# Persistent currents induced by spin-orbit coupling in one-dimensional mesoscopic rings

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The effect of spin-orbit (SO) coupling of the electrons moving in a nonsymmetrical confining potential on the persistent currents in one-dimensional mesoscopic rings is investigated. Our results show that a momentum-dependent effective magnetic field could be induced by the spin-orbit coupling. This leads to a persistent-current contribution. The direction and the amplitude of this current depend on the parameters of SO coupling. An interesting result is that the dependence of this current on the coupling parameters has the steplike characteristics. The condition of adiabatic approximation is also discussed.

The persistent equilibrium current occurring in an isolated mesoscopic ring threaded axially by Aharonov-Bohm (AB) flux was predicted by Büttiker, Imry, and Landauer[1] in 1983, and was demonstrated in three recent experiments.[2–4] Although the phenomenon of persistent current has been extensively studied, a satisfactory explanation for the experimental results has so far not been found. It is generally believed that the amplitudes of current predicted by theoretical models are smaller than the experimental values. The large discrepancy between the measured current amplitude and theories has introduced many open questions. The effect due to spin-orbit (SO) coupling is one of questions in the field. It has been widely accepted that spin-orbit interaction may play an important role in persistent current, for example in the sign and period of the currents.[2,5,6] The magnetoresistance and Berry's phase induced by spin-orbit coupling in low-dimensional or lowered symmetry conductors were discussed by Aronov and Lyanda-Geller,[7] and the latter author also predicted striking topological transitions in Berry's phase interference effects.[8]

Recently, Zhou, Li, and Xue have shown that Aronov's Hamiltonian is not Hermitian, and introduced a more realistic model by adding a confining potential such that one-dimensional (1D) effects are phenomenologically taken into account,[9] thus giving a Hermitian Hamiltonian model for electrons confined in a one-dimensional ring, and a spin-orbit coupling that can be induced by the confining potential. Of course, the persistent current determinated by this Hamiltonian is an interesting question. In addition, an adiabatic approximation is made in Ref. 7, and Stern has obtained a precise criterion to decide whether this approximation is valid.[10] Stern considers an electron with a magnetic moment $\mu$ subject to a magnetic field $B$ in a quasi-one-dimensional ring of the radius $a$. For the electrons at the Fermi level, the condition for the adiabatic approximation is $\mu Ba/V_F \gg 1$, where $V_F$ is the Fermi velocity. In the case discussed in Ref. 7, the adiabatic condition is equivalent to $\omega_0$, $\omega_s \gg 2\omega l$.[11] For the sake of convenience, here we use the notations of Ref. 7, namely $\omega = \hbar/2ma^2$, where $\omega_s$ is the Larmor frequency; $\omega_0 = \hbar\beta/2a$, where $\hbar\beta^2$ is the spin-orbit coupling coefficient in the ring, and $l = n - \frac{1}{2} + \phi/\phi_0 \gg 1$. Estimations[8] show that for an InAs ring of 5-$\mu$m radius and 60-nm width ($m = 0.023 m_0$, $g = 15$, the spin-orbit coefficient $\hbar\beta^2 = 6.0\times 10^{-10}$ eV cm $= 9.6\times 10^{-31}$ J M, the 2D electron density is taken as $N_e = 10^{12}$ cm$^{-2}$, and $E_f = 0.1$ eV), the values of $\omega$, $\omega_s$, and $\omega_0$ are as follows: $\omega \approx 1.0\times 10^8$ Hz, $\omega_0 \approx 9.14\times 10^8$ Hz, and $\omega_s \approx 1.32\times 10^{10}$ Hz (for $g = 15$ and $B = 100$ Gs). Therefore, $\omega_0/2\omega l \approx 4.57/l \ll 1$ and $\omega_s/2\omega l \approx 66.0/l \ll 1$ [because of $l \approx \sqrt{E_f/\hbar\omega} \approx 10^3$ (Ref. 11)]. Thus the adiabatic approximation is invalid in many normal metal or semiconductor rings.

Reference 12 presents an exact solution by diagonalizing a Hamiltonian which is similar to Aronov's Hamiltonian in Ref. 7 using the well-known Bogoliubov transformation. In this paper we obtain an exact solution of the energy eigenvalue problem and persistent current in the system whose Hamiltonian was derived in Ref. 9. Our results show that the spin-orbit interaction makes important contributions to the persistent current. At zero temperature, the persistent currents are the sum of parts $I_{SO}$ induced by spin-orbit coupling and $I_{AB}$ induced by the $A$-$B$ magnetic flux. The results also show that the direction of $I_{SO}$ depends on the parameter of the SO coupling, hence the SO coupling can increase or decrease the total currents. The most interesting result is that $I_{SO}$ varies steplike with the parameters $\omega_1$ and $\omega_2$ increasing, where $\omega_1$ and $\omega_2$ are parameters which express the strength of the spin-orbit coupling. This steplike characteristic of $I_{SO}$ versus $\omega_1$ and $\omega_2$ has a close relationship with the topological transitions predicted in Ref. 8.

We start from the Hamiltonian derived in Ref. 9, the Hamiltonian for a single electron of mass $m$, charge $-e$, and spin $\frac{1}{2}$, as given by

$$\hat{h} = \hbar\omega(-i\partial/\partial\theta + \phi/\phi_0)^2 + \hbar\omega_1(\sigma_x\cos\theta + \sigma_y\sin\theta)(-i\partial/\partial\theta + \phi/\phi_0) + \frac{i\hbar\omega_1}{2}(\sigma_x\sin\theta - \sigma_y\cos\theta) + \hbar\omega_2\sigma_z\left[-i\frac{\partial}{\partial\theta} + \frac{\phi}{\phi_0}\right]. \quad (1)$$

Here $\hbar\omega_1 = \hbar^2\alpha_1/4m^2c^2r$, $\hbar\omega_2 = \hbar^2\alpha_2/4m^2c^2r$, $\alpha_1 = (a' - b')U_0$, and $\alpha_2 = (a - b)U_0$. Using the same approaches as in Ref. 12, Hamiltonian (2) in the second quantization representation is given by

 

$$\hat{H}=\sum_{n\sigma} E_{n\sigma}C^{\dagger}_{n\sigma}C_{n\sigma} + \sum_{n} \Delta_n (C^{\dagger}_{n+}C_{(n+1)-} + C^{\dagger}_{(n+1)-}C_{n+}) , \quad (2)$$

where

$$E_{n\sigma}=\hbar\omega(n+\phi/\phi_0)^2+\sigma\hbar\omega_2(n+\phi/\phi_0)-\mu_{\text{che}} \quad (\sigma=\pm 1) , \quad (3)$$

$$\Delta_n=\hbar\omega_1(n+\tfrac{1}{2}+\phi/\phi_0) . \quad (4)$$

Hamiltonian (3) can be diagonalized by the well-known Bogoliubov transformation

$$\alpha_n=U_nC_{n+}-V_nC_{(n+1)-} , \quad (5)$$

$$\beta_n=V_nC_{n+}+U_nC_{(n+1)-} . \quad (6)$$

The results are

$$\hat{H}=\sum_m A_m\alpha^{+}_m\alpha_m+\sum_n B_n\beta^{+}_n\beta_n , \quad (7)$$

where

$$A_m=(E_{m+}+E_{(m+1)-})/2+\sqrt{g_m^2+\Delta_m^2} ,$$
$$B_n=(E_{m+}+E_{(m+1)-})/2-\sqrt{g_n^2+\Delta_n^2} , \quad (8)$$

and $g_n=(E_{(n+1)-}-E_{n+})/2$. At $T=0$, the total persistent current $I$ is obtained by

$$I=-c\sum_n \partial E_n/\partial\phi=-c\left[\sum_m \partial A_m/\partial\phi+\sum_n \partial B_n/\partial\phi\right] . \quad (9)$$

The summation is taken with energies less than $\mu_{\text{che}}$. At finite temperatures, one can calculate the current from the thermodynamical potential of the system:[13]

$$I=-c\partial F/\partial\phi . \quad (10)$$

Using the diagonalized Hamiltonian (7), we find

$$F=-\frac{1}{\beta}\left\{\sum_m \ln[1+\exp(-\beta A_m)] + \sum_n \ln[1+\exp(-\beta B_n)]\right\} , \quad (11)$$

where $\beta=1/k_BT$, so the persistent current is given by

$$I=-c\left\{\sum_m \frac{\partial A_m/\partial\phi}{1+\exp(-\beta A_m)}+\sum_n \frac{\partial B_n/\partial\phi}{1+\exp(-\beta B_n)}\right\} , \quad (12)$$

with

$$-\partial A_m/\partial\phi=i_{A(m)}+\Delta i_{A(m)} ,$$
$$-\partial B_n/\partial\phi=i_{B(n)}+\Delta i_{B(n)} . \quad (13)$$

Here

$$i_{A(n)}=i_{B(n)}=-2(\hbar\omega/\phi_0)(n+\tfrac{1}{2}+\phi/\phi_0) ,$$
$$\Delta i_{A(n)}=-\Delta i_{B(n)}=-\hbar[(\omega-\omega_2)g_n+\omega_1\Delta_n]/\phi_0\sqrt{g_n^2+\Delta_n^2} . \quad (14)$$

From Eqs. (9), (12), (13), and (14), we can obtain the persistent currents at zero and nonzero temperatures. For the sake of simplicity we will mainly discuss the properties of the current at zero temperature.

At $T=0$, the total persistent current is given as a sum of $I_{AB}$ induced by $A$-$B$ flux and $I_{\text{SO}}$ induced by the effective flux of the spin-orbit coupling, i.e.,

$$I=I_{AB}(\phi)+I_{\text{SO}}(\omega_1,\omega_2) , \quad (15)$$

where

$$I_{AB}(\phi)=\sum_m i_{A(m)}+\sum_n i_{B(n)} , \quad (16)$$

$$I_{\text{SO}}(\omega_1,\omega_2)=\sum_m \Delta i_{A(m)}+\sum_n \Delta i_{B(n)} . \quad (17)$$

It is easy to confirm that the current $I_{AB}$ is persistent current induced by a pure $A$-$B$ flux, because the current $I_{AB}$ is independent of SO coupling parameters $\omega_1$ and $\omega_2$. Let $\kappa=\Delta_n/g_n$, obtained by

$$\kappa=\omega_1/(\omega-\omega_2) \quad (18)$$

and

$$\Delta i=-\Delta i_{A(n)}=\Delta_{B(n)}=(\hbar\omega_1/\phi_0\kappa)\sqrt{1+\kappa^2} . \quad (19)$$

A striking feature of the results is that $\Delta i_{A(n)}$ and $\Delta i_{B(n)}$ are independent of the energy quantum number $n$ and the magnetic flux. Therefore the current $I_{\text{SO}}$ is a persistent current induced by the effective flux. In the following, we give the computation results and some discussions at $T=0$.

Our numerical calculations are shown in Figs. 1–3. The results show that persistent currents induced by spin-orbit coupling vary steplike with the spin-orbit coupling, and that the direction of $I_{\text{SO}}$ depends on the value of $\omega_2$. In the case of $\omega_2<\omega$, the spin-orbit coupling reduces the total persistent currents; conversely, the spin-orbit coupling enhances total persistent currents at $\omega_2>\omega$; i.e., $\omega_2=\omega$ is a critical point, and the critical point does not depend on $\omega_1$.

Figure 1 shows the dependence of the persistent current $I_{\text{SO}}$ on $\omega_1/\omega$ and $\omega_2/\omega$. It is shown that $I_{\text{SO}}$ is an even function of $\omega_1/\omega$. The direction of $I_{\text{SO}}$ has a critical point at the value of $\omega_2=\omega$. If we assume $I_{\text{SO}}$ positive at $\omega_2<\omega$, then $I_{\text{SO}}$ becomes negative at $\omega_2>\omega$. $I_{\text{SO}}$ remains constant when the total electron number $N_e$ increases. $I_{\text{SO}}$ varies differently with $k$, where $N_e=4m+k$ ($k=0$, 1, 2, and 3), and $m$ is an integral.

Figure 2 shows the dependence of $I/I_{AB}$ on $\omega_1/\omega$ at $\omega_2=2.0\omega$ and $\omega_2=-0.5\omega$, respectively. $I/I_{AB}$ is also an even function of $\omega_1/\omega$. The spin-orbit coupling increases or decreases the total persistent currents, depending on the value of $\omega_2$. The value of $I/I_{AB}$ is always smaller than 1 for $\omega_2=-0.5\omega$. This indicates that the spin-orbit effect always reduces the total persistent currents. Conversely, the value of $I/I_{AB}$ is always larger than 1 at $\omega_2=2.0\omega$, which shows that the spin-orbit effect always enhances the total persistent currents. This conclusion is more clearly shown in Fig. 3. The amplitude of $I/I_{AB}$ decreases with the increasing of the electronic number $N_e$, and $I/I_{AB}$ is proportional to $1/N_e$.

Figure 3 shows the dependence of $I/I_{AB}$ on $\omega_2/\omega$ at $\omega_1=2.5\omega$. At $\omega_2<\omega$, the value of $I/I_{AB}$ is always smaller than 1. This means that the direction of $I_{\text{SO}}$ is opposite to that of $I_{AB}$. The value of $I/I_{AB}$ is always larger than 1 at $\omega_2>\omega$. This means that the direction of $I_{\text{SO}}$ is the same as that of the persistent currents induced by the $A$-$B$ flux.

In the following, we study the influence of the spin-

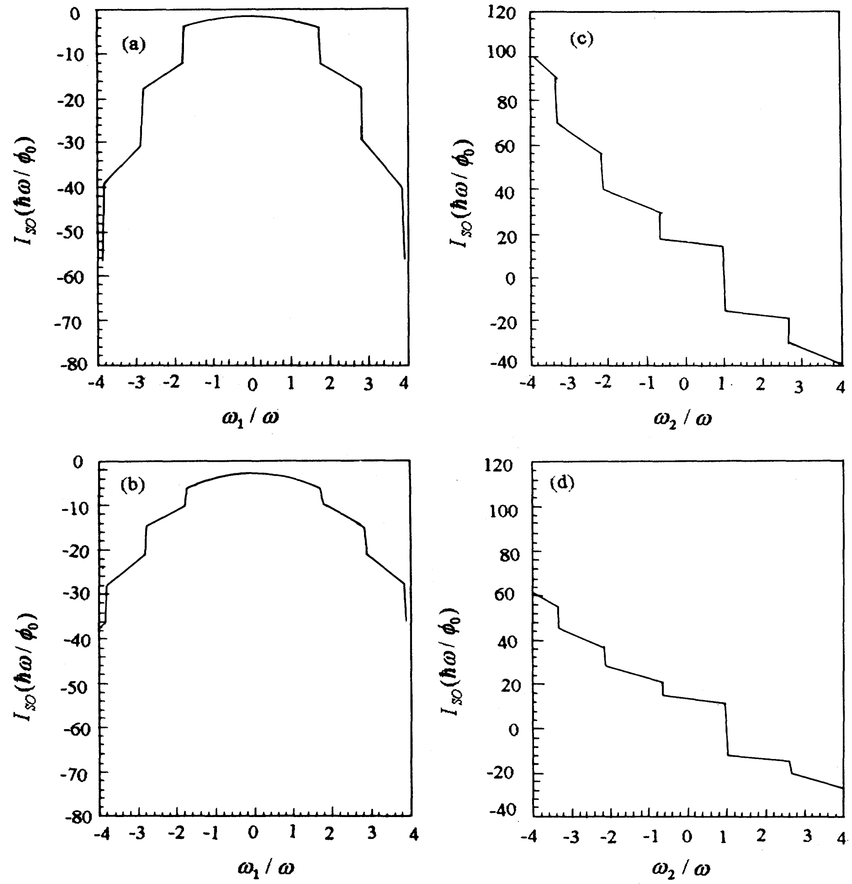


FIG. 1. The dependence of persistent current $I_{SO}$ on parameters of the spin-orbit coupling at $T=0$. (a) $I_{SO}-\omega_1$ at $\omega_2=2.0\omega$ and $N_e=4m+4$, (b) $I_{SO}-\omega_1$ at $\omega_2=2.0\omega$ and $N_e=4m+3$, (c) $I_{SO}-\omega_2$ at $\omega_1=2.5\omega$ and $N_e=4m+4$, (d) $I_{SO}-\omega_2$ at $\omega_1=2.5\omega$ and $N_e=4m+3$. $I_{SO}$ depends on the value of $m$.

orbit coupling on the energy state. If we assume that $n=m+j$, where $j$ is an integral number, then we have

$$A_m-B_{m+j}=2\hbar(|\omega-\omega_2|\sqrt{1+\kappa^2}-\omega j)\times[m+(1+j)/2+\phi/\phi_0] . \quad (20)$$

So we know, when $\omega_1$ and $\omega_2$ satisfy $A_m-B_m\geq 0$ and $A_m-B_{m+1}\leq 0$, that the energy-level sequence is and $A_m-B_{m+2}\leq 0$, the energy-level sequence is $A_m-B_{m+j}\geq 0$ and $A_m-B_{m+j+1}\leq 0$, the energy-level sequence is $\ldots A_m\geq B_{m+j}\geq A_{m-1}\geq B_{m+j-1}\cdot\ldots$

In fact, if the energy-level order varies with the coefficients $\omega_1$ and $\omega_2$, this variance occurs when the effective flux changes a flux quantum $\phi_0=\hbar c/e$. Then the wave function must change by a phase factor $2\pi$. This corresponds to a transition of an energy state $A_n$ with a wave vector $k$ into its neighboring state $|k|+2\pi/L$, and at the same time an energy state $B_n$ with wave vector $k$ changes into its neighboring state $|k|-2\pi/L$, where $L$ is the circumference of the ring. The change of the highest occupied state corresponds to the current change by

$$\Delta I=\begin{cases}4\Delta i & \text{at } N_e=4m \text{ or } 4m+2 ,\\ 2\Delta i & \text{at } N_e=4m+1 \text{ or } 4m+3 .\end{cases} \quad (21)$$

The factor 4 comes from every energy state occupied by two electrons at $N_e=4m+2$ or $4m$. But the highest occupied state is a single occupied state at $N_e=4m+1$ or $4m+3$, so the factor is 2. This is the reason why persistent currents are increased steplike in Figs. 1, 2, and 3; i.e., such a steplike characteristic comes from Eq. (21). The $j$th step happens when $A_m-B_{m+j}=0$, namely $j=|1-\omega_2/\omega|\sqrt{1+\kappa^2}$, and the current amplitude change is $4\Delta i$ or $2\Delta i$, depending on the value of $k$ (where $N_e=4m+k$).

Now we discuss the condition for the adiabatic approximation in our system. We have shown that the condition for the adiabatic approximation in Ref. 7 is $\omega_0$, $\omega_s\gg 2\omega l$, and the condition is not satisfied in normal metals and semiconductors such as InSa. One of the important differences between Hamiltonian (3) in Ref. 7 and (7) in Ref. 9 is that there is the spin $z$-component $\sigma_z$-orbit coupling term in Ref. 9. The existence of this term means that the adiabatic approximation is more easily satisfied than in Ref. 7. Our results show that one can use the adiabatic approximation to analyze Berry's phase with the Hamiltonian in Ref. 9 in many metal and semiconducting rings such as InSa.

In the Fermi level $n\gg 1$, we can neglect the term $(i\hbar\omega_1/2)(\sigma_x\sin\theta-\sigma_y\cos\theta)$ in Hamiltonian (1). Then the operator $-i(\partial/\partial\theta)$, except in the first term is replaced by a number $n$ due to the Hermitian of Hamiltonian (1). Therefore, Hamiltonian (1) becomes[14]

$$\hat{H}=\hbar\omega(-i\partial/\partial\theta+\phi/\phi_0)^2+\hbar\omega_B\mathbf{n}_{(\phi)}\cdot\boldsymbol{\sigma} , \quad (22)$$

where

$$\omega_B=\sqrt{\omega_1^2+\omega_2^2}(n+\phi/\phi_0) , \quad (23)$$

$$\mathbf{n}_{(\phi)}=(\sin\chi\cos\theta,\sin\chi\sin\theta,\cos\chi) , \quad (24)$$

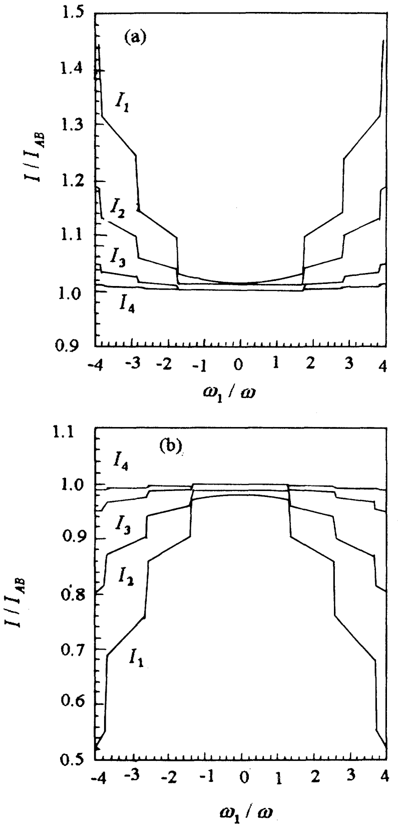


FIG. 2. The dependence of $I/I_{AB}$ on $\omega_1/\omega$ at $\phi/\phi_0=0.25$, $T=0$, $N_e=4m+4$, and (a) $\omega_2=2.0\omega$ and (b) $\omega_2=-0.5\omega$. $I_1$, $I_2$, $I_3$, and $I_4$ express the currents at $m=20$, 50, 200, and 1000, respectively.

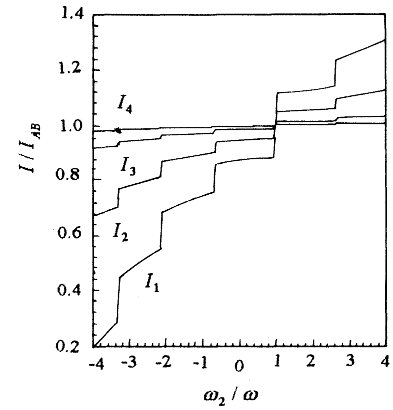


FIG. 3. The dependence of $I/I_{AB}$ on $\omega_2/\omega$ at $\phi/\phi_0=0.25$, $T=0$, $N_e=4m+4$, and $\omega_1=2.5\omega$. $I_1$, $I_2$, $I_3$, and $I_4$ express the currents at $m=20$, 50, 200, and 1000, respectively.

and

$$\sin\chi=\omega_1/\sqrt{\omega_1^2+\omega_2^2}\ ,\quad \cos\chi=\omega_2/\sqrt{\omega_1^2+\omega_2^2}\ .$$

The effective Hamiltonian (22) shows that the nonsymmetric confining potential is equivalent to a momentum-dependent effective magnetic field with a magnitude $B_{\text{eff}}=[\hbar\sqrt{\omega_1^2+\omega_2^2}(n+\phi/\phi_0)]/\mu$ ($\mu$ is the magnetic moment of electron) and a direction angle $\chi$ which describes the deviation of the effective magnetic field from the $Z$ axis. Hence it is not unexpected that this spin-orbit interaction induced by a nonsymmetrical confining potential would provide an effective flux which in turn produces the persistent current in Eq. (17).

Reference 10 gives the adiabatic criterion as $\mu Ba/V_f \gg 1$ ($\hbar=1$). In our system, this is equivalent to $\mu a B_{\text{eff}}/V_f \gg 1$, i.e.,

$$[\sqrt{\omega_1^2+\omega_2^2}(n+\phi/\phi_0)a/V_f] \gg 1 \tag{25}$$

for a free electron, $\hbar\omega[n+(\phi/\phi_0)]^2 \approx V_f^2/2m$, so we can rewrite condition (25) with the inequality

$$\sqrt{\omega_1^2+\omega_2^2}/2\omega \gg 1\ , \tag{26}$$

just as in previous adiabatic analyses we take $\omega_1=\omega_2=\omega_0\approx 9.14\times 10^8$ Hz and $\omega\approx 1.0\times 10^8$ Hz [$\omega_0=\hbar\beta/2a$ in Ref. 7]. In InSa, the adiabatic condition is approximately satisfied because $(\sqrt{\omega_1^2+\omega_2^2})/2\omega \approx 6.4 \gg 1$. This condition for the adiabatic approximation is very different from that in Ref. 7.

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[14]The grand-canonical ensemble Hamiltonian of $N$ electrons in a mesoscopic normal metal ring which is placed in a texture-structure inhomogeneous magnetic field is (Ref. 12)

$$\hat{H}=\sum_{i=1}^{N}[1/2ma^2(P_\theta+eaA_\theta)^2-\gamma B\mathbf{n}_{(\phi)}\cdot\sigma]-\mu_{\text{che}}\hat{N}\ . \tag{A}$$

It can be rewritten as [the Hamiltonian (8) in Ref. 12]

$$\hat{H}=\sum_{n\sigma}E_{n\sigma}C_{n\sigma}^\dagger C_{n\sigma}+\sum_n \Delta(C_{n+}^\dagger C_{(n+1)-}+C_{(n+1)-}^\dagger C_{n+})\ , \tag{B}$$

where

$$E_{n\sigma}=E_n+\sigma\gamma B\cos\chi-\mu_{\text{che}}\quad(\sigma=\pm1)\ , \tag{C}$$

$$\Delta=\gamma B\sin\chi\ . \tag{D}$$

Comparing with (2), (3), and (4), if we assume

$$\gamma B\cos\chi=\hbar\omega_2(n+\phi/\phi_0)\ , \tag{E}$$

$$\gamma B\sin\chi=\hbar\omega_1(n+\tfrac{1}{2}+\phi/\phi_0)\approx\hbar\omega_1(n+\phi/\phi_0)\ , \tag{F}$$

then Hamiltonian (1) can be rewritten as (22), which has a similar formulation to Hamiltonian (A). So we have taken the approximation $[n+\frac{1}{2}+(\phi/\phi_0)]\approx[n+(\phi/\phi_0)]$ in obtaining the approximate Hamiltonian (22).