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Renormalization of the one-dimensional conductance in the Luttinger-liquid model

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Properties of one-dimensional (1D) transport strongly depend on the proper choice of boundary conditions. It has been frequently stated that the Luttinger-liquid (LL) conductance is renormalized by the interaction as ge^2/h. To contest this result I develop a model of 1D LL wire with the interaction switching off at the infinities. Its solution shows that there is no renormalization of the universal conductance while the electrons have a free behavior in the source and drain reservoirs.

There is a great interest in the effect of electron-electron interaction on one-dimensional (1D) transport.[1] Following a paper of Apel and Rice,[2] Kane and Fisher claimed[3] that the universal one-channel conductance e^2/h should be renormalized by a dimensionless constant of the interaction g in the absence of backscattering. A number of authors[4] developed this approach later making it closer to experimental reality, but keeping this renormalization unchanged. There is serious doubt,[5] however, that such a renormalization might occur. It may be argued against using a simple Landauer-Buttiker model,[6] where all statistical properties of the channel transport should be completely determined by the numbers and velocities of the electrons flying from the reservoirs that are connected by the channel. So, it was not surprising to know that the experimentalists do not see this renormalization.[7]

In this paper I will describe 1D wire transport with the model accounting for switching off of the interaction between the electrons inside the reservoirs. The main result of my solution shows that there is not any renormalization of the universal conductance, whatever the length of the wire and the way of imposing of the external field.

Such a model could be specified with the Hamiltonian ($\hbar=1$)

$$\mathscr{H}=\int dx\{-iv\psi^{\dagger}(x)\hat{\sigma}_3\partial_x\psi(x)+\tfrac{1}{2}U\varphi(x)\rho^2(x)\}, \quad (1)$$

where the two-component field $\psi_a(x)$ describes the right and left moving electrons with velocity v through the only conducting channel between two source and drain reservoirs. These reservoirs could be modeled as the adiabatic opening of a number of the channels to the left (right) from $x=0$ ($x=L$), corresponding to a smooth widening of the potential well there. Inside the reservoirs electrons become free. This means that the screened Coulomb interaction, which is a square function of the density $\rho(x)=\Sigma_a\rho_a(x)$ goes to zero for $x<0$ and $x>L$. This switching is ruled by the function $\varphi(x)$ in Eq. (1). Making use of the bosonization technique[8] one could describe such a system with the Langrangian

$$\mathscr{L}_t=\frac{1}{8\pi}\int dx\left[\frac{1}{v}[\partial_t\phi(t,x)]^2-[v+\varphi(x)U/\pi][\partial_x\phi(x,t)]^2\right]. \quad (2)$$

The physical quantities could be calculated from the generating functional

$$\mathscr{Z}=\int D\phi\exp\{iS(\phi)\},\ \ S(\phi)=\int dt\mathscr{L}_t(\phi) \quad (3)$$

due to connection of ϕ with the electron density $\rho=(1/2\pi)\partial_x\phi$ and with the electron current $j=-(e/2\pi)\partial_t\phi$.

To find a current flowing through the channel due to the applied electric field $-\partial_yV(t,y)$ I should add $\mathscr{L}_V=(e/2\pi)\int dy\,\phi(t,y)\partial_yV(t,y)$ to (2) and work out the average of the current with the weight (3). As the action has a Gaussian form the results could be written as

$$\langle j(t,x)\rangle=-\frac{1}{2\pi}\langle\partial_t\phi(t,x)\rangle$$
$$=\int dt'\int dy\,\sigma(t-t',x,y)[-\partial_yV(t',y)], \quad (4)$$

where the conductivity $\sigma(t,x,y)$ is related to the retarded Green function $G(t,x,y)$ of the operator

$$\hat{G}^{-1}=-\frac{1}{v^2}\partial_t^2+\partial_xu^2(x)\partial_x,\ \ u^2(x)=1+U\varphi(x)/(\pi v) \quad (5)$$

in the following way: $\sigma(t,x,y)=-(e^2/\pi v)\partial_tG(t,x,y)$. First, let me check the result for the case of constant u. Reverse transformation of the Fourier representation,

$$\sigma(\omega,k)=\frac{e^2}{\pi u}\frac{i\omega/v'}{(\omega/v')^2-k^2},\quad v'=vu, \quad (6)$$

gives the expression for the free electron conductivity

$$\sigma(t,x)=\frac{e^2}{2u\pi}\int\frac{d\omega}{2\pi}\{\theta(x)e^{-i\omega(t-x/v')}$$
$$+\theta(-x)e^{-i\omega(t+x/v')}\}$$
$$=\frac{e^2}{2u\pi}\theta(t)\{\delta(x-v't)+\delta(x+v't)\}. \quad (7)$$

The last line reveals a simple nature of the current in a linear dispersion approximation,

$$\langle j(t,x)\rangle=-\frac{e^2}{2u\pi}\int dy\{\partial_yV[t_{x+}(y),y]+\partial_yV[t_{x-}(y),y]\}, \quad (8)$$

since all paths of the particles coming into the point (t,x) from the left (right) are equivalent $t_{x\pm}(y)=t\mp(x-y)/v'$. The first line of Eq. (7) in agreement with the prescription of Fisher and Lee[9] shows that the conductance $\sigma_0 = \lim_{\omega\to 0}\sigma(\omega,x)$ at any finite x.

Repeating this method in a general case (5) one finds $\sigma_0=(e^2/\pi v)\lim_{\omega\to 0} i\omega G(\omega,x,y)$, where the retarded Green function

$$\left[\frac{\omega^2}{v^2}+\partial_x u^2(x)\partial_x\right]G(\omega,x,y)=\delta(x-y) \tag{9}$$

should be constructed from two proper mutually independent solutions $[f_{\pm\omega}(x)\propto \exp\pm i(\omega/v)x$ at $x\to\pm\infty]$ of the homogeneous analog of Eq. (9) in a standard way:

$$G(\omega,x,y)=\frac{1}{W(\omega)}[f_{+\omega}(x)f_{-\omega}(y)\theta(x-y) + f_{+\omega}(y)f_{-\omega}(x)\theta(y-x)], \tag{10}$$

$$W(\omega)=u^2(x)[f'_{+\omega}(x)f_{-\omega}(x)-f_{+\omega}(x)f'_{-\omega}(x)]. \tag{11}$$

Only excitation of the waves of the electron density with small k is essential. Therefore, I could make a step-function approximation to $u(x)$. Solution of Eq. (9) with this approximation reveals that $W(\omega)=2i\omega u(\infty)/v$ at small ω. Substituting this into the expression for σ_0 I conclude that the conductance takes its universal meaning $e^2/(2\pi)$ in this model.

Finally, the above considerations prove that the conductance of the 1D wire is not renormalized by the electron-electron interaction in the absence of backscattering while electrons have free behavior in the source and drain reservoirs between which this wire is located.

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