Magnetic properties of disordered Ising systems with various probability distributions of the exchange integrals

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The disordered Ising spin-$\frac{1}{2}$ systems on square and simple cubic lattices are investigated with three types (the Handrich-Kaneyoshi's, the Gaussian, and the rectangular) of probability distribution of the exchange integrals. The general expressions of average magnetization for various distributions are derived by means of the differential operator technique. The Curie temperatures T_C of the systems are calculated with the use of the effective-field theory, the correlated effective-field theory, and the improved correlated effective-field theory (ICEFT), respectively. The dependence of T_C upon the parameter Δ, characterizing the fluctuation of the exchange integrals is given. It is obtained, under ICEFT, that for the case of the square lattice, these distributions give almost the same results and no reentrant phenomena occur, while on the simple cubic lattice, the reentrant phenomena only exist under the discrete distribution.

I. INTRODUCTION

Much attention has been paid to the investigation of disordered Ising systems in which the disorder is represented by the randomness of the exchange integrals. Various methods have been used to discuss disordered ferromagnetic or ferrimagnetic Ising systems, such as the effective-field theory (EFT),[1–4] the finite cluster approximation,[5,6] and the Mastudaira approximation.[7] The differential[8] and the integral[9] operator techniques are also developed. In all these investigations the disorder of the exchange integrals J_{ij} is described by the Handrich-Kaneyoshi (HK) type probability distribution function, according to which J_{ij} may only take two discrete values. Although the EFT is quite simple and better than the molecular-field approximation and has got many useful results for various spin systems, it is still far less satisfactory, because it approximates the average of the product of spin variables to the product of the average of the single spin variable, and thus neglects all the multispin correlation functions. Kaneyoshi *et al.*[10,11] introduced a better approximation—the correlated effective-field theory (CEFT). This theory uses a correlated effective-field parameter to partially consider the spin-spin correlation. Its results are equivalent to the Bethe-Peierls approximation.

Recently more accurate theories, namely the improved correlated effective-field theory (ICEFT) (Ref. 12) and the equivalent crystal transformation (ECT) (Ref. 13) were applied to the disordered Ising systems and obtained some results. Take the reentrant magnetism phenomena, for example; there are interesting conclusions. For a square lattice, the phenomena exist in EFT and CEFT. But with the increase of calculation accuracy, they disappear in ICEFT and ECT.[14] For a simple cubic lattice, the case is just the opposite—they do not exist in EFT and CEFT, but occur in ICEFT.[15] (For the simple cubic lattice, ECT, which is based on the exact result for an ideal system, cannot be used.)

Also recently some continuous probability distribution functions of the random exchange integrals J_{ij} were introduced, namely the Gaussian type[16] and the rectangular type[17] functions. With the help of the integral operator method, these functions were applied on a square lattice. The results were compared with those obtained with the HK function.

In this paper $S=\frac{1}{2}$ disordered Ising systems will be studied on the square and simple cubic lattices with the J_{ij} of the HK type, the Gaussian type, and the rectangular type probability distribution functions, respectively. The differential operator technique will be used. We will derive the general expressions of the average magnetization. The derivation procedure is much simpler than that in previous investigations using integral operator technique. Furthermore, we will calculate the Curie temperatures of the systems in the framework of EFT, CEFT, and ICEFT, respectively, and compare the results obtained.

II. THEORY AND APPLICATIONS TO SQUARE AND SIMPLE CUBIC LATTICES

The Hamiltonian of a random disordered Ising ferromagnetic system is written in the form

$$H=-\frac{1}{2}\sum_{i,j} J_{ij}S_iS_j \ , \qquad (1)$$

where $S_i=\pm 1$ is the spin variable on site i, J_{ij} is the random exchange integral between sites i and j, and is described by the probability distribution function $P\,(J_{ij})$. The summation takes all the nearest-neighboring pairs i and j.

For the disordered Ising spin-$\frac{1}{2}$ system, the exact Callen identity is[18]

$$\langle\langle S_i\rangle\rangle_r=\left\langle\!\!\left\langle \tanh\left[\beta\sum_j J_{ij}S_j\right]\right\rangle\!\!\right\rangle_r \ , \qquad (2)$$

where the summation takes all the nearest-neighboring spin sites of i and $\langle\langle\ \rangle\rangle_r$ refers to double averaging, namely the ensemble $\langle\ \rangle$ and the random configurational $\langle\ \rangle_r$, concerning spin and random exchange integrals, respectively. Introducing the differential operator $D=\partial/\partial x$, we can rewrite Eq. (2) as

$$\langle\langle S_i\rangle\rangle_r=\left\langle\left\langle \exp\left[\sum_{j=1}^{z} J_{ij}S_jD\right]\right\rangle\right\rangle_r \tanh(\beta x)|_{x=0}\ , \tag{3}$$

where z denotes the coordination number of the lattice.

According to the well-known relation for $S=\frac{1}{2}$,

$$\exp(J_{ij}S_jD)=\cosh(J_{ij}D)+S_j\sinh(J_{ij}D)\ , \tag{4}$$

we may write

$$\left\langle\left\langle \exp\left[\sum_{j=1}^{z} J_{ij}S_jD\right]\right\rangle\right\rangle_r =\left\langle\left\langle \prod_{j=1}^{z}[\cosh(J_{ij}D)+S_j\sinh(J_{ij}D)]\right\rangle\right\rangle_r\ . \tag{5}$$

Here we will, as usual, make the following approximations described in Ref. 17: (i) the configurational average of spins and exchange integrals is taken independently and (ii) the exchange integrals J_{ij} for different j are also independent of each other. Thus Eq. (5) can be written in the form

$$\langle\langle S_i\rangle\rangle_r\approx\sum_{n=1}^{z}K_{nz}\sum_{\substack{j_1,j_2,\ldots,j_n=1\\(j_1<j_2,\ldots,j_n)}}\langle\langle S_{j1}S_{j2}\cdots S_{jn}\rangle\rangle_r\ , \tag{6}$$

where all the spins $S_{j1},S_{j2},\ldots,S_{jn}$ are the nearest neighbors of S_i, and the coefficients K_{nz} are defined as

$$K_{nz}=\langle\cosh(DJ_{ij})\rangle_r^{z-n}\langle\sinh(DJ_{ij})\rangle_r^{n}\tanh(\beta x)|_{x=0}\ . \tag{7}$$

Because $\tanh(\beta x)$ is an odd function of x, only odd n appear in K_{nz} for square or simple cubic lattice.

For a square lattice, $z=4$, we have

$$\langle\langle S_i\rangle\rangle_r=4K_{14}\langle\langle S_1\rangle\rangle_r+4K_{34}\langle\langle S_1S_2S_3\rangle\rangle_r\ . \tag{8}$$

For a simple cubic lattice, $z=6$, we have

$$\begin{aligned}\langle\langle S_i\rangle\rangle_r=&6K_{16}\langle\langle S_1\rangle\rangle_r\\&+4K_{36}(3\langle\langle S_1S_2S_3\rangle\rangle_r+2\langle\langle S_1S_3S_5\rangle\rangle_r)\\&+6K_{56}\langle\langle S_1S_2S_3S_4S_5\rangle\rangle_r\ ,\end{aligned} \tag{9}$$

where $S_1,S_2,\ldots,S_5$ are all the nearest neighboring spins of S_i. The spin sites 1, 2, 3, and i are situated on a same plane, but 1, 3, and 5 are not. (See Figs. 1 and 3 in Ref. 12.) Now we will derive the formulas of K_{nz} for different probability distribution functions $P(J_{ij})$. The following derivation will be restricted in odd n and even z.

A. HK probability

In this case

$$P(J_{ij})=\tfrac{1}{2}[\delta(J_{ij}-J-\Delta_1)+\delta(J_{ij}-J+\Delta_1)]\ , \tag{10}$$

where J and Δ_1 represent the mean value and the standard deviation of the exchange integrals J_{ij}, respectively. Although the concrete forms of K_{nz} with HK probability have been obtained in Refs. 14 and 15 for square and simple cubic lattices, we will derive its general form.

With Eq. (10) we have

$$\langle\cosh(DJ_{ij})\rangle_r=\cosh(DJ)\cosh(D\Delta_1)\ ,$$

$$\langle\sinh(DJ_{ij})\rangle_r=\sinh(DJ)\cosh(D\Delta_1)\ .$$

The coefficients K_{nz} can be expressed as

$$K_{nz}=\cosh^{z-n}(DJ)\sinh^{n}(DJ)\cosh^{z}(D\Delta_1)\tanh(\beta x)|_{x=0}\ . \tag{11}$$

The function $\cosh^z(x)$ for even z can be written as the following expansion:

$$\cosh^z(x)=\sum_{q=0}^{z/2}|a_q^{(z)}|\cosh(2qx)\ , \tag{12}$$

where

$$a_q^{(z)}=\frac{2-\delta_{q,0}}{2^z}(-1)^{z/2-q}\begin{bmatrix} z\\ \frac{1}{2}z-q\end{bmatrix}\ , \tag{13}$$

in which

$$\delta_{q,0}=\begin{cases}1, & q=0\\ 0, & q\neq 0\end{cases}$$

and $(_{1/2\ z-q}^{\ \ z})$ is the binomial coefficient.

The function $\cosh^{z-n}(x)\sinh^n(x)$ for even z and odd n can be expressed in the expansion

$$\cosh^{z-n}(x)\sinh^n(x)=\sum_{p=1}^{z/2}b_p^{(n)}\sinh(2px)\ . \tag{14}$$

The coefficients $b_p^{(n)}$ can be obtained by differentiating both sides of Eq. (12) n times with respect to x. Here we only present some of them which will be used later:

$$b_p^{(1)}=\frac{2p}{z}|a_p^{(z)}|\ , \tag{15}$$

$$b_p^{(3)}=\frac{2p}{\prod_{i=0}^{2}(z-i)}[(2p)^2-(3z-2)]|a_p^{(z)}|\ , \tag{16}$$

$$b_p^{(5)}=\frac{2p}{\prod_{i=0}^{4}(z-i)}[(2p)^4-10(z-2)(2p)^2+15z^2-50z+24]|a_p^{(z)}|\ . \tag{17}$$

Using Eqs. (12) and (14), and noticing $e^{D\alpha}f(x)=f(x+\alpha)$, we obtain

$$K_{nz}=\tfrac{1}{2}\sum_{p=1}^{z/2} b_p^{(n)} \sum_{q=0}^{z/2} |a_q^{(z)}| g_{pq} \ , \tag{18}$$

where

$$g_{pq}=\tanh[\beta(2pJ+2q\Delta_1)]+\tanh[\beta(2pJ-2q\Delta_1)] \ . \tag{19}$$

Equation (18) is the general expression of K_{nz} for any even z and odd n in the case of HK type function of J_{ij}. When $\Delta_1 \to 0$ we have $g_{pq}=2\tanh(2\beta pJ)$, irrelevant to q, and

$$\sum_{q=0}^{z/2} |a_q^{(z)}|=1 \ ,$$

thus

$$K_{nz}=\sum_{p=1}^{z/2} b_p^{(n)} \tanh(2\beta pJ) \ . \tag{20}$$

It is nothing but the coefficients K_{nz} for an ideal Ising system.[12]

For a disordered square lattice, the coefficients $a_q^{(z)}$ and $b_p^{(n)}$ are calculated from Eqs. (13) and (15)–(17): $a_0^{(4)}=\frac{3}{8}$, $a_1^{(4)}=-\frac{4}{8}$, $a_2^{(4)}=\frac{1}{8}$ and $b_1^{(1)}=-b_1^{(3)}=\frac{1}{4}$, $b_2^{(1)}=b_2^{(3)}=\frac{1}{8}$; therefore

$$K_{14}=\frac{1}{2^5}[3g_{20}+4g_{21}+g_{22}+2(3g_{10}+4g_{11}+g_{12})] \ , \tag{21}$$

$$K_{34}=\frac{1}{2^5}[3g_{20}+4g_{21}+g_{22}-2(3g_{10}+4g_{11}+g_{12})] \ . \tag{22}$$

For a disordered simple cubic lattice, $a_0^{(6)}=-\frac{10}{32}$, $a_1^{(6)}=\frac{15}{32}$, $a_2^{(6)}=-\frac{6}{32}$, $a_3^{(6)}=\frac{1}{32}$ and $b_1^{(1)}=b_1^{(5)}=\frac{5}{32}$, $b_1^{(3)}=-\frac{3}{32}$, $b_2^{(1)}=-b_2^{(5)}=\frac{4}{32}$, $b_2^{(3)}=0$, $b_3^{(1)}=b_3^{(3)}=b_3^{(5)}=\frac{1}{32}$. Therefore

$$K_{16}=\frac{1}{2^{11}}(5G_1+4G_2+G_3) \ , \tag{23}$$

$$K_{36}=\frac{1}{2^{11}}(-3G_1+G_3) \ , \tag{24}$$

$$K_{56}=\frac{1}{2^{11}}(5G_1-4G_2+G_3) \ , \tag{25}$$

where

$$G_p=10g_{p0}+15g_{p1}+6g_{p2}+g_{p3} \ . \tag{26}$$

Equations (21)–(26) were also obtained in Refs. 14 and 15; now we have derived a more general and concise formula for K_{nz} in Eq. (18).

B. Gaussian probability

In this case

$$P(J_{ij})\equiv P(J-J_{ij},\Delta_2)=\frac{1}{\sqrt{2\pi}\Delta_2}e^{-(J-J_{ij})^2/2\Delta_2^2} \ , \tag{27}$$

where J and Δ_2 are the mean value and the standard deviation of J_{ij}, respectively. With the help of Eq. (27), we may obtain

$$\langle \cosh(DJ_{ij})\rangle_r=\cosh(DJ)\exp(D^2\Delta_2^2/2) \ ,$$

$$\langle \sinh(DJ_{ij})\rangle_r=\sinh(DJ)\exp(D^2\Delta_2^2/2) \ .$$

The coefficients K_{nz} can be written as

$$K_{nz}=\cosh^{z-n}(DJ)\sinh^n(DJ)\exp(zD^2\Delta_2^2/2) \times\tanh(\beta x)|_{x=0} \ . \tag{28}$$

Having in mind the relation

$$\exp(zD^2\Delta_2^2/2)=\frac{1}{\sqrt{2\pi z}\,\Delta_2}\int_{-\infty}^{\infty}\exp\left[-\frac{t^2}{2z\Delta_2^2}+Dt\right]dt$$

and $e^{Dt}\tanh(\beta x)=\tanh[\beta(x+t)]$, we have

$$K_{nz}=\frac{1}{\sqrt{2\pi z}\,\Delta_2}\int_{-\infty}^{\infty}\exp\left[-\frac{t^2}{2z\Delta_2^2}+Dt\right]dt\,\cosh^{z-n}(DJ) \times\sinh^n(DJ)\tanh\beta(x+t)|_{x=0} \ .$$

By the use of Eq. (14),

$$K_{nz}=\tfrac{1}{2}\sum_{p=1}^{z/2} b_p^{(n)}\int_{-\infty}^{\infty}P(t,\sqrt{z}\,\Delta_2)[\tanh\beta(t+2pJ) -\tanh\beta(t-2pJ)]dt \ , \tag{29}$$

where $P(t,\sqrt{z}\,\Delta_2)$ is defined as Eq. (27). From the definition of $P(t,\sqrt{z}\,\Delta_2)$, we know that when $\Delta_2\to 0$ it tends to a δ function $\delta(t)$; K_{nz} thus reduces to Eq. (20) as expected.

For a square lattice, with the value of $b_p^{(n)}$, we have

$$K_{14}=\tfrac{1}{8}\int_0^{\infty}P(t,2\Delta_2)\{\tanh\beta(t+4J)-\tanh\beta(t-4J) +2[\tanh\beta(t+2J) -\tanh\beta(t-2J)]\}dt \ , \tag{30}$$

$$K_{34}=\tfrac{1}{8}\int_0^{\infty}P(t,2\Delta_2)\{\tanh\beta(t+4J)-\tanh\beta(t-4J) -2[\tanh\beta(t+2J) -\tanh\beta(t-2J)]\}dt \ . \tag{31}$$

These expressions are the same as those in Ref. 16 by the use of the integral operator method. Our derivation is not only much simpler, but the general form of K_{nz} (for even z and odd n) is given in Eq. (24) as well. For a simple cubic lattice,

$$K_{16}=\frac{1}{32}\int_0^{\infty}P(t,\sqrt{6}\Delta_2)\{\tanh\beta(t+6J)-\tanh\beta(t-6J)+4[\tanh\beta(t+4J)-\tanh\beta(t-4J)] + 5[\tanh\beta(t+2J)-\tanh\beta(t-2J)]\}dt\ , \tag{32}$$

$$K_{36}=\frac{1}{32}\int_0^{\infty}P(t,\sqrt{6}\Delta_2)\{\tanh\beta(t+6J)-\tanh\beta(t-6J)-3[\tanh\beta(t+2J)-\tanh\beta(t-2J)]\}dt\ , \tag{33}$$

$$K_{56}=\frac{1}{32}\int_0^{\infty}P(t,\sqrt{6}\Delta_2)\{\tanh\beta(t+6J)-\tanh\beta(t-6J)-4[\tanh\beta(t+4J)-\tanh\beta(t-4J)] + 5[\tanh\beta(t+2J)-\tanh\beta(t-2J)]\}dt\ . \tag{34}$$

C. Rectangular probability

The distribution function is[17]

$$P(J_{ij})=\begin{cases}1/(2\Delta_3), & J-\Delta_3<J_{ij}<J+\Delta_3\\ 0, & \text{elsewhere}\ ,\end{cases} \tag{35}$$

where J and $2\Delta_3$ are the mean value and the width of the probability distribution, respectively. With the help of the function $P(J_{ij})$ we have

$$\langle\cosh(DJ_{ij})\rangle_r=\cosh(DJ)\sinh(D\Delta_3)/(D\Delta_3)\ ,$$

$$\langle\sinh(DJ_{ij})\rangle_r=\sinh(DJ)\sinh(D\Delta_3)/(D\Delta_3)\ .$$

The coefficients K_{nz} can be expressed as

$$K_{nz}=\cosh^{z-n}(DJ)\sinh^{n}(DJ)\frac{1}{(D\Delta_3)^z}\sinh^z(D\Delta_3)\tanh(\beta x)|_{x=0}\ . \tag{36}$$

Noticing (for even z)

$$\sinh^z(x)=\sum_{q=0}^{z/2}a_q^{(z)}\cosh(2qx)\ , \tag{37}$$

where $a_q^{(z)}$ are given in Eq. (13) and applying the Laplace transformation for D^{-z},

$$D^{-z}=\frac{1}{(z-1)!}\int_0^{\infty}t^{z-1}e^{Dt}dt\ , \tag{38}$$

we have

$$\frac{1}{(D\Delta_3)^z}\sinh^z(D\Delta_3)\tanh(\beta x)$$

$$=\frac{1}{2(z-1)!\Delta_3^z}\sum_{q=0}^{z/2}a_q^{(z)}\int_{-\infty}^{\infty}\tanh[\beta(x-t)][(t-2q\Delta_3)^{z-1}\theta(t-2q\Delta_3)+(t+2q\Delta_3)^{z-1}\theta(t+2q\Delta_3)]dt\ , \tag{39}$$

where $\theta(t)$ is the Heaviside step function,

$$\theta(t)=\begin{cases}0, & t<0\\ 1, & t>0\ .\end{cases} \tag{40}$$

Substituting Eq. (39) into (36) and using (14) we get

$$K_{nz}=\frac{1}{2}\sum_{p=1}^{z/2}b_p^{(n)}\int_{-\infty}^{\infty}f_z(t)[\tanh\beta(t+2pJ)-\tanh\beta(t-2pJ)]dt\ , \tag{41}$$

where the function $f_z(t)$ is defined as

$$f_z(t)=\frac{1}{2(z-1)!\Delta_3^z}\sum_{q=0}^{z/2}a_q^{(z)}[(t-2q\Delta_3)^{z-1}\theta(t-2q\Delta_3)+(t+2q\Delta_3)^{z-1}\theta(t+2q\Delta_3)]\ , \tag{42}$$

which is an even function of t. Because of the step functions the integration in Eq. (41) actually only takes from $-z\Delta_3$ to $z\Delta_3$, then

$$K_{nz}=\sum_{p=1}^{z/2}b_p^{(n)}\int_0^{z\Delta_3}f_z(t)[\tanh\beta(t+2pJ)-\tanh\beta(t-2pJ)]dt\ . \tag{43}$$

It can be proved that $f_z(t)$ has the following properties: when $\Delta_3 \to 0$, $f_z(t) \to \infty$ and

$$\int_0^\infty f_z(t)dt = 1 , \tag{44}$$

i.e., $f_z(t)$ is a δ function when $\Delta_3 \to 0$. The proof is presented in the Appendix.

From Eq. (43) we know that only the function $f_z(t)$ in the range $0 < t < z\Delta_3$ need to be considered. For $z=4$ and 6 they are, respectively,

$$f_4(t) = \frac{1}{3!(2\Delta_3)^4} \begin{cases} -(t-4\Delta_3)^3 + (t-2\Delta_3)^3, & 0 < t < 2\Delta_3 \\ -(t-4\Delta_3)^3, & 2\Delta_3 < t < 4\Delta_3 \end{cases}$$

and

$$f_6(t) = \frac{1}{5!(2\Delta_3)^6} \begin{cases} -(t-6\Delta_3)^5 + 6(t-4\Delta_3)^5 - 15(t-2\Delta_3)^5, & 0 < t < 2\Delta_3 \\ -(t-6\Delta_3)^5 + 6(t-4\Delta_3)^5, & 2\Delta_3 < t < 4\Delta_3 \\ -(t-6\Delta_3)^5, & 4\Delta_3 < t < 6\Delta_3 . \end{cases}$$

The expression (43) is similar to Eq. (29) in the case of the Gaussian distribution, only the function $P(t, \sqrt{z}\,\Delta_2)$ and the upper limit ∞ of the integration are replaced by $f_z(t)$ and $z\Delta_3$, respectively. Therefore we may easily write down the expression of K_{14}, K_{34}, and K_{16}, K_{36}, K_{56} from Eqs. (30) to (34) with the above replacement.

Although the formulas K_{nz} for $z=4$ have been derived in Ref. 17, here we, by the use of simpler methods, present the general expressions of K_{nz} (43) and $f_z(t)$ (42) for odd n and even z. Of course, it is not difficult to get more general expressions of K_{nz} for any integers n and z by means of similar methods. Having obtained K_{nz} for various probability distribution functions of J_{ij}, we can calculate the average magnetization $\langle\langle S_i \rangle\rangle_r \equiv \mu$ in each case in terms of various spin-spin correlation functions from Eqs. (8) and (9) and investigate the properties of the Ising systems on square and simple cubic lattices.

III. APPROXIMATIONS

From Eqs. (8) and (9) the average of a single spin variable is related to the average of the product of more spin variables. The latter will be related to even more spins according to the following identity:[18]

$$\langle\langle g(S_i)S_i \rangle\rangle_r = \left\langle\!\!\left\langle g(S_i)\tanh\left[\beta \sum_j J_{ij}S_j\right]\right\rangle\!\!\right\rangle_r , \tag{45}$$

where $g(S_i)$ is any function of spin variables except S_i. Hence in order to get a closed set of a few equations, some appropriate approximations must be made. Three decoupling techniques, namely EFT, CEFT, and ICEFT were used to calculate the Curie temperatures of the ideal Ising systems[12] and of the disordered systems in the case of HK type probability on the square[14] and the simple cubic[15] lattices. Now these methods will also be applied to evaluate the Curie temperatures T_C in three probability distributions of the exchange integrals.

(i) EFT. All the multispin correlation functions are approximated as the products of single spin averages. All the $\langle\langle S_i \rangle\rangle_r^n \equiv \mu^n$ for $n > 1$ are neglected when the Curie temperatures are considered.

(ii) CEFT. It is a $z+1$ spin cluster approximation. The cluster consists of a central spin i and all its nearest-neighboring spins. Any spin correlation functions about spins in the cluster are undecoupled and calculated with the use of Eq. (45). Any spin not in the cluster appearing in correlation functions is approximated to only one spin in the cluster according to the formula[10]

$$\begin{aligned} S_k &= \langle\langle S_k \rangle\rangle_r + \lambda(S_j - \langle\langle S_j \rangle\rangle_r) \\ &= \lambda S_j + (1-\lambda)\mu , \end{aligned} \tag{46}$$

where S_j and S_k are the nearest-neighboring spin variables inside and outside the cluster, respectively, and λ is the correlated effective-field parameter. With the help of Eqs. (46) and (45) the set of simultaneous equations for determining the Curie temperatures T_C can be derived and found in Ref. 12.

(iii) ICEFT. It is an improvement on CEFT. If the spin S_k outside the cluster has two equal nearest-neighboring spins S_{j1} and S_{j2}, both inside the cluster, it is better to change Eq. (46) to

$$\begin{aligned} S_k &= \langle\langle S_k \rangle\rangle_r + \tfrac{1}{2}\lambda[(S_{j1} - \langle\langle S_{j1} \rangle\rangle_r) + (S_{j2} - \langle\langle S_{j2} \rangle\rangle_r)] \\ &= \tfrac{1}{2}\lambda(S_{j1} + S_{j2}) + (1-\lambda)\mu . \end{aligned} \tag{47}$$

The set of equations for T_C can also be derived with the use of Eqs. (47) and (45). It can also be found in Ref. 12.

According to Eq. (10) or (11), the closed set of equations for evaluating T_C is the same for each distribution, even for ideal systems, only the concrete forms of K_{nz} are different. These equations are quite lengthy and will not be written here. They can be found in Ref. 12 with only the replacement of various K_{nz} by Eqs. (21)–(25), (30)–(34), and (43) for corresponding cases. These equations can only be solved numerically.

IV. RESULTS AND DISCUSSIONS

For the sake of comparison of the results in three distributions we introduce a unified parameter Δ and discuss its influence upon T_C, where Δ is the standard deviation of each distribution and defined as $\sqrt{\langle (J_{ij}-J)^2 \rangle_r}$. The values of Δ are equal to $\Delta_1=\Delta_2=\Delta_3/\sqrt{3}$. Figure 1 shows the dependence of the reduced Curie temperatures $k_B T_C/J$ upon the parameter Δ on the square lattice. As can be seen, for small Δ, three curves for various distributions and each approximation can hardly distinguish from each other. When $\Delta \rightarrow 0$, $k_B T_C/J$ tend to 3.089, 2.885, and 2.281 for EFT, CEFT, and ICEFT, respectively. These values are the corresponding ones of the ideal Ising systems on the square lattice for three approximation.[12] The behavior of the curves when $\Delta \rightarrow 0$ is due to the fact that any probability distribution function discussed here reduces to the δ function $\delta(J_{ij}-J)$ and the systems become ideal, without randomness.

Under each approximation the Curie temperatures are always the largest in the Gaussian distribution and the smallest in the HK type. When EFT or CEFT is used, with the increase of Δ the difference among various distributions is getting larger. In particular, there appear reentrant phenomena under the discrete distribution, but they do not exist under the continuous ones. This behavior, however, cannot be deduced to the difference between the discrete and continuous distributions. When the more correct approximation ICEFT is applied, no reentrant phenomena exist and the three curves are very close to each other. Therefore it seems unnecessary to distinguish these distributions. In other words, the simplest HK type can be regarded as a good description of the randomness of the exchange integrals in the disordered square Ising systems.

Figure 2 presents the same dependences as Fig. 1, but on the simple cubic lattice. As shown also, for small Δ, three curves for various distributions under each approximation become identical. When $\Delta \rightarrow 0$ they tend to the same values: 5.076, 4.933, and 4.663 for EFT, CEFT, and ICEFT, respectively. They are also the corresponding values of the ideal Ising systems on the simple cubic lattice.[12]

Similar to the result of the square lattice, the Curie temperatures in the Gaussian distribution are mostly the largest. Under EFT and CEFT, the difference between the discrete and continuous distributions is remarkable. The curve of the former first is concave, then changes to convex, while the curves of the latter are always concave. For ICEFT the difference is not very great except within a small range of Δ, $1.5<\Delta<1.549$, where the discrete distribution shows the reentrant phenomena.

In this paper we investigated the disordered Ising systems, in which the disorderness of the exchange integrals

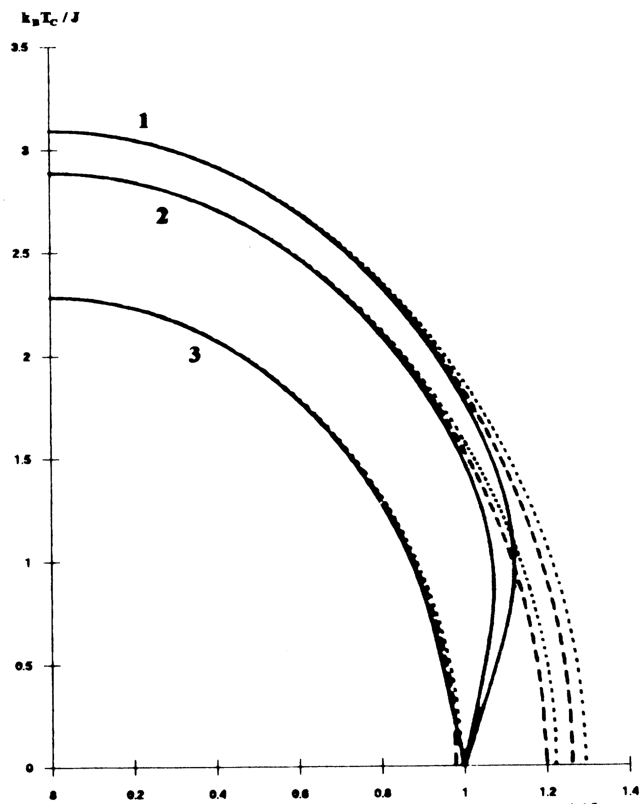

FIG. 1. The dependences of the reduced Curie temperature $k_B T_C/J$ upon the parameter Δ/J on a square lattice. The letters 1, 2, and 3 on the lines refer to EFT, CEFT, and ICEFT, respectively. The HK, Gaussian, and rectangular distributions are represented by solid, dotted, and dashed lines, respectively.

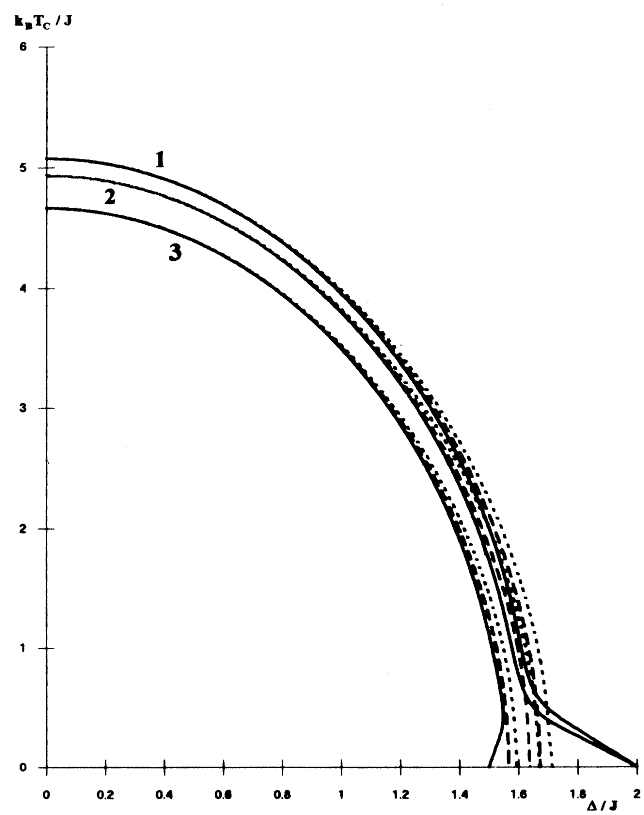

FIG. 2. Similar curves to Fig. 1 but on a simple cubic lattice.

is represented, respectively, by three different probability distribution functions: the HK type, the Gaussian type, and the rectangular type. By the use of the differential operator technique, we derived the general expressions of the coefficients K_{nz} which connect the average of various number of spin variables in each case of distribution. The influence of parameter Δ (standard deviation of each distribution) of the exchange integrals upon the Curie temperatures of the systems is studied. For a square lattice, the results of the ICEFT approximation are almost the same for various distributions. We may conclude that the simplest HK distribution is a quite good description for the randomness of the exchange integrals if ICEFT is used. For a simple cubic lattice the ICEFT is also a good approximation. The discrete and continuous distributions have different influence upon the Curie temperatures. The reentrant phenomena only occur under the discrete distribution within a small range of Δ.

APPENDIX

(i) By differentiating Eq. (37) $2m$ times (m—integer) with respect to x, then letting $x=0$, we may easily get

$$\frac{1}{z!}\sum_{q=0}^{z/2} a_q^{(z)}(2q)^{2m} = \begin{cases} 0, & 0 \le 2m < z \\ 1, & 2m = z \; . \end{cases} \tag{A1}$$

(ii) To prove Eq. (44),

$$\begin{aligned}\int_{-\infty}^{\infty} f_z(t)dt &= \frac{1}{2(z-1)!\Delta_3^z}\sum_{q=0}^{z/2} a_q^{(z)}\left[\int_{2q\Delta_3}^{\infty}(t-2q\Delta_3)^{z-1}dt + \int_{-2q\Delta_3}^{\infty}(t+2q\Delta_3)^{z-1}dt\right] \\ &= \frac{1}{2(z-1)!\Delta_3^z}\int_{z\Delta_3}^{\infty}\sum_{q=0}^{z/2} a_q^{(z)}[(t-2q\Delta_3)^{z-1}+(t+2q\Delta_3)^{z-1}]dt \\ &\quad + \frac{1}{2(z-1)!\Delta_3^z}\sum_{q=0}^{z/2} a_q^{(z)}\left[\int_{2q\Delta_3}^{z\Delta_3}(t-2q\Delta_3)^{z-1}dt + \int_{-2q\Delta_3}^{z\Delta_3}(t+2q\Delta_3)^{z-1}dt\right] .\end{aligned}$$

The first summation of the right-hand side is equal to zero from Eq. (A1). Completing two integrations in the second summation of the right-hand side, we get

$$\int_{-\infty}^{\infty} f_z(t)dt = \frac{1}{2z!}\sum_{q=0}^{z/2} a_q^{(z)}[(z+2q)^z+(z-2q)^z] \; .$$

Expanding $(z+2q)^z+(z-2q)^z$ we know from Eq. (A1) the summation is equal to $2\sum_{q=0}^{z/2} a_q^{(z)}(2q)^z = 2z!$; thus Eq. (44) has been proven.

[1]T. Kaneyoshi, J. Phys. Soc. Jpn. **55**, 1430 (1986).
[2]T. Kaneyoshi, Phys. Rev. B **33**, 7688 (1986).
[3]T. Kaneyoshi, J. Mielnicki, T. Balcerzak, and G. Wiatrowski, Phys. Rev. B **42**, 4388 (1990).
[4]A. Bobak and J. Karaba, J. Magn. Magn. Mater. **127**, 71 (1993).
[5]N. Benayad, A. Benyoussef, and N. Boccara, J. Phys. C **21**, 5417 (1988).
[6]A. Bobak and M. Jascur, J. Phys. Condens. Matter. **2**, 10 053 (1990).
[7]T. Balcerzak, B. T. Cong, J. Mienicki, and G. Wiatrowski, J. Magn. Magn. Mater. **87**, 229 (1990).
[8]R. Honmura and T. Kaneyoshi, J. Phys. C **12**, 3979 (1979).
[9]J. Mielnicki, T. Balcerzak, V. H. Truong, and G. Wiatrowski, J. Magn. Magn. Mater. **51**, 151 (1985).
[10]T. Kaneyoshi, I. P. Fittipaldi, R. Honmura, and T. Manabe, Phys. Rev. B **24**, 481 (1981).
[11]T. Kaneyoshi, Z. Phys. B **60**, 35 (1985).
[12]S. M. Zheng, Phys. Status Solidi B **180**, 463 (1993).
[13]T. Balcerzak, J. Magn. Magn. Mater. **125**, 138 (1993).
[14]S. M. Zheng, Phys. Status Solidi B **185**, K37 (1994).
[15]S. M. Zheng, J. Magn. Magn. Mater. **146**, 133 (1995).
[16]J. Mielnicki, T. Balcerzak, and G. Wiatrowski, Phys. Rev. B **43**, 8564 (1991).
[17]T. Balcerzak, Phys. Rev. B **47**, 15 294 (1993).
[18]H. B. Callen, Phys. Lett. **4**, 161 (1963).