Ginzburg-Landau equations for a *d*-wave superconductor with applications to vortex structure and surface problems

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The properties of a $d_{x^2-y^2}$-wave superconductor in an external magnetic field are investigated on the basis of Gorkov's theory of weakly coupled superconductors. The Ginzburg-Landau (GL) equations, which govern the spatial variations of the order parameter and the supercurrent, are microscopically derived. The single vortex structure and surface problems in such a superconductor are studied using these equations. It is shown that the *d*-wave vortex structure is very different from the conventional *s*-wave vortex: the *s*-wave and *d*-wave components, with the opposite winding numbers, are found to coexist in the region near the vortex core. The supercurrent and local magnetic field around the vortex are calculated. Far away from the vortex core, both of them exhibit a fourfold symmetry, in contrast to an *s*-wave superconductor. The surface problem in a *d*-wave superconductor is also studied by solving the GL equations. The total order parameter near the surface is always a *real* combination of *s*- and *d*-wave components, which means that the proximity effect cannot induce a time-reversal symmetry-breaking state at the surface.

I. INTRODUCTION

Recently, there have been a number of experiments designed to directly probe the pairing state in high-temperature superconductors.[1-4] Wollman *et al.*[1] measured the field-modulated critical currents of corner superconducting quantum interference devices (SQUID's) and junctions, which combine an *s*-wave superconductor with a Y-Ba-Cu-O single crystal. Their results provide a strong evidence for a π-phase shift in the Josephson coupling energy predicted for a *d*-wave pairing state. Mathai *et al.*[2] performed a similar experiment on Y-Ba-Cu-O-Ag-Pb SQUID's using a scanning SQUID microscope, and they claimed that their results provide unambiguous evidence for a $d_{x^2-y^2}$ symmetric order parameter. Tsuei *et al.*[3] used the concept of flux quantization in a tricrystal superconducting Y-Ba-Cu-O ring with grain-boundary Josephson junctions to determine the pairing symmetry. They observed spontaneous magnetization of half a flux quantum, consistent with *d*-wave pairing symmetry. Miller *et al.*[4] proposed a new method of probing the pairing symmetry by measuring the field-modulated critical current of tricrystal devices. Their results in the short junction limit indicate a clear phase shift in the Josephson coupling, suggesting a predominantly *d*-wave pairing symmetry. In short, these recent experiments that directly probe the pairing symmetry of high-T_c superconductors seem to favor a *d*-wave pairing state. Theoretically, it is also suggested[5] that the high-T_c superconductors might possess unconventional pairing symmetry.

In this work, we address the problem of how a *d*-wave superconductor differs from an *s*-wave one. We restrict ourselves to the Ginzburg-Landau (GL) region and consider spatial variations of the *d*-wave order parameter and supercurrent governed by the GL equations. Following the standard procedure for the conventional *s*-wave superconductors, the GL equations are microscopically derived for a *d*-wave superconductor. Based on these GL equations, we study the structure of *d*-wave vortices. It is expected that the structure of a *d*-wave vortex is very different from that of *s* wave[6,7] or *p* wave.[8] We show that the qualitative feature of a single vortex structure in a *d*-wave superconductor can be determined analytically. The *s*-wave and *d*-wave components, with the opposite winding numbers, are found to coexist in the region near the vortex core. Furthermore, the *d*-wave component varies linearly with the distance *r* from the vortex core as $r \rightarrow 0$ and goes to the pure *d*-wave bulk value for large *r*. On the other hand, the induced *s*-wave component has a linear-*r* dependence for small *r* but decays as r^{-2} when *r* is large. The main feature of our results agrees with the suggestion of Volovik[9] and recent numerical calculation of Soininen, Kallin, and Berlinsky.[10] The supercurrent and local magnetic field around the vortex are also calculated analytically. Far away from the vortex core, both of them are found to exhibit a fourfold symmetry, in contrast to an *s*-wave superconductor.

We also study the surface problem in a *d*-wave superconductor by solving the GL equations and find that a small *s*-wave component is induced near the surface. The total order parameter near surface is always a *real* combination of *s*- and *d*-wave components and their relative phase is determined by sign$[-\cos(2\theta)]$, where θ is the angle between *a* axis and the normal direction of the surface. This result suggests that the proximity effect cannot induce a time-reversal symmetry-breaking state at the interface.

In Sec. II, starting from Gorkov's theory of weakly coupled superconductors,[11] the equation for the general gap function is derived, and from which the GL equations are obtained in Sec. III. In Sec. IV, we derive the

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other GL equation for the supercurrent. In Sec. V, we discuss the qualitative features of a single d-wave vortex and using the GL equations. In Sec. VI, we present the result for the supercurrent and local magnetic field around a d-wave vortex. In Sec. VII, we study the proximity effect at the surface of a d-wave superconductor. Section VIII includes conclusion and discussions.

II. GAP EQUATIONS

In this section we shall derive the gap equation for the d-wave order parameter defined through

$$\Delta^*(\mathbf{x},\mathbf{x}')=V(\mathbf{x}-\mathbf{x}')T\sum_{\omega_n}F^{\dagger}(\mathbf{x},\mathbf{x}',\omega_n)\ , \tag{2.1}$$

which allows for more general than conventional s-wave pairing. $V(\mathbf{x}-\mathbf{x}')$ is the effective two-body interaction of the weak-coupling theory. Using Gorkov's[11] description of weakly coupled superconductors it is straightforward to derive the equations of motion for the normal and anomalous Green's functions

$$\left[i\omega_n-\frac{1}{2m}(-i\nabla+e\,\mathbf{A})^2+\mu\right]\widetilde{G}(\mathbf{x},\mathbf{x}',\omega_n)+\int d\mathbf{x}''\Delta(\mathbf{x},\mathbf{x}'')F^{+}(\mathbf{x}'',\mathbf{x}',\omega_n)=\delta(\mathbf{x}-\mathbf{x}')\ , \tag{2.2}$$

$$\left[-i\omega_n-\frac{1}{2m}(i\nabla+e\,\mathbf{A})^2+\mu\right]F^{+}(\mathbf{x},\mathbf{x}',\omega_n)+\int d\mathbf{x}''\Delta^*(\mathbf{x},\mathbf{x}'')\widetilde{G}(\mathbf{x}'',\mathbf{x}',\omega_n)=0\ , \tag{2.3}$$

where μ is the Fermi energy and $\mathbf{A}$ is the vector potential. We define the normal-state Green's function in a magnetic field as

$$\widetilde{G}_0(\mathbf{x},\mathbf{x}',\omega_n)=\left[i\omega_n-\frac{1}{2m}(-i\nabla+e\,\mathbf{A})^2+\mu\right]^{-1}\delta(\mathbf{x}-\mathbf{x}')\approx G_0(\mathbf{x},\mathbf{x}',\omega_n)e^{-ie\,\mathbf{A}(x)\cdot(\mathbf{x}-\mathbf{x}')}\ . \tag{2.4}$$

In the last step of the above equation, we have used the slow variation condition for the magnetic field, i.e., $1/k_F \ll \lambda$, where λ is the London penetration depth of the magnetic field. G_0 in the above equation is the free-electron Green's function in zero field:

$$G_0(\mathbf{x},\omega_n)=\left[i\omega_n+\frac{\nabla^2}{2m}+\mu\right]^{-1}\delta(\mathbf{x})=\frac{1}{(2\pi)^2}\int d\mathbf{k}e^{i\mathbf{k}\cdot\mathbf{x}}\frac{1}{i\omega_n-\xi_{\mathbf{k}}}\ , \tag{2.5}$$

where $\xi_{\mathbf{k}}=\mathbf{k}^2/2m-\mu$ is the single-particle energy with mass m measured from the Fermi energy μ. Using $\widetilde{G}_0$ and by iteration, Eqs. (2.2) and (2.3) can be rewritten in the form

$$\widetilde{G}(\mathbf{x},\mathbf{y},\omega_n)=\widetilde{G}_0(\mathbf{x},\mathbf{y},\omega_n)-\int d\mathbf{x}'d\mathbf{x}''\Delta(\mathbf{x}',\mathbf{x}'')\widetilde{G}_0(\mathbf{x},\mathbf{x}',\omega_n)\times\left[\int d\mathbf{x}_3d\mathbf{x}_4\widetilde{G}_0(\mathbf{x}_3,\mathbf{x}'',-\omega_n)\Delta^*(\mathbf{x}_3,\mathbf{x}_4)\widetilde{G}(\mathbf{x}_4,\mathbf{y},\omega_n)\right]\ , \tag{2.6}$$

$$F^{\dagger}(\mathbf{x},\mathbf{y},\omega_n)=\int d\mathbf{x}'d\mathbf{x}''\widetilde{G}_0(\mathbf{x}',\mathbf{x},-\omega_n)\Delta^*(\mathbf{x}',\mathbf{x}'')\times\left[\widetilde{G}_0(\mathbf{x}'',\mathbf{y},\omega_n)-\int d\mathbf{x}_1d\mathbf{x}_2\widetilde{G}_0(\mathbf{x}'',\mathbf{x}_1,\omega_n)\Delta(\mathbf{x}_1,\mathbf{x}_2)F^{\dagger}(\mathbf{x}_2,\mathbf{y},\omega_n)\right]\ . \tag{2.7}$$

Substituting Eqs. (2.6) and (2.7) into (2.1), and keeping up to third order in Δ, we have

$$\Delta^*(\mathbf{x},\mathbf{y})=\Delta_I^*(\mathbf{x},\mathbf{y})+\Delta_{II}^*(\mathbf{x},\mathbf{y})\ , \tag{2.8}$$

where

$$\Delta_I^*(\mathbf{x},\mathbf{y})=V(\mathbf{x}-\mathbf{y})T\sum_{\omega_n}\int d\mathbf{x}'d\mathbf{x}''\widetilde{G}_0(\mathbf{x}',\mathbf{x},-\omega_n)\Delta^*(\mathbf{x}',\mathbf{x}'')\widetilde{G}_0(\mathbf{x}'',\mathbf{y},\omega_n)\ , \tag{2.9}$$

$$\Delta_{II}^*(\mathbf{x},\mathbf{y})=-V(\mathbf{x}-\mathbf{y})T\sum_{\omega_n}\int d\mathbf{x}'d\mathbf{x}''d\mathbf{x}_1d\mathbf{x}_2d\mathbf{x}_3d\mathbf{x}_4\widetilde{G}_0(\mathbf{x}',\mathbf{x},-\omega_n)\Delta^*(\mathbf{x}',\mathbf{x}'')\times\widetilde{G}_0(\mathbf{x}'',\mathbf{x}_1,\omega_n)\Delta(\mathbf{x}_1,\mathbf{x}_2)\widetilde{G}_0(\mathbf{x}_3,\mathbf{x}_2,-\omega_n)\Delta^*(\mathbf{x}_3,\mathbf{x}_4)\widetilde{G}_0(\mathbf{x}_4,\mathbf{y},\omega_n)\ . \tag{2.10}$$

In the following calculation, we use the cylindrical Fermi surface to relate the physics to high-T_c superconductors. Using the approximation

$$\Delta^*(\mathbf{x}',\mathbf{x}'')\approx e^{\int_x^{x'}\nabla_x\cdot dl+\int_y^{x''}\nabla_y dl}\Delta^*(\mathbf{x},\mathbf{y})\ , \tag{2.11}$$

and introducing central-mass coordinates

$$\mathbf{R}=\frac{1}{2}(\mathbf{x}+\mathbf{y}),\quad \mathbf{R}'=\frac{1}{2}(\mathbf{x}'+\mathbf{x}'')\ ,$$

and relative coordinates

$$\mathbf{r}=\mathbf{x}-\mathbf{y}, \quad \mathbf{r}'=\mathbf{x}'-\mathbf{x}'' ,$$

Eq. (2.9) becomes

$$\Delta_I^*(\mathbf{R},\mathbf{r})=V(\mathbf{r})\int d\mathbf{R}d\mathbf{r}'T\sum_{\omega_n} G_0\left[\mathbf{R}'+\frac{\mathbf{r}'}{2}-\mathbf{R}-\frac{\mathbf{r}}{2},-\omega_n\right]G_0\left[\mathbf{R}'-\frac{\mathbf{r}'}{2}-\mathbf{R}+\frac{\mathbf{r}}{2},\omega_n\right]$$
$$\times\exp[i(\mathbf{R}'-\mathbf{R})\cdot(-i\nabla_R-2e\,\mathbf{A}_R)+i(\mathbf{r}'-\mathbf{r})\cdot(-i\nabla_{\mathbf{r}})]\Delta^*(\mathbf{R},\mathbf{r}) . \tag{2.12}$$

In the above equation we have assumed the slow variation of magnetic field $\mathbf{A}(\mathbf{x})\approx\mathbf{A}(\mathbf{y})\approx\mathbf{A}_{\mathbf{R}}$ or that the magnetic field acts only on the center of mass of the Cooper pairs, not on their relative coordinates. Introducing the operator

$$\mathbf{\Pi}=-i\nabla_R-2e\,\mathbf{A}_R , \tag{2.13}$$

and performing the Fourier transform with respect to the relative coordinate, we obtain

$$\Delta_I^*(\mathbf{R},\mathbf{k})=T\sum_{\omega_n}\int d\mathbf{r}\,e^{-i\mathbf{k}\cdot\mathbf{r}}V(\mathbf{r})\int d\mathbf{R}'d\mathbf{r}'\int\frac{d\mathbf{p}\,d\mathbf{q}\,d\mathbf{k}'}{(2\pi)^6}\exp\left[i\mathbf{p}\cdot\left[\mathbf{R}'-\mathbf{R}+\frac{\mathbf{r}'}{2}-\frac{\mathbf{r}}{2}\right]+i\mathbf{q}\cdot\left[\mathbf{R}'-\mathbf{R}-\frac{\mathbf{r}'}{2}+\frac{\mathbf{r}}{2}\right]\right]$$
$$\times\frac{1}{-i\omega_n-\xi_{\mathbf{p}}}\frac{1}{i\omega_n-\xi_{\mathbf{q}}}e^{i(\mathbf{R}'-\mathbf{R})\cdot\mathbf{\Pi}+i(\mathbf{r}'-\mathbf{r})\cdot\mathbf{k}'}e^{i\mathbf{k}'\cdot\mathbf{r}}\Delta^*(\mathbf{R},\mathbf{r}) . \tag{2.14}$$

Expanding in terms of $\mathbf{\Pi}$ to second order, the above equation can be written in terms of a constant term Δ_{Ic}^* and a gradient term Δ_{Ig}^*:

$$\Delta_I^*(\mathbf{R},\mathbf{k})=\Delta_{Ic}^*(\mathbf{R},\mathbf{k})+\Delta_{Ig}^*(\mathbf{R},\mathbf{k}) , \tag{2.15}$$

where

$$\Delta_{Ic}^*(\mathbf{R},\mathbf{k})=4\int d\mathbf{R}'\frac{d\mathbf{p}\,d\mathbf{k}'}{(2\pi)^4}V(\mathbf{k}'-\mathbf{k})T\sum_{\omega_n}\frac{1}{-i\omega_n-\xi_{\mathbf{p}}}\frac{1}{i\omega_n-\xi_{\mathbf{p}+2\mathbf{k}'}}e^{2i(\mathbf{p}+\mathbf{k}')\cdot(\mathbf{R}'-\mathbf{R})}\Delta^*(\mathbf{R},\mathbf{k}')$$
$$=\int\frac{d\mathbf{k}'}{(2\pi)^2}V(\mathbf{k}'-\mathbf{k})T\sum_{\omega_n}\frac{1}{\omega_n^2+\xi_{\mathbf{k}'}^2}\Delta^*(\mathbf{R},\mathbf{k}') , \tag{2.16}$$

and

$$\Delta_{Ig}^*(\mathbf{R},\mathbf{k})=-2\int d\mathbf{R}'\frac{d\mathbf{p}\,d\mathbf{k}'}{(2\pi)^4}V(\mathbf{k}'-\mathbf{k})T\sum_{\omega_n}\frac{1}{-i\omega_n-\xi_{\mathbf{p}}}\frac{1}{i\omega_n-\xi_{\mathbf{p}+2\mathbf{k}'}}$$
$$\times e^{2i(\mathbf{p}+\mathbf{k}')\cdot(\mathbf{R}'-\mathbf{R})}[(\mathbf{R}'-\mathbf{R})\cdot\mathbf{\Pi}]^2\Delta^*(\mathbf{R},\mathbf{k}')$$
$$=-\int\frac{d\mathbf{p}\,d\mathbf{k}'}{2(2\pi)^2}V(\mathbf{k}'-\mathbf{k})T\sum_{\omega_n}\frac{1}{-i\omega_n-\xi_{\mathbf{p}/2-\mathbf{k}'}}\frac{1}{i\omega_n-\xi_{\mathbf{p}/2+\mathbf{k}'}}\left[\frac{1}{i}\nabla_{\mathbf{p}}\cdot\mathbf{\Pi}\right]^2\delta(\mathbf{p})\Delta^*(\mathbf{R},\mathbf{k}')$$
$$=\int\frac{d\mathbf{k}'}{2(2\pi)^2}V(\mathbf{k}'-\mathbf{k})T\sum_{\omega_n}\left[\left[\frac{1}{i}\nabla_{\mathbf{p}}\cdot\mathbf{\Pi}\right]^2\left[\frac{1}{-i\omega_n-\xi_{\mathbf{p}/2-\mathbf{k}'}}\frac{1}{i\omega_n-\xi_{\mathbf{p}/2+\mathbf{k}'}}\right]\right]_{\mathbf{p}=0}\Delta^*(\mathbf{R},\mathbf{k}')$$
$$=\int\frac{d\mathbf{k}'}{2(2\pi)^2}V(\mathbf{k}'-\mathbf{k})T\sum_{\omega_n}\left[\frac{1}{(2m)^2}\frac{2\xi_{\mathbf{k}}'^2-6\omega_n^2}{(\omega_n^2+\xi_{\mathbf{k}'}^2)^3}[k_x'^2(\Pi_x)^2+k_y'^2(\Pi_y)^2]-\frac{1}{2m}\frac{\xi_{\mathbf{k}'}}{(\omega_n^2+\xi_{\mathbf{k}'}^2)^2}\mathbf{\Pi}^2\right]\Delta^*(\mathbf{R},\mathbf{k}') . \tag{2.17}$$

In the calculation of Δ_{II}^* in Eq. (2.10), the magnetic-field effect can be neglected.[12] Introducing the central-mass and relative coordinates and using the expression of G_0 in Eq. (2.5), we have

$$\Delta^*_{II}(\mathbf{R},\mathbf{r})=-V(\mathbf{r})T\sum_{\omega_n}\int d\mathbf{R}'d\mathbf{r}'d\mathbf{R}_1d\mathbf{r}_1d\mathbf{R}_2d\mathbf{r}_2\Delta^*(\mathbf{R}',\mathbf{r}')\Delta(\mathbf{R}_1,\mathbf{r}_1)\Delta^*(\mathbf{R}_2,\mathbf{r}_2)$$

$$\times\int\frac{d\mathbf{p}\,d\mathbf{q}\,ds\,dt}{(2\pi)^8}\exp\left[i\mathbf{p}\cdot\left[\mathbf{R}'-\mathbf{R}+\frac{\mathbf{r}'}{2}-\frac{\mathbf{r}}{2}\right]+i\mathbf{q}\cdot\left[\mathbf{R}'-\mathbf{R}_1-\frac{\mathbf{r}'}{2}-\frac{\mathbf{r}_1}{2}\right]\right]\frac{1}{-i\omega_n-\xi_{\mathbf{p}}}\frac{1}{i\omega_n-\xi_{\mathbf{q}}}$$

$$\times\exp\left[i\mathbf{s}\cdot\left[\mathbf{R}_2-\mathbf{R}_1+\frac{\mathbf{r}_2}{2}+\frac{\mathbf{r}_1}{2}\right]+i\mathbf{t}\cdot\left[\mathbf{R}_2-\mathbf{R}-\frac{\mathbf{r}_2}{2}+\frac{\mathbf{r}}{2}\right]\right]\frac{1}{-i\omega_n-\xi_{\mathbf{s}}}\frac{1}{i\omega_n-\xi_{\mathbf{t}}}$$

$$=-T\sum_{\omega_n}\int\frac{d\mathbf{k}'}{(2\pi)^2}V(\mathbf{r})e^{-i\mathbf{k}'\cdot\mathbf{r}}\frac{1}{(\omega_n^2+\xi_{\mathbf{k}'}^2)^2}|\Delta^*(\mathbf{R},\mathbf{k}')|^2\Delta^*(\mathbf{R},\mathbf{k}')\ . \tag{2.18}$$

Performing the Fourier transform with respect to the relative coordinate, the above equation takes the following expression:

$$\Delta^*_{II}(\mathbf{R},\mathbf{k})=-\int\frac{d\mathbf{k}'}{(2\pi)^2}V(\mathbf{k}-\mathbf{k}')T\sum_{\omega_n}\frac{1}{(\omega_n^2+\xi_{\mathbf{k}'}^2)^2}|\Delta^*(\mathbf{R},\mathbf{k}')|^2\Delta^*(\mathbf{R},\mathbf{k}')\ . \tag{2.19}$$

In the following section, the GL equations for a d-wave superconductor will be derived from the general gap equations (2.16), (2.17), and (2.19).

III. GL EQUATIONS FOR ORDER PARAMETERS

In order to obtain the generic Ginzburg-Landau equations, which govern the spatial variation of the order parameters, for a d-wave superconductor, we need to specify the form of the interaction. Here we use a model which is reasonable for high-T_c superconductors. Namely, the interaction V contains an on-site repulsion V_0 and a nearest-neighbor attraction V_1. It has been shown that such an interaction gives rise to a pure d-wave superconductivity for a uniform system if the one-site repulsion is large.[13] In the momentum space, this interaction is

$$V(\mathbf{k}-\mathbf{k}')=-V_0+V_1[\cos(k_x-k_x')+\cos(k_y-k_y')]\ , \tag{3.1}$$

which can be rewritten in the form

$$V(\mathbf{k}-\mathbf{k}')=-V_0+\frac{V_1}{2}(\cos k_x+\cos k_y)(\cos k_x'+\cos k_y')+\frac{V_1}{2}(\cos k_x-\cos k_y)(\cos k_x'-\cos k_y')+\sin k_x\sin k_x'+\sin k_y\sin k_y'\ . \tag{3.2}$$

Using cylindrical coordinates, the above equation reduces to

$$V(\mathbf{k}-\mathbf{k}')=-V_s+V_d(\hat{k}_x^2-\hat{k}_y^2)(\hat{k}_x'^2-\hat{k}_y'^2)+V_p\mathbf{k}\cdot\mathbf{k}'\ , \tag{3.3}$$

where $V_s=V_0-2V_1$, $V_d=V_1/8$, and $V_p=V_1$ correspond respectively the s-wave, d-wave, and p-wave channel interactions. For the spin-singlet pairing that we are interested in, the p-wave interaction can be neglected since it does not contribute to the spin-singlet pairing state. Finally the effective interaction responsible for the spin-singlet pairing can be written in the form

$$V(\mathbf{k}-\mathbf{k}')=-V_s+V_d(\hat{k}_x^2-\hat{k}_y^2)(\hat{k}_x'^2-\hat{k}_y'^2)\ . \tag{3.4}$$

By taking both V_d and V_s positive, then V_d corresponds to the attractive interaction responsible for d-wave pairing, and V_s can be regarded as an effective "on-site" repulsive interaction. The generic expression of order parameter that follows Eq. (3.4) is

$$\Delta^*(\mathbf{R},\mathbf{k})=\Delta_s^*(\mathbf{R})+\Delta_d^*(\mathbf{R})(\hat{k}_x^2-\hat{k}_y^2)\ . \tag{3.5}$$

Substituting Eqs. (3.4) and (3.5) into Eq. (2.16), we obtain

$$\Delta^*_{Ic}(\mathbf{R},\mathbf{k})=-V_s\Delta_s^*T\sum_{\omega_n}\int\frac{d\mathbf{k}'}{(2\pi)^2}\frac{1}{\omega_n^2+\xi_{\mathbf{k}'}^2}+V_d\Delta_d^*(\hat{k}_x^2-\hat{k}_y^2)T\sum_{\omega_n}\int\frac{d\mathbf{k}'}{(2\pi)^2}(\hat{k}_x'^2-\hat{k}_y'^2)^2\frac{1}{\omega_n^2+\xi_{\mathbf{k}'}^2}$$

$$=-N(0)V_s\Delta_s^*\ln\frac{2e^{\gamma}\omega_D}{\pi T}+\frac{1}{2}N(0)V_d\Delta_d^*(\hat{k}_x^2-\hat{k}_y^2)\ln\frac{2e^{\gamma}\omega_D}{\pi T}\ , \tag{3.6}$$

where $N(0)$ is the density of states at the Fermi surface, γ is the Euler constant, and ω_D is the cutoff energy for the interactions. Putting (3.4) and (3.5) into (2.17), we have

$$\Delta_{Ig}^*(\mathbf{R},\mathbf{k})=T\sum_{\omega_n}\int\frac{d\mathbf{k}'}{2(2\pi)^2}[-V_s+V_d(\hat{k}_x^2-\hat{k}_y^2)(\hat{k}_x'^2-\hat{k}_y'^2)]$$

$$\times\left[\frac{1}{(2m)^2}\frac{2\xi_{\mathbf{k}'}^2-6\omega_n^2}{(\omega_n^2+\xi_{\mathbf{k}'}^2)^3}(k_x'^2\Pi_x^2+k_y'^2\Pi_y^2)-\frac{1}{2m}\frac{\xi_{\mathbf{k}'}^2}{(\omega_n^2+\xi_{\mathbf{k}'}^2)^2}\Pi^2\right][\Delta_s^*+\Delta_d^*(\hat{k}_x^2-\hat{k}_y^2)]$$

$$=-\frac{N(0)V_s}{4}\int_{-\infty}^{\infty}d\xi_{\mathbf{k}'}T\sum_{\omega_n}\frac{1}{(2m)^2}\frac{2\xi_{\mathbf{k}'}^2-6\omega_n^2}{(\omega_n^2+\xi_{\mathbf{k}'}^2)^3}k_F^2\Pi^2\Delta_s^*$$

$$+\frac{N(0)V_d}{8}(\hat{k}_x^2-\hat{k}_y^2)\int_{-\infty}^{\infty}d\xi_{\mathbf{k}'}T\sum_{\omega_n}\frac{1}{(2m)^2}\frac{2\xi_{\mathbf{k}'}^2-6\omega_n^2}{(\omega_n^2+\xi_{\mathbf{k}'}^2)^3}k_F^2\Pi^2\Delta_d^*$$

$$+\int\frac{d\mathbf{k}'}{2(2\pi)^2}T\sum_{\omega_n}\frac{1}{(2m)^2}\frac{2\xi_{\mathbf{k}'}^2-6\omega_n^2}{(\omega_n^2+\xi_{\mathbf{k}'}^2)^3}(\hat{k}_y'^2-\hat{k}_y'^2)[k_x'^2\Pi_x^2+k_y'^2\Pi_y^2][-V_s\Delta_s^*+V_d\Delta_d^*(\hat{k}_x'^2-\hat{k}_y'^2)]$$

$$=\frac{\lambda_d}{2}\alpha\left[\frac{V_s}{V_d}\Pi^2\Delta_s^*-\frac{1}{2}(\hat{k}_x^2-\hat{k}_y^2)\Pi^2\Delta_d^*\right]+\frac{\lambda_d}{4}\alpha(\Pi_x^2-\Pi_y^2)\left[\frac{V_s}{V_d}\Delta_d^*-(\hat{k}_x^2-\hat{k}_y^2)\Delta_s^*\right], \quad (3.7)$$

where $\alpha=7\zeta(3)/8(\pi T_c)^2$ and $\lambda_d=N(0)V_d/2$. Similarly, we can calculate Δ_{II}^* in Eq. (2.19):

$$\Delta_{II}^*(\mathbf{R},\mathbf{k})=V_s\int\frac{d\mathbf{k}'}{(2\pi)^2}\sum_{\omega_n}\frac{1}{(\omega_n^2+\xi_{\mathbf{k}'}^2)^2}[|\Delta_s|^2\Delta_s^*+(2|\Delta_d|^2\Delta_s^*+\Delta_d^{*2}\Delta_s)(\hat{k}_y'^2-\hat{k}_y'^2)^2]$$

$$-V_d(\hat{k}_y^2-\hat{k}_y^2)\int\frac{d\mathbf{k}'}{(2\pi)^2}(\hat{k}_y'^2-\hat{k}_y'^2)\sum_{\omega_n}\frac{1}{(\omega_n^2+\xi_{\mathbf{k}'}^2)^2}[(2|\Delta_s|^2\Delta_d^*+\Delta_s^{*2}\Delta_d)(\hat{k}_y'^2-\hat{k}_y'^2)+|\Delta_d|^2\Delta_d^*(\hat{k}_y'^2-\hat{k}_y'^2)^3]$$

$$=2\lambda_d(V_s/V_d)\alpha\left[|\Delta_s|^2\Delta_s^*+|\Delta_d|^2\Delta_s^*+\frac{1}{2}\Delta_d^{*2}\Delta_s\right]-\lambda_d\alpha(\hat{k}_y^2-\hat{k}_y^2)\left[\frac{3}{4}|\Delta_d|^2\Delta_d^*+2|\Delta_s|^2\Delta_d^*+\Delta_s^{*2}\Delta_d\right]. \quad (3.8)$$

Comparing both sides of the gap equation for $\hat{k}$-independent terms and terms proportional to $(\hat{k}_x^2-\hat{k}_y^2)$, we obtain

$$\Delta_s^*=-2\lambda_d(V_s/V_d)\Delta_s^*\ln\frac{2e^{\gamma}\omega_D}{\pi T}+2\lambda_d(V_s/V_d)\alpha\left[\frac{1}{4}v_F^2\Pi^2\Delta_s^*+\frac{1}{8}v_F^2(\Pi_x^2-\Pi_y^2)\Delta_d^*+|\Delta_s|^2\Delta_s^*+|\Delta_d|^2\Delta_s^*+\frac{1}{2}\Delta_d^{*2}\Delta_s\right], \quad (3.9)$$

$$\Delta_d^*=\lambda_d\Delta_d^*\ln\frac{2e^{\gamma}\omega_D}{\pi T}-2\lambda_d\alpha\left[\frac{1}{8}v_F^2\Pi^2\Delta_d^*+\frac{1}{8}v_F^2(\Pi_x^2-\Pi_y^2)\Delta_s^*+|\Delta_s|^2\Delta_d^*+\frac{1}{2}\Delta_s^{*2}\Delta_d+\frac{3}{8}|\Delta_d|^2\Delta_d^*\right]. \quad (3.10)$$

The equation for the transition temperature T_c is determined by

$$\lambda_d\ln\frac{2e^{\gamma}\omega_D}{\pi T_c}=1\ . \quad (3.11)$$

A closer examination shows that Eq. (3.9) will lead to unphysical solutions for Δ_s because the convergency of the expansion in terms of order parameters is not established for repulsive interactions. Here, we employ the Padé approximation[14] to avoid this difficulty. It is known that the Padé approximation is typically used when we only know the first few coefficients in the power series expansion of a function and are uncertain whether the power series is convergent. By the Padé approximation, Eq. (3.9) becomes

$$\Delta_s^*=-2(V_s/V_d)\Delta_s^*\left\{1-\frac{\alpha\lambda_d}{\Delta_s^*}\left[\frac{1}{4}v_F^2\Pi^2\Delta_s^*+\frac{1}{8}v_F^2(\Pi_x^2-\Pi_y^2)\Delta_d^*+|\Delta_s|^2\Delta_s^*+|\Delta_d|^2\Delta_s^*+\frac{1}{2}\Delta_d^{*2}\Delta_s\right]\right\}$$

$$\approx-2(V_s/V_d)\Delta_s^*\left\{1+\frac{\alpha\lambda_d}{\Delta_s^*}\left[\frac{1}{4}v_F^2\Pi^2\Delta_s^*+\frac{1}{8}v_F^2(\Pi_x^2-\Pi_y^2)\Delta_d^*+|\Delta_s|^2\Delta_s^*+|\Delta_d|^2\Delta_s^*+\frac{1}{2}\Delta_d^{*2}\Delta_s\right]\right\}^{-1}. \quad (3.12)$$

Finally, we can write the GL equations in a form suitable for finding the GL free-energy functional:

$$2(1+2V_s/V_d)\Delta_s^*+\alpha\lambda_d\left\{\frac{1}{2}v_F^2\Pi^2\Delta_s^*+\frac{1}{4}v_F^2(\Pi_x^2-\Pi_y^2)\Delta_d^*+2|\Delta_s|^2\Delta_s^*+2|\Delta_d|^2\Delta_s^*+\Delta_d^{*2}\Delta_s\right\}=0\ , \quad (3.13)$$

$$-\lambda_d\Delta_d^*\ln(T_c/T)+\alpha\lambda_d\left\{\frac{1}{4}v_F^2\Pi^2\Delta_d^*+\frac{1}{4}v_F^2(\Pi_x^2-\Pi_y^2)\Delta_s^*+2|\Delta_s|^2\Delta_d^*+\Delta_s^{*2}\Delta_d+\frac{3}{4}|\Delta_d|^2\Delta_d^*\right\}=0\ . \tag{3.14}$$

The free energy corresponding to above equations is

$$f=2\alpha_s\lambda_d|\Delta_s|^2-\lambda_d\ln(T_c/T)|\Delta_d|^2+\alpha\lambda_d\left[|\Delta_s|^4+\frac{3}{8}|\Delta_d|^4+2|\Delta_s|^2|\Delta_d|^2+\frac{1}{2}(\Delta_s^{*2}\Delta_d^2+\Delta_d^{*2}\Delta_s^2)\right]$$
$$+\frac{1}{4}\alpha\lambda_d v_F^2[2|\Pi\Delta_s^*|^2+|\Pi\Delta_d^*|^2+(\Pi_x^*\Delta_s\Pi_x\Delta_d^*-\Pi_y^*\Delta_s\Pi_y\Delta_d^*+\text{H.c.})]\ , \tag{3.15}$$

where $\alpha_s=(1+2V_s/V_d)/\lambda_d$. Since the coefficients $2\alpha_s\lambda_d>0$ and $-\ln(T_c/T)<0$, it is easily verified that the pure d-wave solution is stable at infinity, as expected.

IV. GL EQUATION FOR SUPERCURRENT

To calculate the local magnetic field, we need the other GL equation for the supercurrent:

$$\mathbf{j}(\mathbf{x})=-\frac{eT}{im}\sum_{\omega_n}[(\nabla-\nabla')\tilde{G}(\mathbf{x},\mathbf{y},\omega_n)]_{\mathbf{y}=\mathbf{x}}-\frac{2e^2T}{m}\mathbf{A}(\mathbf{x})\sum_{\omega_n}\tilde{G}(\mathbf{x},\mathbf{x},\omega_n)\ . \tag{4.1}$$

Substituting Eq. (2.6) into the above equation, we have

$$\mathbf{j}(\mathbf{x})+\frac{2e^2T}{m}\mathbf{A}(\mathbf{x})\sum_{\omega_n}\tilde{G}(\mathbf{x},\mathbf{x},\omega_n)$$
$$=\frac{eT}{im}\int d\mathbf{x}_1d\mathbf{x}_2d\mathbf{x}_3d\mathbf{x}_4(\nabla_\mathbf{x}-\nabla_\mathbf{y})\Delta(\mathbf{x}_1,\mathbf{x}_2)\Delta^*(\mathbf{x}_3,\mathbf{x}_4)\tilde{G}_0(\mathbf{x},\mathbf{x}_1,\omega_n)\tilde{G}_0(\mathbf{x}_3,\mathbf{x}_2,-\omega_n)\tilde{G}_0(\mathbf{x}_4,\mathbf{y},\omega_n)|_{\mathbf{y}=\mathbf{x}}$$
$$=\frac{eT}{im}\int d\mathbf{x}_1d\mathbf{x}_2d\mathbf{x}_3d\mathbf{x}_4\Delta(\mathbf{x}_1,\mathbf{x}_2)\Delta^*(\mathbf{x}_3,\mathbf{x}_4)\tilde{G}_0(\mathbf{x}_3,\mathbf{x}_2,-\omega_n)$$
$$\times\{-2ie\,\mathbf{A}(\mathbf{x})\tilde{G}_0(\mathbf{x}_4,\mathbf{x},\omega_n)\tilde{G}_0(\mathbf{x},\mathbf{x}_1,\omega_n)$$
$$+e^{-ie\mathbf{A}(\mathbf{x})\cdot(\mathbf{x}-\mathbf{x}_1)-ie\mathbf{A}(\mathbf{y})\cdot(\mathbf{x}_4-\mathbf{y})}(\nabla_\mathbf{x}-\nabla_\mathbf{y})[G_0(\mathbf{x}-\mathbf{x}_1,\omega_n)G_0(\mathbf{x}_4-\mathbf{y},\omega_n)]\}_{\mathbf{y}=\mathbf{x}}\ . \tag{4.2}$$

The relation (2.4) has been used in obtaining the above equation. We note that, to the same order of Δ, the first term on the right-hand side of Eq. (4.2) is exactly the same as the second term on the left-hand side. After this cancellation, using approximtion (2.11) and introducing the center-mass and relative coordinates, we find

$$\mathbf{j}(\mathbf{R})=\frac{eT}{im}\sum_{\omega_n}\int d\mathbf{R}'d\mathbf{r}'d\mathbf{R}''d\mathbf{r}''\left[e^{-i(\mathbf{R}'-\mathbf{R})\cdot\Pi^*+(\mathbf{r}'-\mathbf{r})\cdot\nabla_\mathbf{r}}\Delta(\mathbf{R},\mathbf{r})\right]\left[e^{-i(\mathbf{R}''-\mathbf{R})\cdot\Pi+(\mathbf{r}''-\mathbf{r})\cdot\nabla_\mathbf{r}}\Delta^*(\mathbf{R},\mathbf{r})\right]$$
$$\times G_0\left[\mathbf{R}''-\mathbf{R}'+\frac{\mathbf{r}''}{2}+\frac{\mathbf{r}'}{2},-\omega_n\right]\nabla_\mathbf{r}\left[G_0\left[\mathbf{R}-\mathbf{R}'+\frac{\mathbf{r}}{2}-\frac{\mathbf{r}'}{2},\omega_n\right]G_0\left[\mathbf{R}''-\mathbf{R}-\frac{\mathbf{r}''}{2}+\frac{\mathbf{r}}{2},\omega_n\right]\right]_{\mathbf{r}=0}$$
$$=\frac{eT}{2m}\int\frac{d\mathbf{p}d\mathbf{q}d\mathbf{s}}{(2\pi)^6}(\mathbf{p}+\mathbf{s})\int d\mathbf{R}'d\mathbf{r}'d\mathbf{R}''d\mathbf{r}''e^{-i(\mathbf{R}'-\mathbf{R})\cdot\Pi^*+(\mathbf{r}'-\mathbf{r})\cdot\nabla_r}\int\frac{d\mathbf{k}'}{(2\pi)^2}e^{-i\mathbf{k}'\cdot\mathbf{r}}\Delta^*(\mathbf{R},\mathbf{k})$$
$$\times e^{i(\mathbf{R}''-\mathbf{R})\cdot\Pi+(\mathbf{r}''-\mathbf{r})\cdot\nabla_r}\int\frac{d\mathbf{k}}{(2\pi)^2}e^{i\mathbf{k}\cdot\mathbf{r}}\Delta^*(\mathbf{R},\mathbf{k})e^{i\mathbf{p}\cdot(\mathbf{R}-\mathbf{R}'-\mathbf{r}'/2)+i\mathbf{q}\cdot(\mathbf{R}''-\mathbf{R}'+\mathbf{r}''/2+\mathbf{r}'/2)}$$
$$\times e^{i\mathbf{s}\cdot(\mathbf{R}''-\mathbf{R}-\mathbf{r}''/2)}\sum_{\omega_n}\frac{1}{i\omega_n-\xi_\mathbf{p}}\frac{1}{-i\omega_n-\xi_\mathbf{q}}\frac{1}{i\omega_n-\xi_\mathbf{s}}$$
$$=-\frac{2eT}{m}\int\frac{d\mathbf{k}\,d\mathbf{k}'}{(2\pi)^4}d\mathbf{R}'\mathbf{k}'\Delta^*(\mathbf{R},\mathbf{k})[-i\Pi^*\Delta(\mathbf{R},\mathbf{k}')]\cdot[\nabla_{\mathbf{k}'}e^{2i(\mathbf{k}+\mathbf{k}')\cdot(\mathbf{R}'-\mathbf{R})}]$$
$$\times\sum_{\omega_n}\frac{1}{i\omega_n-\xi_{-\mathbf{k}-2\mathbf{k}'}}\frac{1}{-i\omega_n-\xi_{-\mathbf{k}}}\frac{1}{i\omega_n-\xi_\mathbf{k}}+\text{H.c.}$$
$$=\frac{eT}{2m^2}\sum_{\omega_n}\int\frac{d\mathbf{k}}{(2\pi)^2}\Delta^*(\mathbf{R},\mathbf{k})\frac{1}{\omega_n^2+\xi_\mathbf{k}^2}\left\{\frac{-m\mathbf{k}}{i\omega_n-\xi_\mathbf{k}}[\nabla_{\mathbf{k}'}\cdot\Pi^*\Delta(\mathbf{R},\mathbf{k}')]\bigg|_{\mathbf{k}'=-\mathbf{k}}+\frac{\mathbf{k}}{2(i\omega_n-\xi_\mathbf{k})^2}[\mathbf{k}\cdot\Pi^*\Delta(\mathbf{R},-\mathbf{k})]\right\}+\text{H.c.}$$
$$\tag{4.3}$$

In terms of the expression of order parameter (3.5), the above equation becomes

$$\mathbf{j}(\mathbf{R}) = -\frac{eN(0)\mu\alpha}{2m}\left[\Delta_s \mathbf{\Pi}\Delta_s^* + \frac{1}{2}\Delta_d \mathbf{\Pi}\Delta_d^* + \frac{1}{2}(\Delta_s \Pi_x \Delta_d^* + \Delta_d \Pi_x \Delta_s^*)\hat{\mathbf{x}} - \frac{1}{2}(\Delta_s \Pi_y \Delta_d^* + \Delta_d \Pi_y \Delta_s^*)\hat{\mathbf{y}}\right] + \mathrm{H.c.} \tag{4.4}$$

This equation will be useful in determining the spatial distribution of the local magnetic field and the supercurrent.

V. STRUCTURE OF A SINGLE VORTEX

In this section, we determine the single-vortex structure for a d-wave superconductor by using the GL equations derived in the previous sections. The vortex structure of a superconductor with $d_{x^2-y^2}$ pairing symmetry is of great interest because of its relevance to high-T_c superconductors. It is expected that the structure of a d-wave vortex is very different from that of s wave[6,7] or p wave.[8] This problem was considered by Volovik[9] who studied the density of states produced by the d-wave vortices. From the symmetry consideration, he argued that the core of the vortex in the d-wave superconductor should contain all the possible gap functions that are consistent with the maximal symmetry group of the vortex line. In particular, it should contain the conventional s-wave pairing component with the opposite winding phase. Because of this correction, the total gap function has no lines of gap nodes within the core. Very recently, the d-wave vortex structure has also been studied numerically by Soninen, Kallin, and Berlinsky[10] within the framework of the self-consistent Bogoliubov-de Gennes theory. Their result confirms the existence of the induced s-wave component near the vortex core. However, the asymptotic behavior of order parameters is not clear from their numerical calculation. It is also difficult to identify the exact relative phase between s-wave and d-wave order parameters. The temperature dependence of the order parameters cannot be determined either from their numerical calculation.

To study the structure of a single vortex, the magnetic-field effect could be neglected[12] when we consider the extreme type-II superconductors, such as high-T_c materials. This does not affect our conclusion as long as we confine ourselves in the physically interesting region $r \ll \lambda$. Defining $\xi_0 = \sqrt{\alpha} v_F/2$ which differs with the usual coherent length at zero temperature only by a numerical factor ~ 1 and $\Delta_0 = \sqrt{4/3\alpha}$, we may cast the Ginzburg-Landau equations into dimensionless form $r/\xi_0 \to r, \Delta/\Delta_0 \to \Delta$,

$$\alpha_s \Delta_s^* - \nabla^2 \Delta_s^* - \frac{1}{2}\left[\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right]\Delta_d^* + \frac{4}{3}|\Delta_s|^2\Delta_s^* + \frac{4}{3}|\Delta_d|^2\Delta_s^* + \frac{2}{3}\Delta_d^{*2}\Delta_s = 0 , \tag{5.1}$$

$$-\ln(T_c/T)\Delta_d^* - \nabla^2 \Delta_d^* - \left[\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right]\Delta_s^* + \frac{8}{3}|\Delta_s|^2\Delta_d^* + \frac{4}{3}\Delta_s^{*2}\Delta_d + |\Delta_d|^2\Delta_d^* = 0 . \tag{5.2}$$

In term of cylindrical coordinates, $\mathbf{R} = (r, \theta)$, and noting

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} ,$$

$$\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} = \frac{1}{2}\left[\left[\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right]^2 + \left[\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right]^2\right] ,$$

$$\left[\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right]^2 = e^{2i\theta}\left[\left[\frac{\partial}{\partial r} + \frac{i}{r}\frac{\partial}{\partial \theta}\right]^2 - \frac{1}{r}\left[\frac{\partial}{\partial r} + \frac{i}{r}\frac{\partial}{\partial \theta}\right]\right] ,$$

$$\left[\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right]^2 = e^{-2i\theta}\left[\left[\frac{\partial}{\partial r} - \frac{i}{r}\frac{\partial}{\partial \theta}\right]^2 - \frac{1}{r}\left[\frac{\partial}{\partial r} - \frac{i}{r}\frac{\partial}{\partial \theta}\right]\right] ,$$

the GL equation can be rewritten in terms of the cylindrical coordinates:

$$\alpha_s \Delta_s^* - \left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right]\Delta_s^* - \frac{1}{4}e^{2i\theta}\left[\left[\frac{\partial}{\partial r} + \frac{i}{r}\frac{\partial}{\partial \theta}\right]^2 - \frac{1}{r}\left[\frac{\partial}{\partial r} + \frac{i}{r}\frac{\partial}{\partial \theta}\right]\right]\Delta_d^* - \frac{1}{4}e^{-2i\theta}\left[\left[\frac{\partial}{\partial r} - \frac{i}{r}\frac{\partial}{\partial \theta}\right]^2 - \frac{1}{r}\left[\frac{\partial}{\partial r} - \frac{i}{r}\frac{\partial}{\partial \theta}\right]\right]\Delta_d^* + \frac{4}{3}|\Delta_s|^2\Delta_s^* + \frac{4}{3}|\Delta_d|^2\Delta_s^* + \frac{2}{3}\Delta_d^{*2}\Delta_s = 0 , \tag{5.3}$$

$$-\ln\left[\frac{T_c}{T}\right]\Delta_d^* - \left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right]\Delta_d^* - \frac{1}{2}e^{2i\theta}\left[\left[\frac{\partial}{\partial r} + \frac{i}{r}\frac{\partial}{\partial \theta}\right]^2 - \frac{1}{r}\left[\frac{\partial}{\partial r} + \frac{i}{r}\frac{\partial}{\partial \theta}\right]\right]\Delta_s^* - \frac{1}{2}e^{-2i\theta}\left[\left[\frac{\partial}{\partial r} - \frac{i}{r}\frac{\partial}{\partial \theta}\right]^2 - \frac{1}{r}\left[\frac{\partial}{\partial r} - \frac{i}{r}\frac{\partial}{\partial \theta}\right]\right]\Delta_s^* + \frac{8}{3}|\Delta_s|^2\Delta_d^* + \frac{4}{3}\Delta_s^{*2}\Delta_d + |\Delta_d|^2\Delta_d^* = 0 . \tag{5.4}$$

In general, the full solution to the Ginzburg-Landau equations of a single vortex involves all possible terms that are consistent with the maximal symmetry group of the vortex:

$$\Delta_d^* = \sum_n g_n(r) e^{i(4n+1)\theta} , \tag{5.5}$$

$$\Delta_s^* = \sum_m f_m(r) e^{i(4m-1)\theta} , \tag{5.6}$$

where n,m sum over all integers.

We expect that, far away from the center of the vortex (strictly, in the region of $\xi \ll r \ll \lambda$), the d-wave order parameter takes the form $\Delta_d^* = g_0 e^{i\theta}$. Simple inspection of the Ginzburg-Landau equation shows that the leading-order terms (up to r^{-2}) that are important in our interesting region are

$$\alpha_s \Delta_s^* - \frac{1}{4}\left[e^{3i\theta}\frac{3g_0}{r^2} - e^{-i\theta}\frac{g_0}{r^2}\right] + \frac{4}{3}g_0^2\Delta_s^* + \frac{2}{3}g_0^2 e^{2i\theta}\Delta_s = 0 , \quad (5.7)$$

$$-\ln\left[\frac{T_c}{T}\right]\Delta_d^* - \left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right]\Delta_d^* + |\Delta_d|^2\Delta_d^* = 0 . \quad (5.8)$$

The solutions to the above equations are found to have the following form (up to r^{-2}):

$$\Delta_s^* = \frac{1}{r^2}(-ae^{-i\theta} + be^{3i\theta}) , \quad (5.9)$$

$$\Delta_d^* = g_0 e^{i\theta} + \frac{1}{4r^2}\left[\frac{2}{g_0}e^{i\theta} + (a+3b)(e^{-3i\theta} - e^{5i\theta})\right]$$
$$= e^{i\theta}\left[g_0 - \frac{1}{2g_0 r^2} + \frac{i(a+3b)}{2r^2}\sin 4\theta\right] , \quad (5.10)$$

where

$$g_0 = [\ln(T_c/T)]^{1/2} , \quad (5.11)$$

$$a = \frac{\alpha_s + 10g_0^2/3}{(\alpha_s + 4g_0^2/3)^2 - (2g_0^2/3)^2}\frac{g_0}{4} , \quad (5.12)$$

$$b = \frac{3\alpha_s + 14g_0^2/3}{(\alpha_s + 4g_0^2/3)^2 - (2g_0^2/3)^2}\frac{g_0}{4} . \quad (5.13)$$

Thus the induced s-wave component decays as r^{-2} far away from the core, the $e^{-i\theta}$ and $e^{3i\theta}$ terms combine to give the profile a shape of four-leafed clover (see Fig. 1).

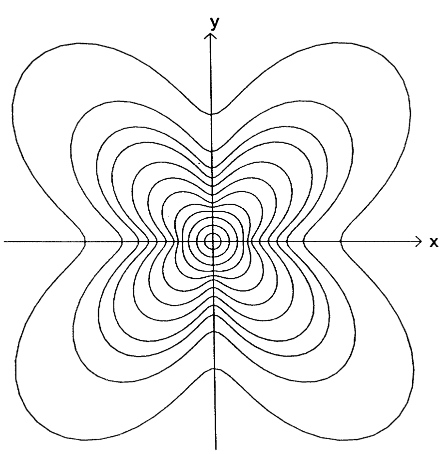

FIG. 1. The equal-interval contours of magnitude of the induced s-wave order parameter $|\Delta_s|^2$.

The d-wave component is also modified by the anisotropic terms proportional to r^{-2}, and shows a fourfold symmetry, as shown in Fig. 2. When temperature approaches T_c, the leading term of d-wave and s-wave components reduce to the following simple form:

$$\Delta_d^* = \sqrt{1 - T/T_c}\, e^{i\theta} , \quad (5.14)$$

$$\Delta_s^* = \frac{1}{4\alpha_s r^2}\sqrt{1 - T/T_c}\,(-e^{-i\theta} + 3e^{3i\theta}) . \quad (5.15)$$

These results indicate that, near T_c, the s-wave and d-wave order parameters have the same temperature dependence, namely $(1 - T/T_c)^{1/2}$. It is also interesting to note that the s-wave component is suppressed by the effective on-site repulsion V_s, since α_s is proportional to V_s. Moreover, our calculation shows that even when $V_s \to 0$, the s-wave component still persists. The characteristic decaying length of the s-wave order parameter measured from the center of the vortex core is $\xi_0/\sqrt{\alpha_s}$, which is also suppressed by V_s.

Near the center of the vortex, to the leading order, our Ginzburg-Landau equations become

$$\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right]\Delta_s^* + \frac{1}{4}e^{2i\theta}\left[\left[\frac{\partial}{\partial r} + \frac{i}{r}\frac{\partial}{\partial\theta}\right]^2 - \frac{1}{r}\left[\frac{\partial}{\partial r} + \frac{i}{r}\frac{\partial}{\partial\theta}\right]\right]\Delta_d^* + \frac{1}{4}e^{-2i\theta}\left[\left[\frac{\partial}{\partial r} - \frac{i}{r}\frac{\partial}{\partial\theta}\right]^2 - \frac{1}{r}\left[\frac{\partial}{\partial r} - \frac{i}{r}\frac{\partial}{\partial\theta}\right]\right]\Delta_d^* = 0 , \quad (5.16)$$

$$\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right]\Delta_d^* + \frac{1}{2}e^{2i\theta}\left[\left[\frac{\partial}{\partial r} + \frac{i}{r}\frac{\partial}{\partial\theta}\right]^2 - \frac{1}{r}\left[\frac{\partial}{\partial r} + \frac{i}{r}\frac{\partial}{\partial\theta}\right]\right]\Delta_s^* + \frac{1}{2}e^{-2i\theta}\left[\left[\frac{\partial}{\partial r} - \frac{i}{r}\frac{\partial}{\partial\theta}\right]^2 - \frac{1}{r}\left[\frac{\partial}{\partial r} - \frac{i}{r}\frac{\partial}{\partial\theta}\right]\right]\Delta_s^* = 0 . \quad (5.17)$$

The general solution to Eqs. (5.16) and (5.17) again has the form

$$\Delta_d^* = g(r)e^{i\theta}, \quad \Delta_s^* = f_1(r)e^{-i\theta} + f_2(r)e^{3i\theta} . \quad (5.18)$$

To the leading order, we find $f_1(r) = c_1 r, f_2(r) = 0$, and $g(r) = c_0 r$. So, near the center of the core,

$$\Delta_d^* \sim c_0 r e^{i\theta} , \quad (5.19)$$

$$\Delta_s^* \sim c_1 r e^{-i\theta} , \quad (5.20)$$

where the constants c_0 and c_1 have to be determined by connecting the solution near the center with the solution far away from the core, just as in the case of conventional

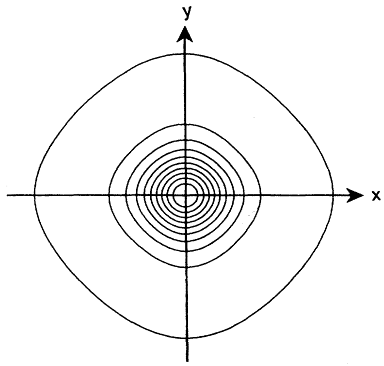

FIG. 2. The equal-interval contours of magnitude of d-wave order parameter $|\Delta_d|^2$. We note that the profile has a fourfold symmetry.

s-wave vortex.[6] Thus near the center of the vortex the induced s-wave component has the exactly opposite winding number relative to the d-wave component.[9] The relative phase between the s- and d-wave components in different regions are shown in Fig. 3.

VI. SUPERCURRENT AND LOCAL FIELD AROUND A VORTEX

In this section, we calculate the distributions of the supercurrent and local magnetic field around a d-wave vortex. In the limit of $\xi_0 \ll \lambda$, the supercurrent of Eq. (4.4) can be rewritten in the following dimensionless form:

$$\mathbf{j}=i\frac{\Phi_0}{2\pi\lambda^2}\left[\Delta_s\nabla\Delta_s^*+\frac{1}{2}\Delta_d\nabla\Delta_d^* \right.$$
$$+\frac{1}{2}\left\{\Delta_s\frac{\partial}{\partial x}\Delta_d^*+\Delta_d\frac{\partial}{\partial x}\Delta_s^*\right\}\hat{\mathbf{x}} \tag{6.1}$$
$$\left. -\frac{1}{2}\left\{\Delta_s\frac{\partial}{\partial y}\Delta_d^*+\Delta_d\frac{\partial}{\partial y}\Delta_s^*\right\}\hat{\mathbf{y}}\right] , \tag{6.2}$$

where Φ_0 is the flux quantum. Then the distribution of the supercurrent around a d-wave vortex is easily determined by putting the results of Δ_s and Δ_d given in the previous section into the above equations. We find, for $r \to 0$,

$$\mathbf{j}=\frac{1}{2}(c_0^2+2c_1^2)r\hat{\theta} , \tag{6.3}$$

and for $\xi_0 \ll r \ll \lambda$:

$$\mathbf{j}=\frac{\Phi_0 g_0^2}{2\pi\lambda^2}\left[\left[\frac{a+3b}{2r^3}\sin 4\theta\right]\hat{\mathbf{r}} - \left[\frac{1}{r}+\frac{1}{g_0^2 r^3}+\frac{a-b}{r^3}+\frac{a+3b}{2r^3}\cos 4\theta\right]\hat{\theta}\right] . \tag{6.4}$$

These results indicate that, near the vortex core, the current flows around the vortex uniformly along the θ direction, while far away from the vortex core, it has both θ and r components and exhibits a fourfold symmetric distribution, as shown in Fig. 4, in sharp contrast to that around a conventional s-wave vortex.

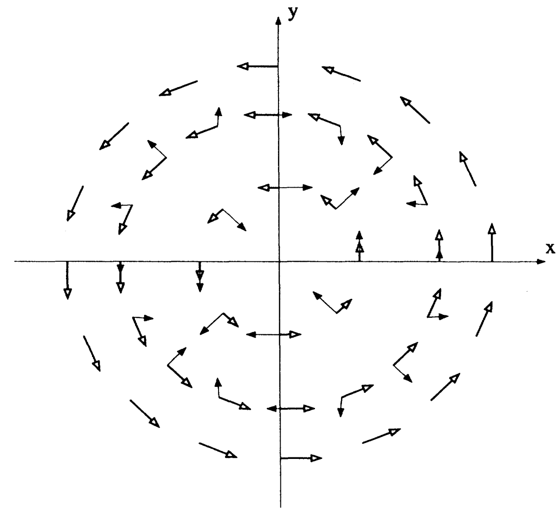

FIG. 3. The relative phase between s-wave and d-wave order parameters in the different regions. The vectors with the outlined arrow represent the d-wave component, while the vectors with filled arrow represent the s-wave order parameter. The angle between the two vectors is the relative phase.

The local magnetic field around a d-wave vortex can be calculated from $\nabla\times\mathbf{B}=4\pi\mathbf{j}$:

$$\mathbf{B}=\begin{cases}\left[B_0-\frac{1}{4}(c_0^2+2c_1^2)r^2\right]\hat{\mathbf{z}}, & r\to 0 ,\\ \frac{\Phi_0 g_0^2}{2\pi\lambda^2}\left[\ln\frac{\lambda}{r}-2-\frac{1}{g_0^2 r^2}-\frac{a-b}{r^2}-\frac{a+3b}{r^2}\cos 4\theta\right]\hat{\mathbf{z}}, & \xi_0\ll r\ll\lambda .\end{cases} \tag{6.5}$$

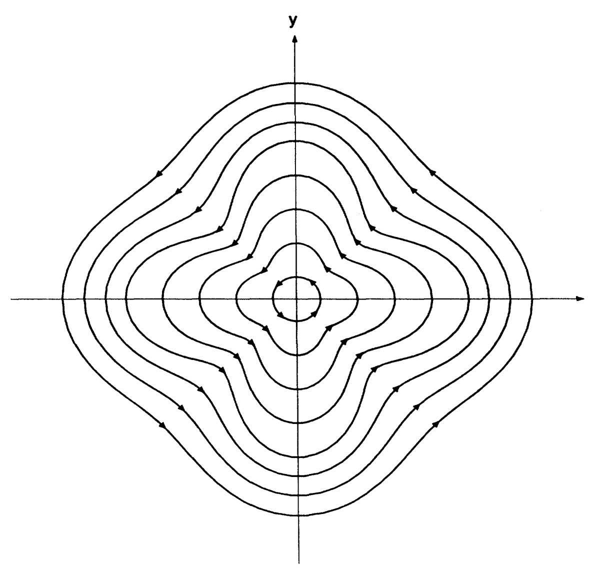

FIG. 4. The streamlines of the supercurrent around a d-wave vortex.

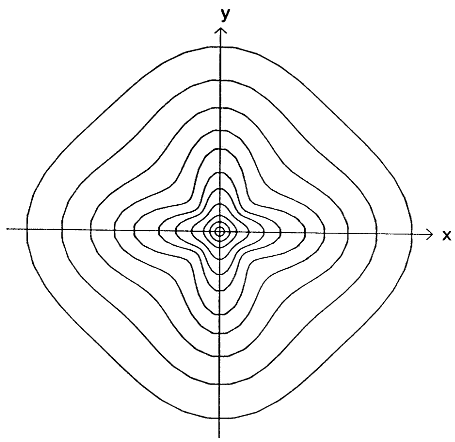

FIG. 5. The equal-interval contours of the local magnetic field $|\mathbf{B}|$ around a d-wave vortex.

The distribution of the local magnetic field around the vortex is plotted in Fig. 5. We clearly see that in the region of $\xi_0 \ll r < \lambda$, the local field, similar to the supercurrent, shows a fourfold symmetry, which should be observable experimentally by the scanning SQUID's.[3]

VII. SURFACE PROBLEM

In this section, we study the distribution of the order parameters at surface of a d-wave superconductor using the GL equations, i.e., the proximity effect. For simplicity, we consider the situation where there are neither currents nor magnetic fields. Suppose the a axis of the superconductor is along x direction which is normal to the surface, we have one-dimensional GL equations:

$$\alpha_s \Delta_s^* - \frac{d^2\Delta_s^*}{dx^2} - \frac{1}{2}\frac{d^2\Delta_d^*}{dx^2} + \frac{4}{3}|\Delta_s|^2\Delta_s^* + \frac{4}{3}|\Delta_d|^2\Delta_s^* + \frac{2}{3}\Delta_d^{*2}\Delta_s = 0 , \quad (7.1)$$

$$-(1-T/T_c)\Delta_d^* - \frac{d^2\Delta_d^*}{dx^2} - \frac{d^2\Delta_s^*}{dx^2} + \frac{8}{3}|\Delta_s|^2\Delta_d^* + \frac{4}{3}\Delta_s^{*2}\Delta_d + |\Delta_d|^2\Delta_d^* = 0 . \quad (7.2)$$

From our numerical calculation (which will be presented below), we find that Δ_s and Δ_d must be real and the s-wave component is always much smaller than the d-wave component. In this case, the Δ_d satisfies approximately the following equation:

$$-(1-T/T_{c})\Delta_d - \frac{d^2\Delta_d}{dx^2} + \Delta_d^3 = 0 , \quad (7.3)$$

with the following boundary conditions:

$$\Delta_d = 0, \ \ x=0; \ \ \Delta_d \to \sqrt{1-T/T_c}, \ \ x \to \infty . \quad (7.4)$$

(Strictly, the boundary condition should be the vanishing of the current at $x=0$. This only complicates the problem, with the qualitative physics unchanged.) The solution is

$$\Delta_d = \sqrt{1-T/T_c}\tanh(\sqrt{1-T/T_c/2}x) . \quad (7.5)$$

The asymptotic behavior of Δ_s can be obtained by using Eqs. (7.1) and (7.5):

$$\Delta_s = \begin{cases} -\dfrac{(1-T/T_c)^2}{2\sqrt{2}\alpha_s}x, & x \to 0 , \\ -\dfrac{(1-T/T_c)^{3/2}}{2\alpha_s}e^{-\sqrt{2(1-T/T_c)}x}, & x \to \infty . \end{cases} \quad (7.6)$$

We have performed numerical calculation of the distribution of the order parameters at the surface for a d-wave superconductor with an arbitrary angle θ with respect to the a axis by solving the GL equations (5.1) and (5.2). The magnitude of the order parameters is shown in Fig. 6 for different temperatures. It is interesting to note that near the surface a small s-wave component is always induced. It depends linearly on x as $x \to 0$ and decays exponentially when x is large. With the increase of the temperatures, the peak of the s-wave component becomes broader and shifts toward to large x side. More remarkably, we find that the total order parameter is a real com-

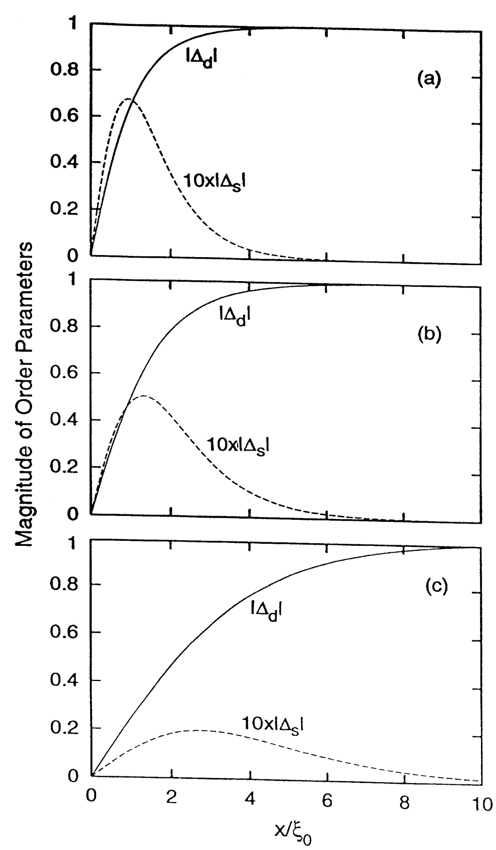

FIG. 6. Spatial variation of the order parameters near a vacuum-d-wave superconductor interface: (a) $T/T_c=0$, (b) $T/T_c=0.5$, and (c) $T/T_c=0.9$.

bination of d-wave and s-wave components. The phase between s- and d-wave order parameters is determined by

$$\phi_d - \phi_s = \text{sign}[-\cos(2\theta)] , \quad (7.7)$$

i.e., $\phi_d - \phi_s$ takes the value either 0 or π, depending on the angle θ between the a axis and the direction normal to the surface. At the first glance, this result seems to be surprising since the coupling term $\Delta_s^{*2}\Delta_d^2 + \Delta_s^2\Delta_d^{*2}$ in the GL free energy favors energetically $\phi_d - \phi_s = \pi/2$, i.e., the $s+id$ state. However, a closer examination shows that this term is only a higher-order correction because the s-wave component itself is small. In fact, the most important term is the mixed-gradient term $\partial x \Delta_s \partial_x \Delta_d^* - \partial_y \Delta_s \partial_y \Delta_d^* + \text{H.c}$. It is this mixed-gradient term that induces the s-wave component. The phase between s- and d-wave components is also primarily determined by this term which gives rise obviously to a phase $\phi_d - \phi_s = 0$ or π. This means that although the proximity effect of a d-wave superconductor can induce a small s-wave component near the surface, a locally time-reversal symmetry-breaking state can never occur in practice.

VIII. CONCLUSIONS

We have established a GL theory for a superconductor with $d_{x^2-y^2}$-wave pairing symmetry. The GL equations obtained in the present work can be easily used to study the properties of nonuniform d-wave superconductors. We have shown that, for a d-wave superconductor with inhomogeneity, the s-wave component is always induced near the inhomogeneous regions. In the other words, this result means that it is impossible to have a pure d-wave state for a nonuniform superconductor.

As an application of our theory, we have studied the single-vortex structure for a $d_{x^2-y^2}$-wave superconductor using the GL equations. The asymptotic behavior of such a vortex has been analytically determined. This most interesting feature is that the s-wave and d-wave components, with the opposite winding numbers, are found to coexist in the region near the vortex core. It has been shown that the d-wave component varies linearly with the distance r from the vortex core as $r \to 0$ and goes to the pure d-wave bulk value for very large r. In the region of $\xi_0 < r \ll \lambda$, it exhibits a fourfold symmetry. On the other hand, the induced s-wave component has a linear-r dependence for small r but decays as r^{-2} when r is large. It shows a fourfold symmetry in the region of $\xi_0 < r \ll \lambda$. Furthermore, for large r, the winding of s-wave component is more complicated and its magnitude exhibits strong anisotropy. The temperature dependence of the s-wave order parameter is complicated, but near T_c, it has the same behavior as the d-wave component, namely $(1-T/T_c)^{1/2}$. This structure could be observable by the scanning tunneling microscopy experiment on a d-wave superconductor in the mixed state, similar to one performed on the conventional s-wave superconductors.[15] We believe that such a complicated d-wave vortex structure will affect the transport properties in the mixed state, such as the resistivity and the Hall effect. The relevant results will be given elsewhere.

The supercurrent and local magnetic field around the d-wave vortex have also been calculated and both of them exhibit a fourfold symmetry, in contrast to those around a conventional s-wave vortex. We believe such a distribution of the local field should be observable experimentally, and could be used as a probe of determining pairing symmetry in high-T_c superconductors.

We have also studied the proximity effect at the surface of a d-wave superconductor using our GL equations, and have found that a small s-wave component is induced near the surface. The total order parameter near surface is always a real combination of s- and d-wave components and their relative phase is determined by sign$[-\cos(2\theta)]$. The immediate consequence of our result is that although the proximity effect of a d-wave superconductor can induce a small s-wave component near the surface, it is impossible to have a locally time-reversal symmetry-breaking state at the surface.

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