Collective pinning and the Hall effect in superconductors

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We calculate the mixed-state longitudinal and Hall resistivity of superconductors based on the perturbation approach of Schmid and Larkin with a frictional force of the form proposed by Vinen and Warren under the influence of randomly distributed weak point pinning centers. The longitudinal resistivity has the Bardeen-Stephen behavior in the flux-flow region. In the weak collective pinning case, for either sample thickness much smaller or much greater than the penetration depth, scaling $\rho_{xy} \propto \rho_{xx}^2$ holds, but no sign reversal in ρ_{xy} is predicted. The Hall conductivity is *independent* of pinning to the first nonvanishing order of the perturbation calculation.

I. INTRODUCTION

The Hall effect in the mixed state has been an important issue for understanding vortex motion in both low-T_c and high-T_c superconductors. Based on a phenomenological approach, Bardeen and Stephen[1,2] (BS) successfully explained some early flux-flow data.[3] However, other Hall-effect measurements[4] on low-T_c superconductors, where the Hall resistivity underwent a sign change or a sudden upturn in the flux-flow region, could not be understood with that phenomenological theory. In particular the BS theory predicts no anomaly in the Hall effect. Subsequently, Noziéres and Vinen[5] (NV) considered more carefully the forces acting on moving vortices but came to similar conclusions.

Recent measurements[6–12] on high-T_c superconductors have shown Hall resistivity sign reversal in various kinds of samples. Freimuth, Hohn, and Galffy[13] attributed the sign reversal to the large thermomagnetic effects in the mixed state. Harris, Ong, and Yan[14] concluded that the sign reversal is a unique feature of vortices parallel to the *ab* plane. However, Hagen *et al.*[10] have argued that the sign reversal is due to the general properties of vortex motion. Several theoretical explanations for this anomaly have been put forward. Wang and Ting[15] considered backflow current due to pinning forces; they derived the longitudinal resistivity ρ_{xx} and the Hall resistivity ρ_{xy} as functions of magnetic field in the flux-flow region, in which ρ_{xy} has a sign change. Recently Wang, Dong, and Ting[16] have further developed their theory. Ferrell[17] considered the effect of the opposing drift of thermally excited quasiparticles; these particles collide quasiclassically with the hydrodynamic superfluid velocity field. His calculated Hall angle has sign opposite to that in the normal state. Dorsey[18] and Kopnin, Ivlev, and Kalatsky[19] showed that the Hall anomaly could be a consequence of the time-dependent Ginzburg-Landau theory. Meilikhov and Farzetdinova[20] interpreted the anomalous Hall effect based on BS and NV models by considering Andreev reflection at the interface between the normal core and the superconducting periphery. Very recently, Feigel'man *et al.*[21] considered an additional Hall effect due to the positive difference between electron density at the center of the vortex core and that far outside the vortex; this contribution has the opposite sign from the conventional one and can cause a sign change.

In spite of many different approaches, the mechanism responsible for the Hall sign reversal remains controversial. Even the basic question of whether pinning has anything to do with the Hall anomaly is confusing. Some experiments[22–24] indicated that strong pinning can make the sign reversal disappear.

A common feature of calculations to date is to treat pinning noncollectively. In order to see what effect collective pinning has on the Hall effect, we calculate the mixed-state longitudinal and Hall resistivity of superconductors based on the perturbation method of Schmid and Hauger[25,26] and Larkin and Ovchinnikov[27,28] with a frictional force of the form proposed by Vinen and Warren[29] under the influence of randomly distributed weak point pinning centers. Based on this approach we find the coefficient of the dissipation part of the Vinen and Warren force is renormalized by the collective-pinning effect, while the coefficient of the nondissipation part is unchanged to the first nonvanishing order of the perturbation calculation. The longitudinal resistivity has the Bardeen-Stephen behavior in the flux-flow region. For either sample thickness much smaller or much greater than the penetration depth, scaling $\rho_{xy} \propto \rho_{xx}^2$ holds.[30,16] The Hall conductivity is *independent* of pinning to the first nonvanishing order of the perturbation correction term. Many theories have now been proposed which predict this scaling, so that observing scaling cannot be taken as proof of the correctness of any theory.

II. VORTEX EQUATION OF MOTION

We consider a superconductor with weak randomly distributed point pinning centers. The external magnetic field $\mathbf{H}$ is applied along the $\hat{z}$ direction. The coordinates of the ith vortex can be expressed as $\mathbf{r}_i(z,t) = \mathbf{R}_i + \mathbf{r}_0(t) + \mathbf{u}_i(\mathbf{r}_i,t) + z\hat{z}$, where $\mathbf{R}_i$ is the original stationary perfect flux-line lattice (FLL) vortex position, $\mathbf{r}_0(t)$ describes that vortex position relative to $\mathbf{R}_i$ in space due to uniform motion of the FLL, and $\mathbf{u}_i(\mathbf{r}_i,t)$ is a two-dimensional displacement vector relative to the perfect vortex lattice position due to pinning forces. The vortex velocity is given by $\mathbf{v}_{iL}=\partial\mathbf{r}_i/\partial t=\partial\mathbf{r}_0/\partial t+\partial\mathbf{u}_i/\partial t\equiv\mathbf{v}_0+\mathbf{v}_{i1}$. For the weak pinning case, we expect $|\mathbf{v}_{i1}|\ll|\mathbf{v}_0|$.

When a transport current with the density $\mathbf{j}_T(j_T>j_c)$ passes through a superconductor, the Lorentz force acting on the vortex line is

$$\mathbf{f}_L=\frac{\phi_0}{c}\mathbf{j}_T\times\hat{z}\ , \tag{2.1}$$

where $\phi_0=hc/2e$ is the flux quantum.

We represent the drag force $\mathbf{f}_v$ acting on the ith vortex with velocity $\mathbf{v}_{iL}$

$$\mathbf{f}_v=-\eta\mathbf{v}_{iL}-\alpha\mathbf{v}_{iL}\times\hat{z}\ . \tag{2.2}$$

A force of this kind was proposed by Vinen and Warren[29] to describe vortex motion in superconductors. More recent work[10,18,30] has used this force for describing the Hall effect. In the absence of pinning,[31] from the Bardeen-Stephen theory we can show (Appendix) $\eta=\phi_0H_{c2}/c^2\rho_n$ and $\alpha=\pm\phi_0H_{c2}H/c^3ne\rho_n^2$, where the + sign is for the hole carrier and the − for the electron carrier. The Hall angle in the absence of pinning is

$$\tan\Theta_H=\frac{\alpha}{\eta}\ . \tag{2.3}$$

The pinning force is due to sample inhomogeneities. We choose the pinning force $\mathbf{f}_{\rm pin}$ of the type[25–28]

$$\mathbf{f}_{\rm pin}(\mathbf{r}_i)=-\nabla U(\mathbf{r}_i)\ , \tag{2.4}$$

where the pinning potential $U(\mathbf{r}_i)=\int d^2r\Phi(\mathbf{r}_i-\mathbf{r})S(\mathbf{r})$, $\Phi=\hbar^2|\Psi|^2/2m$ and Ψ is the order parameter. The dimensionless quantity $S(\mathbf{r})$ is a random quantity for any given point. The correlation function is defined to be

$$\Gamma_U(\mathbf{r}-\mathbf{r}')=\langle U(\mathbf{r})U(\mathbf{r}')\rangle\ , \tag{2.5}$$

where $\langle\cdots\rangle$ denotes the average with respect to the distribution of $\{U(\mathbf{r})\}$.

In the space between the pinning centers the FLL is deformed. For weak point pinning centers, the deformation is weak and may be described by means of the theory of elasticity. The elastic restoring force is[25–28]

$$\mathbf{f}_R=\frac{\phi_0}{B}\left[(c_{11}-c_{66})\nabla_\perp(\nabla_\perp\cdot\mathbf{u}_i)+c_{66}\nabla_\perp^2\mathbf{u}_i+c_{44}\frac{\partial^2}{\partial z^2}\mathbf{u}_i\right]\ , \tag{2.6}$$

where $B=n_\phi\phi_0$ (n_ϕ is the vortex density) is the magnetic induction and c_{11}, c_{44}, and c_{66} are elastic moduli.[32] In the film geometry we have $H\simeq B$.

In the steady state, the sum of all the forces acting on the FLL should be zero. We therefore have the equation of motion of the FLL

$$\mathbf{f}_L+\mathbf{f}_v+\mathbf{f}_{\rm pin}+\mathbf{f}_R=0\ . \tag{2.7}$$

III. SOLUTION OF THE EQUATION OF MOTION

Without pinning, the solution of the rigid lattice motion is

$$\mathbf{v}_0=\frac{1}{\eta}\left[\mathbf{f}_L-\frac{\alpha}{\eta}\mathbf{f}_L\times\hat{z}\right]+O\left[\frac{\alpha^2}{\eta^3}\right]\ . \tag{3.1}$$

The velocity correction term due to pinning satisfies

$$\eta\frac{\partial}{\partial t}\mathbf{u}_i+\alpha\frac{\partial}{\partial t}\mathbf{u}_i\times\hat{z}-(\tilde{c}_{11}-\tilde{c}_{66})\nabla_\perp(\nabla_\perp\cdot\mathbf{u}_i)-\tilde{c}_{66}\nabla_\perp^2\mathbf{u}_i-\tilde{c}_{44}\frac{\partial^2}{\partial z^2}\mathbf{u}_i=\mathbf{f}_{\rm pin}(\mathbf{r}_i)\ , \tag{3.2}$$

where $\tilde{c}_{11}\equiv\phi_0c_{11}/B$, $\tilde{c}_{44}\equiv\phi_0c_{44}/B$, and $\tilde{c}_{66}\equiv\phi_0c_{66}/B$.

A. Two-dimensional (2D) case

We consider first the case where the bending of the vortex along the $\hat{z}$ direction is negligible, so that the $\tilde{c}_{44}$ term in Eq. (3.2) is negligible. This is true when the penetration depth λ is much greater than film thickness d or for considering a single layer of a highly anisotropic superconductor.

In order to get a solution for $\mathbf{u}_i(\mathbf{r}_i,t)$, we define the Green's function $\overleftrightarrow{G}(\mathbf{r}_i-\mathbf{r}_i',t-t')$ as follows:

$$\left[\eta\frac{\partial}{\partial t}\overleftrightarrow{I}-\alpha\hat{z}\times\overleftrightarrow{I}\frac{\partial}{\partial t}-(\tilde{c}_{11}-\tilde{c}_{66})\nabla_\perp\nabla_\perp-\tilde{c}_{66}\nabla_\perp^2\overleftrightarrow{I}\right]\cdot\overleftrightarrow{G}(\mathbf{r}_i-\mathbf{r}_i',t-t')=\overleftrightarrow{I}\delta^2(\mathbf{r}_i-\mathbf{r}_i')\delta(t-t')\ , \tag{3.3}$$

where $\overleftrightarrow{I}$ is the two-dimensional unit tensor of rank two.

It is convenient to make the Fourier transformation of

$$\overleftrightarrow{G}(\mathbf{r}_i-\mathbf{r}_i',t-t')=\frac{1}{(2\pi)^2}\int d^2k\frac{1}{(2\pi)}\int d\omega e^{i\mathbf{k}\cdot(\mathbf{r}_i-\mathbf{r}_i')-i\omega(t-t')}\overleftrightarrow{G}(\mathbf{k},\omega)\ . \tag{3.4}$$

The projection of $\overleftrightarrow{G}(\mathbf{k},\omega)$ in the x-y representation (ignoring terms of order of α^2/η^3) has the following form:

$$
\begin{aligned}
G_{xx}(\mathbf{k},\omega) &= \frac{(\hat{k}\cdot\hat{x})^2}{-i\eta\omega+\tilde{c}_{11}\mathbf{k}^2}+\frac{(\hat{k}\cdot\hat{y})^2}{-i\eta\omega+\tilde{c}_{66}\mathbf{k}^2}\,,\\
G_{yy}(\mathbf{k},\omega) &= \frac{(\hat{k}\cdot\hat{y})^2}{-i\eta\omega+\tilde{c}_{11}\mathbf{k}^2}+\frac{(\hat{k}\cdot\hat{x})^2}{-i\eta\omega+\tilde{c}_{66}\mathbf{k}^2}\,,\\
G_{xy}(\mathbf{k},\omega) &= \left[(\hat{k}\cdot\hat{x})(\hat{k}\cdot\hat{y})-\frac{i\alpha\omega}{(\tilde{c}_{11}-\tilde{c}_{66})\mathbf{k}^2}\right]\left[\frac{1}{-i\eta\omega+\tilde{c}_{11}\mathbf{k}^2}-\frac{1}{-i\eta\omega+\tilde{c}_{66}\mathbf{k}^2}\right],\\
G_{yx}(\mathbf{k},\omega) &= \left[(\hat{k}\cdot\hat{x})(\hat{k}\cdot\hat{y})+\frac{i\alpha\omega}{(\tilde{c}_{11}-\tilde{c}_{66})\mathbf{k}^2}\right]\left[\frac{1}{-i\eta\omega+\tilde{c}_{11}\mathbf{k}^2}-\frac{1}{-i\eta\omega+\tilde{c}_{66}\mathbf{k}^2}\right].
\end{aligned}
\tag{3.5}
$$

If we let $\alpha=0$ the above solution reduces to the known result.[33]

The solution for $\mathbf{u}_i(\mathbf{r}_i,t)$ becomes

$$\mathbf{u}_i(\mathbf{r}_i,t)=-\int d^2r_i'\int dt'\frac{1}{(2\pi)^2}\int d^2k e^{i\mathbf{k}\cdot(\mathbf{r}_i-\mathbf{r}_i')}\overleftrightarrow{\mathbf{G}}(\mathbf{k},t-t')\cdot\nabla U(\mathbf{R}_i+\mathbf{r}_0(t')+\mathbf{u}_i(\mathbf{r}_i',t'))\,. \tag{3.6}$$

Using this equation, we can, in principle, calculate the solution in terms of successive order of weak pinning potential U.[25–28]

Since $\Phi(\mathbf{r})$ in the pinning potential is proportional to the square of the Ginzburg-Landau order parameter for the perfect FLL, we can expand it in terms of reciprocal-lattice vectors $\mathbf{K}_n$ as

$$\Phi(\mathbf{r})=\frac{B}{N\phi_0}\sum_n\Phi(\mathbf{K}_n)e^{i\mathbf{K}_n\cdot\mathbf{r}}\,, \tag{3.7}$$

where we have used Schmid's prefactor convention,[25] N is the total number of vortices in the superconducting sample and $\Phi(\mathbf{K}_n)=\int d^2r\,\exp(-i\mathbf{K}_n\cdot\mathbf{r})\Phi(\mathbf{r})$ with $n=0,\pm1,\pm2,\ldots$.

After getting the second order in terms of pinning potential U, we make the average over pinning centers and sample area. Due to the randomness of the weak pinning centers the first-order term is zero. After the averaging, we can drop the index for each individual vortex. Recall that $\mathbf{v}_1=\partial\mathbf{u}/\partial t$. We then have

$$\mathbf{v}_1=-\frac{i}{4\pi\eta}\sum_{\mathbf{K}_n}\mathbf{K}_n^2\left[\mathbf{K}_n+\frac{\alpha}{\eta}\hat{z}\times\mathbf{K}_n\right]\Gamma_U(\mathbf{K}_n)\int k\,dk\left[\frac{1}{\tilde{c}_{11}\mathbf{k}^2+i\eta\mathbf{K}_n\cdot\mathbf{v}_0}+\frac{1}{\tilde{c}_{66}\mathbf{k}^2+i\eta\mathbf{K}_n\cdot\mathbf{v}_0}\right], \tag{3.8}$$

where $\Gamma_U(\mathbf{K}_n)=|\Phi(\mathbf{K}_n)|^2\Gamma_S(\mathbf{K}_n)$ is the Fourier transform of the correlation function $\Gamma_U(\mathbf{r})$, $\Gamma_S(\mathbf{K}_n)$ is the Fourier transform of the correlation function $\Gamma_S(\mathbf{r})=\langle S(\mathbf{r})S(0)\rangle$.

We further assume that $\Gamma_U(\mathbf{K}_n)$ is independent of direction of $\mathbf{K}_n$, i.e., $\Gamma_U(\mathbf{K}_n)=\Gamma_U(K_n)$. Carrying out the integral and taking the summation over only the six nearest-neighbor reciprocal-lattice vectors yields

$$\mathbf{v}_1=-\frac{1}{16\eta}\left[\frac{1}{\tilde{c}_{11}}+\frac{1}{\tilde{c}_{66}}\right]\mathbf{K}_1^2\Gamma_U(K_1)\times\sum_{i=1}^{6}\left[\mathbf{K}_i+\frac{\alpha}{\eta}\hat{z}\times\mathbf{K}_i\right]\mathrm{sgn}(\mathbf{K}_i\cdot\mathbf{v}_0)\,, \tag{3.9}$$

where $K_i=4\pi/\sqrt{3}a_0$ $[a_0=(2/\sqrt{3})^{1/2}(\phi_0/B)^{1/2}$, $i=1,\ldots,6]$ is the magnitude of the nearest-neighbor reciprocal-lattice vectors.

We next evaluate the vector sum $\sum_{i=1}^{6}\mathbf{K}_i\mathrm{sgn}(\mathbf{K}_i\cdot\mathbf{v}_0)$. Let θ be the angle between the velocity $\mathbf{v}_0$ and one of the nearest-neighbor FLL vectors. Experiment has indicated $\theta=0$,[34] but the perpendicular component (to both $\hat{\mathbf{v}}_0$ and $\hat{z}$) of the vector sum has a discontinuity for $\theta\to+0$ and $\theta\to-0$ (Refs. 25 and 27). However, experimentally the probability of $\theta\to\pm0$ should be equal, therefore the average effect of the perpendicular component is zero. The vector sum is along the direction of $\mathbf{v}_0$.[35] For $\theta=\pi/6$ where $\mathbf{v}_0$ is along the second-nearest-neighbor FLL direction, the vector sum is also simply along the $\mathbf{v}_0$ direction. We now have

$$\sum_{i=1}^{6}\mathbf{K}_i\mathrm{sgn}(\mathbf{K}_i\cdot\mathbf{v}_0)=4K_1\cos\left[\frac{\pi}{6}-\theta\right]\hat{\mathbf{v}}_0\,. \tag{3.10}$$

For transport current in the $\hat{x}$ direction ($\mathbf{j}_T=j_T\hat{x}$) and the choice of $\theta=0$, Eq. (3.9) reduces to

$$\mathbf{v}_1=\frac{\sqrt{3}}{8\eta}\left[\frac{1}{\tilde{c}_{11}}+\frac{1}{\tilde{c}_{66}}\right]K_1^3\Gamma_U(K_1)\left[\hat{y}-2\frac{\alpha}{\eta}\hat{x}\right]. \tag{3.11}$$

The mean electric field associated with the motion of vortex lines is determined by the Josephson relation[36]

$$\mathbf{E}=-\frac{1}{c}\mathbf{v}_L\times\mathbf{B}\,, \tag{3.12}$$

where $\mathbf{v}_L=\mathbf{v}_0+\mathbf{v}_1$ is the total average vortex velocity.

The longitudinal and Hall resistivity are defined as $\rho_{xx}=E_x/j_T$ and $\rho_{xy}=E_y/j_T$ which, using Eqs. (3.1) and (3.11), have the following results:

$$\rho_{xx}=\frac{\phi_0 B}{c^2\eta}\left[1-\frac{\sqrt{3}}{8}\left[\frac{c}{\phi_0 j_T}\right]\left[\frac{1}{\tilde{c}_{11}}+\frac{1}{\tilde{c}_{66}}\right]\times K_1^3\Gamma_U(K_1)\right], \tag{3.13}$$

$$\rho_{xy}=\frac{\phi_0 B}{c^2\eta^2}\alpha\left[1-2\frac{\sqrt{3}}{8}\left[\frac{c}{\phi_0 j_T}\right]\left[\frac{1}{\tilde{c}_{11}}+\frac{1}{\tilde{c}_{66}}\right]\times K_1^3\Gamma_U(K_1)\right]. \tag{3.14}$$

For the weak pinning case we can ignore terms involving squares of the correlation function, therefore we have $\rho_{xy}=(c^2\alpha/\phi_0 B)\rho_{xx}^2$. As long as the phenomenological parameter α has very weak temperature dependence compared to ρ_{xx}, then the scaling $\rho_{xy}\propto\rho_{xx}^2$ holds for fixed magnetic fields as predicted by Refs. 30 and 16. We note that all the data fit to the scaling form were done in temperature-sweep measurements in the thermally activated flux-flow regime.[9,22,37,38] The experimental observation of the scaling law demonstrates that in the thermally activated flux-flow or flux-creep region, the temperature dependence of α is far weaker than the temperature dependence of ρ_{xx}. The perturbation calculation in the thermally activated flux-flow regime has been carried out,[39,33] and has been well justified by many authors.[40,41]

If we choose new phenomenological parameters η' and α' to be as follows:

$$\eta'=\eta\left[1+\frac{\sqrt{3}}{8}\left[\frac{c}{\phi_0 j_T}\right]\left[\frac{1}{\tilde{c}_{11}}+\frac{1}{\tilde{c}_{66}}\right]K_1^3\Gamma_U(K_1)\right], \tag{3.15}$$

$$\alpha'=\alpha, \tag{3.16}$$

then Eqs. (3.13) and (3.14) can be rewritten as $\rho_{xx}=\phi_0 B/c^2\eta'$ and $\rho_{xy}=\phi_0 B\alpha'/c^2\eta'^2$. This is exactly the same solution of the perfect FLL motion of Eq. (3.1) but with renormalized phenomenological parameters. In the first nonvanishing order of the perturbation calculation, η is renormalized, while α is unchanged. The renormalization to higher-order correction remains to be done. Arguments have been made that pinning effects will renormalize the phenomenological coefficient α too,[16] but this does not come out of the first-order perturbation calculation, as we have shown.

The Hall angle is defined as $\tan\Theta_H=\rho_{xy}/\rho_{xx}$ which is given by

$$\tan\Theta_H=\frac{\alpha}{\eta}\left[1-\frac{\sqrt{3}}{8}\left[\frac{c}{\phi_0 j_T}\right]\left[\frac{1}{\tilde{c}_{11}}+\frac{1}{\tilde{c}_{66}}\right]\times K_1^3\Gamma_U(K_1)\right]. \tag{3.17}$$

Based on the experimental fact that the Hall angle in the superconducting state is very small ($\tan\Theta_H=\rho_{xy}/\rho_{xx}\sim10^{-2}-10^{-4}$), the longitudinal and Hall conductivity can be calculated as $\sigma_{xx}=\rho_{xx}/(\rho_{xx}^2+\rho_{xy}^2)\approx1/\rho_{xx}$ and $\sigma_{xy}=\rho_{xy}/(\rho_{xx}^2+\rho_{xy}^2)\approx\rho_{xy}/\rho_{xx}^2$,

$$\sigma_{xx}=\frac{c^2\eta}{\phi_0 B}\left[1+\frac{\sqrt{3}}{8}\left[\frac{c}{\phi_0 j_T}\right]\left[\frac{1}{\tilde{c}_{11}}+\frac{1}{\tilde{c}_{66}}\right]\times K_1^3\Gamma_U(K_1)\right], \tag{3.18}$$

$$\sigma_{xy}=\frac{c^2\alpha}{\phi_0 B}. \tag{3.19}$$

The weak collective-pinning effect will increase the flux motion conductivity as expected. The Hall conductivity is *independent* of pinning to the first nonvanishing order of the perturbation correction term—the pinning correlation function. The field and temperature dependence of the phenomenological coefficient α determines the sign change.[16,30]

B. 3D case

Next we consider an isotropic 3D case. The Fourier transform is three dimensional for the Green's function. The solution of the Green's function, Eq. (3.5) is in the same form except we need to make the substitutions of $\tilde{c}_{11}\mathbf{k}^2\to\tilde{c}_{11}\mathbf{k}_\perp^2+\tilde{c}_{44}k_z^2, \tilde{c}_{66}\mathbf{k}^2\to\tilde{c}_{66}\mathbf{k}_\perp^2+\tilde{c}_{44}k_z^2$ and $\hat{k}\to\hat{k}_\perp$.

The rest of the calculation is similar to Sec. III A. The correction term due to the pinning force to the second order of the pinning potential is given by

$$\mathbf{v}_1=-\frac{d}{4\eta}\frac{1}{\sqrt{2\tilde{c}_{44}}}\left[\frac{1}{\tilde{c}_{11}}+\frac{1}{\tilde{c}_{66}}\right]\sum_{\mathbf{K}_n}\mathbf{K}_n^2|\eta\mathbf{K}_n\cdot\mathbf{v}_0|^{1/2}\Gamma_U(K_n)\left[\mathbf{K}_n+\frac{\alpha}{\eta}\hat{z}\times\mathbf{K}_n\right]\mathrm{sgn}(\mathbf{K}_n\cdot\mathbf{v}_0). \tag{3.20}$$

Again consider the nearest-neighbor summation only

$$\begin{aligned}\sum_{i=1}^{6}|\eta\mathbf{K}_i\cdot\mathbf{v}_0|^{1/2}\mathrm{sgn}(\mathbf{K}_i\cdot\mathbf{v}_0)\mathbf{K}_i=&\,2|\eta K_1 v_0|^{1/2}K_1\hat{\mathbf{v}}_0\left[\sin^{3/2}\theta+\cos^{3/2}\left[\theta-\frac{\pi}{6}\right]+\cos^{3/2}\left[\theta+\frac{\pi}{6}\right]\right]\\&+2|\eta K_1 v_0|^{1/2}K_1\hat{\mathbf{v}}_0\times\hat{z}\left[\cos^{1/2}\theta\sin\theta+\left|\cos\left[\theta+\frac{\pi}{3}\right]\right|^{1/2}\sin\left[\theta+\frac{\pi}{3}\right]\right.\\&\left.-\left|\cos\left[\theta+\frac{2\pi}{3}\right]\right|^{1/2}\sin\left[\theta+\frac{2\pi}{3}\right]\right].\end{aligned} \tag{3.21}$$

Employing the same argument as before for choosing angle $\theta=0$, the correction term for the vortex velocity due to pinning is

$$\mathbf{v}_1=\frac{d}{4\pi\eta}\frac{(\sqrt{3})^{3/2}}{\sqrt{\tilde{c}_{44}}}\left[\frac{1}{\tilde{c}_{11}}+\frac{1}{\tilde{c}_{66}}\right]\left[\frac{\phi_0 j_T}{c}\right]^{1/2} \times K_1^{7/2}\Gamma_U(K_1)\left[\hat{y}-2\frac{\alpha}{\eta}\hat{x}\right] . \tag{3.22}$$

The longitudinal and Hall resistivity are given by

$$\rho_{xx}=\frac{\phi_0 B}{c^2\eta}\left[1-\frac{(\sqrt{3})^{3/2}}{4\pi}\frac{d}{\sqrt{\tilde{c}_{44}}}\left[\frac{c}{\phi_0 j_T}\right]^{1/2} \times\left[\frac{1}{\tilde{c}_{11}}+\frac{1}{\tilde{c}_{66}}\right]K_1^{7/2}\Gamma_U(K_1)\right] , \tag{3.23}$$

$$\rho_{xy}=\frac{\phi_0 B}{c^2\eta^2}\alpha\left[1-2\frac{(\sqrt{3})^{3/2}}{4\pi}\frac{d}{\sqrt{\tilde{c}_{44}}}\left[\frac{c}{\phi_0 j_T}\right]^{1/2} \times\left[\frac{1}{\tilde{c}_{11}}+\frac{1}{\tilde{c}_{66}}\right]K_1^{7/2}\Gamma_U(K_1)\right] . \tag{3.24}$$

Again the scaling $\rho_{xy}\propto\rho_{xx}^2$ holds in the limit of weak pinning.

The Hall angle in the 3D case is given by the following:

$$\tan\Theta_H=\frac{\alpha}{\eta}\left[1-\frac{(\sqrt{3})^{3/2}}{4\pi}\frac{d}{\sqrt{\tilde{c}_{44}}}\left[\frac{c}{\phi_0 j_T}\right]^{1/2} \times\left[\frac{1}{\tilde{c}_{11}}+\frac{1}{\tilde{c}_{66}}\right]K_1^{7/2}\Gamma_U(K_1)\right] . \tag{3.25}$$

The longitudinal and Hall conductivity can be obtained similarly,

$$\sigma_{xx}=\frac{c^2\eta}{\phi_0 B}\left[1+\frac{(\sqrt{3})^{3/2}}{4\pi}\frac{d}{\sqrt{\tilde{c}_{44}}}\left[\frac{c}{\phi_0 j_T}\right]^{1/2} \times\left[\frac{1}{\tilde{c}_{11}}+\frac{1}{\tilde{c}_{66}}\right]K_1^{7/2}\Gamma_U(K_1)\right] , \tag{3.26}$$

$$\sigma_{xy}=\frac{c^2\alpha}{\phi_0 B} . \tag{3.27}$$

IV. DISCUSSION

The exact form of pinning is unknown. We consider the pinning of the form[42] $S(\mathbf{r})=S_0/\pi R_p^2\exp(-r^2/R_p^2)$, where R_p is the radius of the pinning center. Using the spatial and magnetic-field dependence of the order parameter as expressed in Refs. 43 and 44, the correlation function obtained in the high-field range in 2D is

$$\Gamma_U(\mathbf{K}_1)=\frac{S_0^2}{2\pi(2\kappa^2-1)^2\beta_A^2}\left[\frac{\phi_0^2}{8\pi^2\xi^3(0)}\right]^2 \times(1-T/T_c)\frac{1}{b}(1-b)^2e^{-2\pi/\sqrt{3}}e^{-K_1^2r_p^2/4} , \tag{4.1}$$

where $r_p=\sqrt{2}/K_1$ (Ref. 42) is the pinning force range, $b=B/H_{c2}(T)$ is the reduced magnetic field and $\beta_A=1.16$. We choose the following typical high-T_c parameters: $\xi(0)\approx 15$ Å, $\kappa\approx 100$, $H_{c2}(0)\approx 120$ T, $\rho_n\approx 100$ $\mu\Omega$ cm, $n\sim 10^{21}$ cm^{-3}, and with $j_T\approx 10^3$ A/cm^2, $(T_c-T)/T_c\approx 10^{-2}$. In the weak collective-pinning range, the pinning parameters can be estimated as $|S(\mathbf{r})|\sim 10^{-3}$, $R_p\sim 10$ Å, and $S_0\sim 10^{-17}$ cm^2 (Ref. 25).

With the parameters given above, the correction term due to pinning is about two orders of magnitude smaller than 1. Similar results also hold for the 3D high-T_c case and 2D/3D low-T_c cases. The longitudinal resistivities in Eqs. (3.13) and (3.23) behave like the Bardeen-Stephen result $\rho_f/\rho_n=H/H_{c2}$. It is clear from the above estimate that the weak collective pinning due to point pinning centers are not responsible for the Hall resistivity sign reversal.

Earlier estimates of the influence of pinning on the Hall effect have not dealt explicitly with collective effects, but have treated pinning in a nonrigorously averaged way.[15,16,30] Vinokur *et al.*[30] arrive at conclusions similar to ours: scaling occurs, and no sign change is predicted. Other work taking a different approach[15,16] obtains scaling, but predicts a sign change as a *consequence* of pinning. Since the starting point of these two papers is different, it is difficult to draw definite conclusions concerning the correctness of either of these results. We emphasize, however, that the treatment of pinning collectively has gained strong experimental support lately,[40,41,45] and our calculations are more rigorous than previous treatments. It is an experimental fact that in high-T_c materials a sign reversal occurs mostly at temperatures close to T_c, where weak collective pinning is believed to be an adequate description of pinning.[40,41]

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APPENDIX

We adopt the BS theory by simplifying the vortex as a normal core. The quasiparticle with velocity $\mathbf{v}$ inside the core obeys the equation of motion

$$m\frac{d\mathbf{v}}{dt}=q\mathbf{E}_c+\frac{q}{c}\mathbf{v}\times\mathbf{h}-\frac{m}{\tau}\mathbf{v} , \tag{A1}$$

where $q=\pm e$ correspond to hole-electron charge, m is the quasiparticle mass, τ is the quasiparticle relaxation time, $\mathbf{E}_c$ and $\mathbf{h}$ are electric and magnetic field inside the core. The external magnetic field $\mathbf{H}$ is applied along the $+\hat{z}$ direction.

According to the BS theory, the electric field inside the core is a constant

$$\mathbf{E}_c = -\frac{\hbar}{2e\xi^2}(\mathbf{v}_L \times \hat{z}) \ . \tag{A2}$$

In the steady state, the spatial average of $\mathbf{v}$ based on the BS theory is $\langle \mathbf{v} \rangle = \mathbf{v}_T$, where $\mathbf{v}_T$ is the external transport current velocity. We also have $\langle \mathbf{h} \rangle = \mathbf{B} \simeq \mathbf{H}$. The steady-state solution for the quasiparticle equation of motion is

$$\mathbf{v}_T = \frac{(H_{c2}/H)(\omega_c \tau)}{1+(\omega_c \tau)^2}[\mp \omega_c \tau \mathbf{v}_L - \mathbf{v}_L \times \hat{z}] \ , \tag{A3}$$

where we have used $H_{c2} = \phi_0 / 2\pi\xi^2$, and $\omega_c = eH/mc$ is the electron cyclotron frequency. The electron relaxation time τ is related to the normal-state resistivity ρ_n by $\tau = m/ne^2\rho_n$, and n is the normal-state carrier density. For $\omega_c \tau \ll 1$, multiply the above equation by $\phi_0 nq/c$ and cross product of $\hat{z}$ yields

$$\frac{\phi_0}{c} ne \mathbf{v}_T \times \hat{z} = \frac{\phi_0 H_{c2}}{c^2 \rho_n} \mathbf{v}_L \pm \frac{\phi_0 H_{c2} H}{nec^3 \rho_n^2} \mathbf{v}_L \times \hat{z} \ . \tag{A4}$$

The above equation is in the form of $\mathbf{f}_L + \mathbf{f}_v = 0$ with $\eta = \phi_0 H_{c2}/c^2 \rho_n$, $\alpha = \pm \phi_0 H_{c2} H / nec^3 \rho_n^2$.

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