# Josephson-vortex Cherenkov radiation

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We predict the Josephson-vortex Cherenkov radiation of an electromagnetic wave. We treat a long one-dimensional Josephson junction. We consider the wavelength of the radiated electromagnetic wave to be much less than the Josephson penetration depth. We use for calculations the nonlocal Josephson electrodynamics. We find the expression for the radiated power and for the radiation friction force acting on a Josephson vortex and arising due to the Cherenkov radiation. We calculate the relation between the density of the bias current and the Josephson-vortex velocity.

## I. INTRODUCTION

The Josephson vortex is a well-known and an important example of a sine-Gordon soliton in solid state physics. This soliton is a propagating nonlinear wave describing the phase difference $\varphi(\mathbf{r},t)$ between two weakly coupled superconductors and the dynamics of a fluxon residing in this contact.[1] Results concerning the general features of the motion of a Josephson vortex are interesting for different systems in solid state physics where the sine-Gordon soliton exists.

Detailed knowledge of the Josephson-vortex dynamics is important for the flux dynamics and related phenomena in superconductors, e.g., flux creep, flux flow, magnetization relaxation, current-voltage characteristics, etc. Specific features of Josephson-vortex motion are currently under thorough experimental and theoretical study.[2–5] In particular, very fast moving Josephson vortices are observed and treated in annular Josephson tunnel junctions.[2,3]

Josephson-vortex dynamics is very important in the layered high-temperature superconductors due to their crystalline structure. In particular, the most prominent Bi and Tl based copper oxide compounds consist of a periodic stack of weakly coupled two-dimensional CuO layers where the superconductivity presumably resides. In this case a variety of linear crystalline structure defects result from the crossing of the superconducting layers with planar crystalline structure defects, e.g., the grain boundaries, twins, etc. These linear crystalline structure defects can be treated as Josephson junctions.

The critical current density for Josephson junctions in superconducting layers is relatively high, especially for coherent crystalline structure defects, e.g., for low-angle grain boundaries and twins.[6,7] The Josephson penetration length $\lambda_J$ is decreasing if the Josephson critical current density $j_c$ is increasing. If the value of $\lambda_J$ is of the order of or less than the London penetration depth $\lambda$ then the Josephson electrodynamics is nonlocal.[8,9]

Let us consider a superconducting plate with an infinitely long superconductor-insulator-superconductor-(SIS) type Josephson junction parallel to the $x$ axis as shown in Fig. 1. The dynamics of a Josephson vortex in this tunnel contact is described by the sine-Gordon equation for the space and time dependent phase difference $\varphi(x,t)$. Taking into account the damping resulting from the resistance of the junction it reads

$$\varphi_{\tau\tau} - \varphi_{\zeta\zeta} + \eta\varphi_\tau + \sin\varphi = \beta. \tag{1}$$

The subscripts $\tau$ and $\zeta$ are to denote the derivatives over the dimensionless time $\tau = t\omega_J$ and coordinate $\zeta = x/\lambda_J$,

$$\omega_J = \sqrt{\frac{2ej_c}{\hbar C}} \tag{2}$$

is the Josephson plasma frequency, $C$ is the specific capacitance of the junction, $j_c$ is the critical current density of the Josephson junction,

$$\lambda_J = \sqrt{\frac{c\Phi_0}{16\pi^2\lambda j_c}} \tag{3}$$

is the Josephson penetration length, $\Phi_0$ is the flux quantum, $\lambda$ is the London penetration depth,

$$\eta = \frac{1}{\omega_J RC} \tag{4}$$

is the damping constant, $R$ is the specific resistance of the junction, and $\beta = j/j_c$ is the dimensionless density of the bias current across the junction.

The well-known solution of Eq. (1)

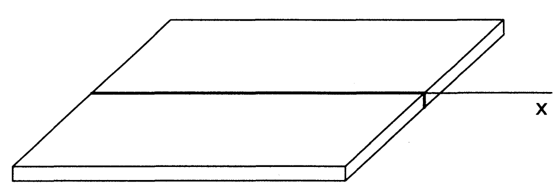


FIG. 1. A superconducting plate with a Josephson junction (thick line).

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$$\varphi_0(x,t) = 4\tan^{-1}\exp\Big[\frac{x-vt}{\lambda_J\sqrt{1-v^2/c_s^2}}\Big], \tag{5}$$

describes the uniform motion of a Josephson vortex with a certain velocity $v$ in the case of zero dissipation and zero driving force ($\gamma = \beta = 0$). It follows from Eq. (5) that in a long Josephson junction a Josephson vortex moves similar to a relativistic particle with the highest possible velocity $c_s = \lambda_J\omega_J$ (Swihart velocity).[1]

An electromagnetic wave with a specific dispersion relation exists in a long one-dimensional Josephson junction.[10] The solution of Eq. (1) in the form of a plane wave with a small amplitude $\varphi_a$,

$$\varphi(x,t) = \varphi_a \exp(-i\omega t + ikx), \qquad |\varphi_a| \ll 1, \tag{6}$$

describes this electromagnetic wave. Let us consider the case of zero dissipation ($\eta = 0$). Then, the relation between the frequency $\omega$ and the wave vector $k$ is given by the formula[1]

$$\omega = \omega_J\,\sqrt{1+\lambda_J^2k^2}. \tag{7}$$

We can determine the phase difference $\varphi(x,t)$ in the framework of the local Josephson electrodynamics, i.e., by the sine-Gordon equation, as long as $\lambda \ll l_\varphi$, where $l_\varphi$ is the characteristic space scale of $\varphi(x,t)$. In particular, Eqs. (5) and (7) are valid if $\lambda \ll \lambda_J$ and $k\lambda \ll 1$.

The phase velocity $v_\varphi$ of an electromagnetic wave in a long Josephson junction is equal to

$$v_\varphi = \frac{\omega}{k} = c_s\,\sqrt{1+\frac{1}{k^2\lambda_J^2}} \tag{8}$$

in the framework of the local Josephson electrodynamics. We show the dependence $v_\varphi(k)$ calculated by means of Eq. (8) by the solid line in Fig. 2. The function $v_\varphi(k)$ monotonically decreases with the increase of the wave vector. It follows from Eq. (8) that for $k\lambda \ll 1$ the value of $v_\varphi(k)$ is higher than $c_s$.

Cherenkov radiation can exist if the radiating particle or quasiparticle can move with the phase velocity of the radiated wave.[11] Since $c_s$ is the highest possible velocity of a Josephson vortex it means that there is no Josephson-vortex Cherenkov radiation of an electromagnetic wave in the framework of the local Josephson electrodynamics.

Josephson-vortex electromagnetic wave radiation has been considered for discrete sine-Gordon systems and Josephson-junction arrays.[12–16] It has been shown that under certain conditions the discreteness of the system results in electromagnetic wave radiation by a moving soliton.[12,13] Small amplitude wave excitation by a moving Josephson vortex has been found in a two-dimensional Josephson-junction array by both semiquantitative models and numerical simulations.[14–16]

In this paper we study analytically the Josephson-vortex Cherenkov radiation of an electromagnetic wave in a long one-dimensional Josephson junction. We consider the wavelength of the radiated electromagnetic wave to be much less than the Josephson penetration depth, i.e.,

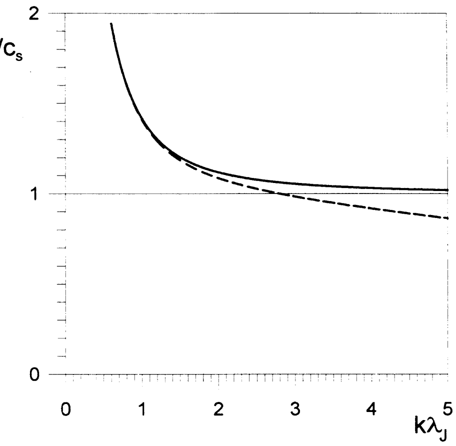


FIG. 2. The dependence of the phase velocity $v_\varphi$ on the wave vector $k$. The solid line represents a plot using Eq. (8); the dashed line represents a plot using Eq. (11).

we treat the case of nonlocal Josephson electrodynamics. We find the amplitude and power of the radiated wave and the radiation friction force acting on a Josephson vortex and arising due to the Cherenkov radiation. We calculate the relation between the density of the bias current across the Josephson junction and the stationary Josephson-vortex velocity. We consider the case of SIS-type Josephson junction with a very high electrical resistivity, i.e., with a very low damping constant.

The paper is organized in the following way. In Sec. II, we consider qualitatively the conditions of Josephson-vortex Cherenkov radiation. We show that the wave vector of the radiated electromagnetic wave, $k$, is from a certain region $k > k_c$ and we find the value of $k_c$. In Sec. III, we use perturbation theory to calculate the amplitude of the radiated electromagnetic wave. We find the radiation friction force acting on a Josephson vortex moving with a constant velocity. We show that dissipation due to this friction force is particularly effective if the Josephson-vortex velocity is approaching the Swihart velocity. In Sec. IV, we summarize the overall conclusions.

## II. QUALITATIVE CONSIDERATION

Let us now consider the dispersion relation $\omega(k)$ for an electromagnetic wave propagating along a Josephson junction in the general case, when the only restriction for the length scale $l_\varphi$ is given by the inequality $\xi \ll l_\varphi$, where $\xi$ is the correlation length. The relation between $l_\varphi$ and $\lambda$ is then arbitrary and we have to take into account the nonlocality of the dependence of the phase difference $\varphi(x,t)$ on the current and magnetic field. As a result in the general case the distribution of $\varphi(x,t)$ is determined by the integro-differential equation[8]

$$\varphi_{\tau\tau} + \eta\varphi_\tau = \frac{\lambda_J^2}{\pi\lambda}\int_{-\infty}^{\infty} K_0\left(\frac{|x-u|}{\lambda}\right)\frac{\partial^2\varphi}{\partial u^2}\,du - \sin\varphi + \beta, \tag{9}$$

where $K_0(x)$ is the zero order modified Bessel function. If the space variation of $\varphi(x,t)$ is slow, i.e., if $\lambda \ll l_\varphi$, then Eq. (9) takes the form of the sine-Gordon equation.

Using Eqs. (6) and (9) we find the dispersion relation $\omega(k)$ for an electromagnetic wave in a long Josephson junction in the framework of nonlocal Josephson electrodynamics. In the case of zero dissipation ($\eta = 0$) it has the form

$$\omega = \omega_J \sqrt{1 + \frac{k^2\lambda_J^2}{\sqrt{1+k^2\lambda^2}}}, \tag{10}$$

and thus the phase velocity of this electromagnetic wave is equal to

$$v_\varphi = \frac{\omega}{k} = c_s \sqrt{\frac{1}{\sqrt{1+k^2\lambda^2}} + \frac{1}{k^2\lambda_J^2}}. \tag{11}$$

The formulas given by Eqs. (8) and (11) for $v_\varphi$ coincide if $k\lambda \ll 1$, i.e., in the range of validity of local Josephson electrodynamics.

It follows from Eq. (11) that the electromagnetic wave phase velocity $v_\varphi$ is a monotonically decreasing function of $k$ and the value of $v_\varphi$ tends to zero when the wave vector $k$ tends to infinity. In particular, in the limiting case $k\lambda \gg 1$ we have[5]

$$v_\varphi \approx \frac{c_s}{\sqrt{k\lambda}} \ll c_s, \qquad k\lambda \gg 1. \tag{12}$$

We show the dependence $v_\varphi(k)$ given by Eq. (11) by the dashed line in Fig. 2. We use for this plot the value $\lambda_J = 5\lambda$, i.e., $\lambda_J \gg \lambda$.

Thus there exists a certain wave vector region $k \geq k_c$ where the phase velocity of an electromagnetic wave in a long Josephson junction is lower than the highest possible velocity of a Josephson vortex. We have the equation $v_\varphi(k_c) = c_s$ to find the value of $k_c$. In the case when $\lambda \ll \lambda_J$ the solution of this equation is given by an approximate formula,

$$k_c \approx \frac{1}{\lambda_J} \sqrt{\sqrt{2}\frac{\lambda_J}{\lambda} + \frac{3}{4}}. \tag{13}$$

The existence of an electromagnetic wave with the phase velocity lower than $c_s$ results in Josephson-vortex Cherenkov radiation. This dissipation mechanism is especially effective when the Josephson-vortex velocity is approaching the highest possible velocity $c_s$.

## III. RADIATION FRICTION FORCE

The Josephson-vortex Cherenkov radiation results, in particular, in a friction force acting on the radiating vortex. In order to find this radiation friction force we solve the following problem.

Let us consider the uniform motion of a Josephson vortex in a long Josephson junction. We treat the velocity of this motion $v$ as a given constant value. We use for calculations the perturbation theory, i.e., we neglect the dissipation arising due to the resistance of the junction while considering the Josephson-vortex Cherenkov radiation of an electromagnetic wave.

In order to find the amplitude of the radiated wave we look for a solution of Eq. (9) with $\eta = 0$ and $\beta = 0$ in the form

$$\varphi(x,t) = \varphi_0(x - vt) + f(x,t), \tag{14}$$

where $\varphi_0(x - vt)$ is the phase difference given by Eq. (5) for a single uniformly moving Josephson vortex. We assume that $|f(x,t)| \ll 1$ and we consider the relation between the values of $|\varphi_0(x,t)|$ and $|f(x,t)|$ to be arbitrary.

The function $f(x,t)$ is described by the linearized equation, Eq. (9), that reads

$$\begin{aligned} &\frac{v^2}{c_s^2} f'' + \left(1 - \frac{v^2}{c_s^2}\right) \cos\varphi_0 \, f \\ &\quad - \frac{\tilde{\lambda}_J}{\pi\lambda} \int_{-\infty}^{\infty} f''(\xi') \, K_0\!\left(\frac{\tilde{\lambda}_J}{\lambda} |\xi - \xi'|\right) d\xi' \\ &\quad = -\varphi_0'' + \frac{\tilde{\lambda}_J}{\pi\lambda} \int_{-\infty}^{\infty} \varphi_0''(\xi') \, K_0\!\left(\frac{\tilde{\lambda}_J}{\lambda} |\xi - \xi'|\right) d\xi', \end{aligned} \tag{15}$$

where the prime is to denote the derivative over the dimensionless variable $\xi = (x - vt)/\tilde{\lambda}_J$ and the characteristic space scale $\tilde{\lambda}_J$ is determined as

$$\tilde{\lambda}_J = \lambda_J \sqrt{1 - v^2/c_s^2}. \tag{16}$$

We present now the dependence $f(\xi)$ as a sum of two terms, i.e., $f(\xi) = f_1(\xi) + f_2(\xi)$. The function $f_1(\xi)$ describes the Josephson-vortex deformation due to the nonlocality of the Josephson electrodynamics. It has the characteristic scale $l_1 \sim 1$ and decays at $\xi \to \pm\infty$, i.e., $f_1(\pm\infty) = 0$. The function $f_2(\xi)$ describes the electromagnetic wave radiated by a moving Josephson vortex. It has the characteristic variation scale $l_2 \sim 1/k\tilde{\lambda}_J \ll 1$. We use the relation $l_2 \ll l_1$ to determine the dependencies $f_1(\xi)$ and $f_2(\xi)$ independently of one another. In other words, we develop a two-scale perturbation theory, which holds with an accuracy of $l_2/l_1 \ll 1$.

The equation determining the function $f_1(\xi)$ follows from Eq. (15) and has the form

$$f_1'' - \cos\varphi_0 \, f_1 = -\alpha^2 \frac{d^4\varphi_0}{d\xi^4}, \tag{17}$$

where the dimensionless parameter $\alpha$ is equal to

$$\alpha = \frac{\lambda}{\sqrt{2}\lambda_J\left(1 - v^2/c_s^2\right)}. \tag{18}$$

We use the relation

$$\frac{\tilde{\lambda}_J}{\pi\lambda} \int_{-\infty}^{\infty} f(\xi') \, K_0\!\left(\frac{\tilde{\lambda}_J}{\lambda} |\xi - \xi'|\right) d\xi' = f(\xi) + \frac{1}{2}\left(\frac{\lambda}{\tilde{\lambda}_J}\right)^2 f''(\xi) + \cdots \tag{19}$$

to derive Eq. (17). The formula given by Eq. (19) is valid for a function $f(\xi)$ with the characteristic variation scale of the order of unity in the case when $\lambda \ll \tilde{\lambda}_J$.

An exact solution of Eq. (17) that satisfies the boundary conditions $f_1(\pm\infty) = 0$ is given by the expression

$$f_1(\xi) = \alpha^2 \left(3\,\frac{\sinh\xi}{\cosh^2\xi} - \frac{\xi}{\cosh\xi}\right). \tag{20}$$

It follows from Eq. (20) that the value of $|f_1(\xi)| \ll 1$ if $\alpha \ll 1$, i.e., if $\lambda/\lambda_J \ll 1 - v^2/c_s^2$. The inequality $\alpha \ll 1$ is therefore the necessary and sufficient condition for the applicability of the perturbation theory approach.

We use now Eq. (15) to calculate the $f_2(\xi)$. The function $f_2(\xi)$ has the characteristic scale $l_2 \ll 1$ and, therefore, with an accuracy of $l_2^2 \ll 1$ we can neglect the second term in the left part of Eq. (15). We solve then the reduced Eq. (15) by means of a Fourier transformation. As a result we find that in the region $\xi \ll 0$ the function $f_2(\xi)$ takes the form

$$f_2(\xi) = f_a \cos q\xi, \qquad \xi \ll 0, \tag{21}$$

where

$$f_a = \frac{8\pi c_s^4}{v^2(v^2+c_s^2)}\exp\left(-\frac{\pi q}{2}\right), \tag{22}$$

$$q = \frac{\tilde{\lambda}_J}{\lambda}\,\frac{\sqrt{c_s^4 - v^4}}{v^2} = \frac{1}{\alpha}\,\frac{c_s\sqrt{v^2+c_s^2}}{\sqrt{2}v^2} \geq \frac{1}{\alpha} \gg 1. \tag{23}$$

The function $f_2(\xi)$ given by Eq. (21) describes an electromagnetic wave radiated by a Josephson vortex moving with a constant velocity $v$. This wave exists only behind the radiating Josephson vortex. The amplitude of this radiated electromagnetic wave $f_a$ is mainly determined by the exponential factor $\exp(-\pi q/2) \leq \exp(-\pi/2\alpha) \ll 1$ and, therefore, the value of $|f_2(\xi)| \ll 1$.

The function $f_2(\xi) = f_2(x,t)$ takes the form

$$f_2(x,t) = f_a \cos(k_p x - \omega_p t), \qquad x \ll vt, \tag{24}$$

in the $x,t$ space. The wave vector $k_p$ and the frequency $\omega_p$ of the radiated electromagnetic wave are equal to

$$k_p = \frac{q}{\tilde{\lambda}_J} = \frac{1}{\lambda}\,\frac{\sqrt{c_s^4 - v^4}}{v^2}, \tag{25}$$

$$\omega_p = \omega_J\,\frac{\lambda_J}{\lambda}\,\frac{\sqrt{c_s^4 - v^4}}{v c_s}. \tag{26}$$

With the accuracy of the perturbation theory approach, i.e., for $\alpha \ll 1$, the values of $k_p$ and $\omega_p$ given by Eqs. (25) and (26) are equal to the values determined by the Cherenkov radiation conditions

$$\frac{\omega(k_p)}{k_p} = v, \qquad \omega_p = \omega(k_p), \tag{27}$$

where the dispersion relation $\omega(k)$ is given by Eq. (10). Note that we use the nonlocal Josephson electrodynamics that is valid if the wavelength is larger than the coherence lenth. It restricts the applicability of this approach to the domain $v \gg c_s/\sqrt{\kappa}$, where $\kappa$ is the Ginzburg-Landau parameter, i.e., the velocity $v$ has to be above a certain threshold given by this inequality.

The amplitude of the radiated electromagnetic wave increases when the velocity of the Josephson vortex tends to the highest possible velocity $c_s$. If the value of the velocity $v$ is close to the value of $c_s$ we can simplify Eqs. (22), (25), and (26) and in the interval $\lambda/\lambda_J \ll 1 - v/c_s \ll 1$ the expressions for $f_a$, $k_p$, and $\omega_p$ take the forms

$$f_a \approx 4\pi \exp\left[-\pi\sqrt{2}\,\frac{\lambda_J}{\lambda}\left(1 - \frac{v}{c_s}\right)\right], \qquad f_a \ll 1, \tag{28}$$

$$k_p \approx \frac{2}{\lambda}\sqrt{1 - v/c_s}, \qquad \frac{1}{\lambda_J} \ll k_p \ll \frac{1}{\lambda}, \tag{29}$$

$$\omega_p \approx 2\omega_J\,\frac{\lambda_J}{\lambda}\sqrt{1 - v/c_s}, \qquad \omega_p \gg \omega_J. \tag{30}$$

Let us now find the radiation friction force $f_r$ acting on a unit length of a uniformly moving Josephson vortex. The radiation energy increase rate is equal to the product $E_w v$, where $E_w$ is the wave energy per unit area of the junction. This value is a sum of two terms. The first one is determined by the power of the friction force and is equal to $f_r v$. The second one is determined by the energy flux due to the radiated electromagnetic wave propagation. This term is equal to $E_w v_g$, where

$$v_g = \left.\frac{\partial\omega}{\partial k}\right|_{k_p} \tag{31}$$

is the group velocity of the wave. As a result, we find for the radiation friction force $f_r$ the formula

$$f_r = E_w\,\frac{v - v_g}{v}. \tag{32}$$

The values of $E_w$ and $v_g$ are given by the expressions

$$E_w = \frac{1}{2}\,C\left(\frac{\hbar\omega_p}{2e}\,f_a\right)^2, \tag{33}$$

$$v - v_g = \frac{v}{2}\left(1 - \frac{v^4}{c_s^4}\right) \tag{34}$$

with the accuracy of $\alpha \ll 1$. Using Eqs. (2), (3), (22), and (26) we find that

$$E_w = \frac{\Phi_0^2}{\pi\lambda^3}\,\frac{c_s^6}{v^6}\,\frac{1 - v^2/c_s^2}{1 + v^2/c_s^2}\exp(-\pi q). \tag{35}$$

Thus the Josephson-vortex Cherenkov radiation of an electromagnetic wave results in a radiation friction force $f_r$ that is given by the formula

$$f_r = \frac{\Phi_0^2}{2\pi\lambda^3}\,\frac{c_s^6}{v^6}\left(1 - \frac{v^2}{c_s^2}\right)^2 \exp(-\pi q). \tag{36}$$

If the value of the Josephson-vortex velocity $v$ is close to the value of $c_s$ we can simplify Eq. (36) and in the interval $\lambda/\lambda_J \ll 1 - v/c_s \ll 1$ the expression for $f_r$ takes the form

$$f_r \approx \frac{2\Phi_0^2}{\pi\lambda^3}\left(1 - \frac{v}{c_s}\right)^2 \exp\left[-2\sqrt{2}\pi\,\frac{\lambda_J}{\lambda}\left(1 - \frac{v}{c_s}\right)\right]. \tag{37}$$

Let us now consider a uniform motion of a Josephson vortex in a Josephson junction with a bias current. In this case the vortex is subjected to the Lorentz $f_L$ and the radiation friction $f_r$ forces. The value of $f_L$ acting per unit length of the vortex is equal to $\Phi_0 j/c$. Equating $f_L$ and $f_r$ we obtain the following relation between the bias current density $j$ and the velocity of uniform motion of the Josephson vortex, $v$:

$$\frac{j}{j_c} = 8\pi \frac{\lambda_J^2}{\lambda^2} \frac{c_s^6}{v^6} \left(1 - \frac{v^2}{c_s^2}\right)^2 \exp(-\pi q). \tag{38}$$

If the value of the Josephson-vortex velocity $v$ is close to the value of $c_s$ we can simplify Eq. (38) and in the interval $\lambda/\lambda_J \ll 1 - v/c_s \ll 1$ it takes the form

$$\frac{j}{j_c} = 32\pi \frac{\lambda_J^2}{\lambda^2} \left(1 - \frac{v}{c_s}\right)^2 \exp\left[-2\sqrt{2}\pi \frac{\lambda_J}{\lambda}\left(1 - \frac{v}{c_s}\right)\right]. \tag{39}$$

A relation analogous to the one given by Eq. (38) and taking into account only the damping due to the resistance of the junction reads[17]

$$\frac{j}{j_c} = \frac{4}{\pi} \frac{\eta v}{\sqrt{c_s^2 - v^2}}. \tag{40}$$

The dependence $j(v)$ given by Eq. (40) is shown in Fig. 3 by the solid line. We use for this plot the value $\eta = 0.05$.

Let us consider the Josephson-vortex Cherenkov radiation for $\eta \neq 0$. It follows from the energy conservation law that for $\eta \ll 1$ the dependence $j(v)$ is a sum of the two dependencies given by Eqs. (38) and (40), i.e.,

$$\frac{j}{j_c} = 8\pi \frac{\lambda_J^2}{\lambda^2} \frac{c_s^6}{v^6} \left(1 - \frac{v^2}{c_s^2}\right)^2 \exp(-\pi q) + \frac{4}{\pi} \frac{\eta v}{\sqrt{c_s^2 - v^2}}. \tag{41}$$

If the value of the Josephson-vortex velocity $v$ is close to the value of $c_s$ we can simplify Eq. (41) and in the interval $\lambda/\lambda_J \ll 1 - v/c_s \ll 1$ it takes the form

$$\frac{j}{j_c} = 32\pi \frac{\lambda_J^2}{\lambda^2} \left(1 - \frac{v}{c_s}\right)^2 \exp\left[-2\sqrt{2}\pi \frac{\lambda_J}{\lambda}\left(1 - \frac{v}{c_s}\right)\right] + \frac{4}{\pi} \frac{\eta v}{\sqrt{c_s^2 - v^2}}. \tag{42}$$

The dependence $j(v)$ given by Eq. (41) is shown in Fig. 3 by the dashed line. We use for this plot the values $\eta = 0.05$ and $\lambda_J = 5\lambda$. It is seen from Fig. 3 that at $j \sim j_c$ the value of $v$ can be significantly less than $c_s$.

The Josephson-vortex velocity tends to a certain maximum, $v_m$, when the current density tends to the critical current density. Using Eq. (42) we can estimate $v_m$ as

$$1 - \frac{v_m}{c_s} \sim \frac{1}{2\sqrt{2}\pi} \frac{\lambda}{\lambda_J}. \tag{43}$$

It follows from Eq. (43) that a noticeable difference between $v_m$ and $c_s$ can be observed even if $\lambda_J > \lambda$.

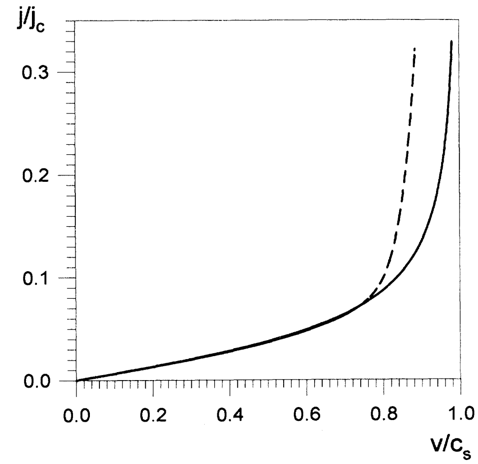


FIG. 3. The dependence of the bias current density $j$ on the Josephson-vortex velocity $v$. The solid line represents a plot using Eq. (40); the dashed line represents a plot using Eq. (41).

Note that, when the Josephson-vortex velocity tends to its maximum value $v_m$, the energy dissipation in the Josephson junction is mainly due to the Josephson-vortex Cherenkov radiation, i.e., the power release happens in the form of electromagnetic radiation.

## IV. SUMMARY

To summarize, we calculate the Josephson-vortex Cherenkov radiation of an electromagnetic wave in a long Josephson junction. This dissipation mechanism results in a radiation friction force and is especially effective if the velocity of the Josephson vortex is approaching the highest possible velocity $c_s$ (Swihart velocity). We find the relation between the density of the bias current across the junction and the Josephson-vortex velocity.

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[1] A. Barone and G. Paterno, *Physics and Applications of the Josephson Effect* (Wiley, New York, 1982).
[2] A.V. Ustivov, T. Doderer, R.P. Huebener, N.F. Pedersen, B. Mayer, and V.A. Oboznov, Phys. Rev. Lett. **69**, 1815 (1992).
[3] A.V. Ustivov, T. Doderer, R.P. Huebener, B. Mayer, and V.A. Oboznov, Europhys. Lett. **19**, 63 (1992).
[4] A. Gurevich, Phys. Rev. B **48**, 12 857 (1993).

[5] R.G. Mints and I.B. Snapiro, Phys. Rev. B **49**, 6188 (1994).
[6] P. Chaudhari *et al.*, Phys. Rev. Lett. **60**, 1653 (1988).
[7] D. Dimos *et al.*, Phys. Rev. Lett. **61**, 219 (1988).
[8] A. Gurevich, Phys. Rev. B **46**, 3187 (1992).
[9] R.G. Mints and I.B. Snapiro, Physica A **200**, 426 (1993).
[10] J.C. Swihart, J. Appl. Phys. **32**, 461 (1961).
[11] L.D. Landau, E.M. Lifshitz, and L.P. Pitaevskii, *Electrodynamics of Continuous Media* (Pergamon Press, Oxford, 1984).
[12] M. Peyrard and M.D. Kruskal, Physica **14**, 88 (1984).
[13] P.A. Bobbert, Phys. Rev. B **45**, 7540 (1992).
[14] U. Geigenmüller, G.J. Lobb, and C.B. Whan, Phys. Rev. B **47**, 348 (1993).
[15] U. Eckern and E.B. Sonin, Phys. Rev. B **47**, 505 (1993).
[16] A.V. Ustinov, M. Cirillo, and B.A. Malomed, Phys. Rev. B **47**, 8357 (1993).
[17] D.W. McLaughlin and A.C. Scott, Phys. Rev. A **18**, 1652 (1978).