Charge transport in junctions between *d*-wave superconductors

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We develop a microscopic analysis of superconducting and dissipative currents in junctions between superconductors with d-wave symmetry of the order parameter. We study the proximity effect in such superconductors and show that for certain crystal orientations the superconducting order parameter can be essentially suppressed in the vicinity of a nontransparent specularly reflecting boundary. This effect strongly influences the value and the angular dependence of the dc Josephson current j_S. At $T \sim T_c$ it leads to a crossover between $j_S \propto T_c - T$ and $j_S \propto (T_c - T)^2$, respectively, for homogeneous and nonhomogeneous distribution of the order parameter in the vicinity of a tunnel junction. We show that at low temperatures the current-phase relation $j_S(\varphi)$ for superconductor–normal-metal–superconductor junctions and short weak links between d-wave superconductors is essentially nonharmonic and contains a discontinuity at $\varphi = 0$. This leads to further interesting features of such systems which can be used for pairing symmetry tests in high-temperature superconductors (HTSC). We also investigated the low-temperature I-V curves of normal-metal–superconductor and superconductor-superconductor tunnel junctions and demonstrated that depending on the junction type and crystal orientation these curves show zero-bias anomalies $I \propto V^2$, $I \propto V^2 \ln(1/V)$, and $I \propto V^3$ caused by the gapless behavior of the order parameter in d-wave superconductors. Many of our results agree well with recent experimental findings for HTSC compounds.

I. INTRODUCTION

In spite of enormous efforts made to understand the physical mechanisms of pairing in various high-temperature superconductors (HTSC) the situation still remains unclear. A key role in understanding of this phenomenon belongs to the question about the symmetry of the order parameter. Quite early after the original discovery of Bednorz and Müller[1] the symmetry of the $d_{x^2-y^2}$ type was suggested for HTSC materials.[2–5] Since then plenty of experiments have been designed to probe the symmetry of the order-parameter in HTSC (see, e.g., Ref. 6 for a review). Although many experimental results are consistent with the picture of d-wave pairing (e.g., the temperature dependence of the penetration depth,[7] NMR and NQR studies,[8] etc.) they still do not allow to rule out other possibilities, like anisotropic s-wave pairing. Moreover the results of some other experiments (see, e.g., Ref. 9) may indicate s-wave rather than d-wave symmetry of the order parameter in HTSC. Therefore it is quite likely that only a set of different and independent experimental tests would allow to make an unambiguous conclusion about the order-parameter symmetry in HTSC compounds.

Important information about the symmetry of superconducting pairing can be obtained from the measurements of both the dc Josephson effect and the quasiparticle current in tunnel junctions between two HTSC. The dc Josephson effect in unconventional superconductors has been discussed by Geshkenbein, Larkin, and Barone[10] and by Sigrist and Rice[11] who demonstrated that the d-wave symmetry of the order parameter may lead to the sign inversion of the Josephson critical current for certain crystal orientations. Under these conditions the tunnel junction becomes the so-called π junction.[12] Being closed by a π junction a superconducting loop with a not very small inductance develops a spontaneous circulating current.[12] As a result the magnetic flux equal to a half of the flux quantum occurs inside a ring and can be easily measured. Measurements of that kind have been carried out for HTSC samples[13–15] and indeed demonstrated the results fully consistent with the above picture. These results in combination with the paramagnetic behavior of granular HTSC compounds[16] and its theoretical interpretation[11,17,18] serve as a serious argument in favor of d-wave pairing symmetry in HTSC.

In contrast to the Josephson effect which is sensitive to the order-parameter phase difference across the junction, low-temperature measurements of the quasiparticle current in tunnel junctions provide information about the quasiparticle density of states in superconducting banks and allow us to distinguish gapless superconductivity from that with a finite gap. The results of numerous experiments vary from a nearly BCS-like to a clear gapless behavior for different HTSC materials (see, e.g., Ref. 6 and references therein). The results of recent tunneling experiments with $Bi_2Sr_2CaCu_2O_8$ samples[19,20] indicate a gapless behavior of the I-V curve at low voltages and

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temperatures (e.g., the dependence $I \propto V^3$ at low V was reported in Ref. 19).

A growing number of experimental data makes it necessary to develop a detailed analysis and specify theoretical predictions concerning both dc Josephson effect and quasiparticle tunneling between d-wave superconductors. Several phenomenological calculations assuming d-wave pairing have been already done (see below) attempting partial understanding of some experimental results. Nevertheless a number of important questions still have to be adressed in this context. E.g., a uniform distribution of the superconducting order parameter on both sides of a tunnel barrier was assumed in many calculations. Being obviously correct for isotropic s-wave superconductors, this assumption may fail for d-wave superconductors depending on their orientation relative to the tunnel barrier plane. Below we will show that the spatial dependence of the order parameter due to the proximity effect becomes particularly important close to the superconducting critical temperature T_c having a strong impact on the dc Josephson effect in d-wave superconductors.

Another problem appears if one applies the tunneling Hamiltonian method to investigation of the charge transport through tunnel junctions in the d-wave case. The momentum dependence of the matrix elements describing tunneling between superconductors becomes particularly important in this case. It is easy to show that the choice of the tunneling matrix elements as being independent on the momentum direction (standard for the s-wave case) leads to confusing results for d-wave superconductors. Furthermore, an unambiguous choice of this dependence cannot be done within this method. This emphasizes the necessity to provide a microscopic description of tunneling between such superconductors based on matching of the electron propagators at the tunnel barrier. This approach leaves no space for ambiguity and, on top of that, it is not confined to the case of low transparency barriers but allows us to study other types of weak links with highly transparent interfaces.

In this paper we will provide an extensive microscopic study of the charge transport in various types of junctions between d-wave superconductors with BCS-like behavior of the density of states. The paper is organized as follows. In Sec. II we develop a detailed study of the proximity effect for d-wave superconductor-insulator and d-wave superconductor–normal-metal structures. We investigate the spatial dependence of the superconducting gap function for various crystal orientations and temperatures and show that for particular orientations the gap at the superconductor-insulator interface can be completely suppressed. The dc Josephson current through tunnel junctions, superconductor–normal-metal–superconductor (SNS) junctions and short weak links between d-wave superconductors is examined in Sec. III. Our analysis allows us to discover several qualitative features of the Josephson effect in such systems which can be used for further experimental tests of the pairing symmetry in HTSC. In Sec. IV we investigate the I-V curves for superconductor-superconductor (SS) and normal-metal–superconductor (NS) tunnel junctions. For most of crystal orientations we found gapless non-Ohmic behavior in the limit of small voltages and $T = 0$. Discussion of our results is presented in Sec. V.

II. PROXIMITY EFFECT IN d-WAVE SUPERCONDUCTORS

The order parameter in bulk superconductors with unconventional pairing depends on the direction of the Fermi momentum $\boldsymbol{p_F}$.[21,22] Beyond that close to the edges of a superconducting piece of metal the order parameter acquires a spatial dependence due to the proximity effect. In this section we present a detailed investigation of this effect for superconductors with d-wave symmetry of the order parameter. We show that the spatial dependence of the order parameter in the vicinity of a low transparency insulating barrier may essentially depend on the crystal orientation relative to the barrier plane. Similar—although quantitatively different—results hold provided a superconductor is in a good electric contact with a normal metal.

In order to describe the proximity effect in d-wave superconductors we make use of the Eilenberger equations for the quasiclassical Green functions.[23] In the case of superconductors with singlet pairing these equations read[23,24]

$$\begin{aligned}
&(2\omega_m + \boldsymbol{v_F}\nabla_{\boldsymbol{R}})f(\hat{\boldsymbol{p}},\boldsymbol{R},\omega_m) \\
&\qquad -2\Delta(\hat{\boldsymbol{p}},\boldsymbol{R})g(\hat{\boldsymbol{p}},\boldsymbol{R},\omega_m)=0, \\
&(2\omega_m - \boldsymbol{v_F}\nabla_{\boldsymbol{R}})f^+(\hat{\boldsymbol{p}},\boldsymbol{R},\omega_m) \\
&\qquad -2\Delta^*(\hat{\boldsymbol{p}},\boldsymbol{R})g(\hat{\boldsymbol{p}},\boldsymbol{R},\omega_m)=0, \\
&\boldsymbol{v_F}\nabla_{\boldsymbol{R}}\, g(\hat{\boldsymbol{p}},\boldsymbol{R},\omega_m)+\Delta(\hat{\boldsymbol{p}},\boldsymbol{R})f^+(\hat{\boldsymbol{p}},\boldsymbol{R},\omega_m) \\
&\qquad -\Delta^*(\hat{\boldsymbol{p}},\boldsymbol{R})f(\hat{\boldsymbol{p}},\boldsymbol{R},\omega_m)=0.
\end{aligned} \tag{1}$$

Here $\omega_m = (2m+1)\pi T$ is the Matsubara frequency, $\hat{\boldsymbol{p}} = \boldsymbol{p_F}/|p_F|$, $\boldsymbol{v_F}(\hat{\boldsymbol{p}}) = \boldsymbol{p_F}/m$ is the Fermi velocity, $\Delta(\hat{\boldsymbol{p}},\boldsymbol{R})$ is the order parameter or the gap function. Anomalous and normal Green functions $f(\hat{\boldsymbol{p}},\boldsymbol{R},\omega_m) = f^{+*}(-\hat{\boldsymbol{p}},\boldsymbol{R},\omega_m)$ and $g(\hat{\boldsymbol{p}},\boldsymbol{R},\omega_m) = g^*(-\hat{\boldsymbol{p}},\boldsymbol{R},\omega_m)$ obey the normalization condition

$$g^2(\hat{\boldsymbol{p}},\boldsymbol{R},\omega_m) + f(\hat{\boldsymbol{p}},\boldsymbol{R},\omega_m)f^+(\hat{\boldsymbol{p}},\boldsymbol{R},\omega_m) = 1. \tag{2}$$

The order parameter Δ is linked to the anomalous Green function by means of the standard self-consistency equation

$$\Delta(\hat{\boldsymbol{p}},\boldsymbol{R}) = -\pi T\sum_m \int \frac{d^2S'}{(2\pi)^3 v_F} V(\hat{\boldsymbol{p}},\hat{\boldsymbol{p'}})f(\hat{\boldsymbol{p'}},\boldsymbol{R},\omega_m), \tag{3}$$

where $V(\hat{\boldsymbol{p}},\hat{\boldsymbol{p'}})$ is the anisotropic pairing potential and the integration is carried out over the Fermi surface. The quasiclassical equations (1) are valid at the scale much larger than the interatomic distance $\sim 1/p_F$ and do not keep track of rapid changes of the system parameters very close to the metal-metal or metal-insulator boundaries. In order to take these boundary effects into account the system of Eqs. (1)–(3) should be supplemented by the boundary conditions[25–27] matching the quasiclassical electron propagators g and f on both sides of the boundary. These boundary conditions may essentially depend on the quality of the interface. In the case of a nonmagnetic specularly reflecting boundary between two metals these conditions read[25]

$$d_-(\hat{\boldsymbol{p}}_-) = d_+(\hat{\boldsymbol{p}}_+),$$
$$d_-(\hat{\boldsymbol{p}}_-)s_-^2(\hat{\boldsymbol{p}}_-) = \left[\left(1+\frac{d_+(\hat{\boldsymbol{p}}_+)}{2}\right)s_+(\hat{\boldsymbol{p}}_+), s_-(\hat{\boldsymbol{p}}_-)\right] \times\frac{1-R(\hat{\boldsymbol{p}}_-)}{1+R(\hat{\boldsymbol{p}}_-)}. \quad (4)$$

Here $[a,b]$ denotes the commutator of matrices a and b, $R(\hat{\boldsymbol{p}})$ is the reflectivity coefficient and the index $+(-)$ labels the electron momentum in the right (left) half space with respect to the boundary plane. The 2×2 matrices d and s are defined by the equations $d(\hat{\boldsymbol{p}}) = \tilde{g}(\hat{\boldsymbol{p}}) - \tilde{g}(\check{\boldsymbol{p}})$, $s(\hat{\boldsymbol{p}}) = \tilde{g}(\hat{\boldsymbol{p}}) + \tilde{g}(\check{\boldsymbol{p}})$, where $\hat{\boldsymbol{p}}(\check{\boldsymbol{p}})$ denotes the incident (reflected) electron momentum and

$$\tilde{g}(\hat{\boldsymbol{p}}) = \begin{pmatrix} g(\hat{\boldsymbol{p}}) & if(\hat{\boldsymbol{p}}) \\ -if^+(\hat{\boldsymbol{p}}) & -g(\hat{\boldsymbol{p}}) \end{pmatrix}. \quad (5)$$

Provided the transparency of the tunnel barrier is equal to zero $D\equiv 1-R=0$ the equations (4) yield[28]

$$g_\pm(\hat{\boldsymbol{p}}) = g_\pm(\check{\boldsymbol{p}}),\;\; f_\pm(\hat{\boldsymbol{p}}) = f_\pm(\check{\boldsymbol{p}}),\;\; f_\pm^+(\hat{\boldsymbol{p}}) = f_\pm^+(\check{\boldsymbol{p}}), \quad (6)$$

where the Green functions are taken at the metal-insulator boundary. In the opposite limiting case of a transparent boundary between two metals $D=1$ the equations (4) reduce to a simple continuity conditions for the Green functions $g_+ = g_-$ and $f_+ = f_-$ at the boundary plane.

For the sake of definiteness let us assume that a d-wave superconductor occupies a half space $x>0$. Provided there is no current flow in this superconductor one can choose the gap function Δ to be real there and define $f_1 = (f+f^+)/2$, $f_2 = (f-f^+)/2$. Then with the aid of (1) we find

$$f_1(\hat{\boldsymbol{p}},x,\omega_m) - \frac{v_x^2}{4\omega_m^2}\partial_x^2 f_1(\hat{\boldsymbol{p}},x,\omega_m) - \frac{\Delta(\hat{\boldsymbol{p}},x)}{|\omega_m|}\left\{1+\frac{v_x^2}{4\omega_m^2}[\partial_x f_1(\hat{\boldsymbol{p}},x,\omega_m)]^2 - f_1^2(\hat{\boldsymbol{p}},x,\omega_m)\right\}^{1/2} = 0, \quad (7)$$

Let us first consider the case of an impenetrable boundary situated at the plane $x=0$. Then the spatial dependence of the gap function $\Delta(x)$ is defined by the combination of Eqs. (1) and (3) with the boundary conditions (6) at $x=0$ and $f_1(x\to\infty) = f_\infty$, $\Delta(x\to\infty) = \Delta_\infty$, where f_∞ and Δ_∞ are the equilibrium values for the anomalous Green function and the order parameter in the bulk superconductor. Provided the order parameter obeys the condition

$$\Delta(\hat{\boldsymbol{p}},\boldsymbol{R}) = \Delta(\check{\boldsymbol{p}},\boldsymbol{R}) \quad (8)$$

the solution of the above equations does not depend on x and thus the functions f_1 and Δ coincide with their equilibrium values far from the boundary. For superconductors with d-wave symmetry of the order parameter $\Delta(\boldsymbol{p_F})$ of the $(p_{x_0}^2 - p_{y_0}^2)$ type the condition (8) is satisfied if one of the principal crystal axes x_0, y_0 or z_0 is perpendicular to the boundary plane.

For other crystal orientations the order parameter $\Delta(\hat{\boldsymbol{p}},x)$ turns out to be spacially inhomogeneous. To proceed further let us assume that one can express the value Δ in the form $\Delta(\hat{\boldsymbol{p}},x) = \psi(\hat{\boldsymbol{p}})\eta(x)$.[29] At temperatures close to T_c and distances larger than the correlation length ξ_0 from the boundary the function $\eta(x)$ obeys the Ginzburg-Landau equations which have a well known solution

$$\eta_{\rm GL}(x) = \eta_\infty \tanh[(x+\beta)/\sqrt{2}\xi(T)], \quad (9)$$

η_∞ is the equilibrium value of $\eta(x)$ far from the boundary and $\xi(T)$ is the temperature-dependent superconducting coherence length. In the case of uniaxial symmetry we have $\xi(T) = [\xi_\parallel^2(T)\cos^2\alpha + \xi_\perp^2(T)\sin^2\alpha]^{1/2}$, where $\xi_\parallel(T)$ and $\xi_\perp(T)$ are the values of the coherence length, respectively, in the basal plane and the transversal direction, α is the angle between the vector normal to the boundary and the basal plane. The value of β is defined by the boundary condition

$$q\eta'(0) = \eta(0)$$

and the parameter q has to be derived from the microscopic theory [Eqs. (3), (6), and (7)].

In the vicinity of the critical temperature $T\sim T_c$ one can linearize Eq. (7) neglecting higher powers of f_1. Then combining (6) and (7) one gets

$$f_1(\hat{\boldsymbol{p}},x,\omega_m) = \frac{1}{|v_x|}\int_0^\infty \left\{\exp\left(-\left|\frac{2\omega_m}{v_x}(x-x')\right|\right)\Delta(\hat{\boldsymbol{p}},x') + \exp\left(-\left|\frac{2\omega_m}{v_x}\right|(x+x')\right)\Delta(\check{\boldsymbol{p}},x')\right\}dx'. \quad (10)$$

Substituting (10) into (3) and setting $\Delta(\hat{\boldsymbol{p}},x) = \psi(\hat{\boldsymbol{p}})\eta(x)$ we arrive at the integral equation

$$\eta(x) = \frac{\pi T\lambda}{\int\psi^2(\hat{\boldsymbol{p}})d^2S}\sum_m\int_0^\infty\int d^2S\,\eta(x')\psi(\hat{\boldsymbol{p}})\left\{\frac{\psi(\hat{\boldsymbol{p}})}{|v_x|}\exp\left(-\left|\frac{2\omega_m}{v_x}(x-x')\right|\right) + \frac{\psi(\check{\boldsymbol{p}})}{|v_x|}\exp\left(-\left|\frac{2\omega_m}{v_x}\right|(x+x')\right)\right\}dx'. \quad (11)$$

The effective coupling constant for an anisotropic superconductor λ is defined by the equation

$$\lambda\psi(\hat{\boldsymbol{p}}) = -\int V(\hat{\boldsymbol{p}},\hat{\boldsymbol{p}}')\psi(\hat{\boldsymbol{p}}')\frac{d^2S'}{(2\pi)^3 v_F}, \tag{12}$$

which yields $\pi T_c \lambda \sum_m |\omega_m|^{-1} = 1$. Equation (11) describes the behavior of $\eta(x)$ at distances $x \lesssim \xi(T)$ from the boundary. This equation coincides with that derived in Ref. 30 within the framework of a different technique for the case of a small gap anisotropy.

A trivial combination of the above equations also allows us to evaluate the Green functions at T close to T_c. E.g., the functions f_1 and f_2 at the superconductor-insulator boundary $x = 0$ and $T \to T_c$ read

$$\begin{aligned} f_1(\hat{\boldsymbol{p}},\omega_m) &= \frac{\psi(\hat{\boldsymbol{p}}) + \psi(\check{\boldsymbol{p}})}{|v_x|} \\ &\quad\times \int_0^\infty \exp\left(-\left|\frac{2\omega_m}{v_x}\right| x\right)\eta(x)dx, \\ f_2(\hat{\boldsymbol{p}},\omega_m) &= \frac{\psi(\check{\boldsymbol{p}}) - \psi(\hat{\boldsymbol{p}})}{v_x}\operatorname{sgn}\omega_m \\ &\quad\times \int_0^\infty \exp\left(-\left|\frac{2\omega_m}{v_x}\right| x\right)\eta(x)dx \\ &= \frac{\psi(\check{\boldsymbol{p}}) - \psi(\hat{\boldsymbol{p}})}{\psi(\hat{\boldsymbol{p}}) + \psi(\check{\boldsymbol{p}})}\operatorname{sgn}(v_x\omega_m) f_1(\hat{\boldsymbol{p}},\omega_m). \end{aligned} \tag{13}$$

The exact solution of (11) can be easily found for the case $\psi(\hat{\boldsymbol{p}}) = -\psi(\check{\boldsymbol{p}})$. Then we have

$$\eta(x) = Cx,$$

i.e., for this particular crystal orientation the gap function vanishes at the boundary $x = 0$ and for any $x > 0$ it is described by the function

$$\eta(x) = \eta_\infty \tanh[x/\sqrt{2}\xi(T)].$$

Combining the equation $\psi(\hat{\boldsymbol{p}}) = -\psi(\check{\boldsymbol{p}})$ with the symmetry condition for the order parameter of the $(p_{x_0}^2 - p_{y_0}^2)$ type we come to the conclusion that the gap function vanishes at the superconductor-insulator interface provided the principal crystal axes x_0 and y_0 constitutes the angle $\pi/4$ with this interface. Note that this conclusion is essentially based on the symmetry arguments and thus remains valid at any temperature below T_c. Indeed, for a pairing potential with the symmetry property $V(\hat{\boldsymbol{p}},\hat{\boldsymbol{p}}') = -V(\hat{\boldsymbol{p}},\check{\boldsymbol{p}}')$ and the boundary condition $f(\hat{\boldsymbol{p}}) = f(\check{\boldsymbol{p}})$ we obtain $\eta(0) = 0$ from the self-consistency equation (3) at any T.

At distances $x \gg \xi_0$ from the boundary the integral equation (11) has a general solution

$$\eta(x) = C(x+q). \tag{14}$$

The parameter q can be easily found from a simple variational procedure[31] which yields

$$\begin{aligned} q = &\left(\frac{\pi^3}{336\zeta(3)T}\int_{v_x>0}[\psi(\hat{\boldsymbol{p}}) + \psi(\check{\boldsymbol{p}})]^2 v_x^3 d^2S \right. \\ &\left. + \frac{7\zeta(3)}{4\pi^3 T}\frac{\left(\int_{v_x>0}[\psi(\hat{\boldsymbol{p}}) + \psi(\check{\boldsymbol{p}})]^2 v_x^2 d^2S\right)^2}{\int_{v_x>0}[\psi(\hat{\boldsymbol{p}}) - \psi(\check{\boldsymbol{p}})]^2 v_x d^2S}\right) \\ &\times \left(\int \psi^2(\hat{\boldsymbol{p}}) v_x^2 d^2S\right)^{-1}, \end{aligned} \tag{15}$$

where integration over the Fermi surface is confined to its part with $v_x > 0$.

Note that with the aid of general symmetry arguments one can unambiguously fix only those crystal orientations for which the parameter q is equal to zero and infinity. The detailed form of q as a function of a crystal orientation relative to the boundary plane for a given pairing symmetry type essentially depends on the particular choice of the basis function $\psi(\hat{\boldsymbol{p}})$. E.g., for the pairing symmetry of the type $(p_{x_0}^2 - p_{y_0}^2)$ for the sake of definiteness one can choose the simplest basis function $\psi(\hat{\boldsymbol{p}}) = \Delta_0(\hat{p}_{x_0}^2 - \hat{p}_{y_0}^2)$ and after an explicit integration in (15) get

$$\begin{aligned} q = &\frac{4\zeta(3)v_F}{5\pi^3 T_c(1+2\sin^2\theta_0)} \\ &\times\left\{\frac{(\cos^2\theta_0 + 3\sin^4\theta_0\cos^2 2\phi_0)^2}{\sin^2\theta_0(1-\sin^2\theta_0\cos^2 2\phi_0)}\right. \\ &\left. + \frac{25}{6\zeta^2(3)}\left(\frac{\pi}{4}\right)^6 (4\cos^2\theta_0 + 19\sin^4\theta_0\cos^2 2\phi_0)\right\}. \end{aligned} \tag{16}$$

Here θ_0, ϕ_0 are the polar and the azimuthal angles of the normal $\boldsymbol{n}$ to the boundary (see Fig. 1). The function $q(\theta_0,\varphi_0)$ is presented in Fig. 2. We see that the parameter q varies from $q = 0$ (or $\eta = 0$) for $\theta_0 = \pi/2$, $\phi_0 = \pi/4$ $[\psi(\hat{\boldsymbol{p}}) = -\psi(\check{\boldsymbol{p}})]$ to $q = \infty$ [i.e., $\eta'(0) = 0$] if one of the main crystal axes x_0, y_0 or z_0 has the angle $\pi/2$ with the interface plane.

Our analysis can be easily modified to take into account the effect of nonmagnetic impurities. In the presence of such impurities close to T_c, Eq. (11) remains valid

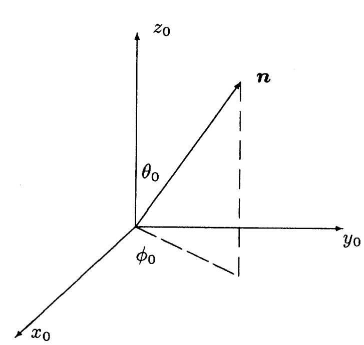

FIG. 1. The relative orientation of the principal crystal axes x_0, y_0, z_0 and the vector $\mathbf{n}$ normal to the boundary plane.

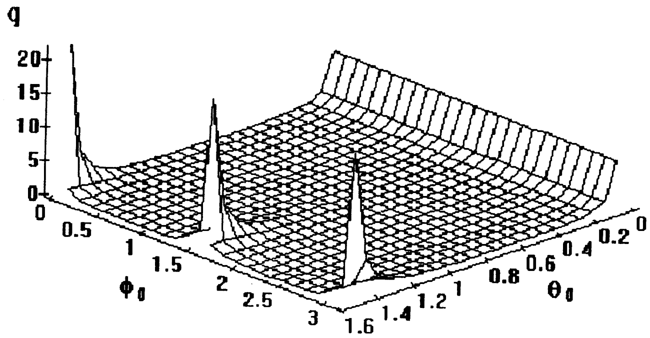

FIG. 2. The parameter q (in units of v_F/T_c) as a function of crystal orientation relative to the boundary plane.

if one substitutes $\omega_m \to \tilde{\omega}_m = \omega_m + \mathrm{sgn}\,\omega_m/2\tau_{\mathrm{imp}}$, where τ_{imp} is the average scattering time. Accordingly the modified critical temperature T_c' is defined by the equation $\pi T_c' \lambda \sum_m |\tilde{\omega}_m|^{-1} = 1$. With the aid of (11) it is easy to check that scattering on nonmagnetic impurities does not lead to qualitative changes in the behavior of the order parameter in the vicinity of a superconductor-insulator interface. E.g., the homogeneous solution $\eta(x) = \eta_\infty$ for $\psi(\hat{\boldsymbol{p}}) = \psi(\check{\boldsymbol{p}})$ and the solution $\eta(x) = Cx$ for $\psi(\hat{\boldsymbol{p}}) = -\psi(\check{\boldsymbol{p}})$ remain valid in the presence of impurities. Similar results hold also for intermediate crystal orientations.

The above analysis shows that in the case of specularly reflecting boundaries the values $\eta(0)$ and η_∞ are of the same order of magnitude only for particular orientations of the normal $\boldsymbol{n}$ within narrow angular intervals $\Delta\phi_0 \sim \Delta\theta_0 \sim [\xi_0/\xi(T)]^{1/2}$ around the crystal axes x_0, y_0, z_0. For other crystal orientations from Eq. (16) one has $\eta(0) \sim \eta_\infty \xi_0/\xi(T) \ll \eta_\infty$, i.e., the d-wave superconducting order parameter turns out to be strongly suppressed in the vicinity of the insulating barrier. In the case of diffusive scattering at the interface one has $q \sim \xi_0$.[30] Thus in this case the parameter $\eta(0) \sim \xi_0\eta_\infty/\xi(T) \ll \eta_\infty$ for $T \to T_c$ and all crystal orientations.

At low temperatures the analysis of the proximity effect becomes more difficult. As we already discussed for particular crystal orientations the form of the order parameter can be described with the aid of symmetry arguments. E.g., for $\psi(\hat{\boldsymbol{p}}) = \psi(\check{\boldsymbol{p}})$ the superconducting order parameter $\Delta(T)$ does not depend on coordinates whereas for $\psi(\hat{\boldsymbol{p}}) = -\psi(\check{\boldsymbol{p}})$ at any T we have $\Delta = 0$ at the superconductor-insulator interface. For other crystal orientations the behavior of the order parameter at $T \ll T_c$ can be qualitatively described by the following estimate.

Let us consider the exact equation (7) and split the frequency range into two intervals: $|\omega_m| \ll \Delta_0$ and $|\omega_m| \gtrsim \Delta_0$. The contribution of the first frequency interval to the self-consistency equation is small in the parameter ω_m/Δ_0. Therefore for our estimate it is sufficient to restrict our consideration to the second frequency interval. For $|\omega_m| \gg \Delta_0$ one can neglect nonlinear terms in (7). Then making use of the condition $\xi_0 \gg v_x/\omega_m$ one gets

$$f_1(\hat{\boldsymbol{p}}, x, \omega_m) = \frac{\eta(x)\psi(\hat{\boldsymbol{p}})}{|\omega_m|} + \frac{\eta(0)[\psi(\check{\boldsymbol{p}}) - \psi(\hat{\boldsymbol{p}})]}{2|\omega_m|} \times \exp\left(-\left|\frac{2\omega_m}{v_x}\right| x\right). \quad (17)$$

Taking (17) as an approximate form for f_1 in the whole frequency interval $\omega_m \gtrsim \Delta_0$, setting $\eta(x)$ =const and expanding in powers of the anisotropy parameter $\int_{v_x>0}[\psi(\hat{\boldsymbol{p}}) - \psi(\check{\boldsymbol{p}})]^2 d^2S / \int \psi^2(\boldsymbol{p}) d^2S$ we find

$$\eta(0) \approx \left(1 - \frac{\int_{v_x>0}[\psi(\hat{\boldsymbol{p}}) - \psi(\check{\boldsymbol{p}})]^2 d^2S}{2\int \psi^2(\boldsymbol{p}) d^2S}\right)\eta_\infty. \quad (18)$$

This estimate provides correct limits $\eta(0) = \eta_\infty$ for $\psi(\hat{\boldsymbol{p}}) = \psi(\check{\boldsymbol{p}})$ and $\eta(0) = 0$ for $\psi(\hat{\boldsymbol{p}}) = -\psi(\check{\boldsymbol{p}})$ and qualitatively describes the low-temperature behavior of $\eta(0)$ for intermediate crystal orientations. It demonstrates that at $T \ll T_c$ the value $\eta(0)$ is of order η_∞ for a relatively wide angular interval $\Delta\phi_0, \Delta\theta_0 \sim 1$. The typical length scale at which the value $\eta(x)$ changes from $\eta(0)$ at the boundary to η_∞ deep in the superconductor is of order ξ_0.

In the case of an ideally transmitting normal-metal–superconductor interface $D(\hat{\boldsymbol{p}}) = 1$ quasiclassical propagators are continuous at this interface: $g_-(x = 0) = g_+(x = 0)$ and $f_-(x = 0) = f_+(x = 0)$. Then imposing the boundary conditions $f_1(x \to \infty) = f_\infty$, $\Delta(x \to \infty) = \Delta_\infty$, $f_1(-\infty) = 0$ and assuming that the order parameter is equal to zero in the normal metal $\Delta(x < 0) \equiv 0$ one can repeat the above analysis and show that at T close to T_c the order parameter is again described by Eqs. (9) and (14), where

$$q = \left(\frac{\pi^3}{336\zeta(3)T}\int \psi^2(\hat{\boldsymbol{p}})|v_x|^3 d^2S + \frac{7\zeta(3)}{4\pi^3 T}\frac{[\int \psi^2(\hat{\boldsymbol{p}})v_x^2 d^2S]^2}{\int \psi^2(\hat{\boldsymbol{p}})|v_x| d^2S}\right) \times \left(\int \psi^2(\hat{\boldsymbol{p}}) v_x^2 d^2S\right)^{-1}. \quad (19)$$

According to this result for NS structures at $T \to T_c$ the parameter q is of order ξ_0 for all crystal orientations. As before at $T \to 0$ the order parameter changes from $\Delta(x = 0)$ to $\Delta(x = \infty)$ at distances of order ξ_0 from the NS boundary. To estimate the value $\eta(0)$ for this temperature interval one can follow the procedure developed in Ref. 32 for the case of isotropic s-wave superconductors. Then similarly to Refs. 32 and 33 one finds $\eta(0) \approx 0.5\eta_\infty$ at $T \to 0$. This estimate appears to hold for any crystal orientation. It also agrees with the results of Ref. 34 in which the order parameter of a d-wave superconductor has been calculated numerically for a particular crystal orientation.

Finally let us note that at T close to T_c and distances $x \lesssim \xi_0$ from the interface we also expect an additional suppression of the order parameter with respect to that described by Eqs. (14), (15), and (19). Indeed, the Ginzburg-Landau equation and its solution (9) apply only at distances $x \gg \xi_0$ from the boundary. For

$x \lesssim \xi_0$ it is necessary to proceed within the framework of a rigorous microscopic analysis.[33] E.g., one can show[33] that the exact value $\eta(0)$ at the boundary between a normal metal and an s-wave superconductor is by the factor ≈ 1.4 smaller and the exact ratio $\eta'(0)/\eta(0)$ is by the factor ≈ 1.6 larger as compared to the corresponding values which follow from the standard Ginzburg-Landau analysis (see, e.g., Ref. 35). We believe that similar situation takes place also for superconductors with anisotropic pairing considered here.

III. JOSEPHSON CURRENT FOR ANISOTROPIC SUPERCONDUCTORS

A. Tunnel junctions

Let us investigate the dc Josephson effect in tunnel junctions between two d-wave superconductors. Assuming that the junction transparency is small $D(\hat{\boldsymbol{p}}) \ll 1$ one can proceed perturbatively and expand the boundary conditions (4) in powers of $D(\hat{\boldsymbol{p}})$. Keeping only the linear terms one gets

$$g_-(\hat{\boldsymbol{p}}_-) - g_-(\check{\boldsymbol{p}}_-) = \frac{1}{2}D(\hat{\boldsymbol{p}}_-)[f_+(\hat{\boldsymbol{p}}_+)f_-^+(\hat{\boldsymbol{p}}_-) - f_-(\hat{\boldsymbol{p}}_-)f_+^+(\hat{\boldsymbol{p}}_+)], \tag{20}$$

where the functions $f_\pm(\hat{\boldsymbol{p}}_\pm)$ and $f_\pm^+(\hat{\boldsymbol{p}}_\pm)$ are the anomalous Green functions calculated on both sides of the tunnel barrier for $D(\hat{\boldsymbol{p}}) = 0$. Substituting this expression into the formula for the superconducting current

$$\boldsymbol{j}(\boldsymbol{R}) = -2\pi i e T \sum_m \int \frac{d^2S}{(2\pi)^3 v_F} \boldsymbol{v}_F(\hat{\boldsymbol{p}}) g(\hat{\boldsymbol{p}}, \boldsymbol{R}, \omega_m), \tag{21}$$

we arrive at the general expression for the Josephson current

$$j_S = 2\pi e T \sin\varphi \sum_m \int_{v_x>0} D(\hat{\boldsymbol{p}}_-) v_x(\hat{\boldsymbol{p}}_-)[f_{1-}(\hat{\boldsymbol{p}}_-)f_{1+}(\hat{\boldsymbol{p}}_+) - f_{2-}(\hat{\boldsymbol{p}}_-)f_{2+}(\hat{\boldsymbol{p}}_+)]\frac{d^2S_-}{(2\pi)^3 v_F}, \tag{22}$$

where φ is the phase difference between two superconductors [i.e., the gap is proportional to $\exp(i\varphi)\psi_+(\hat{\boldsymbol{p}}_+)$ and $\psi_-(\hat{\boldsymbol{p}}_-)$ on the left and on the right, respectively], v_x is the Fermi velocity projection on the normal to the plane interface. The functions f_1, f_2 are calculated for real $\psi_+(\hat{\boldsymbol{p}}_+)$, $\psi_-(\hat{\boldsymbol{p}}_-)$. Provided the functions $f_\pm(\hat{\boldsymbol{p}}_\pm, \omega_m)$ do not depend on space coordinates one can easily evaluate j_S (22) and get

$$j_S = 2\pi e T \sin\varphi \sum_m \int_{v_x>0} \frac{\Delta_+(\hat{\boldsymbol{p}}_+)\Delta_-(\hat{\boldsymbol{p}}_-)D(\hat{\boldsymbol{p}}_-)v_x(\hat{\boldsymbol{p}}_-)}{\sqrt{\Delta_+^2(\hat{\boldsymbol{p}}_+) + \omega_m^2}\sqrt{\Delta_-^2(\hat{\boldsymbol{p}}_-) + \omega_m^2}} \frac{d^2S_-}{(2\pi)^3 v_F}. \tag{23}$$

This result coincides with that obtained in Ref. 27. At low temperatures $T \ll \Delta$ it yields

$$j_S = 4e\sin\varphi \int_{v_x>0} \frac{\Delta_+(\hat{\boldsymbol{p}}_+)\Delta_-(\hat{\boldsymbol{p}}_-)}{|\Delta_+(\hat{\boldsymbol{p}}_+)| + |\Delta_-(\hat{\boldsymbol{p}}_-)|} \times K\left(\frac{|\Delta_+(\hat{\boldsymbol{p}}_+)| - |\Delta_-(\hat{\boldsymbol{p}}_-)|}{|\Delta_+(\hat{\boldsymbol{p}}_+)| + |\Delta_-(\hat{\boldsymbol{p}}_-)|}\right) \times D(\hat{\boldsymbol{p}}_-)v_x(\hat{\boldsymbol{p}}_-)\frac{d^2S_-}{(2\pi)^3 v_F}, \tag{24}$$

where $K(t) = \int_0^{\pi/2}(1 - t^2\sin^2\phi)^{-1/2}d\phi$ is the complete elliptic integral. At $\Delta_-(\hat{\boldsymbol{p}}_-) \simeq \Delta_+(\hat{\boldsymbol{p}}_+) \simeq \Delta(\hat{\boldsymbol{p}})$ and any T we find from (23)

$$j_S = \pi e \sin\varphi \int_{v_x>0} \Delta(\hat{\boldsymbol{p}}) \tanh\left(\frac{\Delta(\hat{\boldsymbol{p}})}{2T}\right) \times D(\hat{\boldsymbol{p}})v_x(\hat{\boldsymbol{p}})\frac{d^2S}{(2\pi)^3 v_F}. \tag{25}$$

This result shows that the Josephson current between identical similarly oriented d-wave superconductors at $T \ll \Delta$ is proportional to the product $\Delta(\hat{\boldsymbol{p}})D(\hat{\boldsymbol{p}})v_x(\hat{\boldsymbol{p}})$ averaged over the momentum directions at the Fermi surface.

With the aid of a standard expression for the normal-state resistance of a tunnel junction

$$R_N^{-1} = 2e^2 \int_{v_x>0} D(\hat{\boldsymbol{p}}_-)v_x(\hat{\boldsymbol{p}}_-)\frac{d^2S_-}{(2\pi)^3 v_F} \tag{26}$$

one can easily reduce Eqs. (24) and (25) to the analogous results for conventional superconductors provided the momentum dependence of the gap function Δ is neglected.

Close to the critical temperature $T \sim T_c$ the expression (23) reduces to

$$j_S = \frac{\pi e \sin\varphi}{2T} \int_{v_x>0} \Delta_+(\hat{\boldsymbol{p}}_+)\Delta_-(\hat{\boldsymbol{p}}_-)D(\hat{\boldsymbol{p}}_-)v_x(\hat{\boldsymbol{p}}_-) \times \frac{d^2S_-}{(2\pi)^3 v_F}. \tag{27}$$

As it was demonstrated in the previous section in the presence of a nontransparent insulating barrier the order parameter and the Green functions of a d-wave superconductor do not depend on coordinates only provided one of the principal axes is perpendicular to the barrier plane. For any other crystal orientation the order parameter Δ as well as g and f functions vary in space and the results (23)–(25), (27) are no longer valid. To derive the corresponding generalization of Eq. (27) let us combine the exact formula (22) with the results obtained in Sec. II. As for a spherical form of the Fermi surface the value $D(\hat{\boldsymbol{p}})$ depends only on $\hat{p}_x$, with the aid of Eq. (13) at T close to T_c we find

$$j_S = 8\pi e T \sin\varphi \sum_m \int_{v_x>0} D(\hat{\boldsymbol{p}}_-) v_x(\hat{\boldsymbol{p}}_-) I_+(\hat{\boldsymbol{p}}_+, \omega_m) \times I_-(\hat{\boldsymbol{p}}_-, \omega_m) \frac{d^2 S_-}{(2\pi)^3 v_F}. \tag{28}$$

The functions $I_\pm$ read

$$I_\pm(\hat{\boldsymbol{p}}_\pm, \omega_m) = \frac{1}{|v_x|} \int_0^\infty \Delta_\pm(\hat{\boldsymbol{p}}_\pm, x) \times \exp\left(-\left|\frac{2\omega_m}{v_x}\right| x\right) dx, \tag{29}$$

x is the distance from the interface.

The results (28), (29) show that the expression for the Josepson current in d-wave superconductors is essentially nonlocal in space: the value j_S is determined by the order parameter at distances $\simeq \xi_0$ from the tunnel barrier. E.g., for a particular crystal orientation $\psi(\hat{\boldsymbol{p}}) = -\psi(\check{\boldsymbol{p}})$ at both sides of the barrier close to T_c we get

$$j_S = (\eta'(0))^2 \frac{\pi e \sin\varphi}{96 T^3} \int_{v_x>0} \psi_+(\hat{\boldsymbol{p}}_+)\psi_-(\hat{\boldsymbol{p}}_-) \times D(\hat{\boldsymbol{p}}_-) v_x^3(\hat{\boldsymbol{p}}_-) \frac{d^2 S_-}{(2\pi)^3 v_F}. \tag{30}$$

This result demonstrates that even if the order parameter of a d-wave superconductor vanishes at the interface $\eta(0) = 0$ the Josephson current remains finite. The nonlocality of j_S (28), (29) results in different temperature dependence of the Josephson current for different crystal orientations. E.g., for a particular case $\psi(\hat{\boldsymbol{p}}) = \psi(\check{\boldsymbol{p}})$ and T close to T_c from (27) we have

$$j_S \propto \eta^2(0) \propto T_c - T, \tag{31}$$

whereas for $\psi(\hat{\boldsymbol{p}}) = -\psi(\check{\boldsymbol{p}})$ the result (30) yields

$$j_S \propto \eta'^2(0) \propto (T_c - T)^2. \tag{32}$$

To evaluate the Josephson current for arbitrary crystal orientations let us combine the results (9), (14) with Eqs. (28) and (29). Then making use of (26) and assuming $D(\hat{\boldsymbol{p}}) \propto p_n^k / p_F^k$ $(k = 0, 2, ...)$,[36] we get

$$j_S R_N = \frac{\sin\varphi}{16 e T p_F^2} \frac{\eta_\infty^2 (k+2)}{\xi_+(T)\xi_-(T)} \int_{v_x>0} \left(q_+ q_- + \frac{7\zeta(3)}{2\pi^3 T}(q_+ + q_-) v_x(\hat{\boldsymbol{p}}_-) + \frac{v_x^2(\hat{\boldsymbol{p}}_-)}{48 T^2} \right) \psi_-(\hat{\boldsymbol{p}}_-)\psi_+(\hat{\boldsymbol{p}}_+) \frac{v_x^{k+1}(\hat{\boldsymbol{p}}_-)}{v_F^{k+1}} d^2 S. \tag{33}$$

Here $q_\pm$ are the values of the parameter q (16) on both sides of the interface. The result (33) is valid provided the parameters $q_\pm$ are not very large $q_\pm \ll \xi_\pm(T)$. According to (16) this implies that the direction of either one of the crystal principal axes should not be very close to that normal to the interface $\Delta\phi_0, \Delta\theta_0 \gg (\xi_0/\xi(T))^{1/2}$. At $\Delta\phi_0, \Delta\theta_0 \lesssim (\xi_0/\xi(T))^{1/2}$ the expression for j_S shows a crossover from (33) to (27). Thus we can conclude that at T close to T_c for a wide range of crystal orientations the proximity effect strongly influences the dc Josephson current in d-wave superconductors leading to the temperature dependence $j_S \propto (\eta_\infty/\xi(T))^2 \propto (T_c - T)^2$. Only if one of the crystal principal axes is (nearly) perpendicular to the junction plane this dependence changes and becomes $j_S \propto T_c - T$. At lower temperatures $T \lesssim \Delta(T)$ the role of the proximity effect becomes less important and the expression (23) qualitatively describes the dc Josephson current for a wide range of crystal orientations.

Sigrist and Rice[11] suggested the following simple phenomenological expression for the Josephson current between two tetragonal superconductors with $p_{x_0}^2 - p_{y_0}^2$ type of pairing near T_c:

$$j_S = j_0 \sin\varphi (n_{+(x_0)}^2 - n_{+(y_0)}^2)(n_{-(x_0)}^2 - n_{-(y_0)}^2). \tag{34}$$

Here $n_{\pm(i)}$ denotes the projection of the unit vector normal to the boundary plane on the i axis.

Since the basis function $\psi(\hat{\boldsymbol{p}})$ is not unique for a given pairing symmetry type, the dependence of the Josephson current on the orientation of the boundary plane relative to the crystal axes cannot be chosen unambiguously from the symmetry arguments only. Equation (34) presents the simplest example for a particular angular dependence of the Josephson current consistent with the pairing symmetry. For more complicated cases higher powers of $n_{\pm(x,y)}$ can appear.

In the case of diffusive scattering at the boundary the Josephson current does not depend on the relative orientation of two superconductors and $\eta(0) \sim [\xi_0/\xi(T)]^2 \eta_\infty$.[10] If we put $\eta(0) \approx 0$, in accordance with (32) and (33) at T close to T_c we have $j_0 \propto (T_c - T)^2$.

As it follows from our analysis the dependence of j_S on the relative orientation of superconductors becomes important for the specular scattering at the insulating barrier. From Eq. (27) one can easily recover the dependence of the Josephson current j_S on the angle ϕ between the z_0 axes of two superconductors. E.g., if $n_{\pm(x_0)} = 1$ and $D \propto p_x^2/p_F^2$ we have

$$j_S R_N = \frac{\Delta_1 \Delta_2 \sin\varphi}{96 e T} \left(9 + \frac{1}{2}\cos 2\phi \right), \tag{35}$$

where Δ_1, Δ_2 are the maximum values of $\Delta(\hat{\boldsymbol{p}}_+)$, $\Delta(\hat{\boldsymbol{p}}_-)$. We see that in the case of Eq. (35)

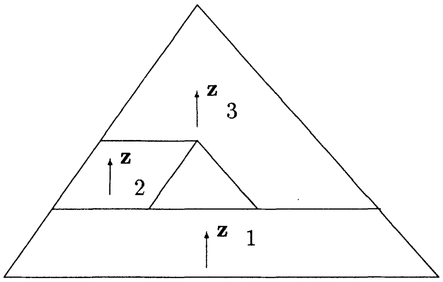

FIG. 3. An example of the circuit which contains the odd number of π junctions.

the Josephson critical current is positive for all values of ϕ (0 contact). However if, for instance, the z_0 axes for both superconductors are perpendicular to the boundary plane, Eq. (27) yields

$$j_S \propto \cos 2\phi, \tag{36}$$

where ϕ is now the angle between the x_0 axes of the superconductors. In the latter case the Josephson critical current turns out to be negative (π junction[12]) for certain values of ϕ. Thus, rotating one of the crystals around its z_0 axis one can turn the 0 junction into the π junction. The latter property remains also for lower temperatures $T \ll T_c$ in which case the ϕ dependence of the Josephson current is more complicated.

Note that the dependence (36) permits to realize a very simple configuration, for which three parts of *the same* superconductor form a closed circuit with the odd number of π junctions (see Fig. 3). The z_0 axes of the grains coincide and are taken to be perpendicular to junction planes. The angles between the x_0 axes of the first and second as well as the second and the third grains are supposed to be equal to $\pi/6$. Then according to (36) the corresponding junctions are of the 0 type. In contrast the junction between the third and the first grains turns out to be of the π type because the angle ϕ is equal to $\pi/3$ for this junction.

B. SNS junctions

Let us now consider the dc Josephson effect in planar structures superconductor-normal-metal-superconductor (SNS). In the case of clean s-wave superconductors this effect was studied in Refs. 37 and 38. Here we provide the generalization to the case of d-wave superconductors with arbitrary orientations relative to the boundaries.

In order to evaluate the supercurrent in SNS junctions one has to solve the Eilenberger equations (1) in superconducting and normal regions and match the solutions at NS interfaces with the aid of the boundary conditions (4). Below we shall consider the case of transparent NS boundaries with $D(\hat{\boldsymbol{p}}) = 1$ and assume that the order parameter in the normal metal is equal to zero $\Delta(-d/2 < x < d/2) \equiv 0$, d is the thickness of a normal layer between two d-wave superconductors which occupy two half spaces $x < -d/2$ and $x > d/2$. To find the Green functions of superconducting banks we shall use the following ansatz:

$$\begin{aligned} f_\pm(\hat{\boldsymbol{p}}_\pm, x, \omega_m) &= e^{i\varphi_\pm(\hat{\boldsymbol{p}}_\pm)} \\ &\mp e^{i\varphi_\pm(\hat{\boldsymbol{p}}_\pm)} \operatorname{sgn} v_x \delta g_\pm(\hat{\boldsymbol{p}}_\pm, x, \omega_m), \\ f^+_\pm(\hat{\boldsymbol{p}}_\pm, x, \omega_m) &= e^{-i\varphi_\pm(\hat{\boldsymbol{p}}_\pm)} \\ &\pm e^{-i\varphi_\pm(\hat{\boldsymbol{p}}_\pm)} \operatorname{sgn} v_x \delta g_\pm(\hat{\boldsymbol{p}}_\pm, x, \omega_m), \\ g_\pm(\hat{\boldsymbol{p}}_\pm, x, \omega_m) &= \delta g_\pm(\hat{\boldsymbol{p}}_\pm, x, \omega_m). \end{aligned} \tag{37}$$

Here $\varphi_\pm(\hat{\boldsymbol{p}}_\pm)$ are the phases of the order parameters $\Delta_\pm(\hat{\boldsymbol{p}}_\pm)$. Equation (37) satisfies the normalization condition (2). The ansatz (37) is correct for $\omega_m \ll \Delta(\hat{\boldsymbol{p}})$. The latter inequality in turn holds for the parameter region $v_F/d \ll T \ll |\Delta_\pm(\theta = 0)|$ or $T \ll v_F/d$, $d \gg \xi_0$ which will be considered below.

Substituting (37) into (1) in the main approximation one obtains

$$|v_x| \nabla_x \delta g_\pm(\hat{\boldsymbol{p}}, x, \omega_m) = \mp 2 |\Delta_\pm(\hat{\boldsymbol{p}}_\pm, x)| \delta g_\pm(\hat{\boldsymbol{p}}_\pm, x, \omega_m). \tag{38}$$

The solution of the Eilenberger equations (1) in the normal metal is trivial. Combining this solution with (37) and making use of the continuity condition for the Green functions at NS interfaces we find

$$g_N(\hat{\boldsymbol{p}}, \omega_m) = \delta g_-(\hat{\boldsymbol{p}}_-, \omega_m) = \delta g_+(\hat{\boldsymbol{p}}_+, \omega_m),$$

$$[e^{i\varphi_-(\hat{\boldsymbol{p}}_-)} + e^{i\varphi_-(\hat{\boldsymbol{p}}_-)} \operatorname{sgn} v_x \delta g_-(\hat{\boldsymbol{p}}_-, \omega_m)] e^{-2\omega_m d/v_x}$$

$$= e^{i\varphi_+(\hat{\boldsymbol{p}}_+)} - e^{i\varphi_+(\hat{\boldsymbol{p}}_+)} \operatorname{sgn} v_x \delta g_+(\hat{\boldsymbol{p}}_+, \omega_m), \tag{39}$$

g_N is the Eilenberger Green function in the normal metal. Similarly to the case of conventional superconductors (see, e.g., Ref. 33) Eqs. (39) yield

$$g_N(\hat{\boldsymbol{p}}, \omega_m) = \operatorname{sgn} v_x \tanh\left\{\frac{i\varphi(\hat{\boldsymbol{p}})}{2} + \frac{\omega_m d}{v_x}\right\}, \tag{40}$$

where $\varphi(\hat{\boldsymbol{p}}) = \varphi$ for $\psi_+(\hat{\boldsymbol{p}}_+)\psi_-(\hat{\boldsymbol{p}}_-) > 0$ and $\varphi(\hat{\boldsymbol{p}}) = \varphi + \pi$ for $\psi_+(\hat{\boldsymbol{p}}_+)\psi_-(\hat{\boldsymbol{p}}_-) < 0$ [as before the gap is chosen to be proportional to $\psi_-(\hat{\boldsymbol{p}}_-)$ on the left side and to $\exp(i\varphi)\psi_+(\hat{\boldsymbol{p}}_+)$ on the right side of the barrier].

Substituting (40) into (21) we arrive at the final expression for the Josephson current in SNS junctions. For $v_F/d \ll T \ll |\Delta_\pm(\theta = 0)|$ we reproduce the standard result

$$j_S = 6en \exp(-2\pi T d/v_F) \sin\varphi'/md,$$

derived before for conventional superconductors.[37,38] Here $\Delta_\pm(\theta = 0) = \Delta(p_{x\pm} = p_F)$ and φ' is the total phase difference between $\Delta_+(\theta = 0)$ and $\Delta_-(\theta = 0)$, $n = p_F^3/3\pi^2$ is the electron concentration. The difference between s- and d-wave superconductors becomes important in the low-temperature limit $T \ll v_F/d$. At $T \to 0$ and $d \gg \xi_0$ we get

$$j_S = \frac{3en}{4\pi md}(C_1[\varphi] + C_2[\varphi + \pi]). \qquad (41)$$

The function $[\varphi]$ defines the standard sawtooth behavior of $j_S(\varphi)$ for s-wave superconductors at $T = 0$ (Ref. 38) (see Fig. 4) and

$$C_1 = \int_{v_x>0} \cos^2\theta d\Omega^+, \quad C_2 = \int_{v_x>0} \cos^2\theta d\Omega^-,$$

$d\Omega^{+,-}$ are the solid angle elements on the Fermi sphere for which the functions $\psi_-(\hat{\boldsymbol{p}}_-)$ and $\psi_+(\hat{\boldsymbol{p}}_+)$ have equal or opposite signs, respectively. The phase dependence of the Josephson current in SNS junctions between d-wave superconductors $j_S(\varphi)$ (41) is presented in Fig. 5(a). In contrast to the analogous dependence for s-wave superconductors (Fig. 4) it contains an additional jump at $\varphi = 0$. This jump is due to the presence of an additional phase shift π acquired by electrons with momentum directions corresponding to different signs of the gap functions in two superconductors.

We believe that the above-mentioned unusual behavior of d-wave SNS junctions can be used to provide an experimental test for the symmetry of the order parameter in high-temperature superconductors. Let us consider a superconducting ring interrupted by an SNS junction with the current-phase relation (41). Rewriting this relation in the form $I = A_1\varphi \mp A_2\pi$, respectively, for $0 < \varphi < \pi$ and $-\pi < \varphi < 0$, one can easily derive the free energy of the system F. In the absence of an external magnetic field we have

$$F(I) = \frac{L}{2}[I^2 + \kappa I^2 - 2I(0)|I|], \qquad (42)$$

where I is the current in the ring, L is the ring inductance, $I(0) = A_2\pi$, $\kappa = 2\pi L A_1/\Phi_0$, Φ_0 is the flux quantum. This expression is valid for $|2LI/\Phi_0| < 1$, for larger values of $|I|$ the two last terms in (42) are periodically continued with the period Φ_0/L. The free energy of the ring with an SNS junction is shown in Fig. 5(b) for the large inductance limit. It has two minima at $\varphi = \pm\pi A_2/A_1$ which correspond to the condition $I_S(\varphi) = 0$. Thus an SNS junction between two d-wave superconductors has a twofold degenerate ground state inside the interval $-\pi < \varphi \leq \pi$. This behavior differs from that for tunnel junctions in which case the system has only one energy minimum at $\varphi = 0$ or $\varphi = \pi$.

Minimizing (42) with respect to I we find the equilibrium value for the current

$$I = \pm I(0)/(1 + \kappa).$$

This result means that an SNS junction described by the current-phase relation (41) *always* induces a spontaneous current in a superconducting ring no matter how small the inductance L is. This result differs from that obtained for a ring with a π junction[12] in which case the spontaneous superconducting current can occur only provided L is sufficiently large.

Without an external magnetic field the ground state of the system is degenerate with respect to the direction of the current I flowing across the ring. This degeneracy is lifted by an external magnetic flux Φ applied to the ring. In this case the value I in the last two terms of (42) should be substituted by $I + (\Phi/L)$ and the energies of the two lowest states differ by $\Delta F = 2I(0)\Phi/(1 + \kappa)$. For $\kappa \gg 1$ and $A_1 \sim A_2$ we obtain a simple estimate $\Delta F \sim \Phi_0\Phi/L$.

If one considers a SQUID configuration with two SNS

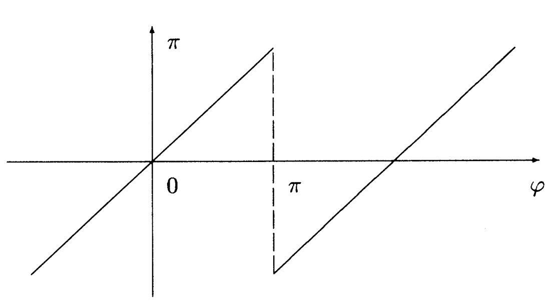

FIG. 4. The sawtooth function $[\varphi]$ of Eq. (41).

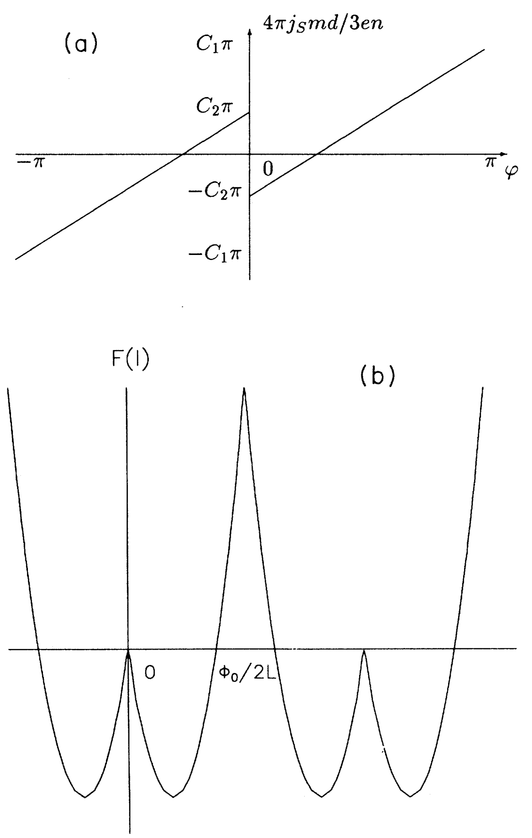

FIG. 5. The current-phase dependence (41) (a) and the energy (b) of an SNS junction between d-wave superconductors at $T = 0$.

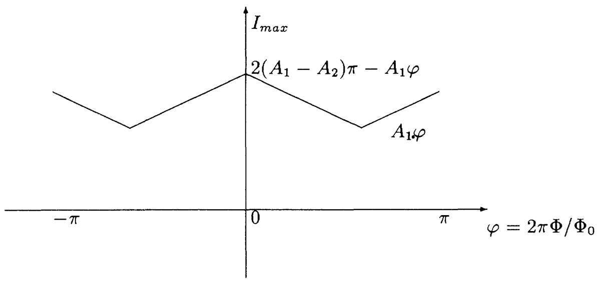

FIG. 6. The maximum current $I_{\max}$ through the SQUID with two SNS junctions [described by the current-phase dependence (41)] as a function of the external magnetic flux Φ.

junctions one can easily see that the critical current through this system $I_{\max}$ may reach its minimum value not only at $\Phi/\Phi_0 = 1/2$ (as in the case of a SQUID with 0 junctions) or at $\Phi/\Phi_0 = 0$ (as for a SQUID with π junction) but at an arbitrary value of Φ/Φ_0 depending on the relation between A_1 and A_2. The dependence $I_{\max}(\Phi)$ for a SQUID with identical SNS junctions and $A_1 \geq 2A_2$ is depicted in Fig. 6. The minimum value of $I_{\max}$ is reached at $\Phi/\Phi_0 = (A_1 - A_2)/2A_1$.

C. Short weak links

In addition to tunnel junctions and SNS structures another type of weak links between d-wave superconductors is of physical interest. Let us consider two superconductors separated by an impenetrable insulating barrier with a small orifice of a typical size $L \ll \xi_0$. Below we shall assume that electrons can freely move (rather than tunnel) through this orifice and put its transparency coefficient equal to one $D(\hat{\boldsymbol{p}}) = 1$. This model describes various geometries (microconstrictions, microbridges, etc.) which provide a direct contact between two metals. The normal-state conductance of such systems depends only on the cross-sectional area of the orifice $\mathcal{A}$ and is given by the well-known expression for the inverse Sharvin resistance

$$1/R_o = e^2 p_F^2 \mathcal{A}/4\pi^2. \tag{43}$$

In the case of conventional superconductors the dc Josephson effect in this type of weak links was studied in detail by Kulik and Omel'yanchuk.[28] It was found in Ref. 28 that at low temperatures the corresponding current-phase relation deviates from the standard $\sin\varphi$ form leading to a somewhat higher Josephson critical current than that for tunnel junctions.[39] Here we briefly discuss the generalization of the theory[28] for the case of d-wave superconductors.

Following Ref. 28 we shall assume that the gap function Δ is not disturbed in superconducting bridges due to the presence of a microconstriction. This assumption is valid everywhere except for a narrow region $\delta r \ll \xi_0$ close to the orifice. It is straightforward to check that the particular form of Δ in this region is not important for calculation of the current through the orifice. Therefore without loss of generality (and also for the sake of definiteness) we stick to the same form of the order parameter $\Delta(x)$ in two superconducting bridges as that discussed before for the case of tunnel junctions.

First let us consider crystal orientations $\psi_\pm(\hat{\boldsymbol{p}}) \approx \psi_\pm(\check{\boldsymbol{p}})$ for which the value Δ is (nearly) uniform in both superconductors. Then following the procedure,[28] one can easily solve the Eilenberger equations in superconductors. Matching the Green functions at $x = 0$ for the electron trajectories passing through the orifice and assuming these functions to be equal to the equilibrum ones far from the weak link $x \to \pm\infty$ similarly to Ref. 28, we obtain the expression for the superconducting current through the orifice $I_S = j_S \mathcal{A}$

$$I_S = 8\pi e T \mathcal{A} \sum_{m>0} \int_{v_x>0} \frac{v_x(\hat{\boldsymbol{p}}_-)\Delta_+(\hat{\boldsymbol{p}}_+)\Delta_-(\hat{\boldsymbol{p}}_-)\sin\varphi}{\omega_m^2 + \{[\omega_m^2 + \Delta_+^2(\hat{\boldsymbol{p}}_+)][\omega_m^2 + \Delta_-^2(\hat{\boldsymbol{p}}_-)]\}^{1/2} + \Delta_+(\hat{\boldsymbol{p}}_+)\Delta_-(\hat{\boldsymbol{p}}_-)\cos\varphi} \frac{d^2S_-}{(2\pi)^3 v_F}. \tag{44}$$

At $T \gg \Delta$ Eq. (44) reduces to Eq. (27) with $D(\hat{\boldsymbol{p}}) = 1$. To analyze the result (44) at lower temperatures it is again convenient to introduce the quantity $\varphi(\hat{\boldsymbol{p}})$. Then for $|\Delta_+(\hat{\boldsymbol{p}}_+)| \approx |\Delta_-(\hat{\boldsymbol{p}}_-)|$ similarly to the case of conventional superconductors[28] from (44) we get

$$I_S = 2\pi e \mathcal{A} \int_{v_x>0} |\Delta_-(\hat{\boldsymbol{p}}_-)| \sin[\varphi(\hat{\boldsymbol{p}})/2] \tanh\left(\frac{|\Delta_-(\hat{\boldsymbol{p}}_-)|\cos[\varphi(\hat{\boldsymbol{p}})/2]}{2T}\right) v_x(\hat{\boldsymbol{p}}_-) \frac{d^2S'}{(2\pi^3)v_F}. \tag{45}$$

Here d^2S' is the element of the Fermi sphere for which the condition $|\Delta_+(\hat{\boldsymbol{p}}_+)| \approx |\Delta_-(\hat{\boldsymbol{p}}_-)|$ is satisfied. As in Ref. 28 the current turns out to be discontinuous at $\varphi = \pi$. In the opposite limit $|\Delta_+(\hat{\boldsymbol{p}}_+)| \gg |\Delta_-(\hat{\boldsymbol{p}}_-)|$ and at $T = 0$ with the logarithmic accuracy we find $[0 \leq \varphi(\hat{\boldsymbol{p}}) \leq 2\pi]$

$$I_S = 4e\mathcal{A} \int_{v_x>0} \sin\varphi(\hat{\boldsymbol{p}}) |\Delta_-(\hat{\boldsymbol{p}}_-)| \ln\frac{|\Delta_+(\hat{\boldsymbol{p}}_+)|}{|\Delta_-(\hat{\boldsymbol{p}}_-)|} v_x(\hat{\boldsymbol{p}}_-) \frac{d^2S''}{(2\pi)^3 v_F}, \quad \text{for } |\varphi(\hat{\boldsymbol{p}}) - \pi| \gg \ln^{-1}\frac{|\Delta_+(\hat{\boldsymbol{p}}_+)|}{|\Delta_-(\hat{\boldsymbol{p}}_-)|},$$

$$I_S = -4\pi e\mathcal{A} \int_{v_x>0} \operatorname{sgn}[\varphi(\hat{\boldsymbol{p}}) - \pi] |\Delta(\hat{\boldsymbol{p}}_-)| v_x(\hat{\boldsymbol{p}}_-) \frac{d^2S''}{(2\pi)^3 v_F}, \quad \text{for } |\varphi(\hat{\boldsymbol{p}}) - \pi| \ll \ln^{-1}\frac{|\Delta(\hat{\boldsymbol{p}}_+)|}{|\Delta(\hat{\boldsymbol{p}}_-)|}, \tag{46}$$

where d^2S'' denotes the element of the Fermi sphere with $|\Delta_+(\hat{\boldsymbol{p}}_+)| \gg |\Delta_-(\hat{\boldsymbol{p}}_-)|$. We see that the magnitude of the current jump at $\varphi(\hat{\boldsymbol{p}}) = \pi$ (46) is by the factor $\sim \ln^{-1}(|\Delta_+|/|\Delta_-|)$ smaller as compared to the case $|\Delta_+| = |\Delta_-|$ (45). For $\varphi(\hat{\boldsymbol{p}})$ close to π and arbitrary ratio $(|\Delta_+|/|\Delta_-|)$ the magnitude of the jump reads

$$I_S = -4\pi e\mathcal{A}\int'_{v_x>0} \mathrm{sgn}\,[\varphi(\hat{\boldsymbol{p}}) - \pi]\frac{|\Delta(\hat{\boldsymbol{p}}_+)||\Delta(\hat{\boldsymbol{p}}_-)|}{|\Delta(\hat{\boldsymbol{p}}_+)| + |\Delta(\hat{\boldsymbol{p}}_-)|}v_x(\hat{\boldsymbol{p}}_-)\frac{d^2S}{(2\pi)^3 v_F}. \tag{47}$$

The integration in (47) runs over the parts of Fermi surface where $\varphi(\hat{\boldsymbol{p}})$ is close to π. As the function $\psi(\hat{\boldsymbol{p}})$ changes its sign on the Fermi surface an additional jump on the $I_S(\varphi)$ dependence takes place at $T \to 0$ similarly to the case of SNS junctions.

Let us emphasize again that the result (44) holds only for a homogeneous distribution of the order parameter in superconducting banks. Within the same framework an analogous result was recently derived by Yip.[40] Provided the condition $|\Delta(\hat{\boldsymbol{p}})| \approx |\Delta(\check{\boldsymbol{p}})|$ is not satisfied, the superconducting order parameter depends on the coordinate and the expression for Josephson current deviates from (44). In this case after a straightforward calculation one can show that at $T \gg \Delta(T)$ the value I_S is given by Eqs. (28), (29) and hence again reduces to the result (33) with $D(\hat{\boldsymbol{p}}) = 1$ $(k = 0)$ and $R_N \to R_o$.

IV. QUASIPARTICLE TUNNELING AND PHASE FLUCTUATIONS

A. Low voltage conductance and I-V curve

A possible way to test the symmetry of the superconducting order parameter is to measure the I-V curve of a tunnel junction in the limit of low temperature and voltage. In the case of isotropic s-wave superconductors at $T \ll \Delta$ only a small number of quasiparticles activated above the gap contributes to the junction conductance G. Therefore in the limit of small voltages we have $G \propto \exp(-\Delta/T)$. At $T = 0$ no quasiparticles exist above the gap and the current across the junction is equal to zero $I = 0$ provided the externally applied voltage V does not exceed the value Δ/e for NS junctions and $2\Delta/e$ for SS junctions. Below we shall show that in the case of d-wave symmetry of the order parameter the I-V curve of a tunnel junction is entirely different in the corresponding temperature and voltage intervals.

Let us assume that the time-independent external voltage V is applied to the tunnel junction between two metals. Then expressing the current in terms of the Green function of the system and making use of the boundary conditions (4) in the lowest order in D after a standard calculation (see, e.g., Ref. 25) one easily finds

$$j_N = e\int\left(\int_{v_x>0}\left[\tanh\left(\frac{\epsilon}{2T}\right) - \tanh\left(\frac{\epsilon - eV}{2T}\right)\right]\right.$$
$$\left.\times g'_+(\epsilon - eV, \hat{\boldsymbol{p}}_+)g'_-(\epsilon, \hat{\boldsymbol{p}}_-)d\epsilon\right)$$
$$\times v_x(\hat{\boldsymbol{p}}_-)D(\hat{\boldsymbol{p}}_-)\frac{d^2S_-}{(2\pi)^3 v_F}. \tag{48}$$

Here j_N is a dissipative contribution to the current across the junction and $g'_\pm(\epsilon, \hat{\boldsymbol{p}}_\pm)$ are the normalized densities of states of two metals in the vicinity of a tunnel barrier. In the case of d-wave superconductors for $\Delta(\hat{\boldsymbol{p}}) = \Delta(\check{\boldsymbol{p}})$ we have

$$g'_\pm(\epsilon, \hat{\boldsymbol{p}}_\pm) = \frac{|\epsilon|\Theta[|\epsilon| - |\Delta_\pm(\boldsymbol{p}_\pm)|]}{\sqrt{\epsilon^2 - \Delta^2_\pm(\boldsymbol{p}_\pm)}}. \tag{49}$$

Let us first calculate the I-V curve of an NS junction. Setting $\Delta_- = 0$ and $\Delta_+ = \Delta(\hat{\boldsymbol{p}})$ and substituting (49) into (48) we obtain at $T = 0$

$$j_N = 2e\int_{v_x>0}[(eV)^2 - \Delta^2(\hat{\boldsymbol{p}})]^{1/2}\Theta[eV - |\Delta(\hat{\boldsymbol{p}})|]$$
$$\times D(\hat{\boldsymbol{p}})v_x(\hat{\boldsymbol{p}})\frac{d^2S}{(2\pi)^3 v_F}. \tag{50}$$

Equation (50) defines the dissipative current across the tunnel junction for crystal orientations with $\Delta(\hat{\boldsymbol{p}}) \approx \Delta(\check{\boldsymbol{p}})$. For other crystal orientations the superconducting density of states in the vicinity of a tunnel barrier deviates from (49). Nevertheless—as in the case of a dc Josephson current—at $T = 0$ the result (50) remains to be valid apart from an unimportant numerical factor of the order of 1. Below we shall neglect this factor and apply the result (50) to any crystal orientation. Then choosing the order parameter in the form $\Delta(\hat{\boldsymbol{p}}) = \Delta_0(p^2_{x_0} - p^2_{y_0})$ from Eq. (50) we have

$$j_N = \frac{ep_F^2}{4\pi^3}\int_0^{2\pi} d\phi\int_0^{\pi/2} d\theta\sin\theta\cos\theta D(\theta)\{(eV)^2 - \Delta_0^2[\sin^2\theta\cos^2\phi - (\sin\theta\sin\phi\cos\theta_0 - \cos\theta\sin\theta_0)^2]^2\}^{1/2}$$
$$\times\Theta(eV - |\Delta(\hat{\boldsymbol{p}})|). \tag{51}$$

Here θ_0 is the angle between the vector $\boldsymbol{n}$ normal to the junction plane and the crystal axis z_0, Δ_0 is the maximal value of $\Delta(\hat{\boldsymbol{p}})$.

It is easy to see that—in contrast to the case of s-wave superconductors—the current (51) does not vanish even for $eV \ll \Delta_0$. In the latter limit the main contribution to j_N comes from quasiparticles with the momentum directions close to the directions for which the order parameter

$\Delta(\hat{\boldsymbol{p}})$ is equal to zero. The integral over these momentum directions can be in turn split into two terms

$$j_N = j_{N1} + j_{N2}. \tag{52}$$

The first term j_{N1} is defined by the integral over the narrow solid angle region around the direction z_0 or, in other words, over the momentum values $p_{z_0} \approx p_F$. With the logarithmic accuracy the corresponding integration in (51) yields

$$j_{N1} = \frac{e v_x(\theta_0) D(\theta_0)(eV)^2 p_F^2}{8\pi^2 v_F \Delta_0} \ln \frac{\Delta_0}{eV}. \tag{53}$$

The second term j_{N2} comes from the integration over momentum directions close to the lines $p_{x_0} = \pm p_{y_0}$. In the vicinity of these lines the gap function is $\Delta(\hat{\boldsymbol{p}}) = (p_2/p_F)\Delta_0 h(p_1)$ where p_1 is the coordinate along the line of zeros and p_2 the one in perpendicular direction. In our case $h(p_1) = 2\sin\theta'$, where θ' is the angle between $\hat{\boldsymbol{p}}$ and z_0. Integrating over p_2 we obtain

$$j_{N2} = e\left[\int_{v_x>0} \frac{D(p_1)v_x(p_1)}{|h(p_1)|} dp_1\right] \frac{(eV)^2 p_F}{8\pi^2 \Delta_0 v_F}. \tag{54}$$

Comparing the results (53) and (54) one can conclude that in the limit $eV \ll \Delta_0$ the current j_{N1} dominates for crystal orientations $\pi/2 - |\theta_0| \gg \ln^{-1/(k+1)}(\Delta_0/eV)$. E.g., for $\theta_0 = 0$ and $D(\theta) \propto \cos^2\theta$ $(k = 2)$ we get

$$j_{N1} = \frac{eV^2}{R_N \Delta_0} \ln \frac{\Delta_0}{eV}. \tag{55}$$

For $\pi/2 - |\theta_0| \ll \ln^{-1/3}(\Delta_0/eV)$ the axis z_0 nearly coincides with the junction plane and the term j_{N1} becomes small. In this case the current j_N is given by the term j_{N2} (54). For $\theta_0 = \pm\pi/2$ and $eV \ll \Delta_0$ it yields

$$j_{N2} = \frac{\pi}{4\sqrt{2}} \frac{eV^2}{R_N \Delta_0}. \tag{56}$$

The zero-temperature I-V curves for an NS tunnel junction with $D \propto p_x^2/p_F^2$ are presented in Fig. 7 for

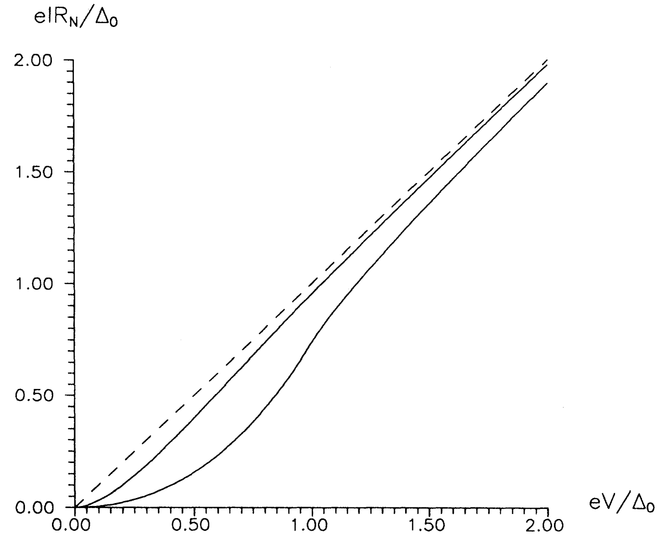

FIG. 7. The I-V curves for a tunnel junction between a normal metal and a d-wave superconductor at $T = 0$. The results are shown for the specific crystal orientations for which the axis z_0 is either perpendicular to the junction plane (upper solid curve) or coincides with this plane (lower solid curve). Ohmic I-V curve is shown by a dashed line.

two particular crystal orientations (one of the principal axes x_0 or z_0 is perpendicular to the barrier plane). At low voltages $eV \ll \Delta_0$ these curves follow the results (55) and (56) [improving the logarithmic accuracy of (55): $\ln(\Delta_0/eV) \to \ln(2.4\Delta_0/eV)$]. At higher voltages $eV \sim \Delta_0$ the I-V curves shows a smooth crossover to the standard Ohmic behavior.

The I-V curve of a tunnel junction between two d-wave superconductors can be calculated analogously. Substituting (49) into (48) for the case of identical superconductors at $T = 0$ we find

$$j_N = 2e \int_{v_x>0} \left(\int_{|\Delta(\hat{\boldsymbol{p}}_-)|}^{eV-|\Delta(\hat{\boldsymbol{p}}_+)|} \frac{\omega(eV-\omega)d\omega}{[\omega^2 - \Delta^2(\hat{\boldsymbol{p}}_-)]^{1/2}[(eV-\omega)^2 - \Delta^2(\hat{\boldsymbol{p}}_+)]^{1/2}} \right) D(\hat{\boldsymbol{p}}_-) v_x(\hat{\boldsymbol{p}}_-) \frac{d^2 S_-}{(2\pi)^3 v_F}. \tag{57}$$

The integration in (57) is made over that parts of Fermi surface where $eV - |\Delta_+(\hat{\boldsymbol{p}}_+)| > |\Delta_-(\hat{\boldsymbol{p}}_-)|$. In a general case the zero lines of the order parameters in two superconductors do not coincide. Assuming that the angle χ between these lines in the point of their intersection $\hat{\boldsymbol{p}}_i$ obeys the condition $eV/\Delta_0|h(\hat{\boldsymbol{p}}_i)|\chi \ll 1$, we can easily obtain the leading order contribution to the quasiparticle current for $eV \ll \Delta_0$:

$$j_N = \frac{e v_x(\hat{\boldsymbol{p}}_i) D(\hat{\boldsymbol{p}}_i) p_F^2 (eV)^3}{24\pi v_F |h_+(\hat{\boldsymbol{p}}_{i+}) h_-(\hat{\boldsymbol{p}}_{i-})| |\sin\chi| \Delta_0^2}. \tag{58}$$

In order to find the total current it is necessary to sum up the contributions from all intersection points. Then one obtains

$$j_N = a \frac{e^2 V^3}{R_N \Delta_0^2}. \tag{59}$$

Here the factor a keeps track on the particular relative orientation of superconductors and is of order one for most of such orientations. This factor vanishes only provided $v_x(\hat{\boldsymbol{p}}_i) D(\hat{\boldsymbol{p}}_i) = 0$, i.e., if the intersection points coincide with the poles of the Fermi surface and the z_0 axes of both superconductors are in the junction plane.

If both superconductors are oriented identically we again arrive with the aid of (57) at the expression for the current j_N defined by the expressions (53)–(56) multiplied by the numerical prefactor

$$\frac{8}{\pi}\int_0^{1/2}\omega K\left(\frac{\omega}{1-\omega}\right)d\omega \approx 0.6,$$

where as before $K(t)$ is the complete elliptic integral. In this case the zero lines of $\Delta_-(\hat{\boldsymbol{p}}_-)$, $\Delta_+(\hat{\boldsymbol{p}}_+)$ coincide if they are drawn on the same Fermi surface.

The zero temperature I-V curves and the differential conductance $G = dI/dV$ for the junction between two d-wave superconductors calculated numerically from the equation (57) are presented in Fig. 8. Curves 1 and 2 of Figs. 8(a) and 8(b) were calculated assuming that, respectively, z_0 and x_0 axes of both superconductors are perpendicular to the boundary plane. Orientation of x_0 and y_0 axes of two superconductors is identical for the curve 1 whereas for the curve 2 their y_0 axes constitute the angle $\pi/2$ between each other. Again in the low voltage limit the numerical curves agree well with the analytic results (59), (55) and allow us to define the corresponding numerical prefactors. E.g., the factor a in Eq. (59) is found to be equal to $a \sim 0.35$ for the above crystal orientation and the logarithmic accuracy of Eq. (55) can be improved by a substitution $\ln(\Delta_0/eV) \to \ln(3.9\Delta_0/eV)$.

For larger voltages of order Δ_0 the differential conductance $G = dI/dV$ has a maximum which position depends on the relative crystal orientation. E.g., for the specific crystal orientations 1 and 2 of Fig. 8(b) the value $G(V)$ reaches its maximum, respectively, at $eV \simeq 1.05\Delta_0$ and $eV = 2\Delta_0$.

In order to understand the physical reasons for this effect let us express the zero-temperature differential conductance in the form:

$$G(V) = \left(eR_N\int_{v_x>0} D(\hat{\boldsymbol{p}}_-)v_x(\hat{\boldsymbol{p}}_-)d^2S\right)^{-1}\left(\int_{v_x>0} D(\hat{\boldsymbol{p}}_-)v_x(\hat{\boldsymbol{p}}_-)d^2S\left[(\pi e/2)\delta(u)|\Delta_+(\hat{\boldsymbol{p}}_+)\Delta_-(\hat{\boldsymbol{p}}_-)|^{1/2} + \Theta(u)\frac{d}{dV}\int_{|\Delta_-(\hat{\boldsymbol{p}}_-)|}^{eV-|\Delta_+(\hat{\boldsymbol{p}}_+)|}\frac{\omega(eV-\omega)d\omega}{\{[\omega^2-\Delta_-^2(\hat{\boldsymbol{p}}_-)][(eV-\omega)^2-\Delta_+^2(\hat{\boldsymbol{p}}_+)]\}^{1/2}}\right]\right), \qquad (60)$$

where we define $u = eV - |\Delta_+(\hat{\boldsymbol{p}}_+)| - |\Delta_-(\hat{\boldsymbol{p}}_-)|$. Due to the presence of the factor $D(\hat{\boldsymbol{p}}_-)v_x(\hat{\boldsymbol{p}}_-)$ the main contribution to the integral over the Fermi surface (60) comes from the velocity directions close to that perpendicular to the junction plane. For the orientation 2 of Fig. 8 the order parameter of both superconductors is equal to Δ_0 in this direction and the value dI/dV reaches its maximum at $eV = 2\Delta_0$. In contrast for the orientation 1 of Fig. 8 the order parameter vanishes along the direction normal to the interface. Accordingly the maximum of $G(V)$ has a much smaller amplitude and takes place at lower voltages $eV \approx \Delta_0$.

For further analytic analysis of the expression (60) let us make use of the identity

$$\int \delta(u)|\Delta_+(\hat{\boldsymbol{p}}_+)\Delta_-(\hat{\boldsymbol{p}}_-)|^{1/2}D(\hat{\boldsymbol{p}}_-)v_x(\hat{\boldsymbol{p}}_-)d^2S = \int\frac{|\Delta_+(\hat{\boldsymbol{p}}_+)\Delta_-(\hat{\boldsymbol{p}}_-)|^{1/2}D(\hat{\boldsymbol{p}}_-)v_x(\hat{\boldsymbol{p}}_-)dl}{|\nabla_{p_-}[|\Delta_+(\hat{\boldsymbol{p}}_+)|+|\Delta_-(\hat{\boldsymbol{p}}_-)|]|}, \qquad (61)$$

where l is the local coordinate along the line $eV = |\Delta_+(\hat{\boldsymbol{p}}_+)| + |\Delta_-(\hat{\boldsymbol{p}}_-)|$. At the extremum point $\hat{\boldsymbol{p}}_- = \hat{\boldsymbol{p}}_0$ of $|\Delta_+| + |\Delta_-|$ the expression in the denominator of (61) is equal to zero and the conductance $G(V)$ suffers a jump or divergence. If the sum of the order parameter reaches its maximum $|\Delta_+| + |\Delta_-| = a - bp_1^2 - cp_2^2$ (p_1 and p_2 are the local euclidean coordinates on the Fermi surface of the "-" metal near the corresponding maximum point) the value of this jump $\delta G = G(eV > a) - G(eV < a)$ reads

$$\delta G = -\frac{\pi^2|\Delta_+\Delta_-|^{1/2}Dv_x|_{\hat{p}=\hat{p}_0}}{2R_N(bc)^{1/2}\int_{v_x>0}Dv_x d^2S}, \qquad (62)$$

e.g., for the curve 2 of Fig. 8(b) we have $a = 2\Delta_0$ and $\delta G = -\pi/3R_N$. No jump occurs if $Dv_x = 0$ in the extremum points of $|\Delta_+| + |\Delta_-|$ [curve 1 of Fig. 8(b)].

Similarly in the local minimum points of $|\Delta_+|+|\Delta_-| = a + bp_1^2 + cp_2^2$ the jump of the differential conductance $G(eV = a)$ has the opposite sign ($\delta G > 0$) and the same absolute value (62). In the case of a saddle point $|\Delta_+| + |\Delta_-| = a + bp_1^2 - cp_2^2$ ($b, c > 0$) the differential conductance logarithmically diverges at $eV \to a$. Within the logarithmic accuracy we get

$$G = \frac{\pi|\Delta_+\Delta_-|^{1/2}Dv_x|_{\hat{p}=\hat{p}_0}}{R_N(bc)^{1/2}\int_{v_x>0}Dv_x d^2S}\ln\left(\frac{rp_F^2}{|a-eV|}\right), \qquad (63)$$

where $r = b$ for $eV < a$ and $r = c$ for $eV > a$.

Let us also point out that the jumps of $G(V)$ at the local maximum or minimum points of $|\Delta_+(\hat{\boldsymbol{p}}_+, T)| + |\Delta_-(\hat{\boldsymbol{p}}_-, T)|$ remain also at $T > 0$. The magnitude of these jumps is given by the expression $\delta G(T) = \delta G(T = 0)[\tanh(\Delta_-(T)/2) + \tanh(\Delta_+(T)/2)]/2$.

Thus, while for isotropic superconductors the differential conductance G has a δ-functional singularity at $eV = |\Delta_+| + |\Delta_-|$, in the case of anisotropic pairing this singularity is washed out due to the momentum dependence of the order parameter. Nevertheless at the extremum points of $|\Delta_+(\hat{\boldsymbol{p}}_+)| + |\Delta_-(\hat{\boldsymbol{p}}_-)|$ the jumps or divergences of $G(V)$ occur as the remnants of the original singularity. The presence of such peculiarities allows for a direct experimental measurement of the extremum values of $|\Delta_+(\hat{\boldsymbol{p}}_+)| + |\Delta_-(\hat{\boldsymbol{p}}_-)|$ and the parameters b and c for any given crystal orientation.

Another interesting feature of the differential conductance $G(V)$ is the "kneelike" behavior in the vicinity of the point $eV = \Delta_0/2$ [curve 2 of Fig. 8(b)]. A rapid change of the slope of $G(V)$ around this point is caused by a topological reason: the number of different solutions of the equation $|\Delta_+| + |\Delta_-| = eV$ at the Fermi hemisphere $v_x > 0$ changes from four at $eV < \Delta_0/2$ to two at $eV > \Delta_0/2$. The curve 1 does not show the kneelike feature because for the corresponding crystal orientation the number of the above solutions remains constant within the interval $0 < eV < 2\Delta_0$.

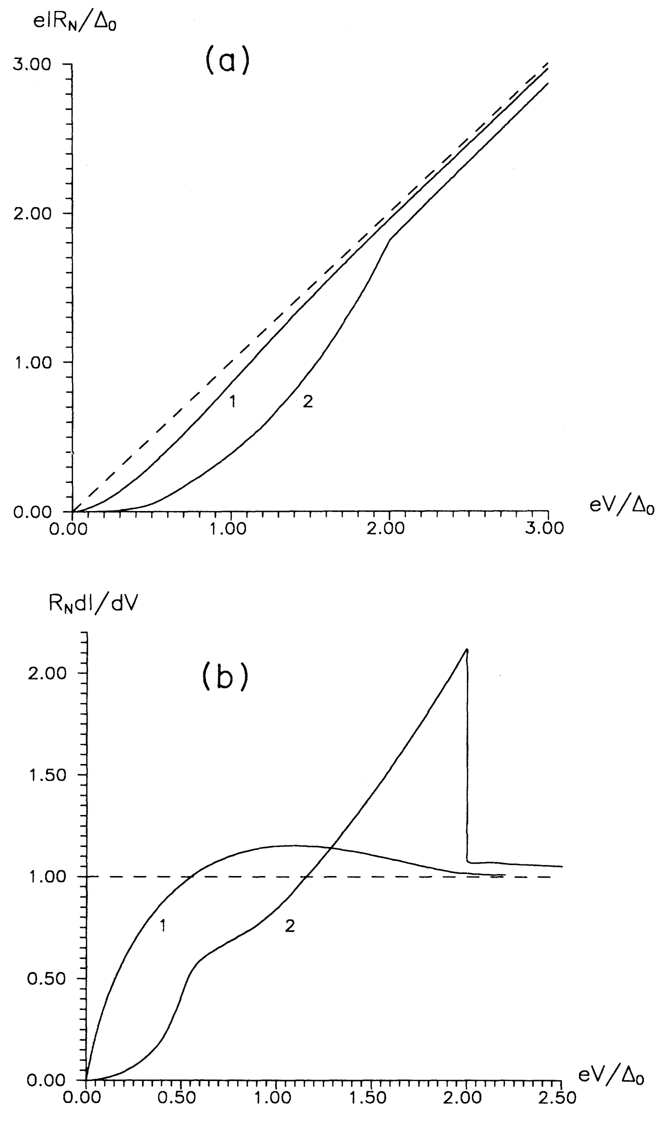

FIG. 8. (a) The I-V curves for a tunnel junction between two d-wave superconductors at $T = 0$. The solid curve 1 corresponds to similarly oriented superconductors with their axes z_0 being perpendicular to the junction plane. For the solid curve 2 the x_0 axes of both superconductors were taken to be perpendicular to the junction plane whereas their y_0 axes were rotated by the angle $\pi/2$ with respect to each other. (b) The zero-temperature differential conductance $G = dI/dV$ for the same crystal orientations.

Note that the behavior of the low voltage conductance $G \propto V^2$ has been detected in recent experiments with SS tunnel junctions.[19] This behavior is in a good agreement with our theoretical predictions (58) and (59). Also for higher voltages our results qualitatively agree with those reported in Ref. 19. At this point it is important to emphasize that the exact value of the conductance jump $|\delta G|$ (62) depends on the detailed form of the function $\Delta(\boldsymbol{p})$ through the parameters b and c. Therefore the discrepancy between theoretical ($|\delta G| = \pi/3R_N$) and experimental[19] values of the jump by a factor of order 2 might be due to deviation of the momentum dependence of the order parameter from the simple form $\Delta(\boldsymbol{p}) = \Delta_0(p_x^2 - p_y^2)$ adopted here.

The low voltage dependence $G \propto V^2$ for a tunnel junction between d-wave superconductors as well as the jump δG at higher voltages have been also discussed in a recent paper by Won and Maki[41] within a different theoretical framework. They evaluated the quasiparticle current by means of the standard tunneling Hamiltonian approach assuming that tunneling matrix elements are independent of the momenta of tunneling electrons and making use of the expressions for the superconducting densities of states averaged over all momentum directions. This approach yields the results which are independent of relative orientation of two superconductors. Although it appears to be quite difficult to justify such an approach microscopically we believe that it might work—at least qualitatively—for diffusive SS boundaries. However it clearly fails for specularly reflecting boundaries in which case the quasiparticle current essentially depends on the relative orientation of d-wave superconductors.

Very recently the case of specularly reflecting boundaries has been independently studied by Bruder, van Otterlo, and Zimanyi.[42] These authors also proceeded within the tunneling Hamiltonian approach completed by a phenomenological assumption about the angular dependence of the tunneling matrix elements. For identically oriented superconductors with z_0 axes being in the barrier plane they also arrived at the result $j_N \propto V^2$ which agrees with our results (54), (56). However for misoriented superconductors at $T = 0$ a vanishing subgap current $I(eV \ll \Delta_0) = 0$ has been found in Ref. 42. In contrast our results (53), (54), and (58) demonstrate that even at $T = 0$ the subgap current does not vanish for all crystal orientations with $v_x(\hat{\boldsymbol{p}}_i)D(\hat{\boldsymbol{p}}_i) \neq 0$,[43] $\hat{\boldsymbol{p}}_i$ is the electron momentum value at the intersection point of the nodal lines for the order parameters Δ_+ and Δ_-. The origin of this disagreement lies in the fact that the authors[42] considered the case of a cylindrical Fermi surface whereas the analysis developed here is based on a picture of a (nearly) spherical Fermi surface. Under certain restrictions the Eilenberger formalism can be also applied to superconductors with nonspherical Fermi surfaces[44,45] and the corresponding generalization of our approach is straightforward. E.g., for cylindrical Fermi surfaces "cut" by the planes at the Brillouin-zone boundaries our results remain valid provided order-parameter zero lines of two superconductors intersect or coincide. In order to find the values h_+ and χ in (58) it is sufficient to express the function $\Delta_+(\hat{\boldsymbol{p}}_+)$ in terms of the $\hat{\boldsymbol{p}}_-$ variable and draw the nodal line of this function on the Fermi surface of the "-" metal. Then it is easy to see that the result (59) (with $a \sim 1$ and R_N being the normal-state resistance of the corresponding junction) remains valid. If the nodal lines of Δ_+ and Δ_- do not intersect each other the quasiparticle current j_N vanishes in the subgap voltage region at $T = 0$. We believe that the results of Ref. 42 correspond to the latter physical situation. Indeed the analysis[42] was developed for cylindrical Fermi surfaces with parallel z_0 axes being in the junction plane. In this case Eq. (57) yields a vanishing subgap current for $V \lesssim \gamma\Delta_0$, γ is the angle between the x_0 axes of two superconductors.

For metals described by the dispersion law $\epsilon(\boldsymbol{p}) = (p_{x_0}^2 + p_{y_0}^2)/2m_1 + p_{z_0}^2/2m_2$, Eqs. (26), (54), and (58) also remain valid. For misoriented superconductors the current is diminished by the factor $(eV/\Delta_0)(m_1/m_2)^{1/2}$ as compared to the case of similarly oriented superconductors. On the other hand, in this case the junction normal-state resistance also increases by the factor $(m_2/m_1)^{1/2}$. Absorbing this factor into the expression for R_N we again arrive at the result (59) with $a \sim 1$.

At low but finite temperatures $T \ll \Delta_0$ the results obtained above for the case $T = 0$ remain valid for not very small voltages $eV \gg T$. In the opposite limit $eV \ll T$ the main contribution to the current j_N comes from quasiparticles thermally activated above the gap. In the limit $eV \ll T \ll \Delta_0$ for the NS junction we find

$$j_N = G(T)V, \tag{64}$$

where the linear conductance $G(T)$ of a tunnel junction between misoriented superconductors is

$$G(T) \sim T^2/R_N\Delta_0^2. \tag{65}$$

Analogously for the crystal orientations described by Eqs. (53) and (54) one gets, respectively, $G(T) \sim (T/R_N\Delta_0)\ln(\Delta_0/T)$ and $G(T) \sim (T/R_N\Delta_0)$.

B. Effective action

Finally let us briefly demonstrate how the above results can be generalized to take into account thermodynamic and quantum fluctuations of the phase difference φ across the tunnel junction between d-wave superconductors. The grand partition function of this junction can be expressed in terms of the path integral over the φ variable (see, e.g., Ref. 46)

$$Z \sim \int \mathcal{D}\varphi(\tau)\exp\{-S_{\text{eff}}[\varphi(\tau)]\}, \tag{66}$$

τ is the imaginary time variable which changes from 0 to $\beta = 1/T$. To evaluate the effective action functional $S_{\text{eff}}[\varphi]$ we make use of the approach developed in Refs. 47 and 46 which allows us to recover $S_{\text{eff}}[\varphi]$ from the expression for the kernel of the current density operator $\boldsymbol{j}(\boldsymbol{r},\tau)$ by means of the integration over the effective "coupling constant" λ. In our case the corresponding formula reads

$$S[\varphi] = \int_0^1 d\lambda \int_0^\beta d\tau\, \varphi(\tau) j[\lambda\varphi(\tau)]\mathcal{A}/2e, \tag{67}$$

where $j[\varphi(\tau)]$ represents the current density through the junction. In the interesting limit of low frequencies the expression for the supercurrent j_S reduces to the standard Josephson relation

$$j_S = j_0 \sin\varphi(\tau), \tag{68}$$

whereas the kernel for the quasiparticle current operator has the form[46]

$$j_N[\varphi(\tau)] = 2e\int_0^\beta d\tau'\alpha(\tau-\tau')\sin\left(\frac{\varphi(\tau)-\varphi(\tau')}{2}\right). \tag{69}$$

Combining (67)–(69) and also taking into account the charging energy term one immediately arrives at the Ambegaokar-Eckern-Schön (AES) effective action[48]

$$S_{\text{eff}} = \int_0^\beta d\tau\left[\frac{C}{2}\left(\frac{\dot{\varphi}}{2e}\right)^2 - E_J\cos\varphi(\tau)\right] - \int_0^\beta d\tau\int_0^\beta d\tau'\alpha(\tau-\tau')\cos\left(\frac{\varphi(\tau)-\varphi(\tau')}{2}\right), \tag{70}$$

C is the junction capacitance and $E_J = j_0\mathcal{A}/2e$ is the Josephson coupling energy which can be positive or negative depending on the relative crystal orientation. The particular form of $\alpha(\tau)$ depends on the form of the I-V curve in the limit of small V. E.g., making use of (69) it is easy to show that for $I \propto V^3$ one has $\alpha(\tau) \propto \tau^{-4}$. More precisely, combining (59) with (69) we obtain at $T \to 0$

$$\alpha(\tau) = 3a/\pi e^2 R_N\Delta_0^2\tau^4. \tag{71}$$

According to the above analysis this result holds for most of crystal orientations. For the orientations described by the I-V curves (55) and (56) we find, respectively, $\alpha(\tau) \sim \ln(\Delta_0\tau)/e^2R_N\Delta_0\tau^3$ and $\alpha(\tau) \sim 1/e^2R_N\Delta_0\tau^3$. The latter dependence has been also obtained in Ref. 42. We believe that the above results might be helpful for a quantitative description of thermodynamic and quantum properties of Josephson junctions and granular arrays composed by d-wave superconductors.

V. DISCUSSION

The microscopic analysis of the charge transport in tunnel junctions and weak links formed by d-wave superconductors allows us to uncover several interesting features of such systems. We demonstrated that the order parameter of a d-wave superconductor can be essentially suppressed in the vicinity of the insulating boundary depending on its orientation relative to the principal crystal axes of such a superconductor. This proximity effect can in turn strongly influence the Josephson current between two superconductors and becomes particularly important at T close to T_c. In the latter case the temperature dependence of the Josephson critical current j_0 varies from $j_s \propto T_c - T$ for a homogeneous order parameter in superconducting bulks (i.e., if one of the principal crystal axes is nearly perpendicular to the junction plane) to $j_0 \propto (T_c - T)^2$ for other crystal orientations. The results of our calculation show a significant dependence of the Josephson current on the relative orientation of the superconductors and are consistent with those of recent experiments[13–15] which indicate the possibility of d-wave pairing symmetry in HTSC compounds.

The current-phase relation for SNS junctions and short superconducting weak links essentially deviates from the standard Josephson relation $j_S = j_0 \sin\varphi$ in the low-temperature limit. In the case of d-wave symmetry of the order parameter at $T = 0$ the current-phase relation $j_S(\varphi)$ for SNS junctions shows an additional jump (as compared to the case of s-wave pairing) at the point $\varphi = 0$. Accordingly the superconducting coupling energy for such junctions $E(\varphi)$ (in contrast to tunnel junctions) has two degenerate minima within the phase interval $-\pi < \varphi \leq \pi$ [see Fig. 5(b)] which correspond to two *different* stable zero current states. Positions of these minima do not coincide with $\varphi = 0$ or $\varphi = \pi$ (as for tunnel junctions) but can be located at any point inside the interval $-\pi < \varphi \leq \pi$. Thus in the analogy with 0 and π junctions one can say that the systems in question provide an interesting example of "whatever junction." Being included into a SQUID ring such a junction induces a spontaneous superconducting current no matter how small the ring inductance is and yields to further features different from those for SQUID's with tunnel junctions. We believe that these features could be used as an additional test for the pairing symmetry in HTSC.

An additional information about the form of the superconducting order parameter and the density of states is contained in the expression for the quasiparticle tunneling current. We evaluated this current for tunnel junctions between a normal metal and a d-wave superconductor as well as between two d-wave superconductors in the low-temperature limit. The corresponding I-V curves show zero-bias anomalies of the type $j_N \propto V^2$, $j_N \propto V^2 \ln(1/V)$ or $j_N \propto V^3$ depending on the junction type and relative crystal orientations. The latter dependence agrees well with the experimental results.[19] At larger voltages the differential conductance of SS junctions has a peak (also detected experimentally[19,20]), which position also depends on the relative crystal orientation.

We would like to point out that one can, in principle, provide an example of a π junction not only between d-wave superconductors but also between s-wave superconductors with multisheet Fermi surfaces and different signs of the order parameter on different sheets. From this point of view an experimental confirmation of π-junction-like properties of HTSC compounds yet cannot completely exclude s-wave pairing. On the other hand, for special types of Fermi surfaces the anisotropic s-wave order parameter [e.g., $\Delta(\boldsymbol{p}) \propto \cos p_{x_0}a + \cos p_{y_0}a$] can be equal to zero for certain momentum directions not due to the symmetry reasons (as it would be for d-wave superconductors). Therefore the low-temperature measurements of a quasiparticle tunneling current rather can be considered as a "gaplessness" test than really distinguish between s- and d-wave types of pairing. Bearing all that in mind one can conclude that it is quite important to combine both dc Josephson effect and quasiparticle tunneling measurements for the same tunnel junctions. Although even demonstration of combined π-junction-like and gapless properties of such systems formally cannot yet exclude other than d-wave types of pairing it would strongly favor the possibility of d-wave pairing in HTSC.

The results derived here are not specific for HTSC compounds and can be also applied to other types of unconventional superconductors, like heavy fermion superconductors. Our analysis holds for an arbitrary form of the Fermi surface. For the sake of definiteness some limiting results were derived for the case of a spherical Fermi surface. The latter is by no means restrictive for any of the conclusions reached in the present paper. E.g., our results also remain valid for the case of a (nearly) cylindrical Fermi surface which appears to be more relevant for several HTSC compounds. The modification of our results for the latter case reduces to an effective renormalization of the junction normal-state resistance provided the zero lines of $\Delta_{\pm}$ intersect or coincide. A special case of nonintersecting zero lines can be also treated easily within the framework of our approach.

It is important to emphasize that our analysis is completely based on the microscopic theory and does not involve model assumptions which are inevitably present in the tunneling Hamiltonian approach. Within the latter approach the correct dependence of the current on the momentum directions and crystal orientations (important for d-wave superconductors) usually cannot be recovered in a unique way and the validity of the final results rather depends on physical intuition of the authors than it is controlled by the method itself.

We recently became aware of a paper[49] where the Josephson current between superconductors with mixed $s+id$ symmetry of the order parameter has been analyzed within the tunneling Hamiltonian approach. In order to evaluate j_S the authors[49] neglected the momentum dependence of tunneling matrix elements and omitted the part of the anomalous Green function which depends on the electron momentum in the x direction [Eqs. (3) and (4) of Ref. 49]. Below we shall demonstrate that these model assumptions lead to incorrect results for j_S.

Let us consider the case of $s + id$ pairing symmetry[49] and express the order parameters for two superconductors in the form $\Delta_\pm(\boldsymbol{p}_\pm) = [\Delta_{s\pm}+i\Delta_{d\pm}(\boldsymbol{p}_\pm)]\exp(\pm\varphi/2)$, $\Delta_{d\pm}(\boldsymbol{p}_\pm) = \Delta_{d\pm}(p^2_{x0\pm} - p^2_{y0\pm})$. Then substituting the anomalous Green functions $f_\pm(\boldsymbol{p}_\pm) = \Delta_\pm(\boldsymbol{p}_\pm)/\sqrt{[|\Delta_\pm(\boldsymbol{p}_\pm)|^2+\omega_m^2]}$ into Eqs. (20) and (21) we easily find

$$j_S = j_{C1}\sin\varphi + j_{C2}\cos\varphi, \tag{72}$$

$$j_{C1,2} = 2\pi e T\sum_m \int_{v_x>0} \frac{\Xi_{1,2}(\boldsymbol{p}_-)D(\hat{\boldsymbol{p}}_-)v_x(\hat{\boldsymbol{p}}_-)}{\sqrt{\Delta_+^2(\hat{\boldsymbol{p}}_+)+\omega_m^2}\sqrt{\Delta_-^2(\hat{\boldsymbol{p}}_-)+\omega_m^2}} \times\frac{d^2S_-}{(2\pi)^3 v_F}, \tag{73}$$

where $\Xi_1 = \Delta_{s-}\Delta_{s+} + \Delta_{d-}(\boldsymbol{p}_-)\Delta_{d+}(\boldsymbol{p}_+)$, $\Xi_2 = \Delta_{s-}\Delta_{d+}(\boldsymbol{p}_+) - \Delta_{s+}\Delta_{d-}(\boldsymbol{p}_-)$. In the case of either s- or d-wave pairing in both superconductors we have $j_{C2}=0$ in agreement with the results derived above. For similarly oriented identical $s+id$-wave superconductors with $\Delta_\pm = \Delta_s + i\Delta_d(\boldsymbol{p})$ we define $\tilde\Delta(\boldsymbol{p}) = \sqrt{\Delta_s^2+\Delta_d^2(\boldsymbol{p})}$ and again have $j_{C2}=0$ and

$$j_{C1} = \pi e\int_{v_x>0}\tilde\Delta(\boldsymbol{p})\tanh\left(\frac{\tilde\Delta(\boldsymbol{p})}{2T}\right)D(\boldsymbol{p})v_x(\boldsymbol{p})\frac{d^2S}{(2\pi)^3 v_F}. \tag{74}$$

For a tunnel junction between s- and d-wave superconductors, Eq. (73) yields $j_{C1}=0$ and $j_{C2}\neq 0$ (see, also, Ref. 50).

These results allow to recover the correct dependence of tunneling matrix elements $T_{p_-p_+}$ on the momentum direction of tunneling electrons $\boldsymbol{p}_\pm$. E.g., it is straightforward to check that within the tunneling Hamiltonian approach the results (72) and (73) are reproduced provided one substitutes

$$|T_{p_-p_+}|^2 \propto v_x(\boldsymbol{p}_-)D(\boldsymbol{p}_-)\delta(p_-^{\parallel}-p_+^{\parallel})/v_F(\boldsymbol{p}_-). \tag{75}$$

Physically this result can be easily understood because the probability of tunneling per unit time $|T_{p_-p_+}|^2$ is given by a number of attempts ($\propto v_x$) multiplied by the barrier transparency $D(\boldsymbol{p})$. The δ function assures the momentum conservation in the direction parallel to the junction plane. For typical specularly reflecting barriers we have $D(\boldsymbol{p}) \propto p_x^2$ and thus for identical metals $T_{pp}\propto p_x^{3/2}$. This dependence and the results (72)–(74) differ drastically from those reported in Ref. 49.

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dental degeneracy cases (the latter corresponds to approximately the same value T_c for two types of pairing). Then one has $\Delta(\hat{\boldsymbol{p}}) = \eta_1(x)\psi_1(\hat{\boldsymbol{p}}) + \eta_2(x)\psi_2(\hat{\boldsymbol{p}})$.

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