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Crossover between different regimes of current distribution in the quantum Hall effect

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The distribution of current density in the quantum Hall effect can be probed by the channel-width dependence of the critical current. Both linear and sublinear dependence have been found in such experiments. We observe a crossover from linear to strong sublinear behavior, in the same samples, upon the increase of carrier density. This crossover suggests the existence of qualitatively different regimes of current distribution. The linear behavior is attributed to percolative transport through numerous, randomly distributed microscopic channels, due to strong density fluctuations within the sample. The sublinear regime, on the other hand, is associated with relatively better homogeneity, where the macroscopic polarization of the channel leads to a current distribution that is weighted towards the sample boundaries. Our conclusions may resolve the conflict between recently reported experiments.

The past few years have seen a renewed interest in the question of current and potential distribution in the quantum Hall effect (QHE). The main features of the QHE,[1] namely, the vanishing of the longitudinal resistance ρ_{xx} and the exact quantization of the Hall resistance $\rho_{xy}=h/ie^2$, i being an integer, are explained by models that differ widely in their picture of the current distribution. The universal nature of the effect makes most experimental measurements insensitive to the spatial distribution of current within the sample. Therefore, discrimination between the different models is not straightforward. The earliest approach[2] pictured the transverse potential gradient, namely, the Hall field, to be uniform in the bulk of the sample, leading to a uniform current-density distribution. A later picture attributes the potential drop, and therefore the current, to the edges of the sample only.[3,4] Yet another view, when inherent random potential fluctuations are taken in account, suggests the predominance of random percolating current paths.[5] The subtleties involved in two-dimensional electrostatics at high magnetic field were first pointed out by Halperin,[3] when evaluating the potential created by a redistribution of charge at the edges of the sample. MacDonald, Rice, and Brinkman[6] showed that the potential has to be calculated in a self-consistent way, their calculations resulting in a nontrivial potential profile, even in the clean sample limit. Recently a model that combines bulk and edge electrostatics[7] was derived. However, a complete picture including potential fluctuations, which are present in any realistic sample, is still unavailable. Similarly to the theory, where the issue of current distribution remained muddled, experiments that attempted to probe the current distribution gave different results, depending on the experimental method used.[8,9]

In a recent set of experiments,[10,11] we have shown an indirect way to probe the current distribution in the QHE regime. The idea was to measure the critical current I_c that breaks the dissipationless flow of the QHE, in samples of different widths. This breakdown of the integer QHE at high currents was discovered shortly after the QHE itself,[12] but its mechanism and dependence on various experimental parameters are still only partially understood. Nevertheless, we argued that the dependence of I_c on W, the width of the channel, is closely linked to the actual distribution of currents across its transverse direction. This dependence was found to be strongly sublinear, apparently logarithmic, as seen in Fig. 1. Similar results on different material samples have been shown in Ref. 10.

We interpreted these results in the framework of a self-consistent calculation of the potential drop across the sample. The condition that the potential has to fulfill, in order to reconcile the electrostatics and the quantum-mechanical density of states, was first derived by MacDonald, Rice, and Brinkman.[6] Following the subsequent works of Thouless[13] and Beenakker[14] that solve this equation under ideal sample conditions, we were able to show[10] that the resulting potential drop across the sample indeed leads to a logarithmic dependence of I_c on W, in agreement with our experiments.

However, this result was in disagreement with recent experiments done in similar conditions by Kawaji, Hirakawa, and Nagata,[15] who found a *linear* dependence of I_c on W. In fact, other experiments also hinted that the relation between I_c and W is not universal. Haug, von Klitzing, and Ploog[16] have found a sublinear dependence in Hall bars of different width, and an approximately linear dependence in corbino geometry samples, whereas Boisen *et al.*[17] have measured a

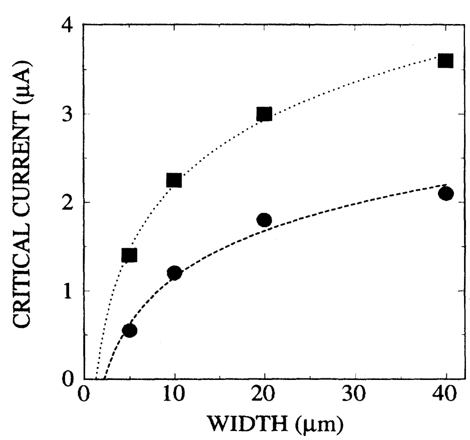

FIG. 1. Critical current vs the channel width measured in the samples of Ref. 11 for filling factors $\nu=1$ (●) and 2 (■). The dashed lines are logarithmic fits.

TABLE I. Summary of the properties of the materials used in different experiments to determine the dependence of I_c vs W: n (10^{11} cm^{-2}), μ (10^5 cm^2/V s), and s (nm). Sublinear dependence is observed in samples having large spacers and high mobility.

	Ref. 11	Ref. 10	Ref. 15	Ref. 17	This work	
					Before illumination	After illumination
n	2.6	2.1	4.7	2.5	1.3	2.2
μ	9.0	7.0	2.1	0.5	1.2	2.1
s	54	36	6		40	40
I_c vs W	sublinear	sublinear	linear	linear	linear	sublinear

linear dependence of I_c on W. Such discrepancy between similar experiments motivated us to seek the causes of this apparent contradiction.

In this paper, we report on further measurements that may explain the difference between these experiments, and help clarify the issues affecting current distribution in the QHE.

One apparent parameter that was different in Refs. 15 and 17 from our experimental values was the significantly lower mobility of their samples (see Table I). In fact, as we shall argue below, the mobility is not the only relevant parameter to differentiate between the samples, but it does underline that the heterostructures used were of different quality. This hint led us to perform a set of measurements with low-mobility material, but with otherwise identical conditions to our previous experiments. The material used had a mobility of $\mu = 1.2\times 10^5$ cm^2/V s, and a sheet carrier concentration of $n = 1.3\times 10^{11}$ cm^{-2} when cooled in the dark. The samples were patterned exactly as in our previous set of experiments,[10] on a single chip, in widths ranging from 10 to 100 μm, and inserted in a dilution refrigerator. I_c was determined for each of the different widths by repeatedly scanning the magnetic field around the integer filling factor, and increasing the current in small steps (Fig. 2), with I_c defined as the value beyond which there was no plateau of $\rho_{xx}=0$.

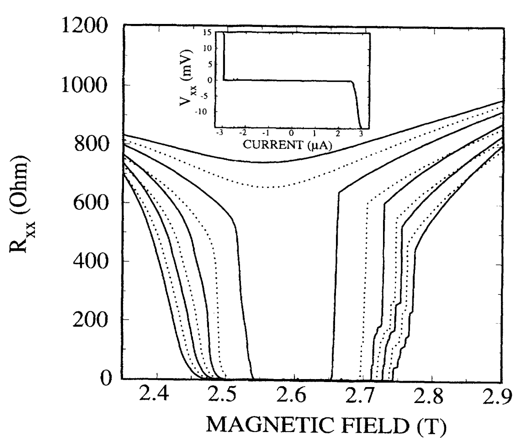

FIG. 2. Determination of I_c by measuring ρ_{xx} vs magnetic field, around $\nu=2$, for a sample of $W=20$ μm, at increasing currents. The current increment between consecutive curves is 0.136 μA; the bottom curve starts at 2.318 μA. I_c is the current at which the $\rho_{xx}=0$ plateau is fully eliminated. Inset: voltage vs current for the same sample at $B=2.6$ T.

The sharpness of the breakdown was also checked by measuring the current-voltage characteristics at fixed magnetic field (inset of Fig. 2).

In contrast to the results we obtained on high-mobility samples, the dependence of I_c on W is found to be *linear*, as shown by the circles and the dashed-line fit in Fig. 3. This result rules out explanations attributing the discrepancies between the different measurements to the sample geometry or the experimental setup,[18] and focuses attention on the intrinsic properties of the materials used. In order to increase *in situ* the mobility, we performed a sequence of brief illuminations of the samples with a light-emitting diode placed inside the dilution refrigerator. Each illumination resulted in a further increase in density and mobility. After each illumination, the values of I_c were measured in the dark, for each width; thus, the effect of the increasing n and μ on $I_c(W)$ was determined.

The illumination of $Al_xGa_{1-x}As/GaAs$ heterostructures at low temperature is known to be able to increase the carrier concentration and the mobility in the two-dimensional electron gas (2DEG).[19] The irreversible ionization of the *DX* centers present in the donor layer results in a persistent higher carrier concentration. After a first short illumination, the carrier concentration, as determined from Shubnikov–de Haas measurements, increased from 1.3 to 1.5×10^{11} cm^{-2}, and the mobility increased from 1.2 to 1.4×10^5 cm^2/V s. As the carrier concentration was higher than before illumination, the magnetic field was increased commensurately in order to

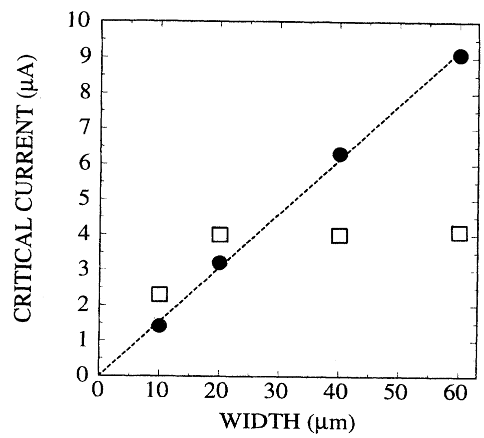

FIG. 3. Critical current vs the channel width measured in the low-quality samples of this work, before illumination (●) and after long illumination (□). The dashed line shows the linear fit.

keep the filling factor constant, therefore increasing the energy gap. The higher energy gap resulted in a somewhat higher I_c,[12] while still keeping its linear dependence on W.

A more prolonged illumination, followed by the same sequence of measurements, gave the following results: μ increased again, from 1.4 to 2.1×10^5 cm^2/V s while n increased from 1.5 to 2.2×10^{11} cm^{-2}. However, this time the critical current developed *sublinear* dependence on W, as shown by the squares in Fig. 3. Note the remarkable fact that, for the wide samples, I_c decreased after illumination despite the higher magnetic field, whereas for the narrow widths I_c still increased.

By modifying the properties of the 2DEG we were thus able to go from linear to strongly sublinear dependence of I_c on W. As this dependence is believed to reflect the distribution of currents in the sample, it suggests that the current distribution had qualitatively changed after illumination.

In order to understand these findings, one must recall that the self-consistent calculations of the Hall potential in the QHE assumed a perfectly homogeneous sample. In a real sample, the random distribution of donors induces substantial fluctuations in the density of the 2DEG. The potential fluctuations from the donor layer will be smeared on a scale comparable to the spacer thickness s, typically hundreds of angstroms, which is the distance between the 2DEG and the donor layer. In order to screen the larger-scale fluctuations, a redistribution of charge in the 2DEG is needed. As long as the required charge redistribution is small compared to the total density of electrons in the 2DEG, screening will be relatively effective.[20] If the fluctuations demand a substantial amount of carrier redistribution, screening will be incomplete and the sample will be divided into regions of varying carrier density. The characteristic length scale R_c of these large charge-density fluctuations has been shown by Efros[20] to be related to the number of ionized impurities N^+ and the 2DEG carrier density n by

$$R_c=\sqrt{N^+}/n. \tag{1}$$

Typical values of N^+ and n lead to a scale of a few thousand angstroms. Fluctuations on comparable or smaller length scales will be rather large and will result in nonuniform carrier concentration. This result is only approximate, as it ignores other space-charge regions such as the surface and the unintentional impurities in the nominally undoped regions. However, numerical calculations by Nixon and Davies[21] have shown that strong fluctuations in the electron density are indeed present on scales of thousands of angstroms at zero magnetic field B. At large B, the screening of potential fluctuations is much less effective, as these fluctuations have to be compared to the number of electrons in the highest occupied Landau level (LL),[20] which is smaller than the total density of electrons at least by a factor of ν. As soon as the fluctuations are strong enough to alter the occupation of the highest LL, the screening, and hence the filling factor itself, will be highly nonuniform in the sample.

Under such conditions, the sample is divided into compressible regions with partial occupation of the highest LL and incompressible regions with full LL occupation. The dissipationless current flows in distinct percolating paths of the incompressible fluid around the compressible islands. This picture of percolating transport was shown by Luryi[5] to explain the main features of the QHE. To help visualize this, we show in Fig. 4 a computer simulation of the density fluctuations in a 2DEG in the QHE. It was produced first by randomly distributing donors in a 2D plane, and averaging the induced potential and carrier density on the scale of the spacer. Translating these density fluctuations in terms of full and partial occupation of the LL's, and presenting them as dark and bright areas, respectively, results in the landscape of Fig. 4, where dark areas represent incompressible regions. This simple approach is not a complete description of the real fluctuations, but it gives a qualitative picture of the division of the current into separate paths. Under conditions of QHE, current flows through the dark regions of incompressible liquid, in many parallel channels that are separated by islands of compressible liquid. In a typical sample of $W=10$ μm, there will be ~50 such channels.

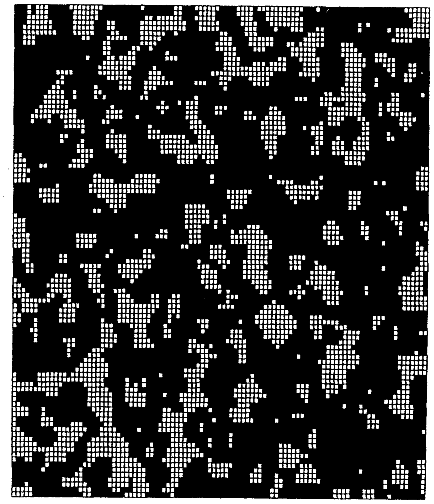

FIG. 4. Simple computer simulation illustrating density fluctuation in a 1×1-μm^2 area of 2DEG with 40-nm spacer thickness, at high magnetic field, as described in the text. The dark areas correspond to incompressible regions with full occupation of the topmost LL. The bright areas indicate compressible regions with different carrier density. Dissipationless transport occurs through the dark areas of incompressible 2DEG, forming a network of parallel microscopic channels. The total current capacity, being the sum of currents in these channels, is linear in the number of such channels, hence linear in W.

We now turn back to the issue of breakdown and the dependence of I_c on the sample width. Clearly, if density fluctuations divide the sample into multiple channels, I_c will just be the sum of the currents in all the participating microscopic channels when each of the latter is carrying the maximum dissipationless current it can sustain (before the onset of dissipation in that particular channel). Note that the mechanism for dissipation in these microscopic channels does not affect this qualitative argument. For macroscopic samples, this statistical sum will have a linear dependence on the number of such channels, and hence on the width. In other words, the linear dependence observed could then be due to the division of the current among multiple paths that percolate through the sample.

As mentioned above, such a picture is possible in the QHE even in samples that are not strongly affected by potential fluctuations at $B=0$. At a filling factor of 2, where the present experiments were done, fluctuations of the order of 25% will drastically alter the occupation of the highest LL. We note that the carrier concentration measured by low-field Hall resistance in various narrow samples and between different probes indeed showed fluctuations of $\sim 20\%$. Furthermore, the latter decreased after illumination. These measurements are certainly not an accurate evaluation of the smaller-scale fluctuations in the samples, as they average out variations across the width of the channel. Still, measurement of such large-scale fluctuations suggests not only that the material used is far from homogeneous, but also that illumination does to a certain extent improve its uniformity. This is in agreement with Eq. (1), where an increase in n will reduce R_c, therefore reducing the amount of unscreened fluctuations. The simulations of Nixon and Davies also indicate that an increase in carrier concentration will lead to a reduction of the potential fluctuations. This reduction of the potential fluctuations at $B=0$ will also decrease the inhomogeneity in the QHE regime by reducing the width of the compressible islands. Furthermore, as the increase in the carrier concentration requires a higher magnetic field to keep the filling factor constant, the larger energy gap will further decrease the amount of fluctuations.

Although μ is the usual parameter used to characterize the quality of the material, there is no one-to-one correspondence between μ and long-range density fluctuations. Nevertheless, the increase in mobility after illumination as $\mu \propto n^{\alpha}$, $\alpha \approx 1.5$, is consistent with the prediction for remote impurity scattering[22] and demonstrates the decreasing influence of the potential fluctuations on the 2DEG.

We can therefore attribute the linear dependence of I_c on W in this experiment to the strong inhomogeneity of the sample that divides the current among microscopic channels, the contributions of which sum in a linear way. The crossover to a regime where the dependence of I_c on W becomes sublinear is achieved by a sufficiently long exposure of the sample to light, which increases n and reduces inhomogeneity, therefore moving the sample closer to the uniform wide-channel model used previously to describe high-quality samples, which indeed predicts such sublinearity.

Thus, we are now in a position to give a plausible explanation for the discrepancy in the dependence of I_c on W in the other experiments mentioned above. In Table I, we list the parameters of the materials measured in the different experiments together with the dependence obtained. The correspondence between the parameters of the samples and the amount of fluctuation is not straightforward, due to competing effects. As argued above, for a *given* impurity potential, high carrier concentration will reduce fluctuations, but, usually, as in the sample of Ref. 15, small spacers are needed to reach high n. On the other hand, the small spacer in this sample means stronger potential fluctuations, so the linear dependence obtained is not altogether surprising. In order to determine the homogeneity of the samples one should therefore include self-consistently the dependence on s, n, and N^+. The latter is usually not accurately known. Still, samples showing linear dependence have at least one of the three hallmarks related to poor homogeneity: low μ, low n, or small s.

In conclusion, the relation between the critical current and the channel width is not universal and reflects the possibility of having very different patterns of current distributions in the QHE regime. By means of *in situ* illumination, we were able to measure in a single material both linear and strongly sublinear dependence of I_c on W. We interpret this as an improvement in the relative homogeneity of the sample after illumination, thus linking the dependence of I_c on W to the quality of the material used. This suggests a resolution of the discrepancy between previous experiments done on different materials, and demonstrates the importance of inhomogeneities in the QHE.

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