Subsurface correlations in the *d*-wave Josephson junction

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A prediction is made for the Josephson vortex structure in the *d*-wave Josephson junction (JJ) for those (special) orientations of the JJ insulating plane when the critical current is expected to be suppressed significantly (or zero) in accordance with the $d_{x^2-y^2}$ symmetry. It is shown that a spontaneous admixture of the d_{xy} wave function in the vicinity of the JJ insulating plane makes the critical current be finite. A conventional magnetic vortex in such a *d*-wave uniform and plane JJ, demonstrating no time-reversal symmetry breaking, splits into two parts, each carrying half of the magnetic flux. The explicit solution for such a pair as well as the lower critical field are obtained. The pair size diverges logarithmically as soon as the JJ orientation approaches the special one. Possible ways of experimental observation of this effect are discussed.

I. INTRODUCTION

The *d*-wave scenario[1] for superconductivity in high-temperature superconducting (HTS) materials predicts that the sign of the Josephson critical current J_c depends on the Josephson-junction (JJ) orientation. As suggested in Ref. 2, the dependence of J_c on the angle θ_1 formed by the tetragonal, e.g., *a* axis of the *d*-wave superconductor occupying the half-space $y<0$ and normal to the JJ plane, with the JJ plane being parallel to the *c* axis, can be represented as

$$J_c = J_0 \cos(2\theta_1) . \tag{1}$$

Here J_0 stands for the factor which does not change sign and can be assumed constant [in (1) and henceforth the orientation of the *a* axis on the other side $y>0$ of the JJ is taken perpendicular to the JJ plane. It, on the one hand, simplifies further consideration of the case of two *d*-wave superconductors, and, on the other hand, allows one to investigate on the same footing the case when the side $y>0$ is occupied by an *s*-wave superconductor].

One can see that current (1) is to be zero at the special angle $\theta_1=\theta_c\equiv\pi/4$. However, such a zero has never been observed experimentally.[1] At the same time, a most intriguing consequence of (1)—the spontaneous magnetic half-flux[2] inside a ring made of three HTS grains for certain orientations—has been observed recently in Ref. 3. The other coherence test suggested by Sigrist and Rice in Ref. 2 and performed in Ref. 4 has proven that the *a*-*c* and *b*-*c* faces of the HTS Y-Ba-Cu-O grain demonstrate opposite signs of the superconducting (SC) gap suggesting $d_{x^2-y^2}$ symmetry.[1] A resolution of such a paradox is given by Sigrist and co-workers in Refs. 5 and 6, where it has been shown that an admixture of a contribution of another symmetry into the pair function should occur close to the JJ. It should make the critical current be finite for all the orientations of the JJ plane. Moreover, under certain conditions this admixture might lead to the time-reversal symmetry breaking and Josephson spontaneous fluxes not restricted by the quantization condition.[5,6] Such fluxes have been observed along the circumference of the Y-Ba-Cu-O grain in Ref. 7.

In this paper it will be shown that, while the condition for the time-reversal symmetry violation[5,6] does not hold, the *d*-wave JJ should still demonstrate another peculiar behavior—splitting of the Josephson integer vortex into two halves. The distance between them diverges logarithmically as soon as the JJ plane orientation approaches the special one, around which the critical current is expected to be zero. In the next section the Ginzburg-Landau (GL) formalism describing bulk and surface properties of the unconventional singlet superconductor is considered. Then, the Josephson's equation taking into account the mixing of the different representations is derived in Sec. III. The explicit solution for the Josephson's composite kink and the correspondent lower critical field are represented in Sec. IV. Then, a discussion and conclusions are given in Sec. V.

II. ENERGY OF THE *d*-WAVE JOSEPHSON JUNCTION

It is worth noting that, even if the SC gap were purely of the $d_{x^2-y^2}$ symmetry in the bulk, an admixture of the d_{xy} representation might occur near the JJ plane (for a review on boundary and the Josephson effects in unconventional superconductors see Ref. 8). Then, one can anticipate the pair function to have the form

$$\psi = \eta_1 d_{k_x^2-k_y^2} + \eta_2 d_{k_x k_y} , \tag{2}$$

where $\eta_1 \equiv |\eta_1|\exp(i\varphi_1)$, $\eta_2 \equiv |\eta_2|\exp(i\varphi_2)$ are amplitudes of the corresponding irreducible representations. These amplitudes depend on the x,y coordinates [uniformity along the *c* axis is assumed in (2) and hereafter], which are understood as coordinates of the Cooper pair center of mass.

It is worth noting that an interpretation of the mixing (2) can be twofold. An admixture of the d_{xy} may be considered as a consequence of rotation of the Cooper pair by some angle $\vartheta(\mathbf{x})$ in the x-y plane, so that in the new frame of reference (rotated by this angle) the form (2) appears to be the pure $d_{x^2-y^2}$. Indeed, originally, one can write

$$d_{k_x^2-k_y^2}=\frac{\cos 2\theta}{\sqrt{\pi}}, \quad d_{k_x k_y}=\frac{\sin 2\theta}{\sqrt{\pi}}, \quad \tan\theta=\frac{k_x}{k_y}, \tag{3}$$

where $\mathbf{k}$ stands for the wave vector of the relative motion of two electrons forming the Cooper pair (this vector belongs mostly to the Fermi surface). Consequently, one arrives at (2) rewritten as

$$\psi(\theta,\mathbf{x})=\eta(\mathbf{x})\frac{\cos 2[\theta-\vartheta(\mathbf{x})]}{\sqrt{\pi}}, \qquad \eta_1(\mathbf{x})=\eta(\mathbf{x})\cos 2\vartheta(\mathbf{x}), \quad \eta_2(\mathbf{x})=\eta(\mathbf{x})\sin 2\vartheta(\mathbf{x}), \tag{4}$$

if the phases φ_1,φ_2 are either the same or shifted by π. In close analogy with the case of the superfluid He^3 considered in Ref. 9, the mixing (2) in the form (4) can be interpreted that the Cooper pair tends to orient itself near the JJ in such a way as to lower the tunneling energy[10] $\hbar J_c/2e$, where $\hbar, e$ stand for the Plank constant and the electron charge, respectively.

However, under certain conditions this interpretation might be incomplete because the term

$$\delta(\eta_1^{*2}\eta_2^2+\text{c.c.}), \tag{5}$$

with $\delta>0$ could appear in the GL free-energy density. As suggested in Ref. 11, such a term favors the relative phase shift $\varphi_1-\varphi_2=\pm\pi/2$, and thereby the time-reversal symmetry violation. This possibility has been explored by Sigrist and co-workers in Refs. 5 and 6 to provide an explanation for the very recent observation[7] of spontaneous magnetic fluxes unrestricted by any quantization condition, and occurring along the circumference of the HTS Y-Ba-Cu-O grain.

In what follows the opposite case $\delta<0$ in (5) will be discussed. If temperature is sufficiently below the SC transition temperature, one can consider (5) to be substantially strong to favor the relative phase shift 0 (or π), and make thereby the representation (4) valid. Consequently, $\vartheta(\mathbf{x})$ has to be found by minimizing the GL functional in the bulk, with the JJ energy added, likewise to how it was done in Refs. 5 and 6 in the case of $\delta>0$. Thus, the total energy per unit length of the c axis (oriented perpendicularly to the x,y plane) can be found as

$$F=\int d^2x \left\{ K|\mathbf{D}\eta_1|^2+K|\mathbf{D}\eta_2|^2+a_1|\eta_1|^2 + a_2|\eta_2|^2+\frac{\beta_1}{2}|\eta_1|^4+\frac{\beta_2}{2}|\eta_2|^4 + \frac{\delta}{2}(\eta_1^{*2}\eta_2^2+\text{c.c.})+\beta_{12}|\eta_1|^2|\eta_2|^2 + \frac{\text{curl}\,\mathbf{A}^2}{8\pi}\right\}+F_J, \qquad \mathbf{D}\equiv\nabla\cdots-\frac{2\pi}{\varphi_0}i\,\mathbf{A}, \quad \delta<0. \tag{6}$$

where $\mathbf{A},\varphi_0$ denote vector potential and the unit flux, respectively; the coefficient $K>0$ before the gradient terms is taken the same for both gradients; the other parameters

$$a_1<0, \quad a_2>(\beta_{12}-|\delta|)|\eta_1|^2>0, \quad \beta_1>0, \quad \beta_2>0 \tag{7}$$

ensure the condensation of the single amplitude η_1 in the bulk; F_J stands for the Josephson energy. To find how F_J depends on $\vartheta(\mathbf{x})$ the following considerations can be employed. The critical current (1) sees the actual orientation of the Cooper pair close to the JJ rather than the orientation of the lattice. Therefore, given (4), one should replace the angle θ_1 in (1) by $\theta_{1s}\equiv\theta_1-\vartheta(x,y=-0)$ at the $y<0$ side, with the JJ insulating plane $y=0$ assumed to be infinitely thin, and either the pure $d_{x^2-y^2}[\vartheta(\mathbf{x})=0]$ or the s-wave state is realized in the $y>0$ half space. Consequently, the tunneling energy[10] per unit length of the c axis can be represented as

$$F_J=\int dx\left[-\frac{\hbar}{2e}J_0\cos(2\theta_{1s})\cos\phi\right], \qquad \theta_{1s}\equiv\theta_1-\vartheta(x,y=-0), \qquad \phi\equiv\varphi(x,y=-0)-\varphi(x,y=+0), \tag{8}$$

where φ stands, as usual, for the phase of the SC order parameter which, as discussed above, is taken the same for both amplitudes in (2).

It appears to be convenient to rewrite (6) in terms of (4) as

$$F_{\mathrm{GL}}=\int d^2x\left\{\frac{1}{8\pi\lambda_{ab}^2}\left[\frac{\varphi_0}{2\pi}\right]^2\left|\partial\varphi+\frac{2\pi}{\varphi_0}\mathbf{A}\right|^2 + \frac{(\text{curl}\,\mathbf{A})^2}{8\pi}+f(\vartheta)\right\}+F_J, \qquad f(\vartheta)\equiv\begin{cases}\frac{\chi}{2}(\partial\vartheta)^2+\frac{\nu}{2}\vartheta^2, & y<0,\\ 0, & y>0\end{cases} \tag{9}$$

where the parameters are defined as

$$\chi\equiv 4K\eta_\infty^2, \quad \nu\equiv 8[a_2+(\beta_{12}-|\delta|)\eta_\infty^2]\eta_\infty^2>0, \qquad \lambda_{ab}^{-2}\equiv 8\pi\left[\frac{2\pi}{\varphi_0}\right]^2 K\eta_\infty^2, \quad \eta_\infty^2\equiv\frac{-a_1}{\beta_1}>0. \tag{10}$$

Deriving (9) from (6), the constant amplitude approximation $\eta=\eta_\infty\exp(i\varphi)$ in (4) and the expansion with respect to $\vartheta(\mathbf{x})\to 0$ have been employed.

III. EQUATION FOR JOSEPHSON VORTEX IN THE d-WAVE JUNCTION

As long as the crystal-field effect is expected to be strong, one can expand (8) in powers of ϑ, and retain the first nontrivial term only. Then, after varying (9) and (8) with respect to ϑ, one arrives at the linearized bulk equation

$$-\Delta\vartheta+\xi^{-2}\vartheta=0,\quad \xi^{-2}\equiv\frac{\nu}{\chi}\ , \tag{11}$$

and the boundary condition

$$y=-0:\ \chi\partial_y\vartheta-\frac{\hbar}{e}J_0\sin2\theta_1\cos\phi=0\ . \tag{12}$$

These can be solved by making use of the Fourier transform along x. Consequently, one arrives at the solution

$$\vartheta_q(y)=\frac{\hbar J_0\sin(2\theta_1)}{e\mu\sqrt{1+(\xi q)^2}}(\cos\phi)_q\exp(Q_q y),\quad y<0\ ,$$
$$Q_q\equiv\sqrt{\xi^{-2}+q^2},\quad \mu\equiv\sqrt{\chi\nu}\ , \tag{13}$$
$$(\cos\phi)_q\equiv\int dx\, e^{iqx}\cos\phi(x),\quad \vartheta_q(y)\equiv\int dx\, e^{iqx}\vartheta(x,y)\ .$$

Then, the variation of (9) and (8) with respect to other variables—the phase and the vector potential—results in the conventional equations

$$\frac{2\pi}{\varphi_0}\partial_y^2 A_x=\frac{1}{\lambda_{ab}^2}\left[\partial_x\varphi+\frac{2\pi}{\varphi_0}A_x\right],$$
$$\frac{2\pi}{\varphi_0}\partial_{yx}^2 A_x=-\frac{1}{\lambda_{ab}^2}\partial_y\varphi \tag{14}$$

describing the Meissner effect. In (14) the gauge $A_y=0$, the uniformity along the c axis, and the same London penetration λ_{ab} length on the both sides of the JJ were employed. The solution for the gauge-invariant phase

$$\tilde{\varphi}=\varphi+\frac{2\pi}{\varphi_0}\sigma,\quad A_x=\partial_x\sigma \tag{15}$$

can be found by making use of the Fourier transformation along x as well. It results in

$$\tilde{\varphi}_q(y)=\begin{cases}\tilde{\varphi}_q(+0)\exp(-\tilde{Q}_q y), & y>0\ ,\\ \tilde{\varphi}_q(-0)\exp(\tilde{Q}_q y), & y<0,\end{cases}$$
$$\tilde{Q}_q\equiv\sqrt{\lambda_{ab}^{-2}+q^2},\quad \tilde{\varphi}_q(+0)+\tilde{\varphi}_q(-0)=0\ , \tag{16}$$
$$\tilde{\varphi}_q(y)\equiv\int dx\,\tilde{\varphi}(x,y)\exp(iqx)\ .$$

Solutions (16) and (13) can be substituted into (9) and (8) and integrated over y (by parts). It yields (9) and (8) rewritten solely in terms of the surface Fourier components of the gauge-invariant phase difference as

$$F_{\rm GL}=\sum_q\left\{\frac{1}{16\pi\lambda_{ab}}\left[\frac{\varphi_0}{2\pi}\right]^2\frac{q^2}{\sqrt{1+(\lambda_{ab}q)^2}}|\phi_q|^2-\frac{\hbar}{2e}\left[J_c(\theta_1)(\cos\phi)_q\delta_{q,0}+J'(\theta_1)\frac{|(\cos\phi)_q|^2}{\sqrt{1+(\xi q)^2}}\right]\right\}, \tag{17}$$
$$J'(\theta_1)\equiv\frac{2\hbar}{e\mu}(J_0\sin2\theta_1)^2\ .$$
$$\phi_q\equiv\int dx\,\phi(x)\exp(iqx)\ .$$

It is worth noting that without the term $\sim J'(\theta_1)$ this expression accounts for energy of a conventional JJ.[10] As soon as the effective Josephson length is expected to be a largest scale in the system, one can make replacements

$$\sqrt{1+(\xi q)^2}\to1,\quad \sqrt{1+(\lambda_{ab}q)^2}\to1\ , \tag{18}$$

in (17), and then rewrite (17) in the x-coordinate space as

$$F_{\rm GL}=\int dx\left[\frac{1}{16\pi\lambda_{ab}}\left[\frac{\varphi_0}{2\pi}\right]^2\partial_x\phi^2+\frac{\hbar}{2e}U(\phi)\right]\ ,$$
$$U(\phi)\equiv|J_c(\theta_1)|\left[1-\frac{J_c(\theta_1)}{|J_c(\theta_1)|}\cos\phi\right]+J'(\theta_1)[1-\cos^2\phi]\ , \tag{19}$$

where the constant energy was added to make the ground-state energy be zero.

As long as the lattice orientation is given by $\theta_1=0$ (or $\pi/2$), the form (19) transforms into the expression,[10] whose variation yields the conventional Josephson's equation. The latter, as it is well known, accounts for integer vortices confined in the JJ.

For the special angle $\theta_1=\theta_c(=\pi/4)$ the term $\sim J_c(\theta_1)$ disappears from (19). It ensures that the lowest energy kink turns out to be the half-vortex (HV). The equation describing the HV corresponding to this particular orientation stems from (19) after varying $\delta F_{\rm GL}/\delta\phi=0$ as

$$-\partial_x^2\phi+\frac{1}{2\lambda_{J_c}^2}\sin2\phi=0\ ,$$
$$\phi(-\infty)=0,\phi(+\infty)=\pm\pi\ , \tag{20}$$
$$\lambda_{J_c}\equiv\frac{c\sqrt{\mu}}{8\sqrt{\pi\lambda_{ab}}J_0}\ ,$$

where λ_{J_c} bears a meaning of the Josephson length at the special orientation of the JJ, and the total flux confined in the JJ gap is given by Ref. 10 and (20) as

$$\Phi=\frac{\phi(+\infty)-\phi(-\infty)}{2\pi}\varphi_0=\frac{\pm\varphi_0}{2}\ . \tag{21}$$

For the JJ orientation different from the special one the HV as a single kink does not exist. However, the two HV solutions of the same sign of the magnetic moment can appear as a bound pair. The correspondent equation describing such a solution takes the form

$$-\frac{d^2}{dx^2}\phi+\lambda_{J_c}^{-2}[\beta\sin\phi+\tfrac{1}{2}\sin(2\phi)]=0\ ,$$
$$\beta\equiv|J_c(\theta_1)|/2J'(\theta_1)\ , \tag{22}$$

after varying (19) for an arbitrary JJ orientation. In (22) the critical current $J_c(\theta_1)$ was taken positive (for a single junction it is always possible to make by a trivial shift of the phase by π).

To understand why the lowest energy kink of (22) can be represented as the two HV's, the following considerations may be employed. If the JJ orientation were exactly the special one, the two HV's carrying the half-flux of the same sign would have repelled each other. It makes the distance D between them increase unlimitedly. However, the term $\sim J_c(\theta_1)$ in (19), given a small deviation of the JJ orientation from the special, yields the energy which is increasing as $\sim|\theta_1-\theta_c|D>0$. Therefore some equilibrium D should exist. It is easy to realize that D should diverge logarithmically as soon as the JJ orientation approaches the special one, and the intervortex forces are exponentially decreasing with $D\rightarrow\infty$. In what follows the solution of (22), and D will be found explicitly.

IV. THE COMPOSITE KINK SOLUTION: THE LOWER CRITICAL FIELD

Taking the first integral of (22), one finds

$$\int\frac{d\phi}{\sqrt{1-\cos 2\phi+4\beta(1-\cos\phi)}}=\frac{1}{\sqrt{2}\lambda_{J_c}}\int dx\ , \qquad (23)$$

where the boundary conditions

$$\phi(-\infty)=0,\quad \phi(+\infty)=2\pi\ , \qquad (24)$$

accounting for the kink carrying the unit flux, were taken into account. Integration of (23) yields the solution

$$\cos(\phi/2)=-\frac{\sinh[\sqrt{1+\beta}(x-x_0)/\lambda_{J_c}]}{\sqrt{\beta^{-1}+\cosh^2[\sqrt{1+\beta}(x-x_0)/\lambda_{J_c}]}}\ , \qquad (25)$$

with x_0 being a coordinate of the center of the kink. This solution is characterized by two scales. The first,

$$\lambda_J=\frac{\lambda_{J_c}}{\sqrt{1+\beta}} \qquad (26)$$

determines behavior of the exponents composing (25). This scale is the Josephson's length, given an arbitrary β in (22). In the limit $\beta\rightarrow\infty$ [achieved because of $\theta_1\rightarrow 0$ in (22)] corresponding to the case of the conventional JJ, solution (25) transforms into the well-known Josephson's vortex solution,[10] with the Josephson's length given as

$$\lambda_{J_\infty}\equiv\lim_{\theta_1\rightarrow 0}\lambda_J=\sqrt{\hbar c^2/16\pi e\lambda_{ab}J_0}\ . \qquad (27)$$

In the opposite limit $\beta\rightarrow 0$ (achieved due to $\theta_1\rightarrow\theta_c$), the other scale greater than (26) emerges. This scale represents the separation D between two parts of (25) each described by the solution of (20), and carrying thereby half of the unit flux. The distance D can be defined as a distance between two points where the phase in (25) acquires values of $\pi/2$ and $3\pi/2$, respectively. Then, (25) yields

$$D=\frac{2\lambda_{J_c}}{\sqrt{1+\beta}}\ln[\sqrt{1+\beta^{-1}}+\sqrt{2+\beta^{-1}}]$$
$$\approx\lambda_{J_c}\ln\frac{4}{\beta}\ . \qquad (28)$$

Employing (20), (11), (10), (22), and (25), one can rewrite (28) as

$$D=\lambda_{J_c}\ln\frac{B}{|\theta_1-\theta_c|},\quad \theta_1\rightarrow\theta_c,\quad \theta_c=\frac{\pi}{4}\ ,$$
$$B\equiv\frac{64\pi e}{\hbar c^2}J_0\lambda_{ab}^2\xi\ , \qquad (29)$$

where all the parameters can be expressed by means of observable quantities.

To find the lower critical field the energy of the composite kink (25) should be calculated. Then an integration of (19) with (25) substituted, yields this field as

$$H_{c1}=\frac{H_{c1}(0)}{2}\sqrt{J_c(\theta_1)/J_c(0)}\left[\sqrt{1+(1/\beta)}+\sqrt{\beta}\ln\frac{1+\sqrt{1+\beta}}{\sqrt{\beta}}\right]\ , \qquad (30)$$

with $H_{c1}(0)$ being the critical field which corresponds to the angle $\theta_1\rightarrow 0$ (limit $\beta\rightarrow\infty$). A simple analysis of (30) shows that the ratio of the critical field at the special orientation to $H_{c1}(0)$ is given as

$$\frac{H_{c1}(\theta_c)}{H_{c1}(0)}=\frac{1}{2}\left[\frac{J_{ce}(\theta_c)}{J_{ce}(0)}\right]^{1/2}\ , \qquad (31)$$

where $J_{ce}(\theta_1)$ stands for the effective critical current as a function of the JJ orientation. One finds that

$$J_{ce}(0)=J_0,\quad J_{ce}(\theta_c)=2J'(\theta_c)\ , \qquad (32)$$

with (19) taken into account. It is worth noting that, if there were no splitting of the Josephson's vortex, relation (31) would not have had a factor 1/2. An explanation for this fact turns out to be straightforward. Indeed, energy of two halves (separated by the distance larger than the Josephson's penetration length) of the unit flux is 1/2 of the unit flux energy. As long as the lower critical field of the JJ is given by the JJ vortex energy, the factor 1/2 appears in (31).

V. DISCUSSION

Now let us focus on how this effect of the Josephson's vortex splitting could be observed. The relation (31) containing the extra 1/2, while compared with the situation of the conventional junction, allows one to make it indirectly.

The technique[12] of Josephson vortices imaging might resolve the splitting of the integer vortex, if the distance D given by (29) becomes larger than the Josephson length. Let us estimate how the JJ orientation should be close to the special one to make D sufficiently large. To this end the following parameters $J_0=10^5$–10^6 A/cm^2, $\lambda_{ab}\approx 2\times 10^{-5}$ cm, can be taken for the HTS compounds. The parameter ξ determines a scale of Eq. (11) describing how far from the JJ the admixture of the representation d_{xy} persists. A magnitude of this length depends on how strong the crystal-field effect is nearby the JJ. Indeed, the magnitude of ξ is determined by the parameter ν [see (11)], which in turn is controlled by the Cooper pair locking to the lattice [see (9) and the discussion following (2)]. Diminishing of this effect ($\nu\rightarrow 0$) because of, e.g., structural defects normally existing close to boundaries, would result in values of this scale significantly larger than the typical correlation length. Presuming that $\xi = 10^{-6}$ cm, one finds $B \leq 0.1$ in (29). The latter estimate implies that the distance D between two half-vortices becomes observably large, if the JJ orientation belongs to the domain $|\theta_1 - \theta_c| \leq 0.01 - 0.1$[rad].

It is worth noting that the fractional SC vortex in the bulk has been predicted by Volovik and Gor'kov in Ref. 13. The possibility of splitting the integer bulk vortex has also been discussed (see Refs. 8 and 14). However, these effects can occur, if the SC order parameter is multicomponent. In the case of the bulk vortex splitting the distance between two half-vortices is fixed, being given by microscopic parameters. Contrary to this, the distance D between the Josephson half-vortices considered above can be made arbitrarily large, in principle, by changing the JJ orientation.

The possibility of HV occurrence in the plane JJ, one part of which has negative critical current, has been considered by Bulaevskii, Kuzii, and Sobyanin.[15] In this case the HV turns out to be spontaneous and attached to the line where J_c changes sign. In the situation $\theta_1 = \theta_c$ considered above the HV cannot exist in the ground state. However, it can appear as a zero mode kink. In other words, it can move along the JJ.

In conclusion, it is shown that the conventional Josephson vortex in the d-wave JJ, demonstrating no violation of the time-reversal symmetry, should split into two half-vortices, if the JJ plane and the tetragonal axis make an angle sufficiently close to the special one $\pi/4$. The distance between the halves diverges logarithmically as soon as the angle approaches the special one. Estimates of the distance between such half-vortices show that the effect of splitting could be observed by the technique.[12]

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[1]D. J. Scalapino, Phys. Rep. **250**, 329 (1995).
[2]M. Sigrist and T. M. Rice, J. Phys. Soc. Jpn. **61**, 4283 (1992).
[3]C. C. Tsuei, J. R. Kirtley, C. C. Chi, Lock See Yu-Jahnes, A. Gupta, T. Shaw, J. Z. Sun, and M. B. Ketchen, Phys. Rev. Lett. **73**, 593 (1994).
[4]D. A. Wollman, D. J. Van Harlingen, J. Giapintzakis, and D. M. Ginzberg, Phys. Rev. Lett. **74**, 797 (1995).
[5]M. Sigrist, D. B. Bailey, and R. B. Laughlin, Phys. Rev. Lett. **74**, 3249 (1995).
[6]K. Kuboki and M. Sigrist (unpublished).
[7]J. R. Kirtley, P. Chaudhari, M. B. Ketchen, N. Khare, S.-Y. Lin, and T. Shaw, Phys. Rev. B **51**, 12057 (1995).
[8]M. Sigrist and K. Ueda, Rev. Mod. Phys. **63**, 239 (1991).
[9]V. Ambegaokar, P. G. de Gennes, and D. Rainer, Phys. Rev. A **9**, 2676 (1974).
[10]A. Barone and G. Paterno, *Physics and Applications of the Josephson Effect* (Wiley, New York, 1982).
[11]G. Kotliar, Phys. Rev. B **37**, 3664 (1988).
[12]M. Hartmann, K. Hipler, D. Koelle, F. Kober, K. Bernhardt, T. Sermet, R. Gross, and R. P. Huebener, Z. Phys. B **75**, 423 (1989); G. M. Fischer, B. Mayer, R. Gross, T. Nissel, K.-D. Husemann, and R. P. Huebener (unpublished).
[13]G. E. Volovik and L. P. Gor'kov, JETP Lett. **39**, 676 (1984).
[14]M. E. Zhitomirskii (unpublished).
[15]L. N. Bulaevskii, V. V. Kuzii, and A. A. Sobyanin, Solid State Commun. **25**, 1053 (1978).