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# Path-dependent conductivity in the regime of floating delocalized states

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We present measurements of the longitudinal conductivity minima in the density–magnetic-field plane of a low-density, strongly disordered two-dimensional electron gas in a gated $GaAs/Al_xGa_{1-x}As$ heterostructure. We find the position in the $n$-$B$ phase diagram of the minimum in $\sigma_{xx}$ to be strongly *path dependent* in the region where the floating of delocalized states is observed. While the position of the minimum in the $n$ sweep is not universal, the position in the $B$ sweep is always found to correspond to an integer value of $nh/eB$. In an attempt to understand the origin of the floating, we offer a simple model, consistent with experiment, that directly links floating to Landau-level mixing.

Several recent experiments, on various GaAs systems,[1–6] have shown that an insulating phase at $B=0$ can undergo a phase transition to the quantum Hall liquid (QHL) phase in an applied magnetic field $B$. The transition was generally interpreted as being consistent with the global phase diagram[7] of the quantum Hall effect, and the theory[8] of the levitation (or floating) of the delocalized states as $B\rightarrow 0$. Direct support for the floating model was advanced in another recent experiment,[9] in which we demonstrated that it is indeed the floating of a delocalized state as $B\rightarrow 0$ that underlies the delocalization transition.

A complete understanding of this remarkable phenomenon clearly requires a reasonable microscopic picture. Toward this end, two recent theoretical papers[10,11] have suggested that Landau-level mixing is at the root of the floating of the delocalized states. In fact, it has already been shown numerically that the energy of the delocalized states can shift upward due to Landau-level mixing.[12,13] Experimentally, a similar determination, with respect to Landau-level mixing, necessitates the examination of the density of states (DOS). In order to obtain the relevant DOS information, we have conducted a systematic study of the conductivity minimum in the vicinity of the insulator–quantum Hall liquid transition. It is well known that, at low temperatures, hopping between localized states is the dominant conduction mechanism for a QHL state,[14] and the minimum in conductivity should directly reflect the minimum in the DOS.[15] In this paper, we present experimental evidence, through an anomalous behavior of the conductivity minimum, that Landau-level mixing is indeed important and directly associated with the floating of the delocalized states.

The measurements for this work were performed on three gated $GaAs/Al_xGa_{1-x}As$ heterostructures from a molecular-beam epitaxy wafer with a low zero-field mobility of $\mu\sim 3\times 10^4$ cm$^2$/V s at a density of $n=5\times 10^{11}$ cm$^{-2}$. Standard magnetotransport measurements were performed. The detailed description of the sample and method of the transport measurement are described in a previous publication.[9]

In a typical quantum Hall effect experiment, all the information on the transport coefficients can be obtained either by scanning the magnetic field while holding the density constant, or by sweeping the density (i.e., gate voltage) while holding the magnetic field constant. One normally expects the two approaches to yield equivalent results. While this expectation appears, for the most part, to have been taken for granted in the literature, our experiment clearly shows that conductivity is a path-dependent quantity.

Figure 1 shows a typical trace of $\sigma_{xx}$ and $\sigma_{xy}$ as functions of gate voltage, with the magnetic field held constant. The top axis of the figure indicates the filling factor, defined by $\nu=nh/eB$. The position of the $\sigma_{xx}$ minimum in the quantum Hall effect generally corresponds to some quantized (plateau) value of $\sigma_{xy}$, irrespective of the sweeping variable ($n$ or $B$). However, the crucial point of this paper is that the respective positions (on the $n$-$B$ phase diagram) of the $n$-swept and the $B$-swept minima in $\sigma_{xx}$ corresponding to the same value of $\sigma_{xy}$ do *not* coincide. In other words, unlike the peak in $\sigma_{xx}$, the minimum is a path-dependent quantity. It is apparent from Fig. 1 that there is a strong deviation of the first minimum from the $nh/eB=2$ line, a weaker deviation of the next minimum from the $nh/eB=4$ line, and almost no

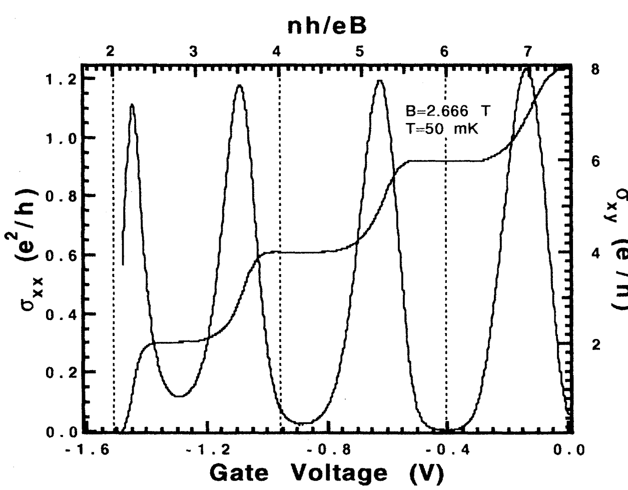


FIG. 1. Typical traces of $\sigma_{xx}$ as a function of the gate voltage. The peaks in $\sigma_{xx}$ are identified as places where the Fermi energy is coincident with delocalized state. The top scale of the figure indicates the filling factor $\nu\equiv nh/eB$. Note the strong deviation of the first minimum from $nh/eB=2$ and the weak deviation of the second minimum from the $nh/eB=4$ line, while the third minimum is coincidental with $nh/eB=6$ line.

 

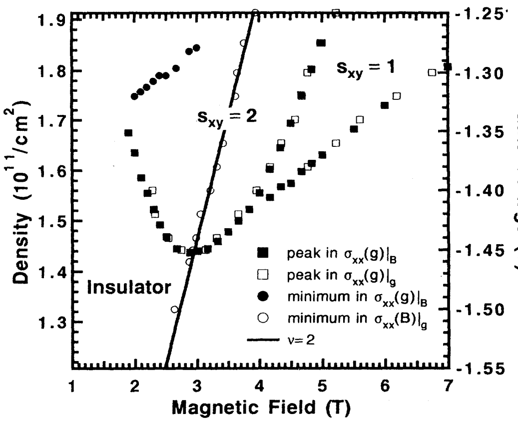


FIG. 2. The $n$-$B$ phase diagram of the insulator–quantum Hall liquid transition for the lowest (spin-unresolved) Landau level. The QHL states $\sigma_{xy}=1$ and $\sigma_{xy}=2$ as well as the insulating phase are shown. The solid circles represent the density-swept minima in $\sigma_{xx}$, and the open circles represent the magnetic-field-swept minima. The solid squares are the gate-voltage-swept peaks in $\sigma_{xx}$, and the open squares are from the magnetic-field-swept peaks. The values of the peak position do not depend on the quantity varied; the values of the minimum are, however, clearly *path* dependent. The solid line is given by $nh/eB=2$.

deviation of the minimum corresponding to the $nh/eB=6$ line. The figure is also highly suggestive of the possibility that this deviation is inherently linked to the floating of extended states, since the peaks in $\sigma_{xx}$ appear to undergo a similar deviation. Thus, a strong upward deviation of a minimum evidently "forces" the peak right above it to float.

Figure 2 shows an insulator-QHL phase diagram, similar to that presented in Ref. 9, for the lowest two Landau levels. This phase diagram is obtained by three evidently equivalent methods: by keeping track of peak positions in sweeping either the gate voltage (as in Fig. 1) or the magnetic field, or by locating the temperature-independent point in the resistivity. Constant-gate and constant-field sweeps are designated by $\sigma_{xx}(B)|_g$ and $\sigma_{xx}(g)|_B$, respectively. Although the peak position of $\sigma_{xx}$ (i.e., the phase boundary) does not depend on the method of sweeping, the minima in $\sigma_{xx}(g)|_B$ and $\sigma_{xx}(B)|_g$ are clearly seen to be nonequivalent. The field-swept minima fall precisely on a line which is given by the expression $n=2eB/h$, corresponding to a filling factor of 2. It is worth noting here that for densities below the lowest delocalized state (i.e., in the insulating phase), there is no longer a minimum in $\sigma_{xx}(B)|_g$ or a plateau in $\rho_{xy}(B)|_g$. Below this point, however, a minimum in $\rho_{xx}(B)|_g$ remains, but the value of $\rho_{xx}(B)|_g$ at the minimum increases with decreasing temperature, as expected of an insulator. The minima for $\rho_{xx}(B)|_g$ in the insulating phase are indicated by open circles.

The robustness of the path-dependence effect is illustrated more directly in Fig. 3. For this graph, the gate voltage was first set to $V_g=-1.80$ V as the magnetic field was varied. A minimum for $\sigma_{xx}(B)|_g$ was found at $B=2.97$ T, independent of the direction of the field sweep, i.e., with no appreciable hysteresis. The magnetic field was then held constant at $B=2.97$ T, and the gate voltage was varied, where a minimum for $\sigma_{xx}(g)|_B$ was found at $V_g=-1.66$ V, also with no

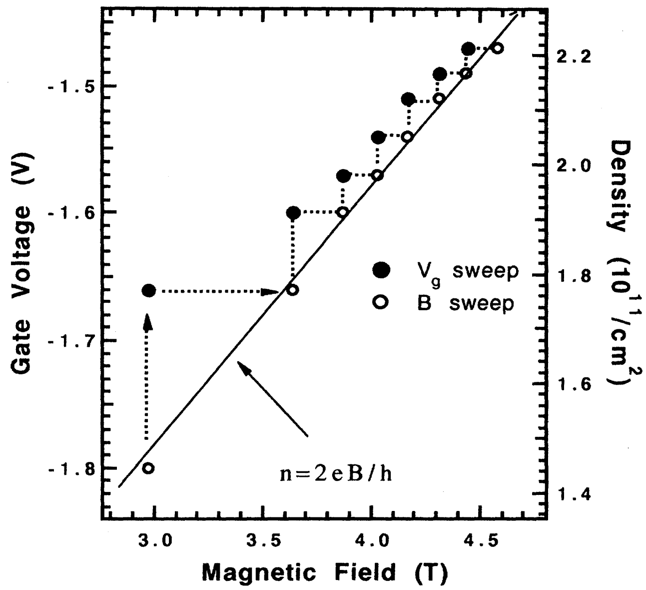


FIG. 3. Path dependence of the conductivity minimum. The position of the minimum in $\sigma_{xx}$ for the $\sigma_{xy}=2$ state. The solid circles were found by sweeping the gate voltage while maintaining a constant magnetic field, and the open circles were found by sweeping the magnetic field at a constant gate voltage. The difference between the two methods is seen to be largest at low magnetic fields and large negative gate voltages.

hysteresis in $V_g$. The curve progressed in a similar zigzag fashion, proceeding upward and to the right. At large magnetic fields or high gate voltages, the minima were too deep and flat, at this temperature, for an exact determination of their positions; hence, no data are plotted beyond $B=4.6$ T. Proceeding above $B=4.6$ T would require a higher temperature. For the range investigated, it is evident that, as the magnetic field is increased, the two methods asymptotically yield similar results. On the other hand, the difference between the two methods is greatest at low densities and low magnetic fields (see both Figs. 2 and 3). Not coincidentally, this region is precisely where the floating of delocalized states is most evident, as shown in Fig. 2.

The fact that our experiment shows a strong path dependence (Figs. 2 and 3), with the density-field relation only for constant-density traces, lends itself to a tantalizingly simple and self-consistent explanation. Let us assume that a minimum in $\sigma_{xx}$ is produced when the Fermi level crosses the minimum of the disorder-broadened density of states.[15] At fixed disorder, this point should be roughly equidistant in energy from the surrounding Landau-level peaks in the DOS. Since disorder is held approximately constant in sweeping the magnetic field,[16] a fixed proportionality between energy and density should be preserved and thus the correct density should be obtained at a given filling factor. On the other hand, since disorder is a strong function of density, a density sweep must inevitably "pass over" adjacent Landau levels with unequal or asymmetric broadening. One would expect the lower-density level to be broadened more, thus shifting the minima in both the DOS and $\sigma_{xx}$ to a higher energy. Due to a substantial Landau-level mixing, some redistribution of states also takes place, with the lower level (band) receiving

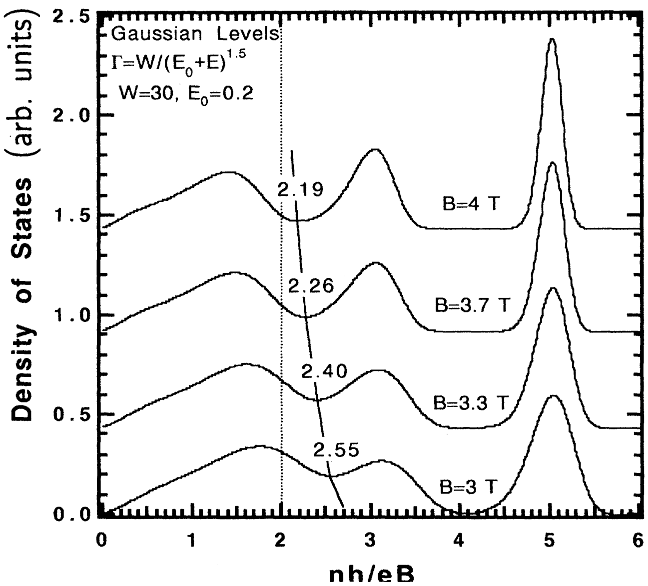


FIG. 4. A graphic illustration of the density of states of the two-dimensional electron gas with strong level mixing of the broadened Landau levels. The resultant DOS is plotted against the filling factor $nh/eB$ for the first three Landau levels for several magnetic fields for a given $\Gamma$. As the value of $B$ decreases, the first minimum shifts to a higher filling factor.

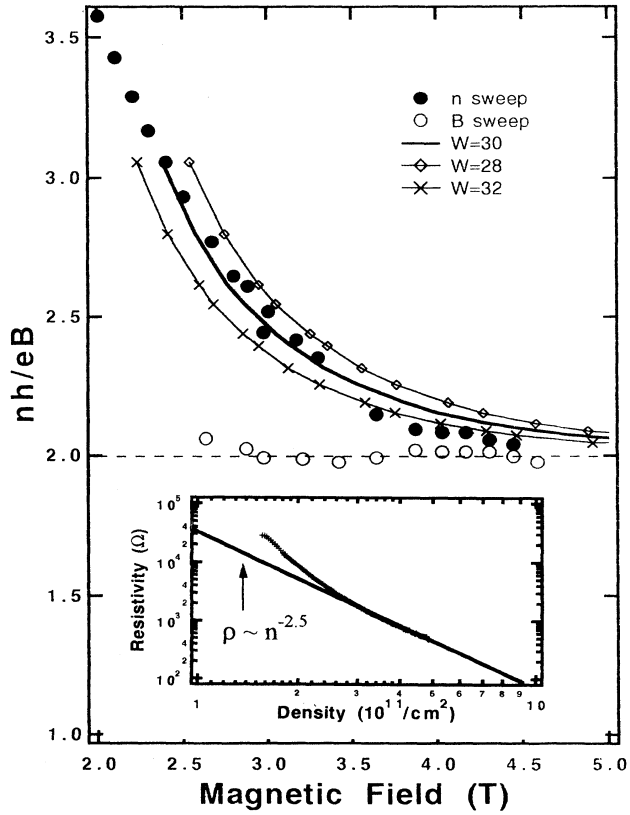


FIG. 5. The filling factor vs $B$. The solid circles are data of the density-sweep minima. The dashed line is a fit of the experimental data by using the Landau-level mixing model. The inset shows the value of the relaxation time determined at $B=0$ and at a high temperature of 4 K.

the larger share. This last effect is what is actually seen in the density–magnetic-field diagram. This argument is also consistent with a recent experiment,[17] which demonstrated that a field-swept minimum in $\rho_{xx}$ corresponded to the correct filling factor, even in the insulating phase, where no quantization in the Hall component was found. This minimum, however, was seen to diverge with decreasing temperature, as expected for the insulating phase. Thus, residual structure in the longitudinal component (i.e., $\rho_{xx}$ or $\sigma_{xx}$) appears to reflect the density of states, even in the absence of delocalized states below the Fermi level.

Figure 4 provides a simple, yet quite revealing, computer-generated demonstration of our model. The uppermost curve represents the numerical sum of several overlapping Landau-type levels, of which only three are shown. The levels are assumed to have the Gaussian form $g_i(E)=C_i\exp\{-(E-E_i)^2/2\Gamma^2\}/\Gamma$, where $E_i=(i+\frac{1}{2})\hbar\omega_c$, $C_i$ is a normalization, and $\Gamma$ is the level width. The normalization ensures that all levels have the same number of states before "mixing" while the width is taken to be $\Gamma=W/(E_0+E)^{3/2}$ (discussed below). The normalization is obtained simply by dividing each individual level by its area (i.e., its integral). The figure clearly shows that, consistent with our experiment, it is indeed the lowest Landau level that is most affected by level mixing and asymmetric broadening. Both the lowest peak and minimum are dramatically shifted up in energy (density) relative to their ideal positions. In terms of the filling factor, the ideal positions of the lowest peak and minimum should be 1 and 2, respectively.

The particular form of the DOS for the width was motivated by the experimental fact that there is no deviation in the $B$ sweep. It is also consistent with the experimental finding by Ashoori and Silsbee in a capacitance measurement that the linewidth is independent of magnetic-field strength.[16] Using the Drude and Heisenberg expressions, $\rho=(m/ne^2\tau)$ and $\Gamma=\hbar/2\tau$, respectively, we get $\Gamma\sim\rho/n$. Now using the experimentally derived dependence $\rho\sim n^{-2.5}$ at a high temperature of 4 K (see inset of Fig. 5), we obtain $\Gamma\sim 1/n^{1.5}$. In the inset, the deviation of the experimental curve from the power-law dependence is due to the strong temperature dependence of the resistivity in the insulating phase (i.e., $n<2.5\times10^{11}$ cm$^2$). A small cutoff parameter $E_0$ is introduced in an *ad hoc* fashion to avoid numerical singularities around $E=0$. The specific value for $E_0$ has no particular significance, as long as it is kept small. The case of sweeping the magnetic field is trivially recovered within the model by simply setting the width $\Gamma$ equal to a constant, in which case no shift is observed.

It should be pointed out that the only adjustable parameter in the model is $W$. In this case, a value of 30 meV$^{5/2}$ was determined by best fitting the experimental data. With a fixed $W$, a series of traces can be generated, as in Fig. 4, which clearly shows how the density of states develops an increasing "asymmetry" in a decreasing magnetic field. Focusing now specifically on the lowest minimum, we generate, after many traces, Fig. 5. This figure exhibits reasonably good agreement of the model-generated data with the experimental data. Another remarkable feature of the figure is the fact that the experimental curve was generated from three different samples (cut from the same wafer). In other words, the position of the minimum seems to exhibit a behavior which evidently depends only on filling factor and magnetic field for a given prescribed sample design parameter.

The fitting parameter $W$ gives a scattering time $\tau = \hbar/2\Gamma$ of about $8\times10^{-13}$ s at a density of $5\times10^{11}$ cm$^{-2}$. This can be compared to the two commonly quoted scattering times, the transport scattering time $\tau_t$, which is found from $\sigma = ne^2\tau_t/m^*$, and the quantum scattering time $\tau_q$, which is associated with Landau-level broadening. At a density of $n=5\times10^{11}$ cm$^{-2}$ the scattering time is $\tau_t = 9\times10^{-13}$ s (determined at $T=10$ K where the zero-field resistivity is only very weakly temperature dependent). On the other hand, a Dingle-type plot[18] of the data at $n=5\times10^{11}$ cm$^{-2}$ gives $\tau_q \approx 4\times10^{-13}$ s. A similar plot at lower density is not possible due to the absence of well-resolved Shubnikov–de Haas oscillations. However, it has been shown[19] that $\tau_q$ depends only weakly on the density, and thus the zero gate value is expected to be a reasonable estimate for those at lower densities. On the basis of these estimates alone, we can only say that the model-based scattering time is not inconsistent with either of the two candidates.

It should also be noted that line shapes other than Gaussian have been suggested in the literature. Ashoori and Silsbee, for instance, suggest that Lorentzian line shapes are more representative than Gaussian ones. In fact, qualitatively similar results were obtained, in our analysis, when Lorentzian levels [of the form $E_i = C_i\Gamma/\{(E-E_i)^2+\Gamma^2\}$] were used instead of Gaussian ones. However, we found that these levels are not as well behaved quantitatively as the Gaussians in the sense that they produce a very slow (and incomplete) convergence towards $nh/eB=2$ at high fields. We believe that in order to make a sensible quantitative comparison with the experimental data, a more realistic DOS should be used. Unfortunately, such a microscopic DOS theory has thus far only been developed in the weak localization limit,[20] for which the level broadening is much smaller than the Landau level spacing.

In conclusion, we have attempted to measure the effect of Landau-level mixing by using a path-dependent $\sigma_{xx}$ minimum as the probe. Using a model for the density of states, we have shown that the floating of the delocalized states can be qualitatively and semiquantitatively linked to Landau-level mixing.

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