Resistance magneto-oscillations of superlattices

B. Laikhtman and D. Menashe
Racah Institute of Physics, Hebrew University, Jerusalem, 91904, Israel
(Received 15 November 1994)

We study theoretically vertical transport in a superlattice in a magnetic field applied along the growth direction. The electron spectrum and the conductivity are characterized by the interplay of the width of the superlattice miniband without magnetic field, 4Λ, the energy separation between Landau levels, $\hbar\Omega$, and the energy uncertainty due to scattering, Γ. When $\hbar\Omega$ is the largest of those energies there are gaps in the electron spectrum and strong magneto-oscillations can be observed. If $\Lambda \gg \Gamma$ the conductivity in the longitudinal direction is calculated with the help of the Boltzmann equation. In the opposite case, we use percolation arguments to estimate the conductivity and prove the existence of a mobility edge. When the magnetic field is small, $\hbar\Omega \ll \Gamma$, the magneto-oscillations are weak and the conductivity is calculated with the help of the self-consistent Born approximation.

I. INTRODUCTION

The extension of research on the quantum Hall effect eventually has shown that very interesting information concerning the properties of a two-dimensional electron gas (2DEG) can be obtained not only from lateral, but also from vertical transport experiments.[1,2] In the study of one or two layers, the effect of a magnetic field applied in the growth direction on tunneling in the same direction is connected mainly with the electron-electron interaction. However, if the number of layers is large, the magnetic field quantization in one well is mixed with the spreading of electron levels, due to interwell tunneling, which affects the electron spectrum and the vertical transport even in the case when the electron-electron interaction is small. In 1986, Störmer *et al.*[3,4] observed longitudinal magneto-oscillations (i.e., the oscillation of the vertical conductivity versus magnetic field in the same direction) and the quantum Hall effect. Piazza *et al.*[5] found evidence of the Landau quantization and localized states in superlattices in optical experiments. On the other hand, some other current-voltage characteristic measurements[6–8] did not report any significant effect of the magnetic field on the longitudinal resistance.

The different results of different experiments appear apparently from different properties of the samples. They suggest that a specific design is necessary for the observation of the magneto-oscillations of the longitudinal conductivity. In this paper, we address the question of what conditions are necessary for the longitudinal magneto-oscillations and discuss details of the oscillation mechanism.

Basically, we will consider n-doped GaAs based superlattices, although the theory can be easily generalized for other cases. The only characteristics of a superlattice important for our purpose are the width of the first miniband 4Λ (in the case of a narrow band, Λ is an overlap integral between two adjacent quantum wells) and the energy uncertainty due to scattering, Γ. We assume that the excitation of electrons to the second and upper minibands can be neglected.

We consider the region of the magnetic field and electron concentration when the filling factor is large, so that the size of most of the electron states in the magnetic field is also large and the Coulomb interaction between electrons appears to be small. In the next section, we discuss qualitatively properties of the electron spectrum necessary to detect a strong magneto-oscillating effect. The density of states is also calculated analytically for a weak and strong magnetic field with the help of the Green's function technique. In Sec. III, the magnetoconductivity is calculated. For a weak magnetic field, we use the Keldysh Green's function technique[9–11] and obtain quantitative results. For the case of a strong magnetic field, where the localization of electron states is important, we use a percolation kind of reasoning. We obtain estimates for the width of the extended states region near the center of Landau level and evaluate the maximum value of the conductivity. In all cases where Green's functions are necessary we use the modified technique, where translationary invariant Green's functions in magnetic field are introduced.[13]

II. ELECTRON SPECTRUM

In superlattices, the quantization of the electron motion in the growth directions results in the formation of minibands. A magnetic field in the same direction leads to the quantization of the in-plane electron motion. The electron spectrum in a strong enough magnetic field ($\hbar\Omega \gg \Gamma$, where Ω is the cyclotron frequency) is characterized by the Landau quantum number, the number of the miniband, and the wave vector in the growth direction (we assume that the field is not strong enough to lift the spin degeneracy). The resulting spectrum can be understood in two ways. One is the splitting of each miniband into Landau levels (compare Ref. 12). The other is the spreading of Landau levels in quantum wells into Landau minibands with the width equal to the original miniband width without a magnetic field, 4Λ.[3,4] As long as the separation between Landau levels, $\hbar\Omega$, is smaller

than 4Λ the Landau minibands overlap with each other and the density of states of the superlattice miniband remains continuous. The magnetic quantization in this case can lead to the longitudinal Shubnikov–de Haas effect. When $\hbar\Omega$ grows larger than 4Λ, the density of states of the original miniband splits into the densities corresponding to different Landau minibands.

In doped superlattices the electron concentration is controlled by the doping and remains constant when the magnetic field is changing. That means that with the increase of the magnetic field, the Fermi level passes from one Landau miniband to another so that more and more Landau minibands appear above the Fermi level. One can expect strong oscillations of the longitudinal magnetoresistance if the condition $\hbar\Omega > 4\Lambda$ is satisfied for the magnetic field when $\hbar\Omega$ is still smaller than the Fermi energy, E_F, so that a few Landau minibands are occupied and are depopulated with the further increase of the magnetic field. For this condition a superlattice with narrow minibands is necessary. Actually, it is enough if the width of the first miniband is a few times smaller than E_F and the gap between the first and the second miniband is so large that the Fermi level is below the bottom of the second miniband.

Another necessary condition for strong oscillations is that the inequality $\hbar\Omega \gg \Gamma$ also has to be satisfied for a magnetic field when $\hbar\Omega < E_F$. This condition means that a smearing of Landau minibands due to scattering is small compared to the energy separation between the minibands. If $\hbar\Omega \lesssim \Gamma$, then the magnetic field leads to a longitudinal Shubnikov–de Haas effect.

We find the electron spectrum with the help of the Green's function technique and the self-consistent Born approximation[14,15] (SCBA). In quantum mechanics, a uniform magnetic field is introduced by a vector potential that is a linear function of the coordinates and is not translationary invariant, $A_j(\mathbf{r}) = A_{jl}r_l$ (here $\mathbf{r}$ is the radius vector in-plane perpendicular to the magnetic field). Green's functions calculated with the help of this potential are not gauge invariant and translationary invariant in spite of the translationary invariance of the problem. However, it is possible to separate a phase factor from a Green's function in such way so that the rest of it does not depend on a specific gauge and is translationary invariant,[13]

$$G(\mathbf{r}_1,\mathbf{r}_2,E) = \exp\left(\frac{ie}{\hbar c}\,\mathbf{r}\cdot\mathbf{A}(\mathbf{R})\right)\tilde{G}(\mathbf{r},E)\ , \qquad (1)$$

where $\mathbf{R} = (\mathbf{r}_1+\mathbf{r}_2)/2$ and $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$. Whatever is the original gauge, the translationary invariant function $\tilde{G}(\mathbf{r},E)$ has a cylindrical symmetry as long as no other physical interaction (e.g., an external in-plane electric field) violates it. That is

$$\tilde{G}(\mathbf{r},E) = \frac{1}{\sqrt{2\pi}l}\sum_N G_N(E)\psi_{N,0}(\mathbf{r})\ , \qquad (2)$$

where $\psi_{N,0}(\mathbf{r})$ is the eigenfunction of an electron in a uniform magnetic field in the state with the Landau quantum number N and zero angular momentum, $l = \sqrt{(c\hbar/eB)}$ is the magnetic length, and B is the magnetic field.

In general, the retarded Green's function in a superlattice has the form

$$G^r_{N,k}(E) = [E - E_N - 2\Lambda\cos k - \Sigma^r_{N,k}(E)]^{-1}\ , \qquad (3)$$

where $E_N = \hbar\Omega\left(N+\frac{1}{2}\right)$ is the energy of the center of the Landau level and k is the dimensionless wave vector in the growth direction. The 2D density of states (the density of states per one period of the superlattice) in terms of this function is expressed as

$$\nu(E) = -\frac{1}{\pi^2 l^2}\sum_N\int_{-\pi}^{+\pi}\mathrm{Im}\ G^r_{N,k}(E)\ \frac{dk}{2\pi}\ . \qquad (4)$$

The main electron scattering mechanisms in superlattices at low temperature are ionized impurity scattering and surface roughness scattering. The correlation length in the growth direction for a random potential resulting from both the surface roughness and impurities is about the superlattice period. This means that in the Wanier representation, the correlation function of the potential has just diagonal matrix elements, $u(\mathbf{r})$, which are all equal.

The in-plane correlation length may be different for impurities and surface roughness. In quantum wells where special measures (growth interrupts) are taken to reduce surface roughness, the typical correlation length of the roughness can be more than 500 Å. Superlattices are usually grown without such precautions and one can expect the in-plane correlation length of the surface roughness to be smaller than 500 Å. The in-plane correlation length of the potential due to impurities is of the order of their distance from the well, i.e., the superlattice period, typically about 100 Å or less. The Fermi wavelength in a 2DEG with a concentration of about 2×10^{11} cm^{-2} is about 500 Å. That is the characteristic scale of $u(\mathbf{r})$ for both scattering mechanisms is about or shorter than the Fermi wavelength.

For the calculation of the self-energy, we need the matrix elements of $u(\mathbf{r})$ between electron wave functions in a magnetic field. The condition $\hbar\Omega < E_F$ means that the magnetic length is larger than the Fermi wavelength. For this case, it is possible to show that the matrix elements of $u(\mathbf{r})$ are reduced to $u_0 = \hbar^2\Omega/2\pi\tau_s$, where τ_s is the single particle relaxation time.[13] Then according to SCBA, $\Sigma^r_{N,k}(E) = \Sigma^r(E-E_N)$ is a periodic function of the energy (with the period $\hbar\Omega$) that satisfy the equation,

$$\Sigma^r(E) = \frac{u_0}{2\pi}\sum_N\int_{-\pi}^{\pi}G^r_{N,k}(E)dk\ . \qquad (5)$$

The k dependence is integrated out and the self-energy is a function of just one argument. The physical reason behind this result is the short range of the scattering potential. Equation (4) now becomes

$$\nu(E) = -\frac{1}{\pi^2 l^2 u_0}\mathrm{Im}\Sigma^r(E)\ . \qquad (6)$$

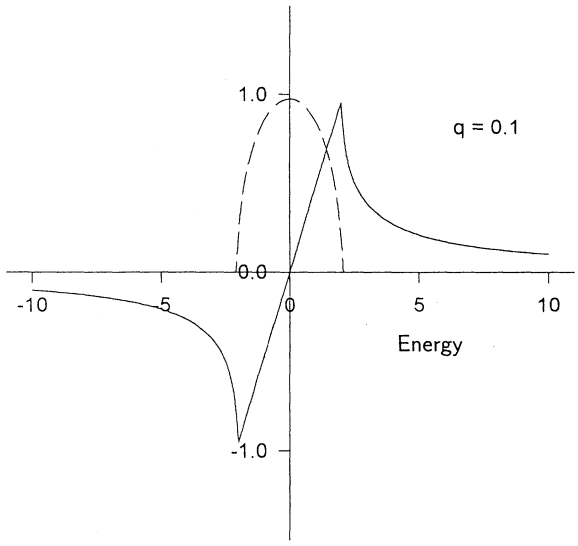

FIG. 1. Typical picture of the real part (solid line) and imaginary part (dashed line) of the self-energy for $q < 1$. All energies are in units of u_0. The imaginary part has a maximum at the center of the band.

In the case of a strong magnetic field, $\hbar\Omega \gg \Lambda$, Γ, the main contribution to the sum in Eq. (5) comes from just one term and

$$\Sigma^r(E) = \frac{u_0}{\sqrt{[E - \Sigma^r(E)]^2 - 4\Lambda^2}} \ . \tag{7}$$

The behavior of $\Sigma^r(E) = \Delta - i\Gamma$ depends on one dimensionless parameter, $q = u_0/4\Lambda^2$. For two values of this parameter, the dependence of Δ and Γ on E is shown in Fig. 1 and Fig. 2. The density of states for a small value

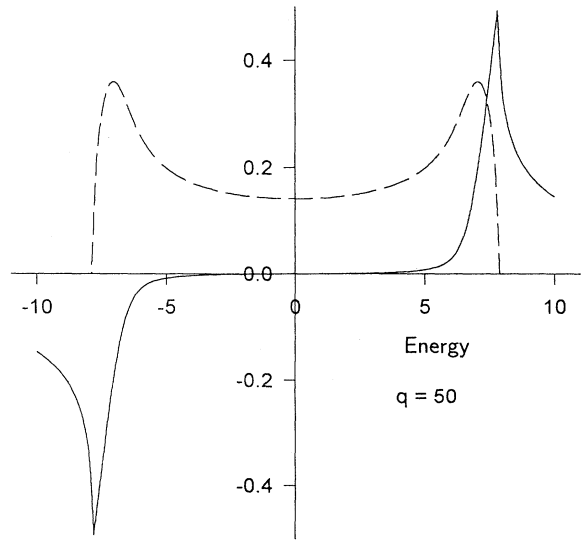

FIG. 2. Typical picture of the real part (solid line) and imaginary part (dashed line) of the self-energy for $q > 1$. All energies are in units of u_0. The imaginary part has a minimum at the center of the band.

of this parameter near $E = E_N$ is

$$\nu(E) = (1/2\pi^2 l^2 u_0)\ \sqrt{4u_0 - (E - E_N)^2}\ , \quad u_0 \gg 4\Lambda^2\ . \tag{8}$$

In the opposite case,

$$\nu(E) = 1/\left[2\pi^2 l^2 \sqrt{4\Lambda^2 - (E - E_N)^2}\right]\ , \quad u_0 \ll 4\Lambda^2\ , \tag{9}$$

in the middle of the Landau miniband and goes to zero near its edges.

In the case of a weak magnetic field, $\hbar\Omega \ll \Gamma$ the part of self-energy depending on the magnetic field is exponentially small. For the degenerate electron gas ($T \ll E_F$, where T is the temperature), the SCBA gives

$$\Delta = \frac{\hbar}{\tau_s}\, e^{-\pi/\Omega\tau_s} J_0\left(\frac{4\pi\Lambda}{\hbar\Omega}\right) \sin\left(\frac{2\pi E}{\hbar\Omega}\right)\ , \tag{10}$$

$$\Gamma = \frac{\hbar}{2\tau_s}\left[1 + 2e^{-\pi/\Omega\tau_s} J_0\left(\frac{4\pi\Lambda}{\hbar\Omega}\right) \cos\left(\frac{2\pi E}{\hbar\Omega}\right)\right]\ . \tag{11}$$

When Λ goes to zero, these expressions are identical with those for the 2D electron gas. The oscillation part of the density of states

$$\nu(E) = \frac{m}{\pi\hbar^2}\left[1 - 2e^{-\pi/\Omega\tau_s} J_0\left(\frac{4\pi\Lambda}{\hbar\Omega}\right) \cos\left(\frac{2\pi E}{\hbar\Omega}\right)\right] \tag{12}$$

is more complicated than in a single quantum well, because of the presence of the Bessel function.

In samples where the electron concentration is constant, the oscillations of the density of states with the magnetic field, lead to an oscillation of the chemical potential, μ. For a large magnetic field, the oscillations of the chemical potential consist of regions of a nearly linear growth with the magnetic field alternating with sharp drops by $\hbar\Omega$, similar to the 2D electron gas. In a low magnetic field, the oscillations are exponentially small and can be described analytically. From Eq. (12), we obtain the following equation for the chemical potential,

$$n = \frac{mT}{\pi\hbar^2}\left[\ln(1 + e^{\mu/T}) - 2\pi J_0\left(\frac{4\pi\Lambda}{\hbar\Omega}\right) \frac{e^{-\pi/\Omega\tau_s}}{\sinh(2\pi^2 T/\hbar\Omega)} \sin\left(\frac{2\pi\mu}{\hbar\Omega}\right)\right]\ . \tag{13}$$

Then assuming that $\mu = \mu_0 + \Delta\mu$, where $\mu_0 = T\ln(e^{E_F/T} + 1) \approx E_F$, we have

$$\Delta\mu = 2\pi T J_0\left(\frac{4\pi\Lambda}{\hbar\Omega}\right) \frac{e^{-\pi/\Omega\tau_s}}{\sinh(2\pi^2 T/\hbar\Omega)} \sin\left(\frac{2\pi E_F}{\hbar\Omega}\right)\ , \tag{14}$$

where terms of the order of $e^{-E_F/T}$ are neglected.

III. MAGNETOCONDUCTANCE

A. Weak magnetic field, $\hbar\Omega \ll \Gamma$

To calculate the conductivity in the vertical direction, it is necessary to obtain an expression for the current in terms of modified Green's functions. With the help of the tunneling Hamiltonian in the Wanier representation,

$$\mathcal{H}^T_{\alpha\beta} = \Lambda(\delta_{\alpha,\beta+1} + \delta_{\alpha+1,\beta}) \ , \tag{15}$$

it is easy to get the expression for the current in terms of the density matrix. The density matrix is expressed in the regular G^{-+} Keldysh Green's function (see Refs. 9–11). The only thing which is left is to express the regular Green's function in the modified, translationary invariant one in the representation of magnetic quantum numbers. For this end, we can start with Eq. (1). For the modified Green's function there is the expansion (2) and the regular Green's function also can be expanded in the electron wave functions in magnetic field,

$$G(\mathbf{r}_1,\mathbf{r}_2,E) = \sum_{NN'\lambda\lambda'} \hat{G}_{N\lambda,N'\lambda'}(E)\psi_{N,\lambda}(\mathbf{r}_1)\psi^*_{N',\lambda'}(\mathbf{r}_2) \ , \tag{16}$$

where the quantum number λ depends on the gauge. Equation (1) now becomes

$$\sum_{NN'\lambda\lambda'} \hat{G}_{N\lambda,N'\lambda'}(E)\psi_{N\lambda}(\mathbf{r}_1)\psi^*_{N'\lambda'}(\mathbf{r}_2) = \exp\left(\frac{ie}{\hbar c}\,\mathbf{r}\cdot\mathbf{A}(\mathbf{R})\right)\ \frac{1}{\sqrt{2\pi l}}\sum_N G_N(E)\psi_{N,0}(\mathbf{r}) \ . \tag{17}$$

Now we make use of the addition theorem,[13]

$$\sum_\lambda \psi_{N,\lambda}(\mathbf{r}_1)\ \psi^*_{N,\lambda}(\mathbf{r}_2) = \frac{1}{\sqrt{2\pi l}}\ \psi_{N,0}(\mathbf{r}_1-\mathbf{r}_2) \times \exp\left(\frac{ie}{\hbar c}\,\mathbf{r}\cdot\mathbf{A}(\mathbf{R})\right) \ . \tag{18}$$

The substitution of this equation in the right-hand side of Eq. (17) and comparison of the expansion in the right-hand side and the left-hand side leads to the relation

$$\hat{G}_{N\lambda,N'\lambda'}(E) = \delta_{NN'}\delta_{\lambda\lambda'}G_N(E) \ . \tag{19}$$

The fact that the regular Green's function in a uniform magnetic field depends only on the Landau quantum number was first shown by Levinson *et al.*[16]

The expression for the current density from the well α to the well $\alpha+1$ in terms of the modified Green's function in the Wanier representation has the form

$$j = \frac{e\Lambda}{2\pi^2 l^2\hbar}\sum_N \int [G^{-+}_{\alpha+1,\alpha}(E) - G^{-+}_{\alpha,\alpha+1}(E)]dE \ . \tag{20}$$

To simplify the notations, we do not show the Landau quantum number as an argument explicitly in the Wanier representation.

To find $G^{-+}_{\alpha\beta}(E)$ for a weak magnetic field we use, as in the 2D electron gas, SCBA.[15] In SCBA in the Wanier representation this function satisfies the following equation (compare Ref. 13):

$$-eFd(\beta-\alpha)G^{-+}_{\alpha\beta} - \Lambda(G^{-+}_{\alpha+1\beta} + G^{-+}_{\alpha-1\beta} - G^{-+}_{\alpha\beta-1} - G^{-+}_{\alpha\beta+1}) = (\Sigma^r_{\alpha\alpha} - \Sigma^a_{\beta\beta})G^{-+}_{\alpha\beta} - \Sigma^{-+}_{\alpha\alpha}G^a_{\alpha\beta} + \Sigma^{-+}_{\beta\beta}G^r_{\alpha\beta} \ , \tag{21}$$

where α and β are numbers of quantum well, F is the electric field and d is the period of the superlattice. Self-energies are diagonal with respect to the well number in SCBA, because we neglect the matrix element of the scattering potential between different wells. We are looking for a translationary invariant in the vertical direction solution to this equation. That is, all Green's functions satisfy the relation

$$G_{\alpha\beta}(E) = G_{\alpha-\beta}\left(E + \frac{\alpha+\beta}{2}\ eFd\right) \ . \tag{22}$$

The self-energies satisfy a similar relation.

Now, if we consider so weak an electric field that eFd is smaller than any other energy scale, then Eq. (21) is reduced to

$$ieFd\left(\frac{\partial G^{-+}_{N,k}}{\partial k} - 2\Lambda\sin k\ \frac{\partial G^{-+}_{N,k}}{\partial E}\right) = (\Sigma^r_N - \Sigma^a_N)G^{-+}_{N,k} + (G^r_{N,k} - G^a_{N,k})\Sigma^{-+}_N \ , \tag{23}$$

where

$$G_{N,k} = \sum_\alpha G_\alpha e^{-ik\alpha} \ . \tag{24}$$

Equation (23) describes, in general, a nonlinear transport with respect to the electric field, because although field corrections to the spectrum are neglected, the electron distribution can be far from equilibrium. We linearize Eq. (23) keeping in mind that all self-energies do not depend on k and cannot contain linear terms in the electric field, because of the invariance with respect to the simultaneous change of the sign of both k and F. That is, in the linear approximation, the last term in the right-hand side of Eq. (23) disappears. This result is similar to the disappearance of the scattering-in term in the Boltzmann equation. It takes place because of the short range scattering. Really, only diagonal in the well number matrix elements of the scattering potential correlation function were kept.

Assuming

$$G^{-+}_{N,k} = G^{(0)}_{N,k} + G^{(1)}_{N,k} \ , \tag{25}$$

where

$$G^{(0)}_{N,k}(E) = -2if_0(E)\ \mathrm{Im}G^r_{N,k} \tag{26}$$

is the equilibrium part of $G^{-+}_{N,k}$ (see Ref. 13), $f_0(E)$ is the

Fermi function, we obtain

$$\Gamma\sum_N G^{(1)}_{N,k} = -ieFd\left(\frac{\partial}{\partial k} - 2\Lambda\sin k\,\frac{\partial}{\partial E}\right) f_0(E) \times\frac{\pi}{\hbar\Omega}\left[1-2\exp\left(-\frac{2\pi\Gamma}{\hbar\Omega}\right) \times\cos\left(2\pi\,\frac{E-\Delta-2\Lambda\cos k}{\hbar\Omega}\right)\right]. \quad (27)$$

Here, only terms of the first order in the small exponent $\exp(-2\pi\Gamma/\hbar\Omega)$ are kept. An exponentially small Δ can be neglected so that in the Wanier representation

$$\sum_N G^{-+}_{\pm 1} = \mp\frac{2\pi eFd\Lambda\tau_s}{\hbar^2\Omega}\, f_0'(E)\left[1-2J_2\left(\frac{4\pi\Lambda}{\hbar\Omega}\right) \times\exp\left(-\frac{\pi}{\Omega\tau_s}\right)\cos\left(\frac{2\pi E}{\hbar\Omega}\right)\right]. \quad (28)$$

The substitution of Eq. (28) into Eq. (20) gives the following expression for the conductivity:

$$\sigma = \frac{2e^2\Lambda^2 md\tau_s}{\pi\hbar^4}\left[\left(1+e^{-\mu/T}\right)^{-1} - \frac{2\pi^2T/\hbar\Omega}{\sinh(2\pi^2T/\hbar\Omega)}\, J_2\left(\frac{4\pi\Lambda}{\hbar\Omega}\right)e^{-\pi/\Omega\tau_s}\,\cos\left(\frac{2\pi\mu}{\hbar\Omega}\right)\right]. \quad (29)$$

The oscillation of the chemical potential results in another oscillating term in the square brackets. However, this term contains a small exponent $e^{-E_F/T}$ and can be neglected. With the same accuracy, the chemical potential in Eq. (29) can be replaced by the Fermi energy and

$$\sigma = \frac{2e^2\Lambda^2 md\tau_s}{\pi\hbar^4}\left[1-\frac{2\pi^2T/\hbar\Omega}{\sinh(2\pi^2T/\hbar\Omega)} \times J_2\left(\frac{4\pi\Lambda}{\hbar\Omega}\right)e^{-\pi/\Omega\tau_s}\,\cos\left(\frac{2\pi E_F}{\hbar\Omega}\right)\right]. \quad (30)$$

The relaxation time τ_s depends on the chemical potential that oscillates with the magnetic field, Eq. (14). These oscillations lead to another oscillating term in the conductivity. However, it is easy to show that compared to the oscillating term in Eq. (30), it contains a small parameter $\hbar\Omega/E_F$. For the case of a wide miniband, $\Lambda \gg \hbar\Omega$, some numerical calculations of the longitudinal magnetoresistance have been recently made by Datars and Sipe.[17]

B. Strong magnetic field, $\hbar\Omega \gg \Gamma, \Lambda$

In this case, at some regions of magnetic field, the Fermi level is pinned at the localized states between Landau level. The pinning cannot be described with the help of the perturbation theory with respect to scattering. In general, the scattering cannot be considered small even near the center of Landau levels and there is no simple analytic description of the conductivity. So for a strong magnetic field, we just prove that although all states in each well of a superlattice are localized, the tunneling between layers leads to the existence of a mobility edge. Extended states are concentrated near the center of the Landau level and we estimate the maximum value of the conductivity in this region.

A random potential resulting from impurities or from the roughness of the interfaces between quantum wells and barriers leads to the smearing of the Landau minibands and the localization of states at the tails of the minibands. The resistance is maximal when the Fermi level is pinned at localized states between different Landau minibands and minimal when the Fermi level is near the middle of the Landau miniband.

It is important to note that here we mean not the localization of two-dimensional states in each of the quantum wells, but a three-dimensional localization that prevents an electron from tunneling between different quantum wells. The existence of both localized and extended states can be understood from simple physical arguments. In each quantum well, electron wave functions in a magnetic field are confined near equipotential lines of the random potential. The width of the confinement strip around equipotential lines is of the order of $l\sqrt{N}$. The energy of the state deviates from $\hbar\Omega(N+1/2)$ by the value of the random potential. (One has to keep in mind that the relevant random potential equals the original random potential averaged over the cyclotron orbit.[18]) Random potentials in adjacent wells, are not correlated and for the tunneling between adjacent wells, two conditions are necessary. (1) An electron can tunnel from an equipotential line in one well to an equipotential line in an adjacent well in the region of their spatial overlap. (2) Tunneling is an elastic process and is possible only if energies of the initial and final states are different by no more than the overlap integral Λ.

The width of the random potential distribution is of the order of Γ. If $\Lambda \ll \Gamma$, then the area where the random potential definitely takes a value different from a given value by no more than Λ is of the order of $a^2\Gamma/\Lambda$ (a is the correlation length of the random potential). The area of the electron confinement is of the order of $tl\sqrt{N}$, where t is the perimeter of the equipotential line. That is if $tl\sqrt{N} \gtrsim a^2\Gamma/\Lambda$, then the area of the electron confinement in one well definitely overlaps a region in the adjacent well, where tunneling is energetically possible. This condition is a limitation for the perimeter of the equipotential line. It is satisfied for states near the center of the Landau level, where equipotential lines are long. The deviation of the potential very far from its average value is a rare occasion and typical perimeters of corresponding regions are small. So that the probability of electron tunneling between the wells is small and such states are localized.

This consideration shows, by the way, that in the case $\Lambda \ll \Gamma$, not only Landau levels in separate quantum wells are smeared but also that the wave vector in the growth direction is not a good quantum number. Really, the wave vector appears as a result of the periodicity in the growth direction. The lack of correlation of the random

potential in different quantum wells violates the periodicity.

Similar arguments allow us to estimate the conductivity in the center of the band. An electron has a chance to tunnel to the adjacent well after traveling the distance about $a^2\Gamma/l\Lambda\sqrt{N}$ along an equipotential line. The velocity of its motion is the Hall velocity in the random potential averaged along the cyclotron orbit.[19] The value of the average potential is of the order of Γ and the velocity $\sim \Gamma/ma\Omega$. That is, each tunneling event takes time $ma^3\Omega/l\Lambda\sqrt{N} = (a/l)^3\hbar/\Lambda\sqrt{N}$. As a result of one tunneling event, an electron moves a distance of the period of the superlattice d. That is the diffusion coefficient along the growth direction $D \sim (l/a)^3 d^2\Lambda\sqrt{N}/\hbar$. The conductivity $\sigma = e^2 g D$, where g is the density of states. The total number of states in one Landau level per unit volume is $1/2\pi l^2 d$ and an estimate for the density of state is $g \sim 1/l^2 d\Gamma$. The same estimate also results from the self-consistent Born approximation. Combining all these estimates, we get the following value for the maximum conductivity:

$$\sigma \sim \frac{\Lambda}{\Gamma}\,\frac{e^2}{\hbar}\,\frac{dl\sqrt{N}}{a^3}. \tag{31}$$

In the case of $\Lambda \gg \Gamma$, the probability to find a state in one well that is not overlapped in energy with another state in an adjacent well is exponentially small. That is, tails of localized states are very narrow compared to the width of the extended states. The conductivity can be found with the help of the Boltzmann equation for each Landau miniband, which gives for zero temperature

$$\sigma = \frac{2e^2\Lambda^2 m d\tau_s}{\pi\hbar^4}\left[1 - \cos\left(\frac{2\pi E_F}{\hbar\Omega}\right)\right], \tag{32}$$

where E_F is the value of the Fermi energy at zero magnetic field. It is worth noting that this expression is not justified near the edges of Landau minibands, where the ratio $E_F/\hbar\Omega$ is close to an integer.

Comparing these two situations, we see that if $\Lambda \ll \Gamma$, the widths of regions of both extended states near a Landau level, where the longitudinal resistance is minimal, and localized states, where the longitudinal resistance is maximal, are of the order of Γ. In the opposite case, when $\Lambda \gg \Gamma$, the regions of a large resistance are very narrow compared to the regions with a small resistance.

IV. CONCLUSION

In conclusion, we have studied the vertical conductivity of superlattices in a magnetic field applied in the same direction. The electron spectrum in a quantizing magnetic field consists of Landau minibands, which can be smeared by scattering. We found conditions necessary for strong magneto-oscillations of the conductivity. These conditions can be easily satisfied in $GaAs/Al_xGa_{1-x}As$ superlattices, with not very wide minibands. If the miniband width and Γ are about 5 meV, then the gaps between Landau minibands in the spectrum appear for the magnetic field of about 4 T.

The conductivity is calculated in two extreme cases of a weak and a strong magnetic quantization. For a weak quantization, the oscillating part of the conductivity is exponentially small, similar to the Shubnikov–de Haas effect in the 2D electron gas. The behavior of the conductivity in the case of a strong quantization resembles the quantum Hall effect. There exists a percolation threshold in each Landau miniband and the conductivity is controlled by the position of the Fermi level with respect to this threshold. We estimated the energy region around the center of each Landau miniband where electron states are extended and evaluated the maximum values of the conductivity corresponding to the position of the Fermi level in the middle of the extended states region.

ACKNOWLEDGMENT

This research was supported by The Israel Science Foundation administered by The Israel Academy of Sciences and Humanities.

[1] R.C. Ashoori, J.A. Lebens, N.P. Bigelow, and R.H. Silsbee, Phys. Rev. Lett. **64**, 681 (1990).
[2] J.P. Eisenstein, L.N. Pheiffer, and K.W. West, Phys. Rev. Lett. **69**, 3804 (1992); Surf. Sci. **305**, 393 (1994).
[3] H.L. Störmer, J.P. Eisenstein, A.C. Gossard, W. Wiegmann, and K.W. Baldwin, Phys. Rev. Lett. **56**, 85 (1986).
[4] H.L. Störmer, J.P. Eisenstein, A.C. Gossard, K.W. Baldwin, and J.H. English, in *Physics of Semiconductors: Proceedings of the 18th International Conference, Stockholm, 1986*, edited by O. Engström (World Scientific, Singapore, 1987), p. 385.
[5] F. Piazza, L. Pavesi, H. Cruz, M. Micovic, and C. Mendoca, Phys. Rev. B **47**, 4644 (1993).
[6] K.K. Choi, B.F. Levine, N. Jarosik, J. Walker, and R. Malik, Phys. Rev. B **38**, 12 362 (1988).
[7] A. Sibille, J.F. Palmier, A. Celeste, J.C. Portal, and F. Mollot, Europhys. Lett. **13**, 279 (1990).
[8] W. Müller, H.T. Grahn, R.J. Haug, and K. Ploog, Phys. Rev. B **46**, 9800 (1992).
[9] L.B. Keldysh, Zh. Eksp. Teor. Fiz. **47**, 1525 (1964) [Sov. Phys. JETP **20**, 1018 (1965)].
[10] E.M. Lifshits, L.P. Pitaevskii, and L.D. Landau, *Physical Kinetics* (Pergamon Press, Oxford, 1981).
[11] G.D. Mahan, *Many-Particle Physics* (Plenum Press, New York, 1990).
[12] M.Ya. Azbel, Phys. Rev. B **26**, 3430 (1982).
[13] B. Laikhtman and E.L. Altshuler, Ann. Phys. **232**, 332 (1994).
[14] T. Ando and Y. Uemura, J. Phys. Soc. Jpn. **36**, 959 (1974).
[15] T. Ando, J. Phys. Soc. Jpn. **37**, 1233 (1974).
[16] I.B. Levinson, A.Yu. Matulis, and L.M. Shcherbakov, Zh. Eksp. Teor. Fiz. **60**, 859 (1971) [Sov. Phys. JETP **33**, 464 (1971)].
[17] A.E. Datars and J.E. Sipe, Phys. Rev. B **51**, 4312 (1995).
[18] M.E. Raikh and T.V. Shahbazyan, Phys. Rev. B **47**, 1552 (1993).
[19] B. Laikhtman, Phys. Rev. Lett. **72**, 1060 (1994).