RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 52, NUMBER 12 15 SEPTEMBER 1995-II

Giant conductance oscillations controlled by supercurrent flow through a ballistic mesoscopic conductor

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(Received 20 July 1995)

We predict giant conductance oscillations for a normal ballistic sample in contact with superconducting elements. The effect is due to quasiparticle interference leading to a resonant sensitivity of the normal current to the superconductor phase difference $\Delta\phi$. In contrast to the case of diffusive transport the amplitude of the oscillations can greatly exceed both e^2/h and the conductance of the system itself measured in absence of superconducting elements. We suggest an experiment where the effect can be revealed most dramatically. A simple relation between the resonant conductance and the Josephson current through the system is derived.

Interest in transport properties of systems with mixed normal and superconducting elements has been continuously growing[1] ever since the theoretical observation[2] that electronic properties of a *normal* mesoscopic system are sensitive to the phase difference $\Delta\phi$ between two superconductors in contact with the normal system. In several recent experiments on metallic systems in the diffusive transport regime[3–5] the conductance was found to oscillate as a function of $\Delta\phi$ with maxima at even multiples of π. The diffusive transport regime has been analyzed theoretically[6,7] and agreement found with experiments as to position and amplitude of the conductance oscillations. In particular, the amplitude of the conductance oscillations of the mixed system was found to be of the same order as the conductance in the absence of superconducting elements. On the other hand, in ballistic superconductor–normal-metal–superconductor (SNS) structures the main focus has been on calculating persistent and Josephson currents.[8–12]

Recently, experimental observations of a phase-sensitive conductance have been made in certain InAs semiconductor heterostructures.[13] In these systems superconducting Nb electrodes are in contact with a two-dimensional electron gas (2DEG). Transport is in the ballistic regime with very little scattering from impurities and little normal scattering at the NS boundaries between InAs and Nb due to the absence of a Schottky barrier between these materials.[14] As a result, distinct magneto-oscillations of the conductance with a period $h/2e$ were observed. In terms of the superconductor phase difference $\Delta\phi$, the period was again 2π, but compared to the results for metallic systems the conductance maxima were shifted by π. While the amplitude of the oscillations was fairly small ($\leqslant 0.1\times 2e^2/h$), they survived when the distance between the superconducting elements was increased.

The results of Ref. 13 give a clear indication that superconductors in contact with a ballistic mesoscopic system have a qualitatively different effect on the transport properties than if the system is in the diffusive regime. The objective of this paper is to show that the difference is much more drastic than what follows from the above comparison. We predict that giant conductance oscillations with an amplitude much bigger than the normal conductance in absence of superconducting elements should be observable in an experiment only slightly different from that carried out in Ref. 13.

A qualitative argument for the giant oscillation amplitudes in the ballistic case starts from a description of the electron transport in terms of resonant tunneling through quantized energy levels of the normal (2D) mesoscopic part of the system. Consider the system shown in Fig. 1. Here the 2DEG is constrained to a quasi-1D channel, separated from the external "reservoirs" L and R by quantum point contacts controlled by the gates G_1, G_1', G_2, and G_2'. The supercon-

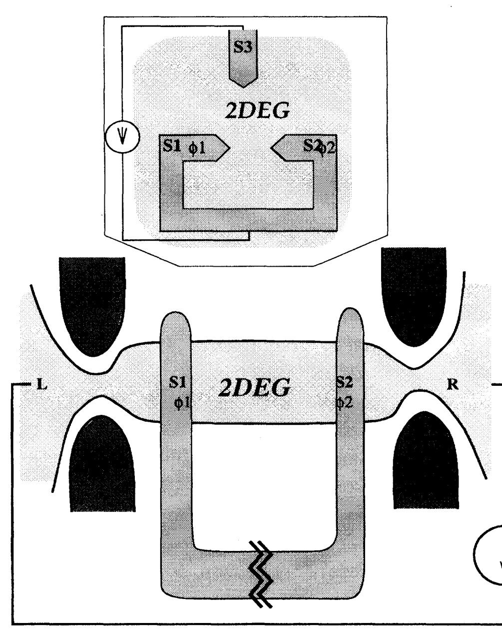

FIG. 1. Suggested experimental layout for measurement of giant conductance oscillations in a ballistic NS system (see text). Inset: configuration realized in the experiment, Ref. 13.

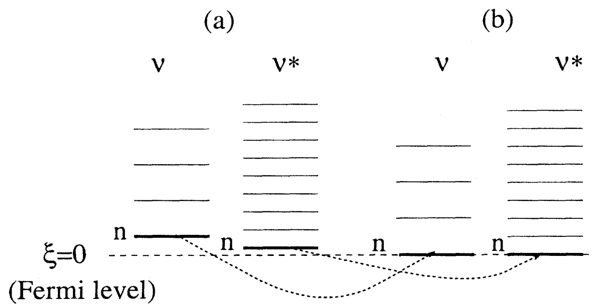

FIG. 2. Positions of the Andreev levels in different (νth and ν^*th) transverse modes for a long channel ($L \gg \xi_0$) [according to Eq.(3)] (a) out of resonance, and (b) in resonance.

ducting electrodes S_1 and S_2 are deposited on top of the system, and a phase difference $\Delta\phi = \phi_1 - \phi_2$ is maintained between them (e.g., by means of a threading magnetic flux). The described configuration contains all the important features of the one experimentally realized in Ref. 13 (see inset): the normal current is carried by quasiparticles, which propagate ballistically in the normal part of the system, and two clean NS boundaries with an adjustable phase difference between the superconductors are present. The gates G allow the coupling strength between the normal reservoirs (bulk 2DEG) and the ballistic channel to be controlled.[15]

Due to confinement both the transverse and longitudinal motion of the electrons in the quasi-1D channel of Fig. 1 are quantized, giving rise to a set of distinct energy levels. These levels can be shifted by means, for instance, of gate voltages or an external magnetic field. Typically only a single level will be lined up with the Fermi energy and hence the conductance will be of order $2e^2/h$. In contrast—when Andreev scattering of electrons traveling in the longitudinal direction is present—the Fermi level itself rather than the bottom of the subbands defined by the transversely quantized levels will serve as a reference energy for the longitudinal quantization giving rise to so-called Andreev levels, which can be driven by the superconductor phase difference $\Delta\phi$ [see Fig. 2(a)]. When $\Delta\phi = (2n+1)\pi$, $N_\perp$ Andreev levels (one in each of $N_\perp$ transverse modes) are simultaneously brought in line with the Fermi energy, thus producing a giant conductance peak [see Fig. 2(b)].[16] The width of the peak is of the order of the single-electron transparency of the barriers between the 2DEG and the electron reservoirs, while its amplitude is equal to $N_\perp 2e^2/h$, reflecting the $N_\perp$-fold degeneracy of the resonant level. This effect demonstrates the extreme sensitivity of ballistic mesoscopic transport to supercurrent flow in the sample. A simple relation between conductance and Josephson current can be obtained in this case [see Eq. (6) below].

In order to consider the giant conductance oscillations quantitatively, let us use the quasi-1D model, presented in Fig. 3. The normal regions are modeled by ideal normal wires, which support $N_\perp$ modes of transverse quantization. Scattering processes occur only at the NS boundaries (A, B) and at the junctions (C, D). Scattering at the junctions is described by real S matrices,[17] which mix neither electrons and holes nor different transverse modes. The latter assumption, as well as the neglect of scattering in the wires themselves, is consistent with the low scattering rate in the 2DEG

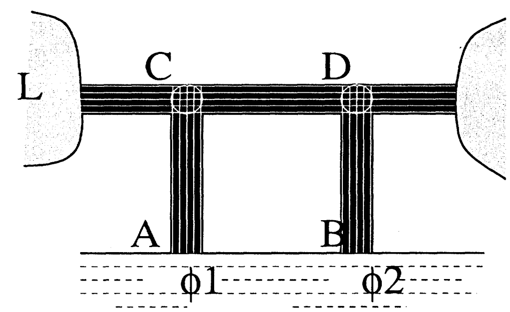

FIG. 3. Quasi-1D model of the system shown in Fig. 1.

of InAs-based heterostructures.[18,19] Moreover, we will assume that only Andreev reflection occurs at NS boundaries; this is reasonably close to the experimental situation.[13] In our analysis, we assume that the size L of the normal part of the system is less than both the phase breaking length $L_\phi = v_F \tau_\phi$ and the normal-metal coherence length $L_T = \hbar v_F / k_B T$ (but exceeds the superconductor coherence length ξ_0). Here τ_ϕ is the inelastic scattering time, v_F is the Fermi velocity of quasiparticles, k_B is the Boltzmann constant, and T is temperature. In the absence of transverse-mode mixing it suffices to calculate the contribution to the conductance from a single transverse mode. The total conductance will then be given by summing over the transverse modes.

We use identical 3×3 S matrices to describe scattering of, respectively, electrons and holes at the junctions A and B. These matrices relate the incoming and outgoing wave amplitudes of the quasiparticles in the wires. They are parametrized by a real number $0 \leqslant \epsilon \leqslant 1/2$,[17] which is the amplitude for transmission of a particle between the system and a reservoir. On the other hand, we assume that normal reflection is absent at the interface between lead 1 (2) and the superconductor at point A (B) and that only Andreev reflection takes place there.

The single-mode conductance can be expressed in terms of the scattering coefficients of the system[20(a)] as

$$G = \frac{2e^2}{h} 2 \int_0^\infty d\xi (T_0^> + R_a^>) \left(-\frac{\partial n_F(\xi)}{\partial \xi} \right) + \eta, \tag{1}$$

where $T_0^>$ ($R_a^>$) is the probability for normal transmission (Andreev reflection) of an electron incident from the left normal reservoir; $n_F(\xi)$ is the Fermi distribution function, and the energy ξ is measured from the Fermi level. The term η in (1) is a rapidly oscillating function of the electron momentum [$\eta \sim \exp(2ip_F L)$, L being the length of the lead $ACDB$][21] and is exactly zero if there is time-reversal symmetry.[20(a)]

The scattering coefficients in (1) can be found by solving the Bogoliubov–de Gennes equations in the normal part of the system[20(b)] (for details see Ref. 21):

$$T_0^>(\xi) \approx R_a^>(\xi) \approx \sum_{\sigma=\pm 1} \frac{\frac{1}{2}\epsilon^2}{1 + 2\epsilon^2 + \cos\left(\Delta\phi + \sigma \frac{2L}{\hbar v_F^\parallel} \xi \right)}. \tag{2}$$

Here $v_F^{\parallel}$ is the longitudinal Fermi velocity in the transverse mode studied, and $\Delta\phi \equiv \phi_1 - \phi_2$ is the superconductor phase difference between the contact points.

In Eq. (2) we have neglected all quickly oscillating terms [such as η in (1)], and all terms of order higher than ϵ^2. The term $2\epsilon^2$ in the denominator must be retained, however, because due to the nonuniform phase dependence of (2), it becomes important close to resonant values of $\Delta\phi$.

Resonance is achieved at energies $\xi_n^{\pm}$ of the Andreev levels in lead $ACDB$ (in the limit $L \gg \xi_0$):

$$\xi_n^{\pm} = \frac{\hbar v_F^{\parallel}}{2L}[(2n+1)\pi \mp \Delta\phi]. \tag{3}$$

Equation (3) reproduces the results for low-energy Andreev levels in a long, clean SNS junction obtained by Kulik.[8] As one can see from (2), due to leakage to the normal reservoirs these levels acquire finite width $\epsilon\hbar v_F^{\parallel}/L$.

In general the energies of Andreev levels in different transverse modes do not coincide. However, for special values, $\Delta\phi = (2n+1)\pi$, of the phase difference $\xi_n^{\pm} = 0$ whatever the value of $v_F^{\parallel}$ is (i.e., irrespective of the transverse-mode number). As a result, at these values of $\Delta\phi$ all $N_\perp$ modes contribute one level to a $N_\perp$-fold degenerate resonant level at the Fermi energy.

At zero temperature the resonant conductance depends only on $T_0^{>}(0)$, $R_a^{>}(0)$. Since the contribution to the resonant conductance of each transverse mode is exactly the same, the total resonant conductance of the system (within accuracy of ϵ^2) is

$$G(\Delta\phi) = N_\perp \frac{2e^2}{h} \frac{2\epsilon^2}{1+2\epsilon^2+\cos\Delta\phi}. \tag{4}$$

The conductance given by (4) is (i) directly proportional to the cross section of the normal lead (i.e., to its conductance without an NS boundary), (ii) independent of the length of the normal lead (as long as it is shorter than L_T), and (iii) the *maxima* of conductance measured as a function of $\Delta\phi$ appear at *odd* multiples of π (see Fig. 4).

The simplest way to calculate the Josephson current is by expressing it in terms of the excitation spectrum of the junction.[9(b),10,11] The contribution from high-energy Andreev states is then exponentially suppressed at low temperatures ($T \ll \Delta$). It suffices, therefore, to use only the low-energy Andreev levels, broadened by coupling to external reservoirs (see above). One finds that

$$I_J^{(\epsilon)}(\Delta\phi) = N_\perp \frac{2e\bar{v}_F^{\parallel}}{\pi L} \sum_{n=1}^{\infty} \frac{(-1)^{n+1} e^{-2|n|\epsilon} \sin n\Delta\phi}{n}, \tag{5}$$

where $\bar{v}_F^{\parallel} = N_\perp^{-1} \Sigma_{\nu=1}^{N_\perp} v_{F,\nu}^{\parallel}$.

In the limit $\epsilon \to 0$ this result reduces to the well-known expression for the Josephson current in a planar SNS junction.[9] Our analytic result (5) agrees with numerical calculations of the Josephson current in a 1D ballistic SNS system,[11] where the normal region was connected to a single normal reservoir.

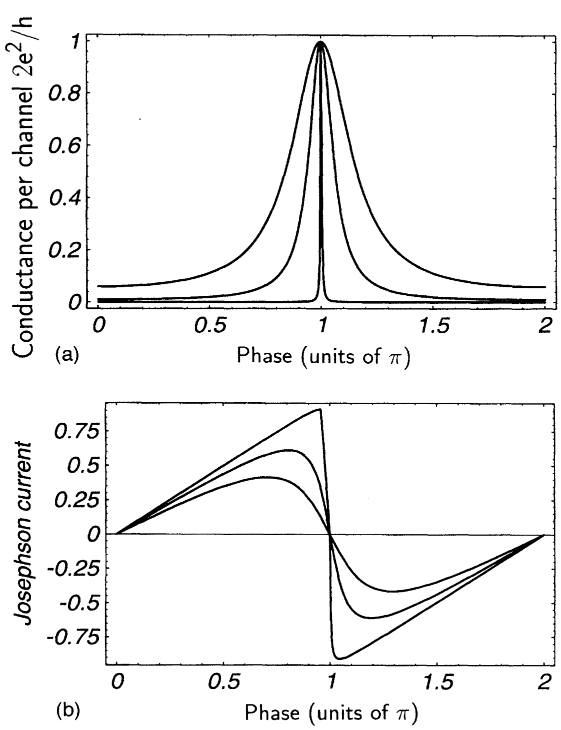

FIG. 4. Normal conductance (a) and the Josephson current (b) in the system as functions of the superconducting phase difference between A and B, $\Delta\phi$, calculated according to Eqs. (4,5) for the values of ϵ=0.005, 0.1, and 0.25.

On comparing the Josephson current (5) and the normal conductance (4) one realizes that indeed, within an accuracy of ϵ^2, there exists a simple relation between them (see Fig. 4),

$$G(\Delta\phi) = \epsilon\left(-\frac{eL}{\hbar\bar{v}_F^{\parallel}} \frac{dI_J^{(\epsilon)}}{d\Delta\phi} + \frac{2e^2}{h} N_\perp \right). \tag{6}$$

This expression is directly proportional to the number of transverse modes. We have again—as in (1) and (2)—neglected quickly oscillating terms, which play a minor role in the many-channel case. The relation (6) reflects the fact that, in the system considered, the same phase-sensitive Andreev states carry both the normal current and the Josephson current.

Impurity scattering decreases the amplitude of the giant oscillations, since in this case a gap opens in the Andreev spectrum in the sense that no Andreev level can reach the Fermi level for any value of $\Delta\phi$.[12] Nevertheless, maximum transparency at zero temperature [$\xi_n^{(\nu)}(\Delta\phi) = \min$] is still achieved at odd multiples of π in all transverse modes simultaneously [i.e., at the inflection points of the function $I_J(\Delta\phi)$].[12] The phase dependence of the oscillations and their relation to the Josephson current will therefore qualitatively be the same as in the purely ballistic case, though their amplitude will be suppressed.

As one can see from Fig. 1, the observation of the giant conductance oscillations discussed above demands only a slight modification of the experimental layout of Ref. 13.

In conclusion, we have shown that in a ballistic SNS system giant oscillations of normal conductance can be observed. Their amplitude can greatly exceed the normal conductance value obtained without superconducting elements present, and is of the order of $N_{\perp}e^2/h \gg e^2/h$. These oscillations are related to the phase derivative of the Josephson current in the system; a simple relation between the oscillating contribution to the conductance and Josephson current was obtained. The effect can be observed using existing experimental techniques.

We are grateful to V. Antonov, C.W.J. Beenakker, T. Claeson, P. Delsing, M. Devoret, D. Esteve, Yu. Galperin, L. Gorelik, V. Petrashov, S. Rashkeev, J. van Ruitenbeek, V. Shumeiko, and A. Slutskin for many fruitful discussions. This work was supported by the Royal Swedish Academy of Sciences (KVA), the Swedish Natural Science Research Council (NFR), and by the Swedish Board for Industrial and Technical Development (NUTEK). One of us (A.K.) gratefully acknowledges the hospitality of the Department of Applied Physics and Department of Physics, Chalmers University of Technology and Göteborg University.

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[15] As we shall see, coupling between the normal reservoirs and the channel should be weak. Therefore it is possible to connect the 2DEG channel with the external circuit by normal-metal (not Nb) contacts, with significant scattering on the interface. Of course, the configuration of Fig. 1 has the advantage that the coupling is *controllable*.

[16] The quasiclassical quantization condition for a closed quasiparticle trajectory that includes sequential Andreev reflections from the two NS interfaces is $[p(\xi)-q(\xi)]L+\pi\pm\Delta\phi=2\pi n$; $n=0,\pm1,\pm2,\ldots$, p,q being the electron (hole) longitudinal momentum. Along this trajectory the quasiparticle switches excitation branch twice (electron$\leftrightarrow$hole). The phases gained along the e (electron) and h (hole) part of the trajectory depend on velocity (i.e., on transverse mode index), but enter with opposite signs and cancel exactly when the corresponding level reaches the Fermi energy [$\xi_n=0$, so that $p(0)=q(0)$]. Therefore the only phases entering the resonance condition $\xi_n=0$ are the Andreev scattering phases, $\pi/2\pm\phi_{1,2}$. Resonance therefore occurs simultaneously in all transverse modes when $\Delta\phi=(2n+1)\pi$.

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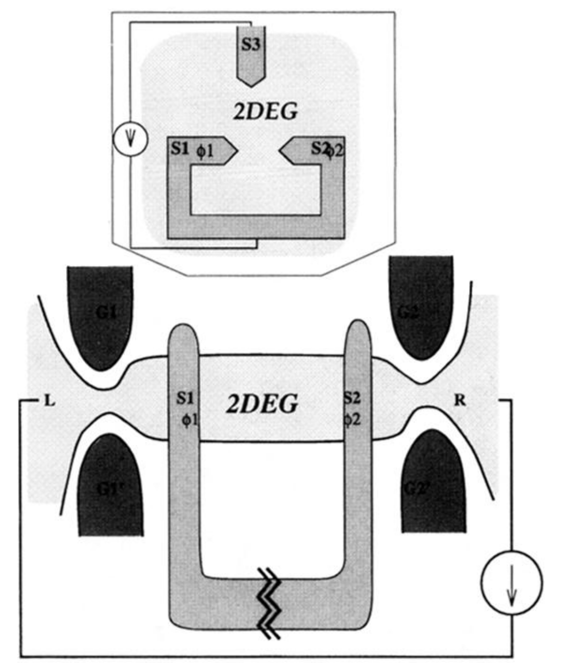
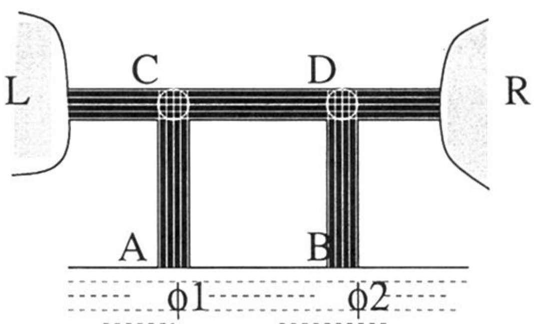


FIG. 1. Suggested experimental layout for measurement of giant conductance oscillations in a ballistic NS system (see text). Inset: configuration realized in the experiment, Ref. 13.



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