Transport through dirty Luttinger liquids connected to reservoirs

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It is shown that the conductance of a weakly disordered Luttinger-liquid quantum wire connected to non-interacting leads is affected by electron-electron interactions in the wire. This is in contrast to the case of a perfect wire the conductance of which is given by e^2/h regardless of interactions in the wire. The disorder-induced correction to the conductance scales with temperature and/or the wire length, the scaling exponent being determined only by the interaction strength in the wire. These results explain recent experiments on quasiballistic GaAs quantum wires.

Transport through quantum wires is commonly believed to be strongly affected by electron-electron interactions. In particular, if only one channel of transverse quantization is open and the electrons are in the Luttinger-liquid state,[1] the conductance of a perfect wire is expected to be Ke^2/h,[2–5] where the parameter K characterizes the sign and the strength of the interactions: $K<1$ for repulsion; $K>1$ for attraction; $K=1$ in the absence of interactions. Moreover, in the case of repulsion, disorder in the wire is predicted to suppress the conductance much more strongly than in the noninteracting case.[2–7] This suppression comes about via temperature- and/or length-dependent corrections to the conductance of a perfect wire that diverge in the limit $T\rightarrow 0$ or $L\rightarrow\infty$ even for an arbitrarily weak disorder, indicating the tendency to an insulating behavior. The exponents of the T and L scalings are also determined by the parameter K. This has to be contrasted with the noninteracting theory, which predicts T-independent conductance (at least for T smaller than the interband energy spacing). The physical explanation of the interaction-induced breakdown of conduction[4] is that the $2k_F$ backscattering due to impurities stimulates the divergence of charge-density-wave fluctuations with the period $2\pi/2k_F$. With all these dramatic differences between the interacting and the noninteracting theories, an experiment seems capable of discriminating between the two.

At first sight, however, a recent experiment on high-mobility GaAs quantum wires[8] has given support to neither theory. Indeed, at higher temperatures ($T>1.2$ K) the observed value of the conductance was e^2/h with a high degree of accuracy, which seems to support the noninteracting theory. On the other hand, reduction of the conductance was observed at lower T, and by fitting the observed T dependence of the conductance into the interacting theory[5] the authors of Ref. 8 obtained a value of $K\approx 0.7$. This seems to support the interacting theory, but also implies that at higher T, where the disorder-induced correction is small, the reduction of the conductance should have been of the order of 30%, which clearly contradicts the data.

A partial resolution to this paradox has been given in two recent papers,[9,10] in which it was emphasized that the model of a homogeneous Luttinger liquid leading to the result Ke^2/h of Refs. 2–5 is not adequate to a typical experimental situation. Indeed, a narrow wire is always connected to wide electron reservoirs by the conducting leads. As the leads and the reservoirs are necessarily *not* one dimensional, the electrons there form not a Luttinger liquid but rather a Fermi liquid which, for present purposes, can be considered as a noninteracting Fermi gas. Consequently, the effective one-dimensional model considered in Refs. 9 and 10 was that of an inhomogeneous Luttinger liquid with the interaction strength varying from some nonzero value in the central part of the system (the "wire") to zero in the outer parts (the "leads"). It has been shown that in the absence of disorder the conductance of such a system is given by e^2/h *regardless* of the interactions in the wire. This explains the absence of the conductance renormalization at higher T observed in Ref. 8. It must be added that remarks supporting this result were made earlier by Kane and Fisher[3] and by Matveev and Glazman.[11]

The question that still remains open is whether the experimentally observed T dependence of the conductance is a signature of a dirty Luttinger-liquid state in the wire itself or is a reflection of the presence of the leads as well. In this paper, I show that the T- and L-dependent corrections to the dc conductance of a weakly disordered wire are determined entirely by the interactions in the wire, and are not affected by the presence of the noninteracting leads. This result, taken together with the results of Refs. 9 and 10, suggests that the experimental observations of Ref. 8 can be seriously considered as an indication of the Luttinger-liquid state in GaAs quantum wires.

The result announced above can be entirely anticipated from the following simple physical picture. Consider a weakly disordered wire containing a Luttinger liquid and adiabatically connected to the Fermi-liquid leads. In the absence of disorder, the finite resistance of a perfect wire ($=h/e^2$) is entirely due to contact resistance:[12–15] some of the electrons coming from the wide leads are reflected as the channel becomes narrower. This reflection takes place outside the wire, where the electrons are in the Fermi-liquid state; therefore, the contact resistance is not affected by the interactions in the wire. Weak disorder in the wire gives rise to an additional contribution to the resistance. This contribution is determined by the scattering *in* the wire, where electrons are in the Luttinger-liquid state, and therefore this contribution has features typical of a Luttinger liquid but not of a Fermi liquid.

As in Refs. 9 and 10, I consider an infinite Luttinger liquid separated into three regions: the wire ($-L/2 < x < L/2$) and the leads ($x \geq L/2$). The interaction parameter K changes abruptly from the value K_W in the wire to the value K_L in the leads. Having in mind the experimental system studied in Ref. 8, I shall focus on the case of repulsive interaction in the wire (i.e., $K_W \leq 1$) and put $K_L = 1$ at the end of the calculation. The length L corresponds to the length of that segment of the original system where the applied electrostatic potential varies most appreciably. Consequently, the electric field is assumed to be zero for $|x| \geq L/2$. I also assume that a random potential $V(x)$ is present only in the wire, i.e., for $|x| < L/2$, and is weak enough to be treated via perturbation theory.[16] The current $I = ej$ is related to the electric field by

$$I(x,t) = \int_{-L/2}^{L/2} dx' \int \frac{d\omega}{2\pi} e^{-i\omega t} \sigma_\omega(x,x') \bar{E}_\omega(x'), \quad (1)$$

where $\bar{E}_\omega(x)$ is the temporal Fourier component of the electric field at point x, and $\sigma_\omega(x,x')$ is the nonlocal ac conductivity, which is given by the Kubo formula[18]

$$\sigma_\omega(x,x') = \frac{ie^2}{\omega} \int_0^\beta \langle T_\tau^* j(x,\tau) j(x',0) \rangle e^{-i\tilde{\omega}\tau} \big|_{\tilde{\omega}=i\omega-\epsilon}. \quad (2)$$

(I set $\hbar = 1$.) In the bosonized form, the particle-number current is given by $j = -i\partial_\tau \phi/\sqrt{\pi}$ and Eq. (2) reduces to[18]

$$\sigma_\omega(x,x') = e^2 \frac{i\tilde{\omega}^2}{\pi\omega} G_{\tilde{\omega}}(x,x') \big|_{\tilde{\omega} \to i\omega - \epsilon}, \quad (3a)$$

$$G_{\tilde{\omega}}(x,x') = \int_0^\beta d\tau \langle T_\tau^* \phi(x,\tau) \phi(x',0) \rangle e^{-i\tilde{\omega}\tau}, \quad (3b)$$

where $G_{\tilde{\omega}}$ is the propagator of the boson field ϕ. The (Euclidean) action of the spinless Luttinger liquid is given by $S = S_0 + S_i$, where

$$S_0 = \int \int_0^\beta dx \, d\tau \frac{1}{2K(x)} \left\{ \frac{1}{v(x)} (\partial_\tau \phi)^2 + v(x)(\partial_x \phi)^2 \right\}, \quad (4)$$

$v(x)$ is the density-wave velocity, and S_i, which describes backscattering due to disorder, is given by

$$S_i = \frac{2}{a} \int dx \int_0^\beta d\tau V(x) \cos(2k_F x + 2\sqrt{\pi}\phi), \quad (5)$$

in which a is the microscopic length cutoff. In the homogeneous model,[2,5] σ_ω in the presence of disorder was found by using the Luther-Peshel formula,[19] which relates the effective mean free time to the $2k_F$ component of the density-density correlator. In the inhomogeneous situation, the meaning of the mean free time is unclear, and I will determine the disorder-induced corrections to σ_ω directly from the Kubo formula (2) via perturbation theory in S_i. In what follows, I will consider only the conductance averaged over the ensemble of disorder realizations. Assuming that $\overline{V(x)} = 0$, where the overbar stands for the ensemble averaging, the first nonvanishing correction to the ensemble-averaged propagator is given by

$$\overline{\delta G(X,X')} = \frac{1}{a^2} \int dX_1 dX_2 \overline{V(x_1)V(x_2)} \cos(2k_F(x_1 - x_2))$$

$$\times \{ \langle \langle \phi(X)\phi(X')Q(X_1,X_2) \rangle \rangle_0$$

$$- \langle \phi(X)\phi(X') \rangle_0 \langle Q(X_1,X_2) \rangle_0 \}, \quad (6)$$

where $X = \{x,\tau\}$, $\langle \cdots \rangle_0$ stands for averaging over Gaussian fluctuations of ϕ with the weight S_0, and

$$Q(X_1,X_2) \equiv e^{i2\sqrt{\pi}[\phi(X_1) - \phi(X_2)]}. \quad (7)$$

At this stage, for the sake of simplicity I choose $V(x)$ in the form of white noise: $\overline{V(x_1)V(x_2)} = n_i u^2 \delta(x_1 - x_2)$, where n_i is the concentration of "impurities" and u is the "impurity strength." The effective elastic mean free path ℓ (in the absence of the interactions) can then be defined as $1/\ell = n_i u^2/a^2 \omega_F^2$, where ω_F is the (nonuniversal) ultraviolet energy cutoff (of the order of the Fermi energy). Although in the real GaAs system the impurity potential is long ranged, the expectation is that this simplification cannot significantly affect the final results, provided that ℓ is replaced by the correct mean free path for a more realistic disorder potential.[17] Thus I find the correction to the nonlocal conductivity:

$$-\overline{\delta\sigma_\omega(x,x')} = \frac{2ie^2 \bar{\omega}^2 \omega_F^2}{\pi\ell\omega} \int_{-L/2}^{L/2} d\bar{x} G_{\tilde{\omega}}^0(x,\bar{x}) G_{\tilde{\omega}}^0(\bar{x},x')$$

$$\times [F_0(\bar{x}) - F_{\tilde{\omega}}(\bar{x})] \big|_{\tilde{\omega} \to i\omega - \epsilon}, \quad (8)$$

where $G_{\tilde{\omega}}^0$ is the propagator in the absence of disorder and $F_{\tilde{\omega}}(x)$ is the τ-Fourier transform of the (inhomogeneous) $2k_F$ density-density correlation function

$$F(x,\tau) \equiv \langle Q(x\tau,x0) \rangle_0$$

$$= \exp\left[-\frac{4\pi}{\beta} \sum_{\tilde{\omega}} (1 - e^{-i\tilde{\omega}\tau}) G_{\tilde{\omega}}^0(x,x) \right]. \quad (9)$$

The propagator $G_{\tilde{\omega}}^0(x,x')$ satisfies the equation

$$\left\{ \frac{\bar{\omega}^2}{v(x)K(x)} - \partial_x \left(\frac{v(x)}{K(x)} \partial_x \right) \right\} G_{\tilde{\omega}}^0(x,x') = \delta(x-x'), \quad (10)$$

and the following boundary conditions: (i) that $G_{\tilde{\omega}}^0(x,x')$ be continuous at $x = \pm L/2$ and $x = x'$; (ii) that $[v(x)/K(x)]\partial_x G_{\tilde{\omega}}^0(x,x')$ be continuous at $x = \pm L/2$, but (iii) discontinous at $x = x'$ so that

$$-\frac{v(x)}{K(x)} \partial_x G_{\tilde{\omega}}^0(x,x') \big|_{x=x'-0}^{x=x'+0} = 1. \quad (11)$$

In addition, I assume that infinitesimal dissipation is present in the leads, so that $G_{\tilde{\omega}}^0(\pm\infty,x') = 0$. To evaluate the function F in Eq. (9), one needs to know $G_{\tilde{\omega}}^0(x,x')$ only for $-L/2 \leq x = x' \leq L/2$. Straightforward, albeit lengthy, algebra leads to the result

$$G_{\tilde{\omega}}^0 = \frac{K_W}{2|\bar{\omega}|} + \frac{K_W}{|\bar{\omega}|} \frac{\kappa_-^2 e^{-L/L_{\tilde{\omega}}} + \kappa_+ \kappa_- \cosh(2x/L_{\tilde{\omega}})}{e^{L/L_{\tilde{\omega}}} \kappa_+^2 - e^{-L/L_{\tilde{\omega}}} \kappa_-^2}, \quad (12)$$

where $L_{\bar{\omega}} \equiv v_W/|\bar{\omega}|$, v_W is the density-wave velocity in the wire, and $\kappa_{\pm} \equiv 1/K_W \pm 1/K_L$. I now consider separately the cases of "high" ($v_W/L \ll T \ll \omega_F$) and "low" ($T \ll v_W/L$) temperatures. (The quotations marks are intended to imply to that the "low" temperature case can be alternatively viewed as the "long" length case and vice versa.)

At "high" temperatures, the second term in Eq. (12) is exponentially small ($\propto \exp[-(L-2x)/L_{\bar{\omega}}]$ unless $\bar{\omega}=0$). The term $\bar{\omega}=0$ gives zero contribution to the sum in Eq. (9), however, as can be seen by performing the infrared regularization of the propagator ($|\bar{\omega}| \to \sqrt{|\bar{\omega}|^2+m^2}$) and then letting $m \to 0$. Thus, only the first term in Eq. (12) has to be taken into account. This term is precisely the same as in the case of a homogeneous Luttinger liquid with parameter K_W. Already at this stage it can be anticipated that the T scaling of the conductance is determined by K_W. The function F is now x independent and is given by

$$F=\left[\frac{2\pi/\omega_F\beta}{\sin\pi\tau/\beta}\right]^{2K_W}. \tag{13}$$

After the analytic continuation $\bar{\omega} \to i\omega - \epsilon$, the limit $\omega \to 0$ can be taken. In this limit the remaining two propagators in Eq. (8) are x and x' independent: $\lim_{\omega\to 0}G_{\omega}^0(x,x') = iK_L/2\omega$.[10] The rest of the calculations proceed exactly as in the homogeneous case. The resulting nonlocal dc conductivity is also x and x' independent from which, with the help of Eq. (1), one finds that the dc conductance is $g=\sigma_0$.[20,10] Recalling the result for the conductance of a perfect wire, $g_0=K_L e^2/h$,[9,10] and restoring $\hbar$, the final result for the conductance can be written as

$$\bar{g}=g_0+\overline{\delta g}=K_L\frac{e^2}{h}\left\{1-CK_L\frac{L}{\ell}\left[\frac{2\pi T}{\hbar\omega_F}\right]^{-2(1-K_W)}\right\}, \tag{14}$$

where

$$C=8\sqrt{\pi}\sin(\pi K_W)\frac{\Gamma(1-K_W)}{\Gamma\left(\frac{1}{2}+K_W\right)}[\Gamma(K_W)]^2. \tag{15}$$

Note that K_L enters only in the prefactors, whereas the exponent of the T scaling is determined by K_W. Tracing back through the calculations, we can now see the reason for this. The K_L dependence comes from the propagators in the prefactor of Eq. (8), which depend on the frequency of the applied field ω. In the limit $\omega \to 0$, they become long ranged and contain only K_L but not K_W. The T scaling comes from the propagator entering the function F [Eq. (9)]. This propagator does not depend on ω. In the "high" temperature limit, it becomes short ranged, and contains only K_W but not K_L.

I now turn to the case of "low" temperatures. In previous work dealing with the homogeneous model,[2,4,5] this case was treated by employing the Lorentzian invariance of the $(1+1)D$ field theory, which ultimately reduces to a simple recipe: to obtain the result at "low" temperatures, one can take the result for "high" temperatures and replace T by v/L, and vice versa. This recipe works, however, only if L has a meaning of the total system size. In our inhomogeneous case, L is the length of the wire, which is only the part of the system, the total system including both the wire and

the leads. Thus, the $T \Leftrightarrow v/L$ equivalence cannot a priori be expected to work. In fact, it will be shown below that such an equivalence exists only if electrons in the leads do not interact, i.e., $K_L=1$.

At "low" temperatures, the sum over $\bar{\omega}$ in Eq. (9) can be replaced by the integral. Performing the integration, I find

$$F(x,\tau)=\frac{1}{[1+(\tau\omega_F)^2]^{K_W}}\exp\left\{-K_W z_0\left[\Phi\left(\frac{\tau v_W}{2L},0,z_0\right)\right.\right.$$
$$\left.\left.+z_0\Phi\left(\frac{\tau v_W}{2L},\frac{L+2x}{2L},z_0\right)\right]\right\}, \tag{16}$$

where $z_0 \equiv \kappa_-/\kappa_+$, and

$$\Phi(x,y,z) \equiv \sum_{n=0}^{\infty}z^{2n}\ln\frac{x^2+(n+1+y)^2}{(n+1+y)^2}. \tag{17}$$

Evaluation of the Fourier integral $\int d\tau(1-e^{i\bar{\omega}\tau})F(x,\tau)$ for arbitrary $\bar{\omega}$ is impossible in this case. As $\bar{\omega}$ has now become a continuous variable ($\beta \to \infty$), however, it is possible to find the asymptotics of the integrand of Eq. (8) for small $\bar{\omega}$ and then analytically continue to $i\omega$. Analysis shows that in the limit $\bar{\omega} \to 0$ the main contribution to the Fourier integral comes from the interval $\tau \gtrsim L/v_W$, where the function Φ [Eq. (17)] can be replaced by its large-x asymptotic form: $\Phi(x,y,z) \approx \ln x^2/(1-z^2)$. It can also be shown that the imaginary part of $F_0-F_{\bar{\omega}}$ is asymptotically larger than its real part in the $\bar{\omega} \to 0$ limit. After simple manipulations one then obtains $F_0-F_{\bar{\omega}}=[(L/L_{\bar{\omega}})^\alpha/i\bar{\omega}]I(\lambda)$, where $\lambda \equiv \omega_F/\bar{\omega}$,

$$I(\lambda) \equiv \int_0^{\infty}dz\frac{z^{-\alpha}\sin z}{[1+\lambda^2 z^2]^{K_W}}, \tag{18}$$

and $\alpha \equiv 2K_L(K_L-K_W)$. As we see, the exponent α, which is going to determine the L scaling of $\overline{\delta g}$, depends both on K_L and K_W and is equal to the exponent of the T scaling [i.e., to $2(1-K_W)$] only if $K_L=1$. Thus, the equivalence of T and L scalings exists only if the electrons in the leads do not interact. To be consistent with the physical content of the model, I will now consider only the case $K_L=1$. We now need to evaluate the asymptotic behavior of the integral (18) for $\lambda \to \infty$. This can be done by using the Parseval formula for Mellin transforms,[21] according to which one has

$$I(\lambda)=\frac{1}{\Gamma(K_W)}\int_{c-i\infty}^{c+i\infty}\frac{ds}{2\pi i}\lambda^{-s}\Gamma(s/2)\Gamma(\bar{s})\sin(\pi\bar{s}/2), \tag{19}$$

where $\bar{s} \equiv 2K_W-1-s$, and c is chosen in the domain of analyticity of the integrand ($0 \leqslant \text{Re}\,s \leqslant 2K_W$). For $\lambda \to \infty$, the leading contribution to the sum over residues is given by the smallest second-order pole at $s=2K_W$. I then find

$$I(\lambda)|_{\lambda\to\infty} \approx \frac{\ln\lambda}{\lambda^{2K_W}}. \tag{20}$$

In the limit being considered, $F_0-F_{\bar{\omega}}$ is x independent, and the integration over $\bar{x}$ in Eq. (8) again gives L. The continuation $\bar{\omega} \to i\omega - \epsilon$ can now be performed. Separating the real and imaginary parts of $\ln\lambda$ in F_0-F_{ω}, I finally obtain the real part of the conductance in the form

$$\overline{\text{Re}g} = \frac{e^2}{h} - \frac{e^2}{h}2\pi^2 \frac{L}{\ell}\left[\frac{2L\hbar\omega_F}{v_W}\right]^{2(1-K_W)} . \tag{21}$$

Similar analysis can be performed in the case of a single potential barrier inserted into the Luttinger-liquid wire which, in the case of a uniform interaction strength, was considered first in Refs. 3 and 4. The result is identical to the case of the extended random potential considered above: in both the weak and strong disorder cases the T and L dependences of the conductance are determined by the parameter K_W of the wire.

The generalization for the case of electrons with spin can readily be performed. As in the homogeneous case,[4,5] the exponent $2 - 2K_W$ is replaced by $2 - K_W^\rho - K_W^\sigma$, or, if the

SU(2) symmetry of the underlying Hubbard model is preserved (i.e., $K_W^\sigma = 1$) by $1 - K_W^\rho$, where $K_W^{\rho/\sigma}$ are the parameters of the charge/spin parts of the Luttinger liquid in the wire.

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