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Composite-fermion effective masses

P. T. Coleridge, Z. W. Wasilewski, P. Zawadzki, A. S. Sachrajda, and H. A. Carmona*
Institute for Microstructural Sciences, National Research Council of Canada, Ottawa, Ontario, Canada K1A OR6

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Fractional quantum-Hall-effect features around filling factor $\nu=\frac{1}{2}$ have been analyzed using the composite-fermion approach. Effective masses deduced from the temperature dependence of the Shubnikov–de Haas (SdH) oscillations, in agreement with other measurements, show a divergence as the filling factor approaches $\nu=\frac{1}{2}$ and scale as $(\text{density})^{1/2}$. The magnetic-field dependence of the amplitude is explained quantitatively in terms of normal impurity scattering and a strong dephasing term associated with density inhomogeneities of order 0.5%. It is pointed out that assumptions made in the derivation of the standard theory used to analyze SdH oscillations are less likely to be satisfied for composite fermions and that some caution should therefore be used in interpreting effective-mass results obtained in this way.

Recently several groups[1–4] have used the composite-fermion (CF) approach[5,6] to analyze fractional quantum-Hall-effect (FQHE) data. In contrast to the hierarchical model this provides a natural explanation for the strength as well as the position of the FQHE features in terms of Landau-level quantization around even denominator filling factors, in particular around $\nu=\frac{1}{2}$. The FQHE energy gaps are then given by the Landau-level spacing $\hbar\omega_c^{\mathrm{CF}}=e\hbar B^*/m^{\mathrm{CF}}$ where $B^*=B-B^{1/2}$ is the deviation of the field from the $\nu=\frac{1}{2}$ value and m^{CF} is the composite-fermion effective mass. For electrons the measured values of m^{CF} are significantly larger than the conventional value ($0.067m_0$) for GaAs and, as predicted,[3,6] scale approximately as $B^{1/2}$. According to gauge arguments[6] the effective mass should diverge logarithmically for $B^*\to 0$: for two-dimensional electron gases (2DEG's) (Ref. 1 and 4) and 2D holes[3] strong divergences have been observed, but in another set of 2DEG samples[2] with similar mobilities, no divergence was visible. Because this phenomena appears sensitive to the disorder in the samples we have investigated it in 2DEG material of different provenance but comparable mobility and present the results here.

The normal Shubnikov–de Haas (SdH) oscillations occur because the conductivity (σ_{xx}) depends on the density of states at the Fermi energy $g(E_F)$, both through the number of carriers and through the scattering rate. In 2D systems, in contrast to the 3D case,[7] σ_{xx} is just proportional to $[g(E_F)]^2$ and the amplitude of the oscillations is then given, for the fundamental component, by[8–10]

$$\Delta\rho_{xx}=4\rho_0[X_T/\sinh(X_T)]\exp(-\pi/\omega_c\tau_q), \quad (1)$$

where ρ_0 is the zero-field resistivity, the term involving X_T describes the thermal damping,[11] and τ_q is the quantum lifetime characterizing the disorder. The disorder can also be expressed in terms of a Dingle temperature $T_D=\hbar/(2\pi k_B\tau_q)$ or a quantum mobility $\mu_q=e\tau_q/m^*$. In the thermal damping term $X_T=2\pi^2k_BT/\hbar\omega_c$ is proportional to m^* and experimental values of m^* are obtained by fitting the temperature dependence of the amplitude. It should be noted, however, that two assumptions made in deriving Eq. (1) are not necessarily valid for composite fermions. First, the thermal damping term is derived assuming all Landau levels (or at least those within a few k_BT of the Fermi energy) have the same shape. The damping then becomes a function only of the ratio $k_BT/\hbar\omega_c$ with the Landau-level shape reflected in the strength of the various harmonics (k), each of which is damped by a factor $kX_T/\sinh(kX_T)$. Experimental results obtained in a series of high-mobility samples show deviations from this expression[12] that are attributed to extra structure in the density of states, at the Fermi level, with an energy scale of order $\hbar\omega_c$.

Second, the term describing the disorder assumes an explicit shape for the Landau levels, Lorentzian (or equivalently Gaussian with a width proportional to $B^{1/2}$). The decay term is essentially the Fourier transform of the Landau-level shape and for different shapes the amplitude will have a different dependence on magnetic field. One example is the potential fluctuations produced by density inhomogeneities. These are particularly important in composite-fermion systems because small variations in density (typically 1%) produce corresponding variations in $B^{1/2}$ but variations in B^* that are, proportionally, an order of magnitude larger. The effect of these can be treated using the technique of phase smearing[13] whereby the reduction in the amplitude of the oscillations is determined by an integration over the spread of phases caused by the density variations.

The two samples used were grown at the National Research Council (NRC).[14] Both had 1200-Å spacer layers, two delta-doped layers separated by 810 Å, a further 100 Å of $Al_xGa_{1-x}As$ and a 120-Å GaAs cap. Sample parameters, after illumination with a red light-emitting diode, are given in Table I. The differences between the samples are attributed to a slightly higher p-type background doping in sample B associated with a small amount of Be contamination. Quantum mobilities quoted in Table I come from low-

TABLE I. Sample parameters.

Sample	Density (10^{11} cm^{-2})	Mobility (10^6 cm^2/V s)	Quantum mobility (μ_q) (10^6 cm^2/V s)
A	1.27	3.5	0.25
B	1.39	2.5	0.23

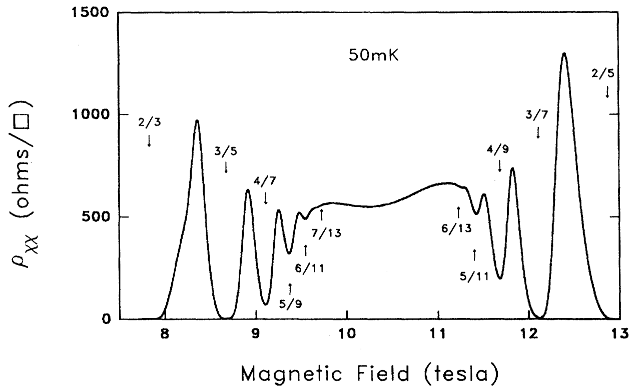

FIG. 1. Overview of composite-fermion fractions around $\nu=\frac{1}{2}$ in sample A at a temperature of 50 mK.

temperature Dingle plots ($T=50$ mK) which had the correct prefactor [$4\rho_0$ in Eq. (1)]. At higher temperatures, because the thermal damping term in Eq. (1) can be incorrect, Dingle plots had incorrect intercepts at $1/B=0$ and gave erroneous values of μ_q.

Figure 1 shows the FQHE features around $\nu=\frac{1}{2}$ in sample A; very similar results are seen in sample B. Composite-fermion effective-mass plots for sample A are shown in Fig. 2. Effective masses m^{CF} are obtained from the slope of $\ln(\Delta\rho_{xx}fB^*/T)$ plotted against T where the factor $f=1-\exp(-2X_T)$, which is usually close to 1, depends weakly on m^{CF} but can be evaluated reiteratively. In all cases a good fit to the data was obtained with only small deviations from the expected, linear, behavior. Results are shown in Fig. 3, with error bars that reflect not only statistical errors but also any systematic deviations. The dashed and solid lines show data from Refs. 1 and 4 scaled by the square root of the density[15] and the agreement between these sets of data, for densities varying by a factor of over 2, confirms the predicted[3,6] variation of m^{CF} as $B^{1/2}$. The results obtained here clearly confirm a divergence of m^{CF} for $B^*\to 0$.

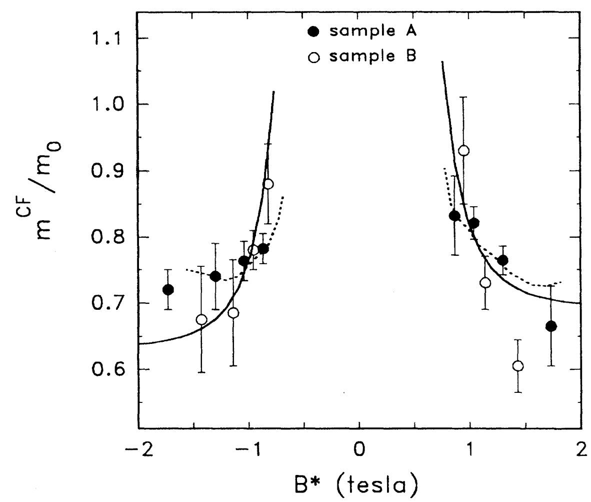

FIG. 3. Composite-fermion effective masses (in units of the free-electron mass m_0) measured in samples A (solid) and B (open). The curves are data from Ref. 1 (dotted) and Ref. 4 (solid) scaled by the square root of the density.

Figure 4 shows a composite-fermion Dingle plot for sample A. The logarithm of the amplitude of the oscillations at $T=0$, taken from the intercepts in Fig. 2, are plotted against $\alpha m^{\mathrm{CF}}/B^*$ where $\alpha=2\pi^2k_B/e\hbar$. If the composite-fermion model is valid and transport can be described by standard theory [Eq. (1)] the intercept at $1/B^*=0$ should be given by $4\rho_0^{\mathrm{CF}}$, i.e., four times the resistivity at $\nu=\frac{1}{2}$. A linear fit, constrained to pass through this point, and through the points with the largest values of B^*, gives a value of

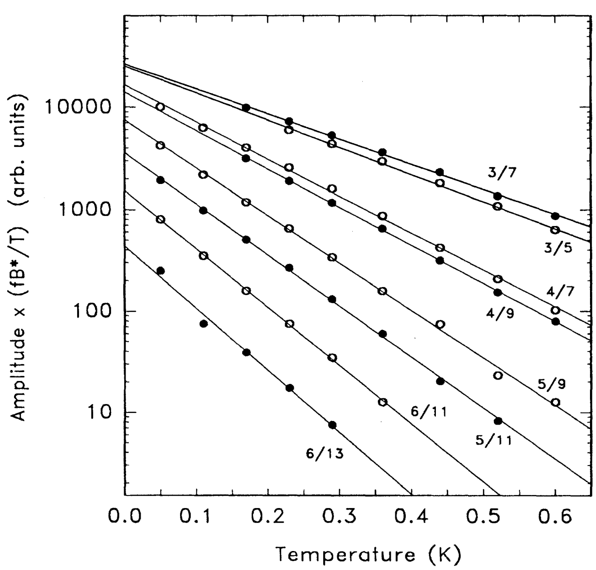

FIG. 2. Composite-fermion effective-mass plots for sample A; the effective mass is proportional to the slope of the lines. The factor f (≈ 1) is defined in the text.

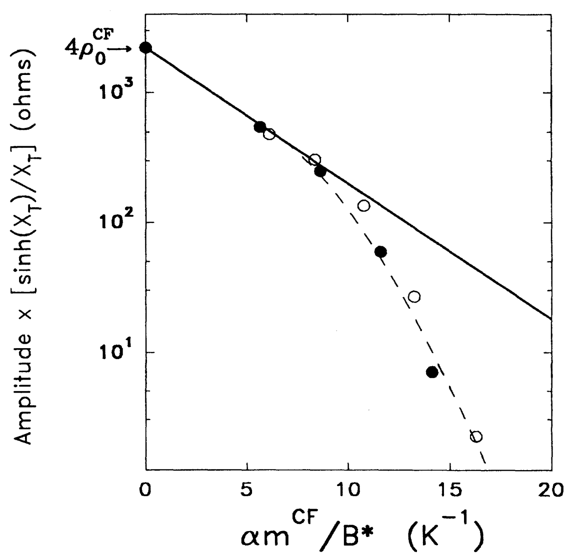

FIG. 4. Dingle plot for composite-fermion amplitudes in sample A. Open symbols denote $B^*<0$, solid symbols $B^*>0$. The solid line, through the point $4\rho_0^{\mathrm{CF}}$, corresponds to a Dingle temperature of 0.24 K, i.e., $\tau_q=5.1$ ps. The dashed line is a fit assuming a Gaussian density distribution with a RMS spread of 0.45%.

$\tau_q^{CF}=5.1$ ps, which corresponds to a Landau-level width (full width $2\Gamma^{CF}=\hbar/\tau_q^{CF}$) of 1.5 K. For these fractions activation energies were also measured, and when plotted against B^* show the linear dependence observed in other samples.[1,3,4] The negative intercept at $B^*=0$ is conventionally interpreted as the Landau-level width. In this case the value 3.1 K is a factor of 2 larger than the width obtained from the Dingle plot. This discrepancy is not unexpected: a well-defined activation plot, with exponential decay over one or more decades, measures the width of the gap in the density of states so the negative intercept in the plot of activation energies against B^* is a measure of the width at the base of the Landau levels, where the density of states goes to zero. By contrast the quantum lifetime extracted from a Dingle plot corresponds to the width at approximately half maximum and, as expected, this is found to be about half the width at the base of the Landau levels. This interpretation of these two sets of data differs somewhat from other results in the literature but the results from sample B are very similar, and we also note that if the data in Ref. 4 are analyzed in the same way, i.e., using a linear fit constrained to go through the point $4\rho_0$ and preferentially weighted towards the larger values of B^*, then the value of τ_q^{CF} in that sample is about 7 ps, corresponding to a width at half maximum of 1.1 K. This is also approximately half the width given by the negative intercept (2.1 K) of the activation energies for that sample.

For the higher fractions, i.e., smaller values of B^* in Fig. 4, there are deviations from the linear Dingle plot similar to those seen in other experiments.[3,4] If these are interpreted in terms of a field-dependent quantum lifetime the scattering rate ($1/\tau_q^{CF}$) diverges as $B^*\rightarrow 0$, but as noted above, the functional form used in Eq. (1) to describe the disorder in terms of a quantum lifetime may well not be correct and some other form of Landau-level shape should probably be considered. In particular, the phase smearing produced by density variations is likely to be very important for composite fermions and will certainly have a different functional form.

If the phase of the SdH oscillations is written as $2\pi F/B$, where the frequency $F=nh/e$ (with n the density), then following Shoenberg,[13] for a field variation ΔB the departure in phase from the standard value is given by $\phi=2\pi F\Delta B/B^2$. If the distribution of phases is given by $D(\phi/\lambda)$ the corresponding amplitude reduction is given by $f(\lambda)/f(0)$, where $f(\lambda)$ is the Fourier transform of D with respect to λ, and assuming a Gaussian spread in density the amplitude decays as $\exp(-\lambda^2/2)$. For composite fermions $\lambda=2\pi F\beta/B^{*2}$, where β represents the equivalent field variation caused by density variations. The dotted curve in Fig. 4 shows a fit to the data using a combination of conventional scattering and phase smearing with a root-mean-square (RMS) density variation of 0.45%. It should be noted that the rather fast falloff, as B^{*-4}, is characteristic of composite fermions. For normal but inhomogeneous systems, a slower falloff, as B^{-2}, is expected and observed[16] and, indeed, the low-field Dingle plot, which is linear within experimental error down to fields of less than 0.04 T, implies a RMS density variation of less than 0.3%. The difference between these two values may just reflect details of the density distribution and differences in screening. Alternatively, it may reflect some additional source of phase smearing present for composite fermions but not as low fields. The most obvious possibility is fluctuations in the "cancellation" magnetic field produced by the two flux quanta attached to each electron. Although the mean value of this field just cancels the external applied magnetic field at $\nu=\frac{1}{2}$, variations about this mean value are to be expected.

There are other possible explanations for the deviations from linearity in the Dingle plot, such as modification to the Landau-level density of states associated with the formation of gaps, but whatever their source the deviations show that the broadening of the composite-fermion Landau levels is not Lorentzian over this field range and that the standard expression for the scattering given in Eq. (1) is not correct for small B^*.

Similar considerations may also apply to effective-mass measurements, if only because the divergences occur over the same field range. Values of m^{CF} are obtained from the temperature dependence because raising the temperature causes the tails of the Fermi function to sample Landau levels away from Fermi energy. At high temperatures the energy scale is $\hbar\omega_c$ but at low temperatures this process will be dominated by contributions from the Landau levels closest to the Fermi level and will depend as much on the shape of the Landau level as on the spacing. If the shape of the Landau levels has no energy dependence then Eq. (1) is valid for the *fundamental*; if there is an energy dependence the apparent measured value of m^* will not be given by eB/ω_c and indeed, deviations attributed to this effect (although in the opposite sense to the divergence observed here) have been observed[12] near $B=0$. Even when Eq. (1) is valid there is the experimental problem of separating the fundamental from other harmonics, or at least ensuring they do not contaminate the measurement. This is conventionally done by harmonic analysis of the oscillations as a function of magnetic field but what is required is the harmonic content as a function of energy. We know the energy spacing of the Landau levels is a function of magnetic field (because there is a divergence in m^{CF}) so it is quite likely these two will not be the same. It can be seen therefore that there are several pitfalls involved in using Eq. (1) to determine composite-fermion effective masses and this suggests caution should be used when interpreting the results of such measurements, particularly for ν near $\frac{1}{2}$.

In summary, analysis of FQHE features in two-high-mobility 2DEG's using a composite-fermion formalism shows that the oscillation amplitudes exhibit the conventional temperature dependence. Measured effective masses are found to diverge for $\nu\rightarrow\frac{1}{2}$ and for different samples the values of m^{CF} scale as (density)$^{1/2}$ as expected. Measurements of the composite-fermion Landau-level broadening are generally in good quantitative agreement with other measurements, and while the field dependence of the amplitude of the composite-fermion oscillations agrees with standard SdH theory for large B^*, this is not so for fractions closer to $\nu=\frac{1}{2}$. The anomalous behavior is explained quantitatively by a RMS density variation of 0.45%. It is pointed out that several assumptions made in deriving the standard theoretical expression for the amplitude of SdH oscillations may very well not be valid for composite fermions and that caution should therefore be used when interpreting results of measurements made in composite-fermion systems using this formalism.

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*Permanent address: Department of Physics, University of Nottingham, Nottingham NG7 2RD, United Kingdom.

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[14] Sample A is NRC1655, sample B NRC1669.

[15] The curve for Ref. 1 (dotted) is derived from a smooth curve drawn through the data points, that for Ref. 4 (solid) uses the fits given in that reference. In both cases the values of m^{CF} have been scaled to a density of $1.33\times10^{11}/\mathrm{cm}^2$, which is the mean of the densities for samples A and B, and B^* is given by the field at which the fractions would occur for this density.

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