PHYSICAL REVIEW B VOLUME 52, NUMBER 24 15 DECEMBER 1995-II

Evidence for deterministic chaos as the origin of electrical tree breakdown structures in polymeric insulation

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(Received 26 May 1995)

Electrical discharges were measured during the propagation stage of electrical tree breakdown in an epoxy resin. An analysis of their number sequence provides strong evidence for the existence of an underlying deterministic chaotic mechanism. The fractal dimension of the tree and that of the reconstructed attractor and Lyapunov exponent were found to be related. The higher fractal dimension tree ($d_t \sim 1.9$) is associated with an attractor dimension $d_f \sim 3.1$ and Lyapunov exponent $\lambda \sim 0.008$ bit/s. The lower fractal dimension tree ($d_t \sim 1.5$) is associated with values of $d_f \sim 3.56$ and $\lambda \sim 0.028$ bit/s. No evidence for the presence of random stochastic processes, an essential ingredient of the dielectric breakdown model, has been found.

Fractal breakdown structures in solids are called electrical trees.[1] They are composed of gas-filled tubules of minimum size ~ 10 μm long, ~ 1 μm radius. During propagation, the gas breaks down in the electrical field, giving electrical discharges which can be monitored. The discharges produce damage around the tree periphery which eventually accumulates to form new tubules which are added to the structure. The structures generated are generally fractal objects and lie in one of two classes: branch trees ($1 < d_t < 2$) and bush trees ($2 < d_t < 3$).[1–3] Here we focus on two models proposed for tree propagation in polymeric solids omitting the possible stochastic process of tree initiation.[4]

Two alternative models have been presented to explain the formation of fractal tree structures. The first of these[5] was based on Niemeyer, Pietronero, and Wiesmann's dielectric breakdown model (DBM).[6] As in the DBM a tubular extension is made by a random stochastic selection from a field-weighted distribution of peripheral directions, but now, no extension is allowed below a threshold field level. In this model the damage either forms a tubule immediately or it does not occur. In contrast, it has been proposed[7] that discharges produce quantifiable subtubule forming damage via avalanches, which is accumulated until it reaches the critical level for tubule formation. Fractal breakdown structures are only produced when mechanism-induced time-dependent fluctuations in the local electric field are introduced.[7] In the absence of these fluctuations, single puncture structures are obtained which exhibited a rapidly accelerating (runaway) propagation rate.

The fundamental difference between the DBM approach and that of Dissado and Sweeney[7] is in the origin of the fluctuations that produce the fractal structure. In the DBM the fluctuations are stochastic in origin, whereas in the discharge-avalanche model[7] they are driven by the mechanism itself. In this latter case it was suggested[7,8] that the local field was modified by the space charge produced as a result of the tube discharges and the avalanche damage process. Both enhancement and reduction of the Laplace field of the discharge could occur, dependent upon the polarity and spatial distribution of the charges. In particular, the charge redistribution during the avalanche damage process will reduce the local field along the avalanche path until the applied field reverses polarity in its ac cycle, when it enhances it. The damage-generating process, which is strongly nonlinear in local field, is therefore a source for both negative and positive feedback effects via modifications to the local field that are self-produced. Such a process fulfills the requirements for the existence of a regime of deterministic chaos,[9] in which the local field would be driven to follow a never-repeating sequence of values as the tree propagates. In this case the sequence trajectory adopted by the local field will lie on the surface of a strange attractor.[9] The DBM, in contrast, implies that the local field will suffer only random fluctuations about a deterministic value. It is therefore possible to distinguish between the two explanations for the tree structures by monitoring the local electric field, or a related feature, during tree growth. This is our intention here.

First, we point out that systems of equations that are capable of exhibiting deterministic chaos have a parameter range in which the system either reaches equilibrium or runs away to infinity. In dielectric breakdown, we would expect a runaway breakdown to occur at high applied fields (when positive feedback dominates) and equilibrium to occur at low fields (when negative feedback dominates).[1,10] Deterministic

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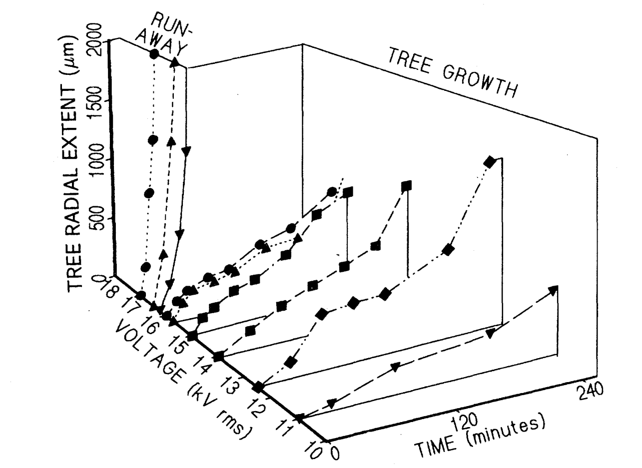

FIG. 1. The length-time plots of breakdown structures grown from a needle electrode in the same epoxy resin at different applied voltages.

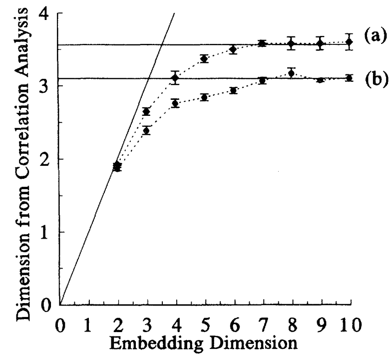

FIG. 2. Plot of the correlation dimension over a range of embedding dimensions for (a) sample A and (b) sample B.

chaos would only occur in an intermediate range where the positive and negative feedback processes are near balance. If fractal trees are a consequence of deterministic chaos in the damage-driving local electric field, we would therefore expect them to lie between a runaway at sufficiently high applied field (voltage), and a nondamaging equilibrium at low field. The experimental results presented in Fig. 1, for Ciba CT1200 epoxy resin, show that this is indeed the case, with the crossover between decelerating tree propagation and accelerating runaway breakdown occurring very sharply at an applied voltage of 16 kV. Repeat experiments using Ciba CT200 epoxy resin show the same systematic behavior with applied voltage but with a crossover to runaway at a voltage of 14.5 kV.[11] We note also that the runaway result obtained in the discharge avalanche model[7] in the absence of field fluctuations is consistent with an interpretation of the fluctuations as due to deterministic chaos. In the DBM, however, it is not possible to determine the rate of propagation, and a crossover such as that observed experimentally can only be obtained by an abrupt change of a field-weighting parameter η to a high value[5] above a critical voltage.

Since it is not possible to measure the local field directly during tree propagation, it is necessary to monitor an observable feature that is field dependent. Electrical discharges are the most obvious of such features. Their magnitude and number will depend upon the fields produced in the tubules by the surrounding space charges and applied voltage.[12] In our case we have chosen to monitor the number of discharges, with magnitude greater than 1.3 pC, over a 1-s interval every 2 s during the propagation of the tree. Pin-plane electrode geometry samples (to model a standard defect) were made by embedding tungsten pins of shank diameter 1 mm and pin-tip radii of 3 μm, into Ciba CY1311 epoxy resin slabs. Tree growth in this material was found to be reproducible and predictable over a range of applied voltage.[13] The fractal dimension, d_t, of the trees grown was varied from $\sim$1.4 to $\sim$1.9 by increasing the ac voltage applied to the pin electrode or reducing the pin-plane separation. The experiments were repeated a number of times. The discharge pattern for trees of a given fractal dimension was reproducible and changed systematically with the applied voltage and pin-plane spacing as described in Ref. 14. The time sequence of discharge numbers was analyzed and described here for two samples: sample A, a lightly branched tree, $d_t = 1.5$, and sample B, a highly branched tree, $d_t = 1.9$, produced by a

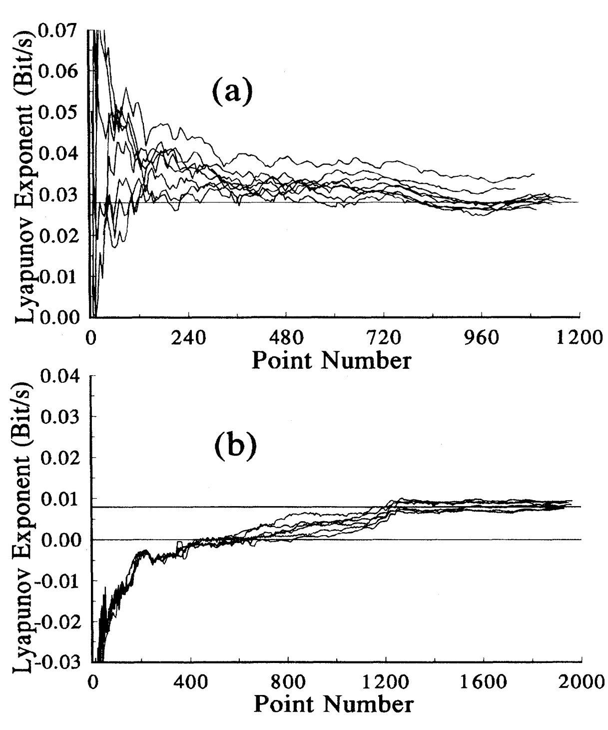

FIG. 3. The running average Lyapunov exponent as the analysis steps through the time series. (a) Sample A, for an embedding dimension of 8. The evolution time, t_e, ranges from four to eight data points in the time series and the lag time, Δ, between components of the pseudovectors, from four to ten. (b) Sample B, embedding dimension $=8$, and values of t_e between four and eight and Δ between four and eight data points.

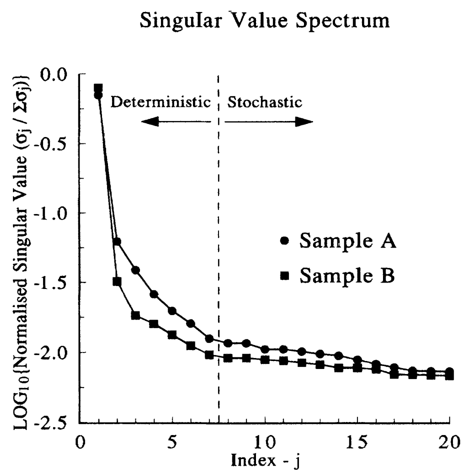

FIG. 4. Singular value spectra for samples A and B. Seven deterministic components are seen to be emerging from the noise floor for both samples.

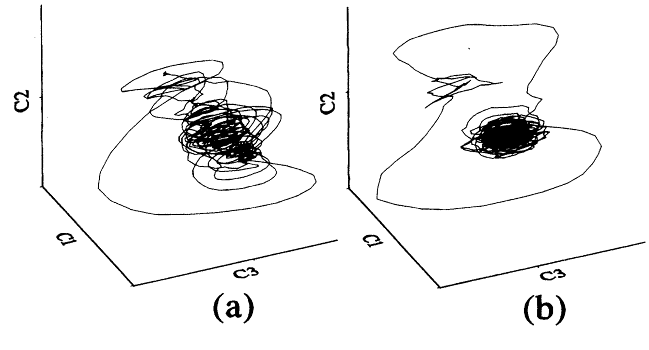

FIG. 5. Three-dimensional representations of the attractors for (a) sample A and (b) sample B, where the time series for the two samples were transformed onto their three most significant deterministic directions.

similar field but higher voltage. To determine whether the discharge activity was governed by a deterministic chaotic or stochastic process, we have applied the technique of Grassberger and Procaccia[15] to obtain the correlation dimension, d_f, and the algorithm of Wolf *et al.*[16] to determine the dominant (most positive) Lyapunov exponent, λ. Estimates of d_f for the two samples are shown in Fig. 2, where they are plotted as a function of the embedding dimension of the pseudovector construction.[15] The correlation dimension was found to saturate to values 3.56 ± 0.10 for sample A, and 3.10 ± 0.05 for sample B, for embedding dimensions greater than 7. If the discharge behavior was the result of a stochastic process, then the estimated correlation dimension should be equal to the embedding dimension[15] and follow the diagonal line in Fig. 2. Dominant Lyapunov exponent analysis was repeated for different embedding dimensions from 2 to 9 and a range of values for the analysis parameters.[16] Convergence occurred for embedding dimensions greater than 7 when the evolution time step, t_e, and lag time between pseudovector components, Δ, were both greater than four data points in the time sequence. In Fig. 3, the running average dominant Lyapunov exponent for the two samples is plotted as the analysis steps through the time sequence for an embedding dimension of 8 and various values of the analysis parameters t_e and Δ. The dominant Lyapunov exponents were $\lambda = +0.028 \pm 0.006$ bit/s for sample A and $\lambda = +0.008 \pm 0.002$ bit/s for sample B. The positive value of λ obtained for the two samples implies chaotic behavior. Finally, we have used the singular system analysis technique described by Broomhead and King[17] to separate the deterministic and stochastic elements of a time sequence of data and to reconstruct the attractor. This also allows us to obtain statistical dimension estimates as defined by Vautard and Ghil[18] for the two time series. The singular spectra for the two samples are shown in Fig. 4, giving a statistical dimension[18] of ~ 7 for the two samples. Diagrams of three-dimensional representations of the attractors for the two samples are shown in Fig. 5, where the trajectories were transformed onto the three most significant deterministic directions. The more complex trajectory for sample A is consistent with its higher correlation dimension and larger dominant Lyapunov exponent. Saturation of λ and d_f for embedding dimensions greater than 7 agrees with the statistical dimension estimates, suggesting that a model with at least seven degrees of freedom will be required to describe the discharge behavior.[18]

The reproducibility of these and other results for trees of intermediate fractal dimensions confirm the following facts: (i) a strange attractor governs the equations describing the propagation of fractal breakdown structures in CY1311 epoxy resin; (ii) the dimension of the attractor decreases from 3.56 to 3.10 and dominant Lyapunov exponent from $+0.028$ bit/s to $+0.008$ bit/s when the fractal dimension of the tree increases from 1.4 to 1.9; (iii) there is no evidence for a random stochastic process in the discharge behavior during tree propagation. The dominant Lyapunov exponent and correlation dimension estimates for the two trees imply that the chaotic aspects of the breakdown process reduced as the applied voltage and tree dimension d_t increased. It must be concluded that fractal breakdown structures in CY1311 and possibly other epoxy resins are the consequence of a deterministic breakdown mechanism operating in a parameter region which leads to deterministic chaos, rather than due to stochastic processes. This conclusion supports the discharge-avalanche model[7] over the DBM (Ref. 5) in polymeric breakdown, with the mechanism-induced field fluctuations of the former being seen to play the role of deterministic chaos in the local field magnitudes. Increases in d_t at high voltages correspond to a trend towards a deterministic mechanism, i.e., an enhancement of the positive feedback processes, which eventually results in runaway behavior when the voltage is high enough.

The authors gratefully acknowledge the financial support of The National Grid Company plc and EPSRC Grant No. GR/J13816 for L.A.D. during the course of this work.

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