tilde{Z}^R(t_4,t_3,t_2,t_1)=\exp\left\{\frac{i}{4}\int_0^\infty d\omega\frac{J(\omega)}{\omega^2}S_\omega(t_4,t_3,t_2,t_1)\right\}, \tag{60}$$

specified by the function

$$\begin{aligned}S_\omega(t_4,t_3,t_2,t_1)=&-\sin\omega(t_2-t_1)-\sin\omega(t_3-t_2)\\&+\sin\omega(t_4-t_2)+\sin\omega(t_3-t_1)\\&-\sin\omega(t_4-t_1)+\sin\omega(t_3-t_4),\end{aligned} \tag{61}$$

and the effective coupling constant given by

$$\lambda=\int_0^\infty d\omega\frac{J(\omega)}{\omega}. \tag{62}$$

The contribution from the correlation bath propagator is similarly given by

$$Z^K(t_4,t_3,t_2,t_1)=\exp\left\{-\frac{1}{4}\int_0^\infty d\omega\frac{J(\omega)}{\omega^2}\coth\frac{\hbar\omega}{2k_BT}C_\omega(t_4,t_3,t_2,t_1)\right\}, \tag{63}$$

where

$$\begin{aligned}C_\omega(t_4,t_3,t_2,t_1)=&2-\cos\omega(t_1-t_2)-\cos\omega(t_2-t_3)\\&+\cos\omega(t_1-t_3)+\cos\omega(t_2-t_4)\\&-\cos\omega(t_1-t_4)-\cos\omega(t_4-t_3).\end{aligned} \tag{64}$$

The two real functions C_ω and S_ω are the real and imaginary parts of the complex function

$$\begin{aligned}f_\omega(t_4,t_3,t_2,t_1)=&2-e^{i\omega(t_2-t_1)}-e^{i\omega(t_3-t_2)}+e^{i\omega(t_3-t_1)}\\&+e^{i\omega(t_4-t_2)}-e^{i\omega(t_4-t_1)}-e^{i\omega(t_4-t_3)}.\end{aligned} \tag{65}$$

Since the environment is in thermal equilibrium the influence function $Z(t_4,t_3,t_2,t_1)$ in fact depends only on three independent variables:

$$\begin{aligned}\tau&=t_2-t_1,\\ \tau'&=t_3-t_4,\\ \tau''&=t_3-t_2,\end{aligned} \tag{66}$$

the above choice being determined by the original time labeling of the Green's functions.

In terms of the independent variables we then obtain for the transition probability per unit time

$$\begin{aligned}w_{\mathbf{p}',l\to\mathbf{p},r}=&\lim_{t\to\infty}\frac{P(t)}{t}\\=&\frac{|V_{\mathbf{p},r}|^2|V_{\mathbf{p}',l}|^2}{\hbar^4}\int_{-\infty}^\infty d\tau''\int_0^\infty d\tau'\int_0^\infty d\tau\ \tilde{Z}(\tau,\tau',\tau'')\\&\times\exp\left\{-\frac{i}{\hbar}(\epsilon_{\mathbf{p},r}-\epsilon_{\mathbf{p}',l})\tau''-\frac{i}{\hbar}(\epsilon_{\mathbf{p}',l}-\tilde{\epsilon}_c-i\Gamma)\tau'\right.\\&\left.+\frac{i}{\hbar}(\epsilon_{\mathbf{p}',l}-\tilde{\epsilon}_c+i\Gamma)\tau\right\},\end{aligned} \tag{67}$$

where the influence function in terms of the three independent variables again is split into the two distinct parts,

$$\tilde{Z}(\tau,\tau',\tau'')=\tilde{Z}^R(\tau,\tau',\tau'')Z^K(\tau,\tau',\tau''), \tag{68}$$

where

$$\tilde{Z}^R(\tau,\tau',\tau'')=\exp\left\{\frac{i}{4}\int_0^\infty d\omega\frac{J(\omega)}{\omega^2}S_\omega(\tau,\tau',\tau'')\right\}, \tag{69}$$

with

$$S_\omega(\tau,\tau',\tau'') = -\sin\omega\tau + \sin\omega\tau' - \sin\omega\tau'' + \sin\omega(\tau''-\tau') + \sin\omega(\tau''+\tau) - \sin\omega(\tau''+\tau-\tau'), \tag{70}$$

and the correlation part

$$Z^K(\tau,\tau',\tau'') = \exp\left\{ -\frac{1}{4}\int_0^\infty d\omega \frac{J(\omega)}{\omega^2} C_\omega(\tau,\tau',\tau'') \times \coth\frac{\hbar\omega}{2k_BT} \right\}, \tag{71}$$

where

$$C_\omega(\tau,\tau',\tau'') = 2 - \cos\omega\tau - \cos\omega\tau' - \cos\omega\tau'' + \cos\omega(\tau''-\tau') + \cos\omega(\tau+\tau'') - \cos\omega(\tau''+\tau-\tau'). \tag{72}$$

The central site energy is shifted downwards according to

$$\tilde{\epsilon}_c = \epsilon_c - \frac{\hbar\lambda}{4}, \tag{73}$$

similar to the negative polaronic energy shift.

For the transmission coefficient in the case of a thermal environment we then get

$$T(\epsilon,\epsilon') = \frac{2}{\pi\hbar^3}\Gamma_l\Gamma_r \int_{-\infty}^\infty d\tau'' \int_0^\infty d\tau' \int_0^\infty d\tau\, \tilde{Z}(\tau,\tau',\tau'') \times \exp\left\{ -\frac{i}{\hbar}(\epsilon-\epsilon')\tau'' - \frac{i}{\hbar}(\epsilon'-\tilde{\epsilon}_c-i\Gamma)\tau' + \frac{i}{\hbar}(\epsilon'-\tilde{\epsilon}_c+i\Gamma)\tau \right\}. \tag{74}$$

This expression has been studied perturbatively in the case of the Einstein model,[4] and also in terms of elastic and inelastic channels.[3] In the following we investigate the total transmission coefficient in the thermal case and obtain explicit expressions for various limiting situations.

1. The total transmission coefficient

If we are not interested in the energetics of the arriving particles in the right electrode, but only in the number of arriving particles, only the total transmission coefficient is relevant. Such a situation arises, for instance, in the case where we can neglect any effect of the Pauli principle in the right electrode, corresponding to the situation of a highly biased left electrode in which case the left-going current is zero. In this situation the current of arriving particles is

$$I = e\int_0^\infty d\epsilon'\, T(\epsilon') f_L(\epsilon'). \tag{75}$$

In such a case we only need the expression for the total transmission coefficient, which, according to Eqs. (74) and (21), is given by

$$T(\epsilon') = \frac{4}{\hbar^2}\Gamma_l\Gamma_r \int_0^\infty d\tau' \int_0^\infty d\tau\, \tilde{Z}(\tau,\tau') \times \exp\left\{ -\frac{i}{\hbar}(\epsilon'-\tilde{\epsilon}_c-i\Gamma)\tau' + \frac{i}{\hbar}(\epsilon'-\tilde{\epsilon}_c+i\Gamma)\tau \right\}, \tag{76}$$

where

$$\tilde{Z}(\tau,\tau') = \tilde{Z}^R(\tau,\tau')Z^K(\tau,\tau') \tag{77}$$

is specified by

$$\tilde{Z}^R(\tau,\tau') = \tilde{Z}^R(\tau,\tau',0) = \exp\left\{ -\frac{i}{4}\int_0^\infty d\omega \frac{J(\omega)}{\omega^2}\sin\omega(\tau-\tau') \right\}, \tag{78}$$

and the correlation part

$$Z^K(\tau,\tau') = Z^K(\tau,\tau',0) = \exp\left\{ -\frac{1}{4}\int_0^\infty d\omega \frac{J(\omega)}{\omega^2}\coth\frac{\hbar\omega}{2k_BT} \times [1-\cos\omega(\tau-\tau')] \right\}. \tag{79}$$

Since the environment is in the thermal equilibrium state, the influence function for the total transmission coefficient only depends on one time variable, and the expression for the total transmission coefficient can be reduced to a single integral by introducing the mean and relative time variables:

$$t_m = \frac{\tau+\tau'}{2}, \qquad t = \tau-\tau'. \tag{80}$$

For the integration region we observe

$$\int_0^\infty d\tau \int_0^\infty d\tau' = \int_{-\infty}^\infty dt \int_{|t|/2}^\infty dt_m, \tag{81}$$

and performing the integration over the mean time we finally obtain

$$T(\epsilon') = \frac{2}{\hbar}\frac{\Gamma_l\Gamma_r}{\Gamma}\int_0^\infty dt\; e^{-(\Gamma t/\hbar)} Z^K(t) \times \left[\tilde{Z}^R(t)\exp\left\{ \frac{i}{\hbar}(\epsilon'-\tilde{\epsilon}_c)t \right\} + \text{c.c.} \right], \tag{82}$$

where

$$\tilde{Z}^R(t) = \exp\left\{ -\frac{i}{4}\int_0^\infty d\omega \frac{J(\omega)}{\omega^2}\sin\omega t \right\}, \tag{83}$$

and

$$Z^K(t) = \exp\left\{ -\frac{1}{4}\int_0^\infty d\omega \frac{J(\omega)}{\omega^2}\coth\frac{\hbar\omega}{2k_BT}(1-\cos\omega t) \right\}. \tag{84}$$

In the case where the oscillators all have the same frequency, the Einstein model, the spectral function takes the form

$$J(\omega)=\frac{4\lambda_0^2}{\hbar^2}\delta(\omega-\omega_0), \tag{85}$$

relevant, e.g., to optical phonons. In this case, we deduce from Eqs. (82)–(84) the total transmission coefficient

$$\begin{aligned}T(\epsilon')=\frac{2\Gamma_l\Gamma_r}{\hbar\Gamma}\int_0^\infty dt\ \exp\left\{-\left(\frac{\lambda_0}{\hbar\omega_0}\right)^2[1+2n(\omega_0)]\right\}\\ \times\left[\exp\left\{\left(\frac{\lambda_0}{\hbar\omega_0}\right)^2[1+n(\omega_0)]e^{-i\omega_0 t}\right\}\right.\\ \times\exp\left\{\left(\frac{\lambda_0}{\hbar\omega_0}\right)^2 n(\omega_0)e^{i\omega_0 t}\right\}\\ \left.\times\exp\left\{-\frac{1}{\hbar}(\Gamma-i[\epsilon'-\tilde{\epsilon}_c)]t\right\}+\text{c.c.}\right],\end{aligned} \tag{86}$$

where $n(\omega)$ is the Bose function

$$n(\omega)=\frac{1}{e^{\hbar\omega/k_BT}-1}. \tag{87}$$

Expanding the exponential functions containing $e^{i\omega_0 t}$ and $e^{-i\omega_0 t}$, and performing the integration over t, we obtain

$$T(\epsilon')=4\Gamma_l\Gamma_r\sum_{n=-\infty}^{\infty}\frac{P_n(T)}{(\epsilon'-\tilde{\epsilon}_c-n\hbar\omega_0)^2+\Gamma^2}, \tag{88}$$

where the temperature dependence is specified by

$$\begin{aligned}P_n(T)=\exp\left\{-\left(\frac{\lambda_0}{\hbar\omega_0}\right)^2[1+2n(\omega_0)]\right\}\sum_{n_1,n_2=0}^{\infty}{}'\ \frac{1}{n_1!n_2!}\\ \times\left[\left(\frac{\lambda_0}{\hbar\omega_0}\right)^2[1+n(\omega_0)]\right]^{n_1}\left[\left(\frac{\lambda_0}{\hbar\omega_0}\right)^2 n(\omega_0)\right]^{n_2},\end{aligned} \tag{89}$$

the prime restricting the summation to the terms for which $n_1-n_2=n$. Transmission can take place with absorption or emission of oscillator quanta giving rise to additional resonance peaks, the function $P_n(T)$ determining the relative weight of the peaks. The energy dependence of the transmission coefficient is illustrated in Fig. 3. In particular, in the low-temperature limit, $k_BT<\hbar\omega_0$, where the Bose function vanishes, $n(\omega)\to 0$, we can ignore all terms containing powers of $n(\omega)$ in Eq. (89), and obtain at zero temperature

$$P_n(0)=\begin{cases}\exp\left\{-\left(\frac{\lambda_0}{\hbar\omega_0}\right)^2\right\}\frac{1}{n!}\left(\frac{\lambda_0}{\hbar\omega_0}\right)^{2n}, & n\geqslant 0\\ 0, & n<0,\end{cases} \tag{90}$$

reflecting the possibility of an electron off resonance to tunnel from the left to the right reservoir via the central site by spontaneously emitting a phonon, and the impossibility of gaining energy from a zero-temperature environment. We note that $P_n(0)$ is a Poisson distribution characterized by its mean value

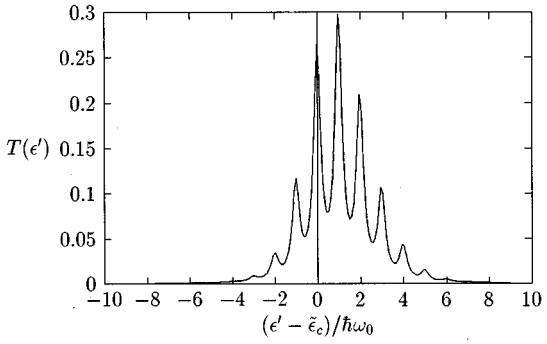

FIG. 3. The total transmission coefficient (in units of $4\Gamma_l\Gamma_r/\Gamma^2$) for the Einstein environment case. The parameters are $\Gamma=0.2\hbar\omega_0$, $\lambda_0=\hbar\omega_0$, and $k_BT=\hbar\omega_0$.

$$\langle n\rangle=\sum_{n=0}^{\infty} nP_n(0)=\left(\frac{\lambda_0}{\hbar\omega_0}\right)^2. \tag{91}$$

2. Approximations conserving the integrated transmission probability

To illustrate the general features of the systematic and fluctuation influences of the environment we choose the spectral function in the further calculations to have the form

$$J(\omega)=\eta\omega\left(\frac{\omega}{\omega_c}\right)^{s-1}\exp\left(-\frac{\omega}{\omega_c}\right). \tag{92}$$

We note that η is a dimensionless constant describing the coupling strength between the central level and the environment, and ω_c is the cutoff frequency for the oscillators.

There are several limits in which simple expressions for the transmission coefficient may be worked out. These limits are the broad resonance limit, $\Gamma>\hbar\omega_c$, the strong-coupling limit, $\eta>1$, the high-temperature limit, $k_BT>(1+1/\eta)\hbar\omega_c$, and the weak-coupling and low-temperature limit, $\eta<1$, $k_BT<\hbar\omega_c/\eta$.

a. Broad resonance limit. In the broad resonance limit, $\Gamma>\hbar\omega_c$, a short time expansion of the influence function is sufficient. The correlation part of the influence function is therefore given by

$$Z^K(t)=\exp\left\{-\frac{\kappa(T)}{8}t^2\right\}, \tag{93}$$

which controls the other part to become

$$\tilde{Z}^R(t)=\exp\left\{-i\frac{\lambda}{4}t\right\}. \tag{94}$$

Here κ is defined as

$$\kappa(T)=\int_0^\infty d\omega\ J(\omega)\coth\frac{\hbar\omega}{2k_BT}. \tag{95}$$

The total transmission coefficient, Eq. (82), is therefore in the broad resonance limit given by

$$T(\epsilon')=\frac{2\Gamma_l\Gamma_r}{\hbar\Gamma}\int_0^\infty dt[e^{-[\kappa(T)/8]t^2-(1/\hbar)[\Gamma-i(\epsilon'-\epsilon_c)]t}+\text{c.c.}]. \tag{96}$$

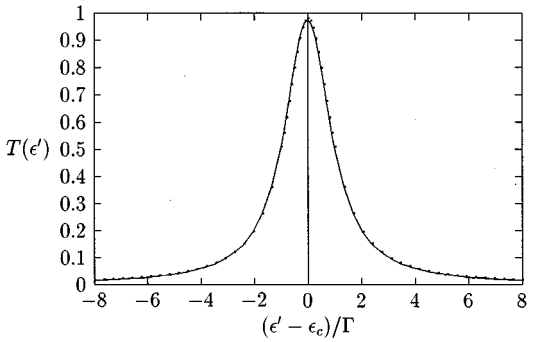

FIG. 4. Transmission coefficient (in units of $4\Gamma_l\Gamma_r/\Gamma^2$) for the Ohmic environment case in the broad resonance limit, $\Gamma>\hbar\omega_c$. The broad resonance approximation (solid curve) is compared to the exact result (dots). The parameter choice is $\eta=1$, $2k_BT=\Gamma$, and $\hbar\omega_c=0.1\Gamma$, which yields $\hbar^2\kappa(T)=0.101\Gamma^2$.

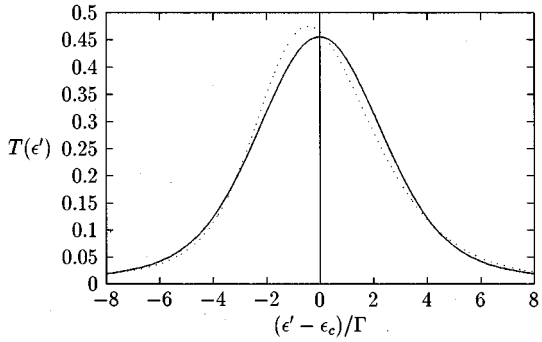

FIG. 5. Transmission coefficient (in units of $4\Gamma_l\Gamma_r/\Gamma^2$) for the Ohmic environment case in the strong-coupling limit, $\eta>1$. The strong-coupling approximation (solid curve) is compared to the exact result (dots). The parameter choice is $\eta=10$, $\hbar\omega_c=\Gamma$, $2k_BT=\Gamma$, and $\hbar^2\kappa(T)\approx 14.27\Gamma^2$.

We note that the polaron shift has canceled out, reflecting that the escape rate out of the central site is so fast that any real part environmental self-energy dressing effect is absent.

The integral in Eq. (96) can now be performed and we obtain

$$T(\epsilon')=\sqrt{\frac{2\pi}{\kappa(T)}}\frac{2\Gamma_l\Gamma_r}{\Gamma\hbar}\left\{e^{[2/(\kappa(T)\hbar^2)][\Gamma-i(\epsilon'-\epsilon_c)]^2}\times\left[1-\Phi\left(\sqrt{\frac{2}{\kappa(T)\hbar^2}}[\Gamma-i(\epsilon'-\epsilon_c)]\right)\right]+\text{c.c.}\right\}, \tag{97}$$

where $\Phi(z)$ is the probability integral, the error function. We note that in the above case the resonant line shape is no longer of the Lorentz type. The energy dependence in the broad resonance limit of the transmission coefficient in the Ohmic case, $s=1$, is illustrated in Fig. 4.

b. Strong-coupling limit. In the strong-coupling limit, $\eta>1$, we only need to consider the short-time limit, $\omega_c t<1$, because at larger times the influence function is exponentially small due to the short range of the correlation part, $Z^K(t)$. Therefore, the correlation part of the influence function is for the present case the same as in Eq. (93), and similarly the retarded part of the influence function is specified by Eq. (94). The expression for the total transmission coefficient is therefore the same as the one given in Eq. (97). However, the validity condition here is the strong-coupling criteria, $\eta>1$, and there is no requirement on the escape rate Γ. We note again that in the short-time-limit approximation the retarded part of the influence function has no effect, and the correlation part of the influence function is for the present case the relevant one. In Fig. 5 the strong-coupling approximation for the transmission coefficient is plotted versus energy for the case of an Ohmic bath. We get in the strong-coupling approximation for typical parameter values, $\eta=10$, $\hbar\omega_c=\Gamma$, and $2k_BT=\Gamma$, a 5% too low transmission maximum that is displaced to a slightly higher energy as compared to the exact result.

c. High-temperature limit. When the temperature is high enough, $k_BT>(1+1/\eta)\hbar\omega_c$, we can again use a short-time approximation

$$Z^K(t)=\exp\left\{-\frac{\kappa'(T)}{8}t^2\right\}, \tag{98}$$

where now

$$\kappa'(T)=\frac{2k_BT}{\hbar}\int_0^\infty d\omega\,\frac{J(\omega)}{\omega}, \tag{99}$$

which again controls the retarded part of the influence function to be

$$\tilde{Z}^R(t)=\exp\left\{-i\frac{\lambda}{4}t\right\}. \tag{100}$$

Similar to the two previous cases, the expression for the total transmission coefficient is

$$T(\epsilon')=\sqrt{\frac{2\pi}{\kappa'(T)}}\frac{2\Gamma_l\Gamma_r}{\Gamma\hbar}\left\{e^{[2/(\kappa'(T)\hbar^2)][\Gamma-i(\epsilon'-\epsilon_c)]^2}\times\left[1-\Phi\left(\sqrt{\frac{2}{\kappa'(T)\hbar^2}}[\Gamma-i(\epsilon'-\epsilon_c)]\right)\right]+\text{c.c.}\right\}, \tag{101}$$

which has the same form as Eq. (97), except that κ is replaced by κ'. The high-temperature approximation is compared to the exact result in Fig. 6 for the parameter values $\eta=1$, $\hbar\omega_c=\Gamma$, and $2k_BT=10\Gamma$. We note that the high-temperature approximation gives a slightly lower transmission maximum than the exact calculation yields.

d. Weak-coupling and low-temperature limit. When the coupling is weak, $\eta<1$, and the temperature is low, $k_BT<\hbar\omega_c/\eta$, the situation is different from the previous cases. The weak-coupling condition, $\eta<1$, forces the part of the influence function, $\tilde{Z}^R$, to equal unity at all times, $\tilde{Z}^R(t)\sim 1$. We also note that there is no contribution from the correlation part in the short-time limit due to the weak-coupling condition. We need therefore to consider the long-time behavior of the correlation part. The reason for the condition $k_BT<\hbar\omega_c/\eta$ is to avoid the high-temperature regime where only short times give a contribution.

The argument of the exponential in the expression for the correlation part of the influence function

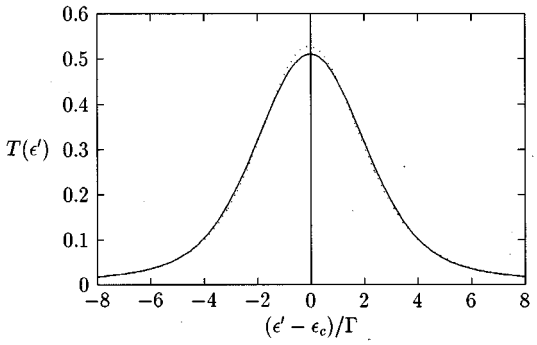

FIG. 6. Transmission coefficient (in units of $4\Gamma_l\Gamma_r/\Gamma^2$) for the Ohmic environment case in the high-temperature limit, $k_BT>(1+1/\eta)\hbar\omega_c$. In the figure the high-temperature approximation (solid curve) is compared to the exact result (dots). The parameter choice is $\eta=1$, $\hbar\omega_c=\Gamma$, $2k_BT=10\Gamma$, and $\hbar^2\kappa'(T)=10\Gamma^2$.

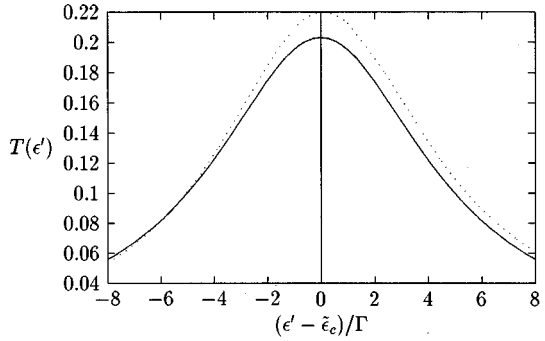

FIG. 7. Transmission coefficient (in units of $4\Gamma_l\Gamma_r/\Gamma^2$) for the Ohmic environment case in the weak-coupling and low-temperature limit, $\eta<1$ and $k_BT<\hbar\omega_c/\eta$. The weak-coupling and low-temperature approximation (solid curve) is compared to the exact result (dots). The parameter choice is $\eta=0.1$, $\hbar\omega_c=100\Gamma$, and $2k_BT=100\Gamma$.

$$I_s(t)\equiv\int_0^\infty d\omega\,\frac{J(\omega)}{\omega^2}\coth\frac{\hbar\omega}{2k_BT}(1-\cos\omega t) \tag{102}$$

approaches in the long-time limit, $t>\max(1/\omega_c,\hbar/2k_BT)$, the expression

$$I_s(t)=\eta\frac{2k_BT}{\hbar}t(\omega_ct)^{1-s}\int_0^\infty dx\;x^{s-3}(1-\cos x), \tag{103}$$

for the exponent region $0<s<2$. If $s>2$, then $I_s(t)$ approaches a constant in the long-time limit, and there is therefore no contribution from the correlation part in this case. To be more specific, we perform the integration for the Ohmic case, $s=1$. In the long-time limit, $I_1(t)=\eta(\pi k_BT/\hbar)t$, and the Ohmic correlation part becomes

$$Z^K(t)=\exp\left\{-\frac{\pi\eta k_BT}{4\hbar}t\right\}. \tag{104}$$

Therefore, the total transmission coefficient is

$$T(\epsilon')=\frac{2\Gamma_l\Gamma_r}{\hbar\Gamma}\int_0^\infty dt\left[\exp\left\{-\frac{1}{4}\eta\frac{\pi k_BT}{\hbar}t-\frac{1}{\hbar}[\Gamma-i(\epsilon'-\tilde{\epsilon}_c)]t\right\}+\text{c.c.}\right]. \tag{105}$$

The polaronic shift is no longer canceled as $\tilde{Z}^R\sim1$. Performing the integration we find for the total transmission coefficient

$$T(\epsilon')=\frac{\Gamma_{\text{eff}}}{\Gamma}\frac{4\Gamma_l\Gamma_r}{(\epsilon'-\tilde{\epsilon}_c)^2+\Gamma_{\text{eff}}^2}, \tag{106}$$

where

$$\Gamma_{\text{eff}}=\Gamma+\frac{\pi\eta k_BT}{4}. \tag{107}$$

We note that the Lorentzian character of the resonant transmission coefficient is preserved in this limit, but the width and the height of the resonance peak are different compared to the case where the bath is absent. Clearly, at high enough temperatures, $T>4\Gamma/\pi\eta k_B$, the resonance peak will be strongly reduced. Therefore, although the coupling is weak, the result is highly nonperturbative. The weak-coupling approximation in the temperature range $4\Gamma/\pi\eta<k_BT<\hbar\omega_c/\eta$ is compared to the exact result in Fig. 7, for parameter values $\eta=0.1$, $\hbar\omega_c=100\Gamma$, and $2k_BT=100\Gamma$. For this parameter choice the resonance peak is broadened approximatively five times compared to the case where the environment is absent. Even though in the case considered the coupling constant is not that small and the temperature is not excessively low, the deviation of the approximately calculated transmission maximum compared to the exact value is no more than 10%.

We notice that the zeroth moment of the transmission coefficient,

$$\int_{-\infty}^\infty d\epsilon'\,T(\epsilon')=\frac{4\pi\Gamma_l\Gamma_r}{\Gamma}, \tag{108}$$

is preserved exactly in all the approximations considered above. We also notice that the first moment of the transmission coefficient,

$$\frac{\Gamma}{4\pi\Gamma_l\Gamma_r}\int_{-\infty}^\infty d\epsilon'\epsilon'\;T(\epsilon')=\epsilon_c \tag{109}$$

is preserved exactly by all the approximations except for the weak-coupling and low-temperature approximation where the transmission coefficient is symmetric around $\tilde{\epsilon}_c$, which is anyhow close to ϵ_c because the coupling is weak.

B. The fluctuating level model

When discussing specific properties of a physical system, it is often possible to neglect the quantum nature of the environment and represent the effect of the environment by a collective classical variable, which is appropriate when quantum fluctuations can be neglected. A well-known example is the excellent account of the atomic spectra obtained by disregarding the quantum fluctuations of the electromagnetic environment, except for cases where degeneracies are only lifted by radiative corrections. A counterexample where the

effect of the environment is of pure quantum nature is of course as easily recalled, that of stimulated emission.

In the case where the degree of freedom X represents environmental degrees of freedom collectively in the form of an external classical potential, we can still use the presented method for calculating the influence. In this case the quantity X just corresponds to a classical potential, and is not an operator but just a c number. We shall in this section discuss the case where the potential X is a fluctuating quantity, the fluctuating level model.

We assume further that the fluctuations are Gaussian, and therefore characterized by the lowest-order correlations

$$\langle X(t)\rangle = c,$$

$$\langle [X(t)-\langle X(t)\rangle][X(t')-\langle X(t')\rangle]\rangle = K(t-t'). \quad (110)$$

The influence of the now classical environmental degree of freedom X is given by the generating functional expression, Eq. (44), however, now the brackets simply denote the Gaussian average with respect to the fluctuating quantity X. We therefore have for the generating functional, which in this case is just the probability theory contour characteristic functional,

$$Z[f] = \exp\left\{ -\frac{i}{\hbar}\int_c d\tau\, f(\tau)\langle X(\tau)\rangle - \frac{1}{2\hbar^2}\int_c d\tau \int_c d\tau' f(\tau)K(\tau-\tau')f(\tau')\right\}. \quad (111)$$

The reason we made the distinction in the previous section in the effects of the environment as expressed in the split

$$Z = Z^R Z^K, \quad (112)$$

where we distinguish between the retarded and correlation contributions, is that they represent two distinct influences of the environment. The term Z^R represents the systematic friction type and energy renormalization influence and Z^K the fluctuating part, including thermal as well as quantum fluctuations. For the thermal environment discussed in the previous subsection, the two types of influence were not independent, but related through the fluctuation-dissipation theorem.

We now compare the thermal quantum environment with the classical stochastic environment introduced above. It follows directly in the presented real-time approach that the fluctuating level model is in one-to-one correspondence with the fluctuational aspect of the thermal case with the prescription for the correlation part

$$D^K(t,t') \mapsto -2iK(t-t'), \quad (113)$$

and substitution of one for $\tilde{Z}^R$, $\tilde{Z}^R \mapsto 1$. In addition, we see that the polaronic shift corresponds to the average displacement of the environment

$$\frac{\hbar\lambda}{4} \mapsto -c, \quad (114)$$

obtained from Eq. (76).

The transmission coefficient for the stochastic environment is obtained from

$$T(\epsilon') = \frac{4\Gamma_r\Gamma_l}{\hbar^2}\int_0^\infty d\tau' \int_0^\infty d\tau\, Z^K(\tau,\tau') \times \exp\left\{ \frac{i}{\hbar}(\epsilon' - \tilde{\epsilon}_c + i\Gamma)\tau - \frac{i}{\hbar}(\epsilon' - \tilde{\epsilon}_c - i\Gamma)\tau'\right\}, \quad (115)$$

where

$$Z^K(\tau,\tau') = \exp\left\{ -\frac{2}{\hbar^2}\int_{-\infty}^\infty dt \int_{-\infty}^\infty dt' f_-^0(t)K(t-t')f_-^0(t')\right\}, \quad (116)$$

and

$$f_-^0(t) = \tfrac{1}{2}\{[\theta(\tau-t)-\theta(-t)]-[\theta(\tau-t)-\theta(-\tau'+\tau-t)]\} \quad (117)$$

and the correlation part Z^K now depends on the explicit form of the correlator K. For example, the high-temperature form of the correlation function D^K for a thermal Ohmic environment, $J(\omega) = \eta\omega$, corresponds to a δ-correlator

$$K(t-t') = \frac{\pi\hbar\,\eta k_B T}{2}\delta(t-t'), \quad (118)$$

the white noise case. The fluctuating level model thus only captures the correlation part Z^K of the thermal environment and neglects the systematic retarded part, except for its average influence.

For the fluctuating level model we therefore get for the influence function in the Ohmic case,

$$Z^K(\tau,\tau') = \exp\left\{ -\frac{1}{4\hbar}\pi\,\eta k_B T|\tau-\tau'|\right\}, \quad (119)$$

and for the transmission coefficient

$$T(\epsilon') = \frac{\Gamma_{\text{eff}}}{\Gamma}\frac{4\Gamma_l\Gamma_r}{(\epsilon'-\tilde{\epsilon}_c)^2+\Gamma_{\text{eff}}^2}. \quad (120)$$

Comparing the result to the uncoupled case, we notice, besides the average energy shift and a reduction of the peak,[12] that the resonance width Γ of Eq. (41) is broadened according to

$$\Gamma \to \Gamma_{\text{eff}} = \Gamma + \frac{\pi}{4}\eta k_B T. \quad (121)$$

We note that the transmission coefficient in this case is the same as the one obtained for the thermal case in the weak-coupling and low-temperature limit.

C. The external field case

Just as in the previous section we shall here consider the case where the environment can be described classically. In addition we shall assume that in contrast to the previous section we know not only the probability distribution of the potential, but in fact the actual potential. In the following we shall therefore investigate the situation where we are able to

couple the electron to an external field at the central site. In the present case of linear coupling it is sufficient to consider the case where the central site energy level changes harmonically in time. The external potential is therefore given by

$$X(t)=X_0\cos\omega_0 t. \tag{122}$$

Such a situation could, for instance, be realized in the case where the Hamiltonian represents a small metallic grain with a single active level whose energy can be changed by an external electric potential, and the grain being coupled to large metallic electrodes.

In the case where the influence of the environment is represented by the external potential given by Eq. (122) we obtain for the influence function

$$\begin{aligned} Z(t_4,t_3,t_2,t_1)&=\langle T_C e^{-(i/\hbar)\int_c d\tau f(\tau)X(\tau)}\rangle|_{f=f^0}\\ &=e^{-(i/\hbar)X_0\int_{-\infty}^{\infty}dt[f_1(t)-f_2(t)]\cos\omega_0 t}\\ &=\exp\left\{\frac{iX_0}{\hbar\omega_0}(\sin\omega_0 t_1-\sin\omega_0 t_2\right.\\ &\quad\left.+\sin\omega_0 t_3-\sin\omega_0 t_4)\right\}. \end{aligned} \tag{123}$$

This corresponds to the fluctuating level model for the case where the potential $X(\tau)$ is known with certainty to be given by the expression in Eq. (122).

For the transition probability per unit time we then obtain the expression

$$\begin{aligned} w_{\mathbf{p}',l\to\mathbf{p},r}=&\frac{|V_{\mathbf{p},r}|^2|V_{\mathbf{p}',l}|^2}{\hbar^4}\int_{-\infty}^{\infty}d\tau''\int_0^{\infty}d\tau'\int_0^{\infty}d\tau\\ &\times\exp\left\{-\frac{i}{\hbar}(\epsilon_{\mathbf{p},r}-\epsilon_{\mathbf{p}',l})\tau''\right.\\ &\left.-\frac{i}{\hbar}(\epsilon_{\mathbf{p}',l}-\epsilon_c-i\Gamma)\tau'+\frac{i}{\hbar}(\epsilon_{\mathbf{p}',l}-\epsilon_c+i\Gamma)\tau\right\}\\ &\times\lim_{t\to\infty}\frac{1}{t}\int_0^t dt_1\exp\left\{\frac{i}{\hbar}\frac{X_0}{\omega_0}(\mathrm{Re}f^{\omega_0}_{\tau,\tau',\tau''}\sin\omega_0 t_1\right.\\ &\left.+\mathrm{Im}f^{\omega_0}_{\tau,\tau',\tau''}\cos\omega_0 t_1)\right\}, \end{aligned} \tag{124}$$

where we have introduced the function

$$f^{\omega_0}_{\tau,\tau',\tau''}=1-e^{i\omega_0\tau}+e^{i\omega_0(\tau+\tau'')}-e^{i\omega_0(\tau+\tau''-\tau')}. \tag{125}$$

We now calculate the integral over t_1 in Eq. (124) by expanding the exponential before performing the integration, and therefore assume that ω_0 is nonzero. The case of zero ω_0 corresponds to a constant external potential, $X(t)=X_0$, which only yields a trivial shift of the resonant energy. The transition probability per unit time can now be expressed as an integral involving the zeroth-order Bessel function, J_0.[11]

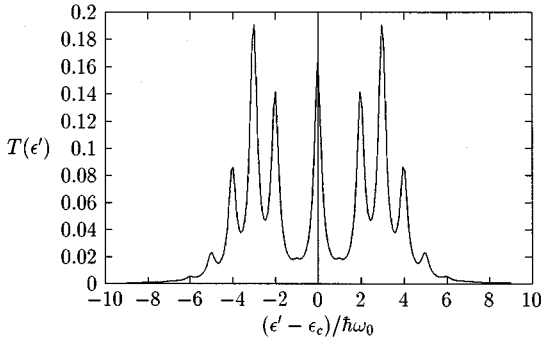

FIG. 8. Transmission coefficient (in units of $4\Gamma_l\Gamma_r/\Gamma^2$) for the external field case, $X(t)=X_0\cos\omega_0 t$. The parameter choice is $\Gamma=0.2\hbar\omega_0$ and $X_0=4\hbar\omega_0$.

$$\begin{aligned} T(\epsilon,\epsilon')=&\frac{2\Gamma_l\Gamma_r}{\pi\hbar^3}\int_{-\infty}^{\infty}d\tau''\int_0^{\infty}d\tau'\int_0^{\infty}d\tau\, J_0\left(\frac{X_0}{\hbar\omega_0}|f^{\omega_0}_{\tau,\tau',\tau''}|\right)\\ &\times\exp\left\{-\frac{i}{\hbar}(\epsilon-\epsilon')\tau''-\frac{i}{\hbar}(\epsilon'-\epsilon_c-i\Gamma)\tau'\right.\\ &\left.+\frac{i}{\hbar}(\epsilon'-\epsilon_c+i\Gamma)\tau\right\}. \end{aligned} \tag{126}$$

If we had assumed that the external potential was a sine function, $X(t)=X_0\sin\omega_0 t$, we would have obtained the same transmission coefficient for nonzero ω_0, but we would of course not have obtained a shift of the resonant energy in the case of $\omega_0=0$. The total transmission coefficient is

$$\begin{aligned} T(\epsilon')=&\frac{4\Gamma_l\Gamma_r}{\hbar^2}\int_0^{\infty}d\tau\int_0^{\infty}d\tau' J_0\left(\frac{X_0}{\hbar\omega_0}|f^0_{\tau,\tau',\tau''=0}|\right)\\ &\times\exp\left\{-\frac{i}{\hbar}(\epsilon'-\epsilon_c-i\Gamma)\tau'+\frac{i}{\hbar}(\epsilon'-\epsilon_c+i\Gamma)\tau\right\}. \end{aligned} \tag{127}$$

Noting that $|f^{\omega_0}_{\tau,\tau',\tau''=0}|^2=4\sin^2(\omega_0(\tau-\tau')/2)$, and using the summation formula for the Bessel function J_0 (Ref. 12) we obtain the total transmission coefficient[13]

$$T(\epsilon')=4\Gamma_l\Gamma_r\sum_{n=-\infty}^{\infty}\frac{J_n^2(X_0/\hbar\omega_0)}{(\epsilon'-\epsilon_c-n\hbar\omega_0)^2+\Gamma^2}. \tag{128}$$

The total transmission coefficient has Lorentzian side bands at all harmonics of $\hbar\omega_0$ with a relative weight determined by the Bessel functions, and maximal peaks in the spectrum at $n\sim\pm X_0/\hbar\omega_0$. The transmission coefficient is plotted for the parameters $\Gamma=0.2\hbar\omega_0$ and $X_0=4\hbar\omega_0$ in Fig. 8. For this parameter choice the first sideband is almost missing because $X_0/\hbar\omega_0$ is close to the first zero of the first-order Bessel function.

D. Coherent state environment

In the following we shall investigate the model for the environment in the coherent state. Such a model may describe transport through a quantum dot coupled to an environment of coherent phonons.[15]

The coherent state has the following representation in terms of the vacuum state $|0\rangle$ of the environment:

$$|\phi\rangle = N_\phi^{-1/2} \exp\{\phi b_0^\dagger\}|0\rangle, \tag{129}$$

where

$$N_\phi = \exp\{|\phi|^2\} \tag{130}$$

is the normalization factor.

The state of the environment is then a pure state and the statistical operator for the environment reduces to the projection operator

$$\rho_b = P_\phi = |\phi\rangle\langle\phi|. \tag{131}$$

To calculate the influence of the environment in the case of a coherent state we therefore need to evaluate the following expression for the influence function:

$$\begin{aligned} Z(t_4,t_3,t_2,t_1) &= Z[f^0] \\ &= N_\phi^{-1}\langle 0|\exp\{\phi^* b_0\} \\ &\quad \times (T_C e^{-(i/\hbar)\int_c d\tau f^0(\tau)X(\tau)}) \exp\{\phi b_0^\dagger\}|0\rangle. \end{aligned} \tag{132}$$

The bracket in the definition of the generating functional corresponds therefore in this case to taking the expectation value with respect to the coherent state:

$$\begin{aligned} Z[f] &= \langle T_C e^{-(i/\hbar)\int_c d\tau f(\tau)X(\tau)}\rangle = \langle\phi|T_C e^{-(i/\hbar)\int_c d\tau f(\tau)X(\tau)}|\phi\rangle \\ &= N_\phi^{-1}\langle 0|e^{\phi^* b_0} T_C e^{-(i/\hbar)\int_c d\tau f(\tau)X(\tau)} e^{\phi b_0^\dagger}|0\rangle. \end{aligned} \tag{133}$$

The matrix element appearing in Eq. (133) is most easily calculated by introducing the single mode generating functional

$$Z[f,f^*] = \langle T_C e^{-(i/\hbar)\int_c d\tau \lambda_0 (f^*(\tau) b_0(\tau) + f(\tau) b_0^\dagger(\tau))}\rangle. \tag{134}$$

We then notice that we can rewrite the generating functional of interest as

$$Z[f] = \langle\phi|T_C e^{-(i/\hbar)\int_c d\tau f(\tau)X(\tau)}|\phi\rangle = Z[\tilde f, \tilde f^*] \tag{135}$$

provided we substitute into Eq. (134)

$$\tilde f(\tau) = f(\tau) + i\hbar \frac{\phi}{\lambda_0}\delta^u(\tau) \tag{136}$$

and

$$\tilde f^*(\tau) = f(\tau) - i\hbar \frac{\phi^*}{\lambda_0}\delta^l(\tau), \tag{137}$$

where $\delta^{u(l)}(\tau)$ is a δ function on the upper (lower) part of the contour, and vanishes on the lower (upper) part.

The single-mode generating functional involves a Gaussian average and is given by the quadratic form

$$Z[f,f^*] = e^{-(i/\hbar^2)\int_C d\tau \int_C d\tau' \lambda_0^2 f^*(\tau) B(\tau,\tau') f(\tau')}, \tag{138}$$

where B is the single-mode Green's function

$$B(\tau,\tau') = -i\langle 0|T_C(b_0(\tau) b_0^\dagger(\tau'))|0\rangle, \tag{139}$$

the zero-temperature limit of the previously introduced bath propagator $D(\tau,\tau')$ for the considered mode. If we therefore insert the proper "force" f^0 according to the prescription Eqs. (136) and (137) we obtain the influence function for an environment in a coherent state

$$\begin{aligned} Z(t_4,t_3,t_2,t_1) &= Z[f_0] \\ &= Z_{T=0}(t_4,t_3,t_2,t_1) \\ &\quad \times \exp\Bigg[2i \frac{\lambda_0}{\hbar\omega_0} \mathrm{Im}\{\phi(e^{-i\omega_0 t_2} - e^{-i\omega_0 t_1} \\ &\quad + e^{-i\omega_0 t_4} - e^{-i\omega_0 t_3})\}\Bigg]. \end{aligned} \tag{140}$$

Here $Z_{T=0}$ denotes the influence function for the thermal case with the temperature set equal to zero, that is, the ground-state case.

Exploiting the observations already made for the external field case we obtain the following expression for the transmission coefficient:

$$\begin{aligned} T(\epsilon,\epsilon') &= \frac{2\Gamma_l\Gamma_r}{\pi\hbar^3}\int_{-\infty}^{\infty} d\tau'' \int_0^\infty d\tau' \int_0^\infty d\tau\, \tilde Z_{T=0}(\tau,\tau',\tau'') \\ &\quad \times J_0\left(\frac{2\lambda_0|\phi|}{\hbar\omega_0}|f^{\omega_0}_{\tau,\tau',\tau''}|\right)\exp\Bigg\{ -\frac{i}{\hbar}(\epsilon-\epsilon')\tau'' \\ &\quad - \frac{i}{\hbar}(\epsilon' - \tilde\epsilon_c - i\Gamma)\tau' + \frac{i}{\hbar}(\epsilon' - \tilde\epsilon_c + i\Gamma)\tau\Bigg\}, \end{aligned} \tag{141}$$

where the function $f^{\omega_0}_{\tau,\tau',\tau''}$ is the same as the function introduced in Eq. (125).

The coherent state case shares features with both the thermal and the external field cases. The coherent state influence function consists of a factor equal to the zero-temperature Einstein model influence function, which describes the systematic influence and quantum zero-point fluctuations, and in addition a factor identical to the external field influence function.

For the total transmission coefficient we get

$$\begin{aligned} T(\epsilon') &= \frac{4\Gamma_l\Gamma_r}{\hbar^2}\int_0^\infty d\tau \int_0^\infty d\tau' J_0\left(\frac{2\lambda_0|\phi|}{\hbar\omega_0}|f^{\omega_0}_{\tau,\tau',\tau''=0}|\right) \\ &\quad \times \exp\left\{ -\left(\frac{\lambda_0}{\hbar\omega_0}\right)^2 (1 - e^{-i\omega_0(\tau-\tau')})\right\} \\ &\quad \times \exp\Bigg\{ -\frac{i}{\hbar}(\epsilon' - \tilde\epsilon_c - i\Gamma)\tau' + \frac{i}{\hbar}(\epsilon' - \tilde\epsilon_c + i\Gamma)\tau\Bigg\}. \end{aligned} \tag{142}$$

Performing a calculation similar to the one for the oscillatory level model we get a simple formula for the total transmission coefficient

$$T(\epsilon') = 4\Gamma_l\Gamma_r \sum_{n=-\infty}^{\infty} \frac{Q_n}{(\epsilon' - \tilde\epsilon_c - n\hbar\omega_0)^2 + \Gamma^2}, \tag{143}$$

where

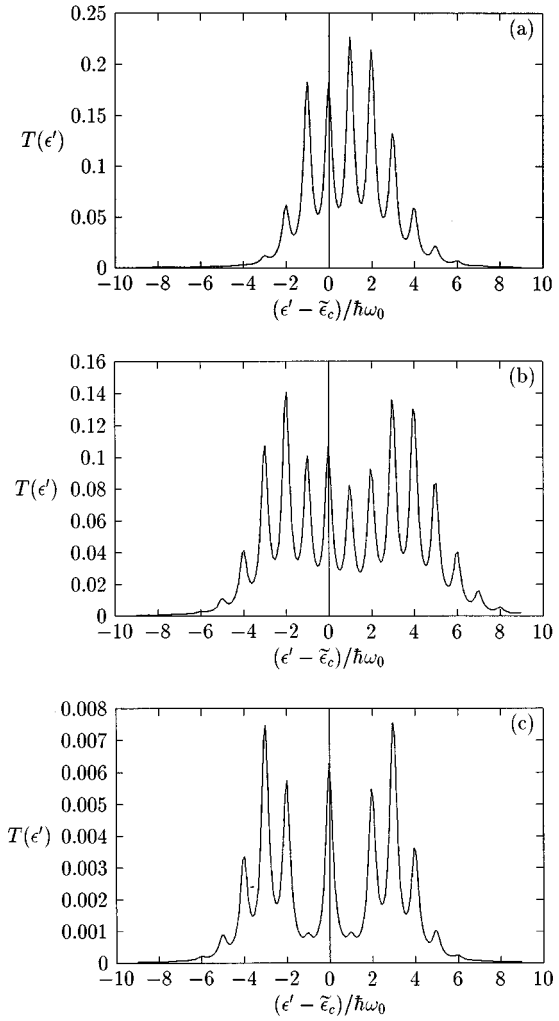

FIG. 9. Transmission coefficient (in units of $4\Gamma_l\Gamma_r/\Gamma^2$) for the coherent state environment. In diagram (a) we have chosen $\Gamma=0.2\hbar\omega_0$, $\lambda_0=\hbar\omega_0$, and $|\phi|=1$. In this case the transmission curve resembles the one obtained for the Einstein model; see Fig. 3. In (b) $\Gamma=0.2\hbar\omega_0$, $\lambda_0=\hbar\omega_0$, and $|\phi|=2$. In (c) $\Gamma=0.2\hbar\omega_0$, $\lambda_0=0.2\hbar\omega_0$, and $|\phi|=10$. The shape of the curve in diagram (c) resembles the one obtained in the external field case; see Fig. 8.

$$Q_n=e^{-(\lambda_0/\hbar\omega_0)^2}\sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{\lambda_0}{\hbar\omega_0}\right)^{2k}J_{n-k}^2\left(\frac{2\lambda_0|\phi|}{\hbar\omega_0}\right). \tag{144}$$

In the limit $\phi=0$, we, of course, recover the zero-temperature Einstein model result, Eq. (90). In the limit of $\lambda_0\to 0$, $\phi\to\infty$, and $\lambda_0|\phi|=\text{const}$, we recover the classical oscillatory level model result, Eq. (128). This crossover behavior is illustrated in Fig. 9.

VII. SUPPRESSION OF QUANTUM INTERFERENCE

In the following section we investigate how the different environments influence the phase coherence in a quantum interference setup. We envisage the situation where two resonant levels coupled to environments are placed in parallel and transport can take place through either.[14] A physical realization could be double-barrier structures situated on the two arms of a ring coupled to two reservoirs. The Hamiltonian still has the form

$$H=H_e+H_b+H_i, \tag{145}$$

but now the sample Hamiltonian corresponds to two levels:

$$H_s=\sum_{c=1,2}\epsilon_c a_c^\dagger a_c. \tag{146}$$

The tunneling can take place through either level with different couplings

$$H_t=\sum_{\mathbf{p},\sigma,c}\{V_{\mathbf{p},\sigma,c}a_c^\dagger a_{\mathbf{p},\sigma}+\text{H.c.}\}. \tag{147}$$

The interaction takes the form

$$H_i=\sum_{c=1,2}a_c^\dagger a_c X_c, \tag{148}$$

where the environment operator depends on the level through the coupling constant

$$X_c=\sum_\alpha \lambda_{\alpha,c}\{b_{\alpha,c}^\dagger+b_{\alpha,c}\}, \tag{149}$$

and we assume a situation where the double-barrier structures are coupled to separate environments,

$$H_b=\sum_{c=1,2;\alpha}\hbar\omega_\alpha\{b_{\alpha,c}^\dagger b_{\alpha,c}+\tfrac{1}{2}\}. \tag{150}$$

In order to have an external parameter to vary we envisage an Aharonov-Bohm type situation by piercing the ring with a magnetic flux Φ so that the propagators change according to propagation around the different arms of the ring according to

$$G_1^R\mapsto e^{(i/2)\Phi/\Phi_0}G_1^R,\quad G_2^R\mapsto e^{-(i/2)\Phi/\Phi_0}G_2^R. \tag{151}$$

The transmission probability given by Eq. (11) now consists of transmission through either arm and to accommodate this two-level situation Eq. (14) is changed into

$$\begin{aligned}P_{\mathbf{p}',l\to\mathbf{p}r}(t)=&\frac{1}{\hbar^4}\sum_{c_1,c_2,c_3,c_4}V_{\mathbf{p}',l,c_1}V_{\mathbf{p},r,c_2}^*V_{\mathbf{p},r,c_3}V_{\mathbf{p}',l,c_4}^*\\&\times\int_0^t dt_1\int_0^t dt_2\int_0^t dt_3\int_0^t dt_4\\&\times\exp\left\{\frac{i}{\hbar}\epsilon_{\mathbf{p},r}(t_2-t_3)+\frac{i}{\hbar}\epsilon_{\mathbf{p}',l}(t_4-t_1)\right\}\\&\times\langle\hat{G}_{c_4,c_3}^A(t_4,t_3)\hat{G}_{c_2,c_1}^R(t_2,t_1)\rangle,\end{aligned} \tag{152}$$

where

$$\hat{G}_{c_2,c_1}^R(t_2,t_1)=-i\theta(t_2-t_1)\langle 0|[\hat{a}_{c_2}(t_2),\hat{a}_{c_1}^\dagger(t_1)]|0\rangle \tag{153}$$

and

$$\begin{aligned}\hat{G}_{c_4,c_3}^A(t_4,t_3)&=[\hat{G}_{c_3,c_4}^R(t_3,t_4)]^*\\&=i\theta(t_3-t_4)\langle 0|[\hat{a}_{c_4}(t_4),\hat{a}_{c_3}^\dagger(t_3)]|0\rangle.\end{aligned} \tag{154}$$

In the following we shall neglect all terms except those for which $c_4=c_3$ and $c_2=c_1$. This is justified if the two resonant levels have an energy difference larger than the width of the levels. Here we simply implement it corresponding to propagation taking place through either arm.

The transmission probability given by Eq. (11) then consists of the transmission probabilities for transmission through either arm and an interference contribution:

$$P_{\mathbf{p}',l\to\mathbf{p},r}(t)=\sum_{c=1,2}P_c(\mathbf{p},\mathbf{p}',t)+P_{\rm int}(\mathbf{p},\mathbf{p}',t), \tag{155}$$

where

$$P_c(\mathbf{p},\mathbf{p}',t)=\frac{|V_{\mathbf{p}',l,c}|^2|V_{\mathbf{p},r,c}|^2}{\hbar^4}\int_0^t dt_1\int_0^t dt_2\int_0^t dt_3\int_0^t dt_4\exp\left\{\frac{i}{\hbar}\epsilon_{\mathbf{p},r}(t_2-t_3)+\frac{i}{\hbar}\epsilon_{\mathbf{p}',l}(t_4-t_1)\right\}$$
$$\times G_c^A(t_4,t_3)G_c^R(t_2,t_1)Z_c(t_4,t_3,t_2,t_1) \tag{156}$$

and the interference term is given by

$$P_{int}(\mathbf{p},\mathbf{p}',t)=\frac{1}{\hbar^4}V_{\mathbf{p}',l,1}V^*_{\mathbf{p},r,1}V_{\mathbf{p},r,2}V^*_{\mathbf{p}',l,2}\int_0^t dt_1\int_0^t dt_2\int_0^t dt_3\int_0^t dt_4\exp\left\{\frac{i}{\hbar}\epsilon_{\mathbf{p},r}(t_2-t_3)+\frac{i}{\hbar}\epsilon_{\mathbf{p}',l}(t_4-t_1)\right\}$$
$$\times G_2^A(t_4,t_3)G_1^R(t_2,t_1)Z_1(0,0,t_2,t_1)Z_2(t_4,t_3,0,0)+\text{c.c.} \tag{157}$$

We now have an influence function for each of the two different environments, Z_1 and Z_2, corresponding to the two different arms of the interferometer. For the transmission coefficient we now have according to Eqs. (18) and (19)

$$T(\epsilon,\epsilon')=\lim_{t\to\infty}\frac{h}{t}\sum_{\mathbf{p},\mathbf{p}'}P_{\mathbf{p}',l\to\mathbf{p},r}(t)\,\delta(\epsilon'-\epsilon_{\mathbf{p}',l})\,\delta(\epsilon-\epsilon_{\mathbf{p},r})$$
$$=\sum_{c=1,2}T_C(\epsilon,\epsilon')+T_{\rm int}(\epsilon,\epsilon'). \tag{158}$$

The calculation of the total transmission coefficient, specified in Eq. (21), in various limits is similar to what has been calculated in Sec. VI. For example, in the absence of an environment we get for the total transmission coefficient

$$T(\epsilon')=T_1(\epsilon')+T_2(\epsilon')+2\sqrt{T_1(\epsilon')T_2(\epsilon')}\frac{|\Gamma_{12r}||\Gamma_{21l}|}{\sqrt{\Gamma_{1l}\Gamma_{1r}\Gamma_{2l}\Gamma_{2r}}}$$
$$\times\cos\left(\phi-\arctan\frac{\Gamma_1}{\epsilon'-\epsilon_1}+\arctan\frac{\Gamma_2}{\epsilon'-\epsilon_2}\right.$$
$$\left.+\arg\Gamma_{12r}\Gamma^*_{21l}\right), \tag{159}$$

where $\phi=2\pi\Phi/\Phi_0$ is the relative phase difference due to the external flux, and T_1 and T_2 are the total transmission coefficients for the individual arms, and specified in the previous sections. The decay rates, whose energy dependence can be neglected in the wide-band limit, are defined as

$$\Gamma_c=\Gamma_{\rm cr}+\Gamma_{\rm cl} \tag{160}$$

and

$$\Gamma_{c_1c_2\sigma}(\epsilon)=\pi\sum_{\mathbf{p}}V^*_{\mathbf{p}\sigma c_1}V_{\mathbf{p}\sigma c_2}\delta(\epsilon-\epsilon_{\mathbf{p}\sigma}). \tag{161}$$

To illustrate the special features of the suppression of phase coherence in the present model, we discuss the cases of thermal bath, fluctuating level, classical field, and coherent state environment below.

A. Thermal bath and fluctuating level cases

The transmission coefficient through either arm we obtained previously. For the interference term, we have the expression

$$T_{\rm int}(\epsilon,\epsilon')=\frac{2}{\pi\hbar^3}e^{i\phi}\Gamma_{12r}\Gamma_{21l}\int_{-\infty}^{\infty}d\tau''\int_0^{\infty}d\tau'\int_0^{\infty}d\tau\,\tilde{Z}_1(\tau,0,0)\tilde{Z}_2(0,\tau',0)$$
$$\times\exp\left\{-\frac{i}{\hbar}(\epsilon-\epsilon')\tau''\right\}\exp\left\{-\frac{i}{\hbar}(\epsilon'-\tilde{\epsilon}_2-i\Gamma_2)\tau'\right\}\exp\left\{+\frac{i}{\hbar}(\epsilon'-\tilde{\epsilon}_1+i\Gamma_1)\tau\right\}+\text{c.c.}, \tag{162}$$

where the influence function corresponding to site c enters as

$$\tilde{Z}_c(\tau,0,0)=\exp\left\{-\frac{i}{4}\int_0^\infty d\omega\,\frac{J_c(\omega)}{\omega^2}\sin\omega\tau\right\}\exp\left\{-\frac{1}{4}\int_0^\infty d\omega\,\frac{J_c(\omega)}{\omega^2}\coth\frac{\hbar\omega}{2k_BT}(1-\cos\omega\tau)\right\}, \tag{163}$$

and the central site energies for each arm are shifted downwards according to

$$\tilde{\epsilon}_c=\epsilon_c-\frac{\lambda_c}{4}, \tag{164}$$

with the negative polaronic energy shift

$$\lambda_c=\int_0^\infty d\omega\,\frac{J_c(\omega)}{\omega}, \tag{165}$$

being site-dependent through the spectral function

$$J_c(\omega)=\frac{4}{\hbar^2}\sum_\alpha\,\lambda_{\alpha,c}^2\,\delta(\omega-\omega_\alpha). \tag{166}$$

In the course of the derivation we have noted that

$$\tilde{Z}_c(\tau,0,0)=\tilde{Z}_c^*(0,\tau,0), \tag{167}$$

which follows from Eq. (17). The interference contribution to the transmission coefficient is seen to have the form

$$\begin{aligned}T_{\rm int}(\epsilon,\epsilon')=&\frac{4}{\hbar^2}e^{i\phi}\Gamma_{12r}\Gamma_{21l}\delta(\epsilon-\epsilon')\int_0^\infty d\tau'\tilde{Z}_2(0,\tau',0)\\&\times\exp\left\{-\frac{i}{\hbar}(\epsilon'-\tilde{\epsilon}_2-i\Gamma_2)\tau'\right\}\int_0^\infty d\tau\;\tilde{Z}_1^*(0,\tau,0)\\&\times\exp\left\{\frac{i}{\hbar}(\epsilon'-\tilde{\epsilon}_1+i\Gamma_1)\tau\right\}+\text{c.c.}\end{aligned} \tag{168}$$

We notice that the interference term vanishes unless the energy of the particle is conserved. Any energy exchange with the thermal environment thus destroys the interference. In the present model interference is thus hypersensitive to the presence of the thermal environment, and in a dramatic fashion displays the equivalence of dissipation and loss of phase coherence.

We can also calculate the transmission coefficient for the case of fluctuating levels. We assume that each site, $c=1$ and $c=2$, is coupled to separate classical fluctuating environments, represented by the variables $X_c(t)$, $c=1,2$, respectively. Analogously to the previous thermal bath case there are contributions to the transmission coefficient from transmission through either arm, and there is an interference contribution specified by Eq. (168), except for the thermal influence function being replaced by

$$\tilde{Z}_c(\tau,0,0)=\tilde{Z}_c(0,\tau,0)=\exp\left\{-\frac{1}{4\hbar^2}\int_0^\tau dt\int_0^\tau dt'\,K_c(t-t')\right\}, \tag{169}$$

where

$$K_c(t,t')=\langle[X_c(t)-\langle X_c(t)\rangle][X_c(t')-\langle X_c(t')\rangle]\rangle. \tag{170}$$

As in the thermal case, the interference contribution to the transmission coefficient is completely suppressed by dissipation.

B. External field and coherent state cases

In the case where the energy levels change harmonically in time with the same frequency

$$X_c(t)=X_c\cos\omega_0 t, \tag{171}$$

but with different coupling strength, we obtain for the interference term

$$\begin{aligned}T_{\rm int}(\epsilon-\epsilon')=&\frac{2e^{i\phi}}{\pi\hbar^3}\Gamma_{12r}\Gamma_{21l}\int_{-\infty}^\infty d\tau''\int_0^\infty d\tau'\int_0^\infty d\tau\,\exp\left\{-\frac{i}{\hbar}(\epsilon-\epsilon')\tau''\right\}\exp\left\{-\frac{i}{\hbar}(\epsilon'-\epsilon_2-i\Gamma_2)\tau'\right\}\\&\times\exp\left\{\frac{i}{\hbar}(\epsilon'-\epsilon_1+i\Gamma_1)\tau\right\}\lim_{t\to\infty}\frac{1}{t}\int_0^t dt_1\exp\left\{\frac{i}{\hbar}(\mathrm{Re}\,g^{\omega_0}_{\tau,\tau',\tau''}\sin\omega_0 t_1+\mathrm{Im}\,g^{\omega_0}_{\tau,\tau',\tau''}\cos\omega_0 t_1)\right\}+\text{c.c.},\end{aligned} \tag{172}$$

where we have introduced the function

$$g^{\omega_0}_{\tau,\tau',\tau''}=\frac{X_1}{\hbar\omega_0}(1-e^{i\omega_0\tau})+\frac{X_2}{\hbar\omega_0}(e^{i\omega_0(\tau+\tau'')}-e^{i\omega_0(\tau+\tau''-\tau')}). \tag{173}$$

Upon performing the integral over t_1 we obtain

$$\begin{aligned}T_{\rm int}(\epsilon,\epsilon')=&\frac{2e^{i\phi}}{\pi\hbar^3}\,\Gamma_{l2r}\Gamma_{21l}\int_{-\infty}^\infty d\tau''\int_0^\infty d\tau'\int_0^\infty d\tau\,J_0(|g^{\omega_0}_{\tau,\tau',\tau''}|)\\&\times\exp\left\{-\frac{i}{\hbar}(\epsilon-\epsilon')\tau''-\frac{i}{\hbar}(\epsilon'-\epsilon_2-i\Gamma_2)\tau'\right\}\\&\times\exp\left\{\frac{i}{\hbar}(\epsilon'-\epsilon_1+i\Gamma_1)\tau\right\}+\text{c.c.}\end{aligned} \tag{174}$$

Change in energy of the particle due to interaction with the external field does not destroy the interference in accordance with the notion that such a nondissipative process does not lead to suppression of phase coherence when time reversal symmetry is unbroken.

Carrying out a similar calculation for the coherent state case, we find a similar qualitative behavior for the interference term. Exchange of one quantum does not change the coherent state and interference is not destroyed.

VIII. SUMMARY AND CONCLUSION

We have developed a functional method to study the influence of various environments on quantum tunneling, and shown that the effect of the environment on the transmission probability, specified by the influence function, is described by the value of a certain generating functional. The state of the environment only occurs explicitly in the expression for the influence function through the appearance of its statistical operator, thereby allowing a unified discussion of the influence of environments on phase coherence and thereby on the transmission properties. An advantage of the approach is that different environments are treated on an equal footing, thereby simplifying comparison of similarities and differences.

We have calculated the influence function for various environments and parameter regimes using the developed nonequilibrium generating functional technique. In order to obtain analytical results we have concentrated on simple resonant tunneling, and invoked the wide-band approximation. In the thermal case we obtained analytical results for the broad resonance limit, the strong-coupling limit, the high-temperature limit, and the weak-coupling and low-temperature limit. The short-time approximation used in the broad resonance, strong-coupling, and high-temperature limits, and the long-time approximation invoked in the weak-coupling and low-temperature limit were numerically demonstrated to be good approximations. For the thermal case we discussed the closed expression obtainable for the Einstein model.

We studied the effect of a classical fluctuating environment on the tunneling process, the fluctuating level model. The environment is here described by a Gaussian distributed stochastic variable. The fluctuating level model was shown to only capture the correlation part of the influence of the thermal environment, as the fluctuating level model neglects the systematic part except for its average influence. We find in this case that the total transmission coefficient is a Lorentzian with its width temperature broadened.

We studied the effect of a harmonically varying external field, and found the heights and widths of the Lorentzian sidebands at all the harmonics of the classical force.

For the coherent state case we also obtain a closed expression for the influence function, and we showed that the coherent state case is an intermediate situation sharing features of both the zero-temperature Einstein case and the external field case.

Finally we studied the suppression of quantum interference by considering a simple model of two resonant levels situated on the two arms of a ring connected to two external reservoirs. The quantum interference was monitored by having the ring enclosing a magnetic flux. When calculating the transmission probability we obtained contributions corresponding to tunneling through either arm and an interference contribution. We found that the interference term vanishes in the thermal bath and fluctuating level cases even for the slightest energy exchange between system and environment. The model thus represents a case where dissipation completely destroys quantum interference. In the case of a nondissipative environment, e.g., a classical force, there is no loss of phase coherence, and there is therefore always an interference contribution, even if energy is not conserved in the transition.

In conclusion, we have demonstrated the efficiency of the developed nonequilibrium generating functional technique for evaluating the environmental influence on coherent quantum processes.

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