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Stochastic resonance and nonlinear response in double-quantum-well structures

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Coherent motion of a wave packet in a double-quantum-well structure can be induced by irradiating it with a weak resonant laser field. The amplitude of this motion exhibits a "stochastic resonance" as a function of the relaxation rate in the double well and of the field amplitude. The maximum oscillation amplitude at each temperature does not depend on the field amplitude, demonstrating the failure of linear-response theory to describe the quantum stochastic resonance.

Stochastic resonance (SR) (Ref. 1) is an archetypal example of a phenomenon where order is caused by random noise. So far SR has been observed and described for overdamped bistable classical systems and predicted for overdamped quantum-mechanical double wells.[2] As long as any quantum-mechanical coherence is suppressed, both classical and quantum SR are equally well described by the classical rate equation approach,[3] the only difference being the mechanism of incoherent transitions (classical or tunneling) between the stable attractors. The feasibility of noise-induced enhancement of the response of quantum-mechanical systems with persistent coherences is a very important question in connection with quantum-mechanical control of coherent transport or chemical reactions in condensed matter or in polyatomic molecules, where the energy sharing between different normal modes seems to obviate use of tailored laser pulses to achieve the desired thermally inaccessible state of the system.[4] Quantum SR can be defined as enhancement of response of a driven quantum-mechanical system by *quantum* noise. The strength of quantum noise depends on two factors: the strength of coupling of the system to its environment and the temperature, with only the coupling strength being variable at zero temperature.

In this paper we report a quantum SR in underdamped bistable potentials, such as double-quantum-well (DQW) semiconductor heterostructures, where coherent oscillations of excitonic wave packets have recently been predicted[7] and observed.[8,9] Optical control of such oscillations has been the subject of recent experimental studies.[10,11] We will show that several features make the SR in these systems different from the conventional SR: (i) The SR is achieved by varying not the temperature but the coupling to the heat bath; (ii) the quantum SR coexists with a regular resonance such that the response is strongly enhanced at the resonant driving frequency; and (iii) the SR also reveals itself as a maximum response as a function of the driving field. The last property indicates the breakdown of linear-response theory and suggests that a large response may be achieved using weak fields; in fact, we find that the optimal enhancement of the response is obtained at a field amplitude that is proportional to the relaxation rate of the DQW.

The system we study is a symmetric double well coupled linearly to a continuum of harmonic oscillators. For a typical DQW structure the splitting in the lowest tunneling doublet is 10–100 cm^{-1}, while the energy spacing between different doublets is on the order of $10^3\ \mathrm{cm}^{-1}$, such that near or below room temperature the upper energy levels are not populated. We chose the laser excitation to be (nearly) resonant with the splitting of the tunneling doublet such that it also does not induce transitions to higher-lying doublets. Under these circumstances the two-level system (TLS) description of tunneling is appropriate,[5,6] so that the Hamiltonian is of the form

$$H=-\hbar\Delta\sigma_x+\sum_j p_j^2/2m_j+\tfrac{1}{2}m_j\omega_j^2x_j^2+c_jx_js_0\sigma_z+V_0\sigma_z\cos(\omega t), \qquad (1)$$

where σ_x and σ_z are the Pauli spin matrices for the two-state representation of the double-well coordinate s, $2s_0$ is the distance between the two wells, and the amplitude of the laser-electron interaction is given by $V_0=eEs_0$, E being the electric-field strength. We chose parameters similar to those reported in Ref. 8 for a GaAs/$\mathrm{Al}_x\mathrm{Ga}_{1-x}$As DQW structure: $s_0=80$ Å, $\Delta=12.7\ \mathrm{cm}^{-1}$.

In what follows we study the steady-state dynamics of the average position of the wave packet, $\langle s(t)\rangle=s_0\langle\sigma_z(t)\rangle$. At long times the wave packet oscillates with frequency ω and amplitude $s_{\lim}$. In the absence of coupling to the reservoir the dynamics of $\langle s(t)\rangle$ depend on the initial condition. If the DQW is at Boltzmann equilibrium at the moment the field is turned on, then, using the rotating wave approximation,[12] $\langle s(t)\rangle$ will exhibit quantum beats that result from the interference of fast oscillations with frequency ω and slow oscillations with the Rabi frequency $\Omega=V_0/\hbar$. The maximum amplitude that can be achieved at a given temperature is $s_{\max}=s_0\tanh(\hbar\Delta/k_BT)$.

Adding weak dissipation changes this picture dramatically. It is natural to expect that the oscillatory behavior will be strongly suppressed; for example, using the optical Bloch equations for the Pauli operators in the rotating frame,[12,13]

$$\langle\dot{\tilde\sigma}_x\rangle=-(V_0/\hbar)\langle\tilde\sigma_y\rangle-(1/\tau_1)(\langle\tilde\sigma_x\rangle-\langle\sigma_x^{\mathrm{eq}}\rangle),$$

$$\langle\dot{\tilde\sigma}_y\rangle=(2\Delta-\omega)\langle\tilde\sigma_z\rangle-(1/\tau_2)\langle\tilde\sigma_y\rangle+(V_0/\hbar)\langle\tilde\sigma_x\rangle, \qquad (2)$$

$$\langle\dot{\tilde\sigma}_z\rangle=-(2\Delta-\omega)\langle\tilde\sigma_y\rangle-(1/\tau_2)\langle\tilde\sigma_z\rangle,$$

for resonant pumping, $\omega=2\Delta$, one obtains the steady-state solution to be $\langle\tilde\sigma_z\rangle=\langle\tilde\sigma_y\rangle=0$ in the limit of $\tau_2^{-1}\rightarrow 0$. Here

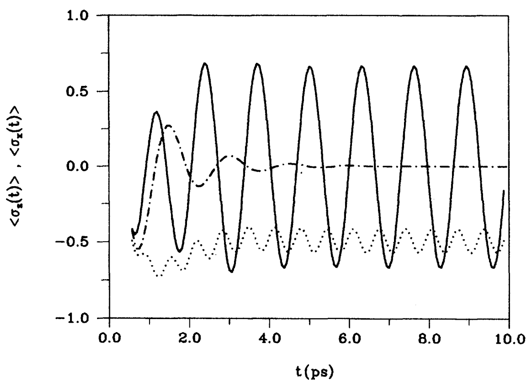

FIG. 1. Mean position of the wave packet in units of s_0 (solid line) and the population difference between the upper and lower eigenstates (dotted line) in a DQW structure with the parameters described in the text, driven by an electric field $E=492$ V/cm with frequency $\omega=2\Delta$ at temperature $T=5$ K. The Kondo parameter is $\alpha=0.16$, the bath cutoff frequency $\omega_c=95.25$ cm^{-1}. Chain-dotted line, mean position of the wave packet in the absence of the field.

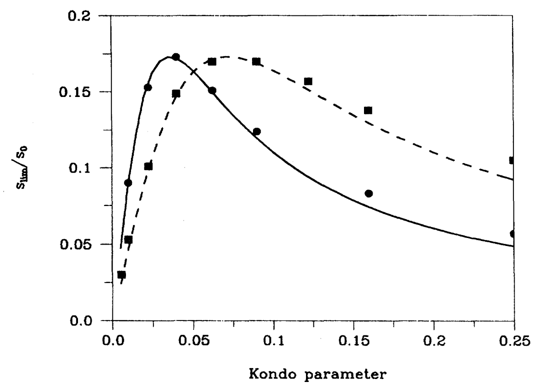

FIG. 2. Wave-packet oscillation amplitude (in units of s_0) plotted as a function of the Kondo parameter for $\omega=2\Delta$, $E=492$ V/cm [circles, simulation data; solid line, Eq. (7)], and $E=984$ V/cm [squares, simulation data; dashed line, Eq. (7)]. The temperature is 72 K, the bath cutoff frequency $\omega_c=95.25$ cm^{-1}.

$\tau_1^{-1}=2\hbar^{-1}s_0^2J(2\Delta)\coth(\hbar\Delta/k_BT)$ and $\tau_2=2\tau_1$ are, respectively, the population relaxation and dephasing times,[14] where $J(\omega)=(\pi/2)\Sigma_j c_j^2\delta(\omega-\omega_j)/m_j\omega_j$ is the bath spectral density, $\langle\sigma_x^{\rm eq}\rangle=\tanh(\hbar\Delta/k_BT)$ is the equilibrium value of $\langle\sigma_x\rangle$ in the absence of the field, and the tilde denotes the rotating frame representation.

We demonstrate below that at certain conditions large-amplitude charge oscillations can be obtained with weak resonant fields. In order to obtain an accurate numerical solution for the dynamics of $\langle s(t)\rangle$ we have utilized our recently proposed tensor multiplication scheme.[15] This path-integral scheme exploits the fact that for smooth spectral densities bath correlations decay during a finite (though not necessarily very short) time, and allows calculation of the reduced dynamics of a system coupled to a harmonic bath all the way to thermal equilibrium, having started from a non-equilibrium initial condition. For the present purpose, the scheme of Ref. 15 was modified to include the effects of a time-dependent field, and the details will be reported elsewhere.[16]

Starting from an arbitrary initial state [for example, $\langle s(0)\rangle=s_0$], the reduced density matrix was propagated until the steady state was reached. A typical result of the calculations is shown in Fig. 1, where $\langle s(t)\rangle$ performs oscillations of considerable amplitude (solid line). In the absence of the field (chain-dotted line) the oscillations decay during a few picoseconds. In the calculations presented in Fig. 1 a standard model of an Ohmic spectral density[5,6] was used, $J(\omega)=(\pi\alpha\hbar/2s_0^2)\omega\exp(-\omega/\omega_c)$, where α is the Kondo parameter and ω_c the bath cutoff frequency. As will be seen below, the results obtained are universal and do not depend on the details of the spectral density. Therefore we expect that although different mechanisms of energy relaxation and dephasing such as electron-phonon or exciton-exciton interactions[17] may dominate at a given temperature and under the particular experimental conditions, the phenomenon of quantum SR should be ubiquitous and observable whenever a suitable field is applied.

Figure 2 presents the dependence of the steady-state oscillation amplitude, $s_{\lim}\equiv s_0\langle\sigma_z\rangle_{\lim}$, on the Kondo parameter for two different intensities of the driving field. It is seen that the maximum steady-state amplitude is independent of field strength. This maximum as a function of the dissipation parameter can be thought of as a stochastic resonance, in analogy with the classical SR. Weaker dissipation requires weaker fields to achieve maximum response. To the right of the maximum, linear-response theory holds, that is, $s_{\lim}$ is proportional to the field E, while for weaker coupling, to the left of the maximum, a weaker field produces stronger output. The breakdown of linear-response theory is demonstrated in Fig. 3 where the dependence of $s_{\lim}$ on E is presented.

These features can be explained using the following rea-

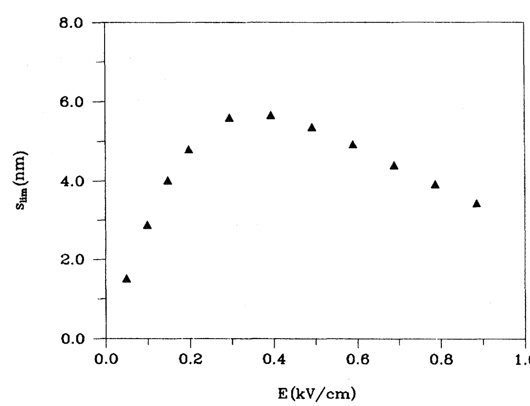

FIG. 3. Wave-packet oscillation amplitude plotted as a function of electric field at $T=5$ K. The Kondo parameter is $\alpha=0.10$, the frequency of the field $\omega=2\Delta$, and the bath cutoff frequency $\omega_c=95.25$ cm^{-1}.

soning: Starting from the Heisenberg equations of motion for the evolution of the Pauli matrices,

$$\begin{aligned}\dot{\sigma}_x(t)&=-2\sigma_y(t)f(t)/\hbar,\\ \dot{\sigma}_y(t)&=2\Delta\sigma_z(t)+2\sigma_x(t)f(t)/\hbar,\\ \dot{\sigma}_z(t)&=-2\Delta\sigma_y(t),\end{aligned}\tag{3}$$

where $f(t)=V_0\cos(\omega t)+s_0\Sigma_j c_j x_j(t)$ is the instantaneous energy bias of the TLS, one obtains

$$\begin{aligned}\langle\ddot{\sigma}_z\rangle+(2\Delta)^2\langle\sigma_z\rangle=&-4\Delta\langle\sigma_x\rangle(V_0/\hbar)\cos(\omega t)\\ &-4\Delta(s_0/\hbar)\sum_j c_j\langle\sigma_x x_j(t)\rangle.\end{aligned}\tag{4}$$

According to Ref. 18, the last term in Eq. (4) can be converted to a damping term in the weak system-bath coupling limit, assuming free bath dynamics and using a Markovian approximation. As a result, a harmonic-oscillator equation is obtained with the periodic force proportional to the population difference $\langle\sigma_x\rangle$ between the two eigenstates of the TLS,

$$\langle\ddot{\sigma}_z\rangle+(2\Delta)^2\langle\sigma_z\rangle+(2/\tau_2)\langle\dot{\sigma}_z\rangle=-4\Delta\langle\sigma_x\rangle(V_0/\hbar)\cos(\omega t).\tag{5}$$

The weak system-bath coupling condition[18] used in the derivation of Eq. (5) can in principle be relaxed using Dakhnovskii's kinetic equation,[19] resulting in a more general, albeit less tractable, theory.

To obtain an approximate solution to Eq. (5) we use the fact that the population difference between the TLS eigenstates, $\langle\sigma_x\rangle$, stays practically constant in the steady state, as shown in Fig. 1. This observation suggests using the steady-state value obtained from the rotating-wave approximation of Eq. (2):

$$\langle\sigma_x\rangle_{\lim}\approx\tanh(\hbar\Delta/k_BT)\left(1+\frac{V_0^2\tau_1\tau_2/\hbar^2}{1+\tau_2^2(2\Delta-\omega)^2}\right)^{-1},\tag{6}$$

where we have used the fact that the transformation to the rotating frame does not affect the operator σ_x.

Using Eqs. (5) and (6), the origin of the maximum of $s_{\lim}$ as a function of the field E is readily understood: as the field is increased, the population difference (which results from the competition of resonant pumping of the TLS and energy relaxation) decreases, leading to a nonmonotonic dependence of the right-hand-side of Eq. (5) on V_0.

With the approximation of Eq. (6), Eq. (5) is the standard equation of motion for a forced harmonic oscillator with dissipation. The steady-state solution oscillates with amplitude

$$s_{\lim}/s_0=\frac{4\Delta V_0/\hbar}{[(\omega^2-4\Delta^2)^2+\omega^2/\tau_1^2]^{1/2}}\langle\sigma_x\rangle^{\mathrm{eq}},\tag{7}$$

which exhibits maxima as a function of τ_1, V_0, and ω. At $\omega=2\Delta$ this result is also obtainable from the Bloch equations, Eq. (2). However, away from the resonance condition the Bloch equations fail to predict, even qualitatively, the correct frequency dependence of $s_{\lim}$.[16] The theoretical curves given by Eq. (7) are also presented in Fig. 2 and are in good agreement with the results of the path-integral simulation.

For resonant pumping, $\omega=2\Delta$, the maximum oscillation amplitude is achieved for $V_0=2^{-1/2}\hbar/\tau_1$ and is equal to $2^{-1/2}s_0\tanh(\hbar\Delta/k_BT)$, which differs from that in the absence of dissipation by only a factor of order unity.[20] This is by a factor of approximately $2\Delta\tau_1$ larger than the static response to the field V_0. With weak dissipation, $2\Delta\tau_1\gg 1$, showing that the SR results in large "signal" amplification. Using typical values of τ_1 for GaAs DQW structures,[8,17] $\tau_1=1-5$ ps, the field required to achieve SR is $E=0.3$-1.5 kV/cm in the case of resonant pumping.

To conclude, as demonstrated in this paper, the role of dissipation in a driven nonlinear quantum system does not necessarily amount to the loss of control over the system due to destruction of coherence. Another evidence of this kind is the noise-induced localization in a double well driven by a resonant field.[21] The noise-induced *enhancement* of order in quantum systems may be a practical way to overcome the difficulties that plague experimental laser control of complex systems.

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