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Flux creep in inhomogeneous superconductors

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We suggest a theoretical model of the flux creep in inhomogeneous superconductors taking into consideration the interaction between magnetic vortices. The flux line's interaction results in changes of the pinning potential. The pinning potential begins to depend on this interaction, which results in an additional term in the average flux of vortices (or in the diffusion coefficient). This results in additional terms in the diffusion equation for the vortices. Using a microscopic model, we have found the nonlinear equation to describe the flux line's density variations in these conditions. Using the relevant empirical parametrization for the height of the potential barriers we have found the numerical solutions of the diffusion equation in a one-dimensional superconductor. The flux diffusion is shown to be enhanced when the intervortex interaction is significant.

INTRODUCTION

There are different approaches to describing a magnetic flux creep in superconductors. One well known model is the model of a one-particle flux line.[1] In this model the independent motion of every vortex caused by the effect of the transport current is considered, but the intervortex interactions are neglected. In the other limit a superconductor is supposed to be rather regular and the magnetic vortex structure is that of a regular lattice.[2] Moreover, the pinning potential is considered to be a perturbation.[3] In the latest papers superconductors are considered to have small inhomogeneities and the intervortex interactions are supposed to be stronger than the interactions of vortices with the pinning potential.

In space-inhomogeneous high-temperature superconductors (HTSC's) in magnetic fields $H \geq H_{c1}$ (where H_{c1} is the first critical field) a periodic vortex lattice does not occur since the pinning potential can be rather deep and nonperiodic (see also Ref. 4). The location of vortices in these conditions is caused by their interactions with the space-inhomogeneous pinning potential.

In the general case the pinning potential is not small and it cannot be regarded as a perturbation, but at the same time the intervortex interaction must not be neglected. The present communication deals with just such a situation. We suggest a microscopic model to describe flux creep motion in type-II superconductors. We will suppose the potential pinning to be strong enough so the magnetic vortex motion is connected with jumps of vortices from one potential well to another. However, in contrast with the one-particle flux creep model[1] we will take into consideration the fact that more than one vortex can fit in every potential well. The interaction between vortices changes their average energy and, hence, the probability to jump to the neighboring wells. We show that the interaction of vortices pinned in the potential wells can drastically enhance their diffusion. This results in changes of the flux line kinetics even at low flux density.

MICROSCOPIC MODEL

Let us consider magnetic field values $H > H_{c1}$. The kinetics of the vortices is determined by the interactions between the vortices and the interactions of vortices with both an electric current and a pinning potential. For the vortex concentration n we have the equation

$$\frac{\partial n}{\partial t} = -\mathrm{div} \langle \mathbf{q} \rangle \tag{1}$$

($B = n\phi_0$, where B is the magnetic induction, and ϕ_0 is the elementary quantum of magnetic flux), where $\langle \mathbf{q} \rangle$ is the average flux of vortices.

To find $\langle \mathbf{q} \rangle$ let us consider the following simple model (see Fig. 1). There is a pinning potential caused by the crystalline lattice inhomogeneities; the average depth of the potential wells is F_0. Suppose there is some distribution of concentration $n(x,y,t)$ on the Oxy plane. We will consider variations of both the vortex concentration and the pinning potential on much longer characteristic scales than the average jump scale of a pinned vortex, l.

Let suppose a vortex concentration gradient ∇n exists. Since mutual repulsion increases the average vortex energy (see also Ref. 1), the latter rises together with the vortex concentration in the pinning well. So the pinning potential decreases below that of the lattice, F_0, by the amount of the interaction energy

$$F(n) = F_0 - \langle \varepsilon_{\mathrm{int}}(n) \rangle, \tag{2}$$

where $\langle \varepsilon_{\mathrm{int}}(n) \rangle$ is the average interaction energy calculated for one vortex.

Suppose that the vortex concentration is not high, so the intervortex interaction is greatest inside every pin-

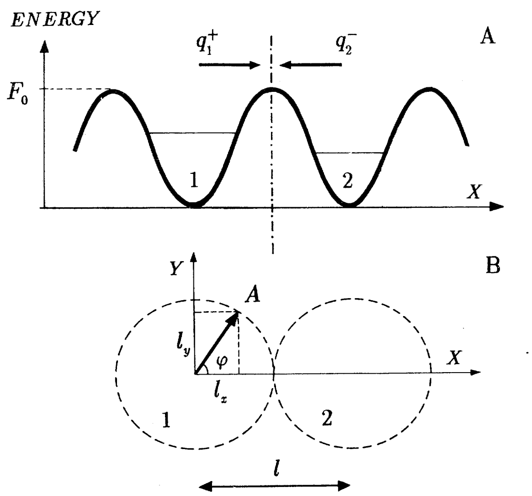

FIG. 1. Scheme of microscopic model illustrating calculation of the average flux of vortices $\langle \mathbf{q} \rangle$. (a) The pinning potential caused by the lattice inhomogeneities. The average energy level of vortex in every well $\langle \varepsilon(n) \rangle$ is shown. The average length of jumps l is the distance between the wells. (b) Regions of the potential wells 1 and 2 on the Oxy plane.

ning potential well. Let us consider two neighboring wells 1 and 2 (see Fig. 1). The frequency for a vortex to jump from the well in any direction on the Oxy plane is[1]

$$\nu = \nu_0 \exp(-F/T), \tag{3}$$

where ν_0 is the typical oscillation frequency for the pinned vortex, and F is the pinning energy barrier. The average height of the barrier F_0 formed by the lattice inhomogeneities in any direction is just the same. But an average flux $\langle \mathbf{q} \rangle \neq \mathbf{0}$ exists when a concentration gradient ∇n exists. Suppose that $n_1 > n_2$, where $n_{1(2)}$ is the concentration of vortices in the first (second) well. Then in accordance with Eq. (2) $F_1 < F_2$, where $F_{1(2)}$ is the height of the energy barrier for vortices from the first (second) well. Therefore $\nu_1 > \nu_2$, where $\nu_{1(2)}$ is the frequency of jumps determined in accordance with Eq. (3). The average energy levels for one vortex in the wells 1 and 2 are depicted schematically in Fig. 1. The energy level in well 1 is higher than in well 2, so a vortex can jump more easily from well 1 than from well 2. Therefore, jumps from well 1 into well 2 will occur more often than from well 2 into well 1, which accounts for the existence of some average flux motion.

For calculation of the average flux of vortices in some direction (Ox, for example) one has to consider two neighboring wells (1 and 2) and to calculate the average flux over the cross section between them. Let us consider the vortex in well 1 first and set the origin of coordinates Oxy at the well [see Fig. 1(b)]. Suppose the vortex is jumping into some point A situated on the circle with radius $\sim l/2$. Then a flux with the following components is connected with the jump:

$$q_x(\varphi) = n v_x^{\text{jump}} = n\nu l_x = \frac{1}{2} n\nu l \cos\varphi,$$

$$q_y(\varphi) = n v_y^{\text{jump}} = n\nu l_y = \frac{1}{2} n\nu l \sin\varphi, \tag{4}$$

where $v_x^{\text{jump}} = \nu l_x$ and $v_y^{\text{jump}} = \nu l_y$ are the x and y components of the jumping rate. The angle φ between the direction of the jump and the axis Ox can be absolutely arbitrary. Therefore, for calculation of the average vortex flux connected with jumps in the positive direction of the Ox axis, one has to integrate over the angle φ in the limits from $-\pi/2$ to $\pi/2$:

$$q_x^+ = \int_{-\pi/2}^{\pi/2} q_x(\varphi) d\varphi = n\nu l;$$

by analogy,

$$q_y^+ = \int_0^{\pi} q_y(\varphi) d\varphi = n\nu l. \tag{5}$$

Note that the flux caused by jumping out of well 1 in the direction opposite to the Ox axis on average is just the same, $q_x^- = q_x^+$, but it has the opposite direction. Therefore the average flux connected with jumping out of one well (1, for instance) is

$$\langle q_1 \rangle = q_{x1}^+ - q_{x1}^- = n_1\nu_1 l - n_1\nu_1 l = 0.$$

For calculation of the flux along the Ox axis one has to find the difference between the "positive" flux from the first well q_{x1}^+ [see Fig. 1(a)] and the "negative" flux from the second well q_{x2}^-:

$$\langle q_x \rangle = q_{x1}^+(n_1, F_1) - q_{x2}^-(n_2, F_2) = n_1\nu_1 l - n_2\nu_2 l. \tag{6}$$

So if the concentration gradient is absent and $n_1 = n_2$, $\nu_1 = \nu_2$ and $\langle q_x \rangle = 0$. But in the case shown in Fig. 1(a) $n_1\nu_1 > n_2\nu_2$ and $\langle q_x \rangle > 0$ is directed along the Ox axis.

We denote the concentration $n_1 = n$ and the energy barrier value $F_1 = F$ for the first well. We believe the characteristic scales of the concentration variations to be much larger than the average length of jumps, l. That is why one can represent both the concentration and the average energy barrier for the second well in the form of Taylor-series expansions:

$$n_2 \simeq n_1 + \left.\frac{\partial n}{\partial x}\right|_{x=x_1} \Delta x = n + l\frac{\partial n}{\partial x}, \tag{7}$$

$$F_2 \simeq F_1 + \left.\frac{\partial F}{\partial n}\frac{\partial n}{\partial x}\right|_{x=x_1} \Delta x = F + l\frac{\partial n}{\partial x}\left(\frac{\partial F}{\partial n}\right), \tag{8}$$

where F is the average value of the energy barrier.

Substituting Eq. (7) and Eq. (8) into Eq. (6) and calculating the average flux of vortices along the Ox axis in first-order terms over the concentration gradient (for $\partial n/\partial x$ here), we have

$$\langle q_x \rangle = l(n_1\nu_1 - n_2\nu_2)$$
$$= l\left\{n_1\nu_0 \exp\left(-\frac{F_1}{T}\right) - n_2\nu_0 \exp\left(-\frac{F_2}{T}\right)\right\}$$
$$\cong -l^2\nu_0 \exp\left(-\frac{F}{T}\right)\left\{1 - \frac{n}{T}\frac{\partial F}{\partial n}\right\}\frac{\partial n}{\partial x}. \quad (9)$$

All directions are equivalent on the Oxy plane; therefore, by analogy with Eq. (9),

$$\langle q_y \rangle \approx -l^2\nu_0 \exp\left(-\frac{F}{T}\right)\left\{1 - \frac{n}{T}\frac{\partial F}{\partial n}\right\}\frac{\partial n}{\partial y}. \quad (10)$$

Finally we get the following expression for the average flux of vortices in the linear approximation over ∇n:

$$\langle \mathbf{q} \rangle = \langle q_x \rangle \mathbf{e}_x + \langle q_y \rangle \mathbf{e}_y$$
$$\approx -l^2\nu_0 \exp\left(-\frac{F}{T}\right)\left\{1 - \frac{n}{T}\frac{\partial F}{\partial n}\right\}\nabla n. \quad (11)$$

Note that $\partial F/\partial n < 0$ as follows from Eq. (2) for the repulsive interaction of vortices.

We get for the vortex concentration from Eq. (1) and Eq. (11)

$$\frac{\partial n}{\partial t} = l^2\nu_0 \nabla\left\{\left(1 - \frac{n}{T}\frac{\partial F}{\partial n}\right)\exp(-F/T)\nabla n\right\}. \quad (12)$$

Note that we did not suppose the term $(n/T)(\partial F/\partial n)$ to be small compared with unity when deriving Eq. (12). So the additional terms that we have obtained in Eq. (12) are not an expansion of $\exp(-F/T)$ over the interaction energy value. The nonlinear Eq. (12) describes the magnetic flux creep kinetics.

The influence of the discussed mechanism increases with increasing $|\partial F/\partial n|$. The greatest influence should take place for superconductors when the inhomogeneity size $L_p \sim \lambda$, where λ is the vortex size. At this condition the vortices are localized in the pinning potential wells (when $L_p \ll \lambda$ the vortex cannot fit in the well). The spatial structure of HTSC's can be quite suitable for the proposed model (see, e.g., Ref. 5, where the scale of inhomogeneities $L_p \leq 1$ μm). Thus $L_p \sim \lambda_L$, where λ_L is the scale of the Abrikosov vortex, so intervortex interaction is significant.

The modern experimental research on HTSC's shows an important role of large-scale inhomogeneities for the transport properties of the superconductors.[6] The inhomogeneity scale $L_p \sim d \sim 1000$ Å was found to be significant for the transport properties. This is just the same order of scale as the scale λ_L. Note that the typical scales of inhomogeneities can be even larger in polycrystalline HTSC samples. Thus it was shown in Refs. 7 and 8 that the spatial structure of polycrystalline samples of Y-Ba-Cu-O type contains a wide spectrum of characteristic inhomogeneity scales $L_p \sim 1$–10 μm.

The changes of the kinetic equation that we have obtained lead to changes in the magnetic flux creep. To illustrate this fact we consider the magnetic field penetration in a one-dimensional superconductor. We use the following phenomenological formula for the $F = F(n)$ dependence (see, e.g., Ref. 9):

$$F = F(n) = \frac{G(T)}{n^\alpha} \propto \frac{(1 - T/T_c)^\gamma}{n^\alpha}, \quad (13)$$

where γ and α are some parameters ($\alpha \approx 1$–3). The singularity at $n = 0$ is connected with the choice of approximation (13) only. Really F is a finite value at $n = 0$.

Although the real dependence $F(n)$ can differ from Eq. (13) the changes in Eq. (12) that we have obtained are important even from the viewpoint of the simple model of the dependence $F = F(n) \propto 1/n^\alpha$. This parametrization (13) for the height of the potential barriers is empirical. We have used it because it describes some experimental data rather well. So one can believe that the parametrization is connected with reality in some way. It is important to note that the choice of the dependence (13) does not influence the main result of our work, the form of Eq. (12) for the flux creep where intervortex interaction is taken into account. The dependence is used only for illustration of the importance of consideration of the interaction of vortices in the localized states. To investigate the exact dependence $F = F(n, T)$ is indeed an interesting problem for both theory and experiment. The real dependence is obviously connected with the type of lattice inhomogeneities and it cannot be general in this sense.

ONE-DIMENSIONAL MODEL

Introducing the dimensionless variables $x/l \to x$, $\nu_0 t \to t$, and $n(T/G(T))^{1/\alpha} \to n$ we get

$$\frac{\partial n}{\partial t} = \frac{\partial}{\partial x}\left\{(1 + \alpha/n^\alpha)\exp(-1/n^\alpha)\frac{\partial n}{\partial x}\right\}. \quad (14)$$

The nonlinear diffusion coefficient in Eq. (14)

$$D(n) = (1 + \alpha/n^\alpha)\exp(-1/n^\alpha) \quad (15)$$

consists of two parts. $D_1 = \exp(-1/n^\alpha)$ corresponds to diffusion without interaction between vortices. The second part in the preexponential multiplier (15) $\alpha/n^\alpha \propto (n/T) \mid \partial F/\partial n \mid$ corresponds to the interaction between vortices. When $\alpha > 1$ $D(n)$ is a nonmonotonic function. The dependences $D_1(n)$ (curves 1) and $D(n)$ (curves 2) for $\alpha = 1$ (a) and $\alpha = 3$ (b) are plotted in Fig. 2. From formula (15) and Fig. 2 one can see that the interaction between vortices essentially enhances their diffusion in the entire interval of concentration where jumping creep is important [$0 < n \leq 1$, since there is no localization of vortices on pinning centers when $n \gg 1$ and $\exp(-F/T) \sim 1$]. At large vortex concentration $(n/T) \mid \partial F/\partial n \mid \to 0$ (at $\alpha > 0$) and $D(n) \approx D_1(n)$. This physically means a shallow depth of wells in high magnetic fields, which results in easy vortex jumps out of the wells, and, hence, negligible influence of intervortex interaction on diffusion.

The important influence of interaction between vortices on the magnetic flux creep can be understood under these conditions from the following qualitative considerations, taking into account formula (3) for the frequency of jumps $\nu(F)$. The interaction between vortices results in changes of the energy barrier F. Under strong localization of vortices ($F/T \gg 1$) the dependence $\nu(F)$ is an

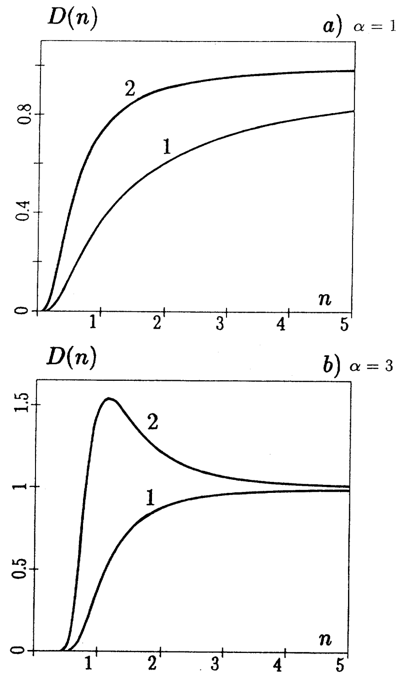

FIG. 2. Curves of the diffusion coefficient versus the concentration given for two values of α [see Eq. (14)]: (a) $\alpha = 1$; (b) $\alpha = 3$. Curve 1 is the diffusion coefficient $D_1 = \exp(-1/n^\alpha)$ without the interaction between vortices; curve 2 is $D(n)$ determined by the formula (15).

exponential one. Therefore it results in a sharp change of ν. In calculation of the average flux of vortices it results in an additional term in the kinetic equation, which is not small in the general case. Thus the interaction of vortices that was not taken into account before is important in fields $H \geq H_{c1}$ in the conditions discussed above.

We now consider numerical solutions of Eq. (14) to illustrate the diffusion enhancement caused by the vortex interactions. Let us consider Eq. (14) on the positive half-line $x > 0$:

$$n = n(x,t), \quad n(x,0) = n_1, \quad n(0,t) = n_2. \tag{16}$$

The stationary state for solutions $n(x,t)$ of Eq. (14) in these conditions is

$$n = n(x \geq 0, t \to +\infty) = n_2 = \text{const.} \tag{17}$$

The numerical solutions of the nonlinear equation (14) are displayed in Fig. 3. The displayed curves $n(x)$ correspond to various time values t. One can see from Fig. 3 that the vortex interaction enhances the magnetic vortex diffusion essentially. An additional illustration of this effect is Fig. 4 where the temporal changes of the magnetic flux are depicted (the case corresponds to the situation in Fig. 3). Figure 4 displays the temporal changes of a dimensionless magnetic flux $\delta\phi(t)$:

$$\delta\phi(t) = \int_0^\infty [n(x,t) - n_1]dx \tag{18}$$

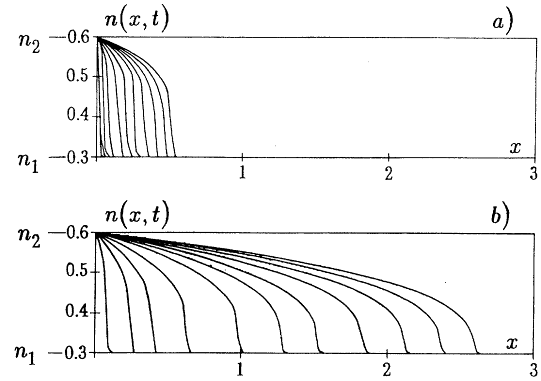

FIG. 3. Curves of the function $n(x,t)$ versus x displayed for different time values $t = 0.1, 0.5, 1, 2, 4, 6, 8, 11, 14, 17$, and 20. $n(x \geq 0, t = 0) = n_1$; $n(x = 0, t > 0) = n_2$; $\alpha = 2$. (a) Numerical solutions of the nonlinear equation (14) for the case without intervortex interaction (the term $\propto \partial F/\partial n$ in the preexponent is not taken into account). (b) Solutions of the nonlinear equation (14) given for the same time values t.

for the cases (a) without interaction of vortices and (b) where the interaction between vortices is taken into account.

We compare the solutions of the nonlinear equation (14) with the analytical solution we have obtained in the limit of small values of the gradient ∇n. Consider the case when $n(x,t) \approx n_1 \approx n_2$, and linearize Eq. (14). For small gradients ∇n from Eq. (14) we get the linear equation

$$\frac{\partial n}{\partial t} = D\frac{\partial^2 n}{\partial x^2}, \tag{19}$$

where the diffusion coefficient $D \approx D(n_1) = (1 + \alpha/n_1^\alpha)\exp(-1/n_1^\alpha)$. The solution of Eq. (19) in conditions (16) is

$$n(x,t) = n_2 + (n_1 - n_2)\ \text{erf}(x/\sqrt{4Dt}), \tag{20}$$

where

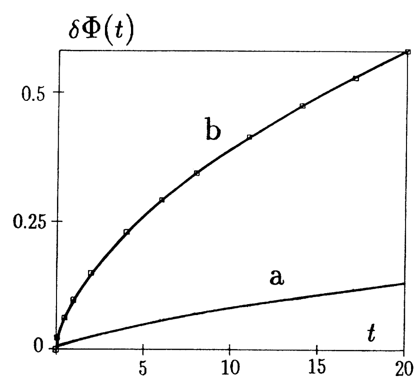

FIG. 4. Dependences of the dimensionless magnetic flux $\delta\phi(t)$ on time, determined by Eq. (18). Curve a is without intervortex interaction [corresponds to Fig. 3(a)]; curve b is with intervortex interaction [corresponds to Fig. 3(b)].

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z \exp(-y^2)dy \qquad (21)$$

is the error function. The solution (20) for the linear equation (19) is shown in Fig. 5. For both the cases (a) without interactions of vortices and (b) where the interactions are taken into account the picture is analogous to Fig. 3 where the solutions of the nonlinear equation are displayed (note that the curves in Fig. 5 are displayed for the same time values as in Fig. 3). The two figures (Fig. 3 and Fig. 5) differ from each other in the form of the curves. Moreover, the scale over the Ox axis in Fig. 5 is ten times smaller than in Fig. 3 where the solutions of the nonlinear equation are displayed. So the diffusion for the linear equation is much smaller. Therefore consideration of the nonlinear equation (12) can be important for the problem of magnetic flux diffusion in superconductors. The nonlinear diffusion results in changes of the spatial profile of the magnetic induction. It is well known that the linearized diffusion equation has often been used when considering the dynamics of flux in superconductors. But it should be pointed out that even estimation of the characteristic diffusion time can be made only rather approximately because of the sharp exponential dependence of the diffusion coefficient $D = D(n)$. Moreover, the spatial distribution of the concentration of vortices in the nonlinear case is quite different from the linear solutions. That is why one should take into account the nonlinearity of the diffusion equation for magnetic flux.

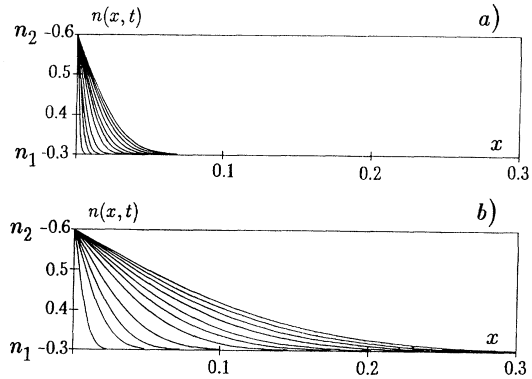

FIG. 5. Solutions (20) of the linear diffusion equation (19): (a) without interaction of vortices; (b) the interaction is taken into account. Parameters are just the same as for solutions of the nonlinear equation depicted in Fig. 3 ($\alpha = 2$). Curves are given for just the same time values $t = 0.1,\ 0.5,\ 1,\ 2,\ 4,\ 6,\ 8,\ 11,\ 14,\ 17$, and 20.

Finally, we have offered a generalization of the well known Kim-Anderson model[1,10] for inhomogeneous superconductors with large-scale inhomogeneities. We have shown that in these conditions one must take into consideration the intervortex repulsion in the potential wells of the pinning potential.

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