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Supercurrent force on Andreev-reflected quasiparticles and excess currents

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The current-carrying condensate of a superconductor exerts a force parallel to the current direction on Andreev-reflected quasiparticles. As a consequence, in a point contact on a normal-metal–superconductor junction the excess current due to Andreev reflections from the supercurrent-carrying layer is predicted to decrease with increasing Cooper pair momentum $\mathbf{p}_s$. The supercurrent force can be compensated, and thus directly measured, by a weak applied external field $\mathbf{B}_a \perp \mathbf{p}_s$ in the normal metal of thickness l: Analytical and numerical computations of quasiparticle-wave-packet trajectories show that the excess current as a function of $\mathbf{p}_s$ and $\mathbf{B}_a$ is maximum when $B_a = B_{ac} \equiv p_s / le$.

I. INTRODUCTION

Thirty years ago Andreev[1] discovered that spatial variations of the superconducting pair potential scatter electrons into holes and vice versa. Within ten years from this discovery it was shown that electron $\leftrightarrow$ hole scattering is responsible for the Tomasch effect[2,3] and Josephson currents through mesoscopic normal regions in superconductor–normal-metal–superconductor (SNS) junctions where the pair potential may vanish over very many coherence lengths.[4–7] Beautiful experimental demonstrations of Andreev scattering in junctions involving conventional[8–10] and high-temperature superconductors[11,12] were performed; they are based on the excess current which results from reflected holes carrying the same current as incident electrons. The excess current is a direct consequence and evidence of the presence of Cooper pairs in the superconductors.

Nevertheless, until recently Andreev scattering has remained a phenomenon more in the backwaters than in the mainstream of superconductivity physics. This has changed dramatically in the past couple of years. Many papers are now dedicated to the explanation of hitherto not well understood transport phenomena in terms of (multiple) electron $\leftrightarrow$ hole scattering and the associated Cooper pair production and destruction.[13,14]

Andreev scattering considered so far is due to the spatial variation of the modulus of the superconducting pair potential Δ. The associated (off-diagonal) force

$$\mathbf{f}_{\Delta 1} = -2\,\mathrm{Re} \int u^* v (\nabla |\Delta|) e^{i\varphi} d^3 r \tag{1}$$

is finite if the electron and hole wave functions u and v belong to quasiparticles with energies below the maximum value of $|\Delta|$. Recently it has been shown that also spatial variations of the phase φ of the pair potential exert a force

$$\mathbf{f}_{\Delta 2} = \frac{4m}{\hbar} \int \mathbf{v}_s \mathrm{Im}(u^* v \Delta) d^3 r \tag{2}$$

on Andreev scattered quasiparticles, if a supercurrent with the gauge-invariant Cooper pair velocity $\mathbf{v}_s$ flows in the boundary between a normal-metal and a superconducting region.[15] This force explains microscopically the interface force in the Nozières-Vinen-Warren theory of vortex motion.[16] In the present paper we look into another direct manifestation of this off-diagonal supercurrent force.

II. QUASIPARTICLE TRAJECTORIES

Let us consider a normal-metal–superconducting (NS) bilayer with a point contact on top of the N layer; see Fig. 1. A supercurrent with Cooper pair velocity $\mathbf{v}_s = v_s \mathbf{e}_y$ and Cooper pair momentum $\mathbf{p}_s = 2m\mathbf{v}_s$ is flowing in the S layer. The force $\mathbf{f}_{\Delta 2}$ from this supercurrent will cause deviations of the Andreev-reflected holes from the ballistic trajectories they would follow, if $\mathbf{v}_s$ were zero and $\mathbf{f}_{\Delta 2}$ would not change their momentum parallel to the NS interface by an amount proportional to $\mathbf{v}_s$. Because of the deviations at finite $\mathbf{v}_s$ not all Andreev-reflected holes return to the point contact, and the

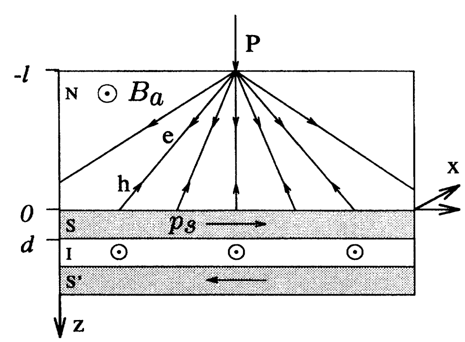

FIG. 1. Trajectories of electrons (e), injected by a point-contact P into a normal layer N, and of holes (h) produced by Andreev reflection in the superconducting layer S with Cooper pairs of momentum $\mathbf{p}_s$. Also shown are an insulating layer I and a second superconducting film S′ with a supercurrent of same magnitude as in S but opposite direction. The magnetic fields from the supercurrents in S and S′ compensate each other in the N layer. (The extensions of the sample in x and y directions are assumed to be so large that all surface effects can be neglected.) For compensation measurements an external magnetic field $\mathbf{B}_a$ may be applied. The deviations of the trajectories due to the supercurrent force and the magnetic field are smaller than the thickness of the trajectory lines.

excess current is reduced. This reduction is a quantitative measure of $\mathbf{f}_{\Delta 2}$. Alternatively, on may measure the supercurrent force by compensating its effect on the quasiparticle trajectories with the help of an applied weak external magnetic field $\mathbf{B}_a = \mathbf{e}_x B_a$. We will discuss both situations.

The quasiparticle trajectories in the normal metal with and without magnetic field *before* and *after* Andreev reflection are calculated quasiclassically, similarly to the work of Benistant *et al.*[9] In order to obtain the trajectory shifts *by* Andreev reflection from the current carrying N-S interface we compute quasiparticle-wave-packet solutions of the time-dependent Bogoliubov–de Gennes equations (TDBdGE), where we neglect any magnetic field. (Recently, using time-dependent density-functional theory, the TDBdGE have been extended to strongly correlated superconductors[20] so that Andreev scattering can be analyzed for these systems, too.) We use the effective mass approximation, the δ-function-like interface potential, the steplike change of Fermi energy from N to S, and the matching conditions of Schüssler and Kümmel.[21] The pair potential of the current carrying S layer has the form

$$\Delta(\mathbf{r}) = \Theta(z)\Delta_0 \exp(i p_s y/\hbar). \tag{3}$$

In Andreev reflection the quasiparticles decay exponentially over the coherence length ξ_0. Since we assume that the London penetration depth λ is much larger than ξ_0, it is a good approximation to use a constant Cooper pair momentum in the phase of the pair potential; this momentum, as a function of the applied supercurrent and magnetic field, is given by Eq. (13).

Proceeding as Hofmann and Kümmel[15] we find that Eq. (2) for $\mathbf{f}_{\Delta 2}$ is not affected by the effective mass approximation. Solving the TDBdG equations exactly by an arbitrarily shaped wave packet we find that the change of the quasiparticle momentum expectation value because of $\mathbf{f}_{\Delta 2}$,

$$\mathbf{p}_{\Delta 2} = \int_{-\infty}^{\infty} \mathbf{f}_{\Delta 2}\, dt = -\mathbf{p}_s \mathscr{P}_{\mathrm{AR}}, \tag{4}$$

is equal to the negative Cooper pair momentum $\mathbf{p}_s$ times the probability of Andreev reflection $\mathscr{P}_{\mathrm{AR}}$ (which depends on the material parameters): the momentum, required in order that the Cooper pair generated in Andreev reflection can merge into the current carrying condensate, is missing from the Andreev-reflected hole. This, of course, also shows in the quasiparticle wave packets of the incident electron and the reflected hole. Computed with Gaussians in Andreev approximation[1] they are the same as the ones given by Eqs. (29) and (30) in Ref. 15 with the modification that in the hole wave packet of Eq. (30) the y component of the momentum, $\hbar k_y$, is changed by $-p_s$. [Furthermore, the normalization factor is altered because of electron $\leftrightarrow$ electron scattering due to the surface potential and the changes of effective mass and Fermi energy at the N-S interface (Ref. 17).]

With these wave packets the trajectories described by the centers of mass of the electrons and holes are calculated as the expectation values of the position operator. If from the point contact in $(x_0, y_0, z_0 = -l)$ at time $t=0$ an electron with effective mass m_N, energy $E < \Delta_0$, and wave vector $\mathbf{k} = (k_x, k_y, k_z)$, i.e., group velocity $(v_x, v_y, v_z) = \hbar\mathbf{k}/m_N$, is injected into the normal metal, then the trajectory of the

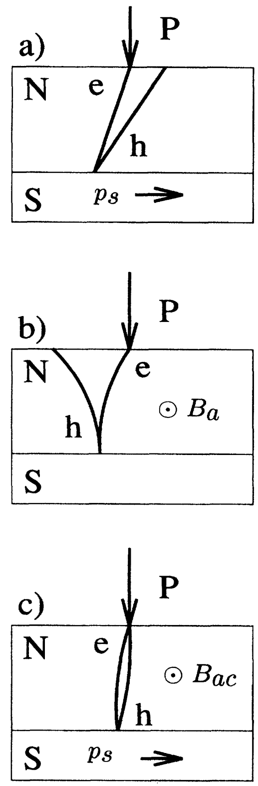

FIG. 2. Schematic plot of quasiparticle trajectories including the deviations due to (a) supercurrent force, (b) external magnetic field, and (c) both. In (c) the magnetic field B_{ac} compensates the effect of the supercurrent force. The deviations of the trajectories due to the supercurrent force and the magnetic field are extremely exaggerated.

Andreev-reflected hole (with group velocity opposite to momentum) is given by the time-dependent expectation values

$$\langle x\rangle(t) = -v_x(t - l/v_z), \tag{5}$$

$$\langle y\rangle(t) = (-v_y + p_s/m_N)(t - l/v_z), \tag{6}$$

$$\langle z\rangle(t) = -[v_z^2 - (p_s/m_N)^2 + 2v_y p_s/m_N - 4E/m_N]^{1/2} \times (t - l/v_z). \tag{7}$$

This trajectory is plotted schematically in Fig. 2(a), together with the one of the incident electron. The deviation of the return path of the hole from the electron path is extremely exaggerated.

If a weak external magnetic field B_a is applied in the N layer, the quasiparticle trajectories are qualitatively like the ones shown in Fig. 2(b). They are computed using the Ehrenfest theorem.[15] The calculation, including the effects of Andreev reflection, yields the deviation $(\delta x, \delta y)$ of the position, where the returning hole hits the plane in $z = -l$, from the position where the emitted electron left the point contact. There are three contributions to $(\delta x, \delta y)$, labeled by 1,2,3. The first one is due to $\mathbf{f}_{\Delta 1}$, which changes the z component of the quasiparticle momentum, the second one is caused by $\mathbf{f}_{\Delta 2}$, which changes the y component of momentum, and the

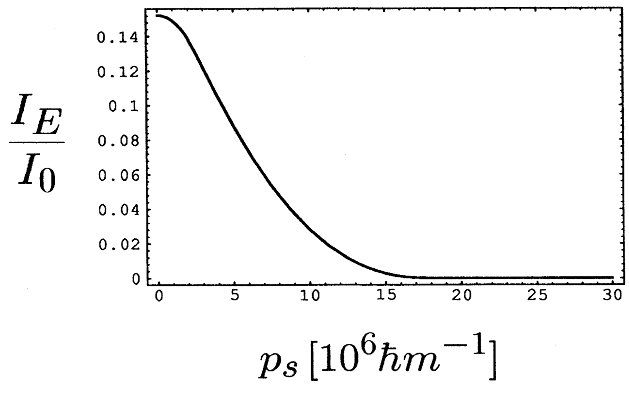

FIG. 3. Excess current I_E, normalized to the injected current I_0, as a function of the Cooper pair momentum p_s (in units of $10^6\hbar$/m) for the case of zero external magnetic field.

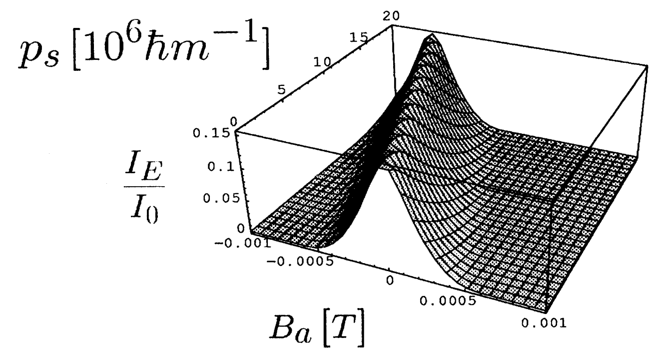

FIG. 4. Excess current I_E, normalized to the injected current I_0, as a function of the Cooper pair momentum p_s and the applied magnetic field B_a. For every value of p_s the maximum of I_E/I_0 defines the compensation field B_{ac}.

third one results from the magnetic field. Taking into account that Δ_0 is much smaller than the Fermi energies we obtain the deviations as

$$(\delta x,\delta y)_1=-\frac{2lE}{m_N v_z^3}(v_x,v_y), \tag{8}$$

$$(\delta x,\delta y)_2=\frac{lp_s}{m_N v_z^3}(v_xv_y,v_y^2+v_z^2), \tag{9}$$

$$(\delta x,\delta y)_3=-\frac{l^2eB_a}{m_N v_z^3}(v_xv_y,v_y^2+v_z^2). \tag{10}$$

From Eqs. (9) and (10) we see that the deviations due to $\mathbf{f}_{\Delta 2}$ and the magnetic field compensate each other, if

$$B_a=B_{ac}\equiv\frac{p_s}{le}. \tag{11}$$

In the next section we will see from numerical computation that the excess current, as a function of the applied field B_a and the applied supercurrent momentum p_s, is maximum, if the magnetic field B_a satisfies Eq. (11). Then the quasiparticle trajectories should be as indicated in Fig. 2(c). Thus, by determining B_{ac}, one measures the effect of the supercurrent force $\mathbf{f}_{\Delta 2}$.

III. EXCESS CURRENT

Andreev-reflected holes, originating from electrons which are injected with a certain velocity (v_x,v_y,v_z) by the point contact of circular area πb^2, hit the $z=-l$ plane in a circle shifted from the center of the point contact. This shift, in units of the contact radius b, is given by

$$\delta=[(\delta x)^2+(\delta y)^2]^{1/2}/b, \tag{12}$$

with $\delta x=\delta x_1+\delta x_2+\delta x_3$ and $\delta y=\delta y_1+\delta y_2+\delta y_3$. Inserting this δ into Eq. (2) (and the preceding one) of Ref. 10 we compute the excess current numerically. We assume equal effective masses in N and S and vanishing scalar potentials so that the Andreev reflection probability is practically one for (1 meV=)$E<\Delta_0$. The results, shown in Figs. 3 and 4, are obtained with the following set of parameters[10]: point-contact radius $b=15$ nm, thickness of the normal metal $l=20$ μm, mean free path $=l$, and Fermi wave number (of silver) $k_F=1.2\times10^8$ cm^{-1}. The thickness d of the superconducting plate S is assumed to be equal to the London penetration depth $\lambda=200$ nm.

Figure 3 for the case of zero external magnetic field shows how the supercurrent force $\mathbf{f}_{\Delta 2}$ results in the decrease of the excess current I_E with the Cooper pair momentum p_s of the applied supercurrent.

Figure 4 shows the combined effect of an applied external magnetic field B_a and the applied supercurrent on the excess current. For each Cooper pair momentum p_s there is one magnetic field B_{ac} for which the excess current is maximum, i.e., the situation of Fig. 2(c) is realized. The numerically obtained dependence of B_{ac} on p_s, indicated by the dots in Fig. 5, follows very close the analytical result of Eq. (11).

IV. DISCUSSION

Measurement of the decrease of the excess current with p_s and of the compensating magnetic field B_{ac} will be direct evidence of the supercurrent force $\mathbf{f}_{\Delta 2}$ on Andreev-reflected quasiparticles. Normal metals with large mean free paths and type-II superconductors which can carry supercurrents with large Cooper pair momenta p_s should be convenient junction materials. The sensitivity of the experiment depends upon the sensitivity of the shift δ, Eq. (12), with respect to changes in p_s and B_a. We note from Eqs. (8)–(10) and (12) that the smaller the effective mass m_N in the N layer the smaller the Cooper pair momentum p_s which produces a sufficiently large shift δ and the correspondingly large decrease of the excess current. Thus replacing the normal metal by a (modulation) doped, degenerate semiconductor with a (possibly quasi-two-dimensional) electron gas of small effective mass m_N and sufficiently large mean free path may be an appropriate way of enhancing experimental sensitivity. In addition, the smaller Fermi velocity of the semiconductor enforces the sensitivity-enhancing effect of the smaller m_N according to Eqs. (9) and (10). What matters is the ratio of the momenta p_s and $m_N v_z$. Detailed calculations have shown

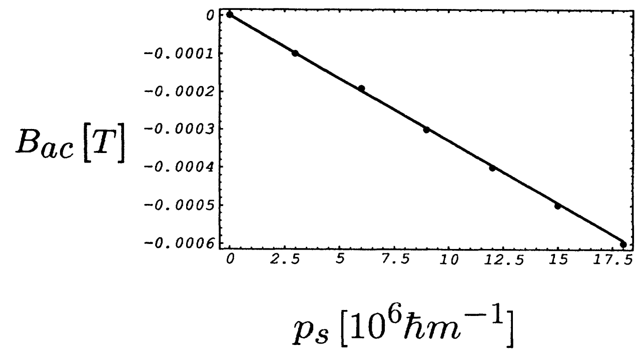

FIG. 5. Compensation field B_{ac} as a function of the Cooper pair momentum p_s in the superconducting layers S. Dots: numerical results from Fig. 4; straight line: analytical result of Eq. (11).

that different Fermi energies and effective masses in N and S only reduce somewhat the magnitude of the excess current.[17] As a consequence, in Fig. 3 one would have a smaller I_E, and the Cooper pair momentum p_s would be reduced. The latter has also the advantage that the Cooper pair momentum stays out of the value range where pair breaking may occur in the S layer. According to Eq. (11) the compensating magnetic field B_{ac} will decrease with p_s.

An appropriate experimental setup is indicated in Fig. 1. For the key parameters we assume (i) the N layer thickness l is about equal to or less than the quasiparticle mean free path so that the quasiparticles travel ballistically in the normal metal; (ii) the S layer thickness d is of the order of the London penetration depth λ so that the supercurrent density at the N-S interface is still comparable with its maximum value at the S-I (insulator) interface; and (iii) d exceeds the BCS coherence length ξ_0 so that Andreev reflection is not reduced by normal backscattering from the S-I interface.

If the two thin, flat superconducting plates (separated by the insulator) extend between $-\infty \leqslant x,y \leqslant +\infty$, the density of the supercurrent $\mathbf{j}_s = \mathbf{e}_y j_s(z) = -\mathbf{e}_y |e| n_c v_s$ flowing in the N-S interface is uniform with respect to x. From London's and Maxwell's equations this current density has been computed for the configuration of Fig. 1. At $z=0$ the corresponding momentum of the Cooper pairs of density n_c, mass $2m$, and charge $2e=-2|e|$ becomes[17]

$$\mathbf{p}_s = \mathbf{e}_y \frac{m}{e n_c \lambda \sinh(d/\lambda)} \left[\int_0^d j_s(z) dz - \frac{B_a}{\mu_0} \left(1 - \cosh \frac{d}{\lambda} \right) \right]. \tag{13}$$

Here $\int_0^d j_s(z) dz$ is the total applied supercurrent (per unit length in x direction). We are confident that the uniformity of the current density will be preserved for plates of finite x,y extensions, too, because of the similarity of the considered experimental setup with the configuration of a finite, thin superconducting plate separated from a much larger ground plate discussed in Ref. 18. Theoretically, uniformity of the current density was shown for this plate configuration.[19]

Finally, the decrease of the point-contact excess current in supercurrent carrying NS layers by the supercurrent force may also be useful in probing the existence of the Fulde-Ferrell-Larkin-Ovchinnikov state in the heavy fermion superconductor UPd_2Al_3.[16,22]

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