Magnetic-field effect on sound propagation as a probe of electronic structure and electron interaction in an itinerant-electron ferromagnet

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We carry out a systematic formulation and a model numerical calculation on the effect of a magnetic field on the sound velocity and attenuation of a ferromagnetic metal based on the Stoner model. We find the magnetic-field effect to be strongly exchange enhanced and to depend very sensitively on the details of the electronic structure, both above and below the Curie point T_C. According to our result, for $T>T_C$ the magnetic-field effects on the sound velocity and attenuation are proportional to, respectively, χ^3 and χ^4, where χ is the (Stoner) magnetic susceptibility, with coefficients very sensitively reflecting the electronic structure near the Fermi surface. We also obtain similar features of the magnetic-field effects for $T<T_C$; the magnetic-field effects reflect very sensitively the electronic structure and electron interaction of a ferromagnetic metal. The mechanism of such magnetic effect on sound is that the screening of the ion-ion interaction is affected by the (additional) spin splitting of the conduction electron bands induced by a magnetic field. These findings suggest that the magnetic-field effects on sound propagation can be a useful probe of the electronic structure and electron interaction of a metal, ferromagnetic one, in particular.

I. INTRODUCTION

It is well known that the effects of a magnetic field on the velocity and attenuation of sound vary quite widely among different metals, especially magnetic ones. If we put

$$s(H)=s+\Delta s(H) \tag{1.1}$$

for the sound velocity under a magnetic field **H** in the paramagnetic state, we have $\Delta s(H)/H=k(\mu_B H/\varepsilon_F)^2$, as we will see below. Then we have $|k|=O(10^6)$ for Fe-Ni alloys, while for an ordinary nonmagnetic metal, $|k|=O(1)$.[1] Not all of the ferromagnetic metals have such large value of k; in Ni, for instance, the size of k is much less.[1] The mechanism of such magnetic-field effect on the velocity (and attenuation) of sound in a metal, particularly a magnetic one, however, is not yet well understood. It is not understood why the values of k can be so different for Ni and Fe-Ni, for instance.

In a metal, the ion-ion interaction which determines phonon frequency is screened by conduction electrons. A magnetic field can have an effect on sound velocity or phonon frequency because this screening behavior is modified by the magnetic field; the magnetic field induces the spin splitting of the conduction electron bands. The principal purpose of this paper is to formulate this mechanism of the magnetic-field effects on sound velocity and attenuation, for both above and below T_C, by treating the screening with the mean-field approximation within the Stoner model of itinerant electron magnetism. As an illustration of our results, we also carry out a numerical calculation on those magnetic-field effects by using a simple model electronic density of states. Thus we find that those magnetic-field effects depend very sensitively on the electronic structure and electron interaction in a metal. We show that the large difference in the values of k between Ni and Fe-Ni, for instance, can be understood from the difference in their electronic structures.

In Sec. II we present a systematic derivation of various magnetic-field effect coefficients. Although some of them were previously obtained by us,[2–4] here we explicitly derive all of them in a unified way. Then, in Sec. III we present the results of our numerical examples. Concluding remarks will be given in Sec. IV.

II. GENERAL FORMULATION

In a jelliumlike model, if we treat the screening of the ion-ion interaction by the generalized random-phase approximation, which takes into account the effect of the exchange interaction between electrons, the frequency of a longitudinal acoustic phonon, the only kind which we consider in this paper, is given by[5]

$$\omega_{\mathbf{q}}^2=\Omega_{\mathbf{q}}^2-|g(\mathbf{q})|^2\tilde{\chi}_e(\mathbf{q},\omega_{\mathbf{q}}) , \tag{2.1}$$

where $\Omega_{\mathbf{q}}$ is the bare-phonon frequency, $g(\mathbf{q})$ is the electron-phonon interaction constant, and

$$\tilde{\chi}_e(\mathbf{q},\omega)=\frac{\tilde{F}_+(\mathbf{q},\omega)+\tilde{F}_-(\mathbf{q},\omega)}{1+v(\mathbf{q})[\tilde{F}_+(\mathbf{q},\omega)+\tilde{F}_-(\mathbf{q},\omega)]} \tag{2.2}$$

is the electron charge response, where

$$\tilde{F}_\sigma(\mathbf{q},\omega)=\frac{F_\sigma(\mathbf{q},\omega)}{1-\tilde{V}F_\sigma(\mathbf{q},\omega)} \tag{2.3}$$

is the exchange-enhanced Lindhard function of σ spin electrons, with the ordinary Lindhard function given by

$$F_\sigma(\mathbf{q},\omega)=\sum_{\mathbf{k}}\frac{f(\varepsilon_{\mathbf{k}\sigma})-f(\varepsilon_{\mathbf{k}+\mathbf{q}\sigma})}{\varepsilon_{\mathbf{k}+\mathbf{q}\sigma}-\varepsilon_{\mathbf{k}\sigma}-\hbar\omega-i0^+} . \quad (2.4)$$

Here $\tilde{V}$ and $v(\mathbf{q})=4\pi e^2/Vq^2$ are, respectively, the exchange and the Coulomb interactions between electrons, V being the volume of the system, $\varepsilon_{\mathbf{k}\sigma}=\varepsilon_{\mathbf{k}}-\tilde{V}n_\sigma$ is the one-particle energy of an electron with a wave vector $\mathbf{k}$ and spin σ; $-\tilde{V}n_\sigma$ is the exchange self-energy, with n_σ the total number of σ spin electrons in the system.

The result of Eqs. (2.1) and (2.2) is valid for both the ferromagnetic and paramagnetic states. If we consider only the paramagnetic state, where we have $F_+(\mathbf{q},\omega)=F_-(\mathbf{q},\omega)=F(\mathbf{q},\omega)$, and neglect the effect of the exchange interaction on the screening, which amounts to replacing all the exchange-enhanced Lindhard function $\tilde{F}_\sigma(\mathbf{q},\omega)$ by $F(\mathbf{q},\omega)$, Eq. (2.1) reduces to the familiar textbook result[6]

$$\omega_{\mathbf{q}}^2=\Omega_{\mathbf{q}}^2-|g(\mathbf{q})|^2\frac{2F(\mathbf{q},\omega_{\mathbf{q}})}{1+2v(\mathbf{q})F(\mathbf{q},\omega_{\mathbf{q}})} . \quad (2.5)$$

The Lindhard function, Eq. (2.4), for $\omega\neq 0$ is a complex quantity:

$$F_\sigma(\mathbf{q},\omega+i0^+)\equiv R_\sigma(\mathbf{q},\omega)+iI_\sigma(\mathbf{q},\omega) , \quad (2.6)$$

$$R_\sigma(\mathbf{q},\omega)=\sum_{\mathbf{k}}P\frac{f(\varepsilon_{\mathbf{k}\sigma})-f(\varepsilon_{\mathbf{k}+\mathbf{q},\sigma})}{\varepsilon_{\mathbf{k}+\mathbf{q},\sigma}-\varepsilon_{\mathbf{k}\sigma}-\hbar\omega} \quad (2.7)$$

$$\cong\sum_{\mathbf{k}}P\frac{f(\varepsilon_{\mathbf{k}\sigma})-f(\varepsilon_{\mathbf{k}+\mathbf{q},\sigma})}{\varepsilon_{\mathbf{k}+\mathbf{q},\sigma}-\varepsilon_{\mathbf{k}\sigma}}=F_\sigma(\mathbf{q},0)\equiv F_\sigma(\mathbf{q}) , \quad (2.7a)$$

$$I_\sigma(\mathbf{q},\omega)=\pi\sum_{\mathbf{k}}[f(\varepsilon_{\mathbf{k}\sigma})-f(\varepsilon_{\mathbf{k}+\mathbf{q}\sigma})]\times\delta(\varepsilon_{\mathbf{k}+\mathbf{q}\sigma}-\varepsilon_{\mathbf{k}\sigma}-\hbar\omega) \quad (2.8)$$

$$\cong\frac{\pi}{2}N_\sigma(0)\frac{\omega}{v_{F\sigma}q} , \quad (2.8a)$$

where P indicates taking the principal part in integrating and $\delta(x)$ is the Dirac δ function; $N_\sigma(0)$ and $v_{F\sigma}$ are, respectively, the density of states at the Fermi surface and the Fermi velocity of σ spin electrons. The approximations of Eqs. (2.7a) and (2.8a) (Ref. 6) are valid for $|\omega|/v_{F\sigma}q\ll 1$.

Corresponding to Eq. (2.6), Eq. (2.1) is rewritten as

$$\omega_{\mathbf{q}}^2=\Omega_{\mathbf{q}}^2-|g(\mathbf{q})|^2\,\mathrm{Re}\tilde{\chi}_e(\mathbf{q},\omega_{\mathbf{q}})-i|g(\mathbf{q})|^2\,\mathrm{Im}\tilde{\chi}_e(\mathbf{q},\omega_{\mathbf{q}}) . \quad (2.9)$$

The phonon frequency to be obtained from Eq. (2.9) should be complex,

$$\omega_{\mathbf{q}}\equiv\bar{\omega}_{\mathbf{q}}-i\gamma_{\mathbf{q}} . \quad (2.10)$$

By putting Eq. (2.10) into Eq. (2.9) we have

$$\bar{\omega}_{\mathbf{q}}^2-\gamma_{\mathbf{q}}^2=\Omega_{\mathbf{q}}^2-|g(\mathbf{q})|^2\,\mathrm{Re}\tilde{\chi}_e(\mathbf{q},\bar{\omega}_{\mathbf{q}}) \quad (2.11)$$

$$\cong\bar{\omega}_{\mathbf{q}}^2 , \quad (2.11a)$$

$$2\gamma_{\mathbf{q}}\bar{\omega}_{\mathbf{q}}=|g(\mathbf{q})|^2\,\mathrm{Im}\tilde{\chi}_e(\mathbf{q},\bar{\omega}_{\mathbf{q}}) , \quad (2.12)$$

where, by assuming $0<\gamma_{\mathbf{q}}\ll\bar{\omega}_{\mathbf{q}}$, we put $\mathrm{Re}\tilde{\chi}_e(\mathbf{q},\bar{\omega}_{\mathbf{q}}-i\gamma_{\mathbf{q}})=\mathrm{Re}\tilde{\chi}_e(\mathbf{q},\bar{\omega}_{\mathbf{q}})$, etc.

In the pure jellium model, the bare-phonon frequency is given by the ionic plasma frequency, $\Omega_{\mathrm{pl}}=\{4\pi Z^2e^2N/(M_IV)\}^{1/2}$, where M_I, Z, and N are, respectively, the mass and charge of an ion and the total number of ions. Then, from the phonon frequency given by Eq. (2.5) with $\Omega_{\mathbf{q}}=\Omega_{\mathrm{pl}}$, we obtain[6]

$$s=\left[\frac{V\Omega_{\mathrm{pl}}^2}{8\pi e^2N(0)}\right]^{1/2}=\frac{1}{\sqrt{3}}\left[\frac{mZ}{M_I}\right]^{1/2}\equiv s_0 , \quad (2.13)$$

$$\gamma_{\mathbf{q}0}=\frac{\pi}{4v_Fq}\bar{\omega}_{\mathbf{q}}^2 , \quad (2.14)$$

where we put $F(0)=N(0)$ [see Eq. (2.23) below], $N(0)$ being the electronic density of states per spin at the Fermi surface in the paramagnetic state, and noted $N(0)=3n/4\varepsilon_F$, with n the total number of electrons for the free-electron energy dispersion, $\varepsilon_k=\hbar^2k^2/2m$, m being the electron mass.

As for the attenuation constant, we often use the spatial energy attenuation constant $\alpha_{\mathbf{q}}$, in place of the temporal one, $\gamma_{\mathbf{q}}$; they are related by

$$\alpha_{\mathbf{q}0}=\frac{2\gamma_{\mathbf{q}0}}{s_0} . \quad (2.15)$$

In an experiment, what is given is the frequency of sound wave, rather than the wave number. Thus, in the following we use $\alpha_{\omega 0}$ or $\gamma_{\omega 0}$, etc., in place of $\alpha_{\mathbf{q}0}$ or $\gamma_{\mathbf{q}0}$, etc.

By applying Eqs. (2.11a) and (2.12) to Eqs. (2.1) and (2.2) we obtain[3]

$$\left[\frac{s}{s_0}\right]^2=\xi+2N(0)\Big/\sum_\sigma\tilde{F}_\sigma(0) , \quad (2.16)$$

$$\frac{\alpha_\omega}{\alpha_{\omega 0}}=2\left[\frac{s_0}{s}\right]^2N(0)\sum_\sigma\frac{v_F}{v_{F\sigma}}\frac{F_\sigma(0)}{(1-\tilde{V}F_\sigma(0))^2}\frac{1}{\left[\sum_\sigma\tilde{F}_\sigma(0)\right]^2} , \quad (2.17)$$

where we put

$$\Omega_{\mathbf{q}}^2-\Omega_{\mathrm{pl}}^2=\xi s_0^2q^2 . \quad (2.18)$$

The parameter ξ is considered to represent deviations from the pure jellium model; for the pure jellium, we have $\xi=0$. The physical origin of $\xi s_0^2q^2$ is the direct interaction between ion cores which is outside of the Coulomb interaction between separated ionic charges. Since such direct ion-core interaction is repulsive, ξ is considered to have a positive sign and a magnitude of $O(1)$.

Our concern in this paper is how the velocity and attenuation of sound would change with an applied magnetic field $\mathbf{H}$. The effect of a magnetic field is to change the screening of the ion-ion interaction, and such an effect on phonon frequency can be incorporated by changing all the $F_\sigma(q,\omega)$ in Eqs. (2.1) and (2.2) by $F_\sigma(q,\omega{:}H)$, which is obtained by replacing $\varepsilon_{k\sigma}=\varepsilon_k-\tilde{V}n_\sigma$ by

$$\varepsilon_{k\sigma}(H)=\varepsilon_k-\tilde{V}n_\sigma(H)+\sigma\mu_B H \tag{2.19}$$

in $F_\sigma(q,\omega)$; $n_\sigma(H)$ is the number of σ spin electrons under the magnetic field H. In this paper, for the ferromagnetic state, we assume such state as shown in Fig. 1, with $n_+<n_-$, that is, $M=-\mu_B(n_+-n_-)>0$. Then a positive magnetic field in the direction of the z axis further spin split the electronic density of states.

The effect of a magnetic field on velocity and attenuation is obtained by replacing all the $F_\sigma(0)$'s appearing in Eqs. (2.16) and (2.17) by $F_\sigma(0;H)=F_\sigma(0,0;H)$. Thus, for the sound velocity, corresponding to Eq. (2.16) we have

$$\left[\frac{s(H)}{s_0}\right]^2 =\xi+2N(0)\Big/\left[\frac{F_+(0;H)}{1-\tilde{V}F_+(0;H)}+\frac{F_-(0;H)}{1-\tilde{V}F_-(0;H)}\right]. \tag{2.20}$$

$\alpha_\omega(H)/\alpha_{\omega 0}$ is similarly obtained from Eq. (2.17). Here, note that v_F or $v_{F\sigma}$ is not uniquely determined from a given $N(\varepsilon)$ or $N_\sigma(\varepsilon)$. Thus we assume the following relation, which is valid for the free-electron-like energy dispersion:

$$N_\pm(0)/v_{F\pm}=N(0)/v_F . \tag{2.21}$$

In exploring $F_\sigma(0;H)$, we put the spin splitting of the bands induced by the external field equal to $2\eta=\tilde{V}\Delta M/\mu_B+2\mu_B H$, where ΔM is the change in magnetization due to the external field. The electron system may be in the ferromagnetic state. In that case, 2η represents the *additional spin splitting* of the bands due to the external magnetic field. With η, Eq. (2.19) is rewritten as $\varepsilon_{k\sigma}(\eta)=\varepsilon_{k\sigma}+\sigma\eta$, and then the Fermi distribution of electrons is given as

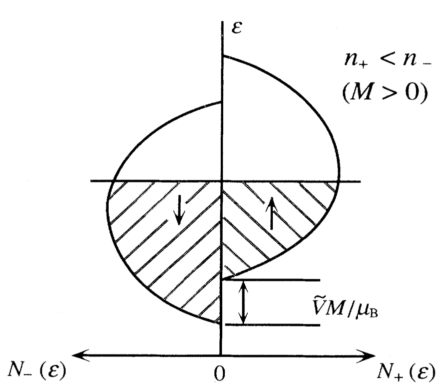

FIG. 1. The ferromagnetic state is assumed to be with $n_->n_+$ or $M=-\mu_B(n_+-n_-)>0$.

$$f(\varepsilon_{k\sigma}(\eta))=[1+\exp\{\beta((\varepsilon_{k\sigma}-\mu)+(\sigma\eta-\Delta\mu))\}]^{-1} \equiv f_\sigma(\varepsilon_k;\eta) , \tag{2.22}$$

where $\Delta\mu$ is the change in the chemical potential due to the magnetic field. The Lindhard function for $\mathbf{q}=0$ under a magnetic field is then given by

$$F_\sigma(0;H)=\int d\varepsilon N(\varepsilon)\left[-\frac{\partial f_\sigma(\varepsilon;\eta)}{\partial\varepsilon}\right] . \tag{2.23}$$

Expanding the right-hand side in terms of $\sigma\eta-\Delta\mu$, we obtain

$$F_\sigma(0;H)\equiv F_\sigma(0;\eta) =F_\sigma(0)+F'_\sigma(0)(\sigma\eta-\Delta\mu) +\frac{1}{2!}F''_\sigma(0)(\sigma\eta-\Delta\mu)^2+\cdots , \tag{2.24}$$

where

$$F_\sigma^{(n)}(0)\equiv-\int d\varepsilon N(\varepsilon)\frac{\partial^{(n+1)}f_\sigma(\varepsilon;\eta)}{\partial\varepsilon^{n+1}}\Bigg|_{\eta=0} . \tag{2.25}$$

$\Delta\mu$ is obtained by requiring the conservation of electron number,

$$\sum_{\mathbf{k},\sigma}f(\varepsilon_{k\sigma}(\eta))=\sum_{\mathbf{k},\sigma}\left[f(\varepsilon_{\mathbf{k}\sigma})+f'(\varepsilon_{\mathbf{k}\sigma})(\sigma\eta-\Delta\mu) +\frac{1}{2!}f''(\varepsilon_{\mathbf{k}\sigma})(\sigma\eta-\Delta\mu)^2+\cdots\right] =\sum_{\mathbf{k},\sigma}f(\varepsilon_{\mathbf{k}\sigma}) . \tag{2.26}$$

Retaining terms up to $O(\eta^2)$, we have

$$\Delta\mu=\frac{\Pi}{P}\eta+\frac{2}{P^3}[F_+^2(0)F'_-(0)+F_-^2(0)F'_+(0)]\eta^2+\cdots , \tag{2.27}$$

where for convenience we put

$$P=\sum_\sigma F_\sigma(0) , \quad \Pi=\sum_\sigma \sigma F_\sigma(0) . \tag{2.28}$$

By putting the above $\Delta\mu$ into Eq. (2.29), we obtain

$$F_\sigma(0;\eta)=F_\sigma(0)\left[1+b_{1\sigma}\left[\frac{\eta}{W}\right] +b_{2\sigma}\left[\frac{\eta}{W}\right]^2+\cdots\right] , \tag{2.29}$$

$$b_{1\sigma}=2\sigma\frac{F'_\sigma(0)F_{-\sigma}(0)}{F\sigma(0)P}W , \tag{2.30}$$

$$b_{2\sigma}=\left[\frac{F''_\sigma(0)}{2P}\left\{3\frac{F_{-\sigma}(0)}{F_\sigma(0)}-1\right\} -2\frac{F'_\sigma(0)}{F_\sigma(0)}\frac{\sum_{\sigma'}F^2_{\sigma'}(0)F'_{\sigma'}(0)}{P^3}\right]W^2 , \tag{2.31}$$

where W is the width of the electron energy band; we may use ε_F in place of W. Then if we rewrite Eq. (1.1) as $s(\eta)=s+\Delta s(\eta)$, from Eq. (2.20) we obtain

$$\frac{\Delta s(\eta)}{s}=f_1\left[\frac{\eta}{W}\right]+f_2\left[\frac{\eta}{W}\right]^2+\cdots , \tag{2.32}$$

$$f_1=\left[\frac{s_0}{s}\right]^2\frac{2N(0)}{\tilde{P}^2}\sum_\sigma D_\sigma^2F_\sigma(0)b_{1\sigma} , \tag{2.33}$$

$$f_2=-\frac{1}{2}\left[\frac{s_0}{s}\right]^2\frac{2N(0)}{\tilde{P}}\times\left[\frac{1}{\tilde{P}}\sum_\sigma D_\sigma^2F_\sigma(0)\{\tilde{V}F_\sigma(0)b_{1\sigma}^2+b_{2\sigma}\}-2\frac{1}{\tilde{P}^2}\left\{\sum_\sigma D_\sigma^2F_\sigma(0)b_{1\sigma}\right\}^2\right], \tag{2.34}$$

where we put

$$\tilde{P}=\sum_\sigma \tilde{F}_\sigma(0) , \quad D_\sigma=1/[1-\tilde{V}F_\sigma(0)] . \tag{2.35}$$

For the paramagnetic state, D_σ reduces to $D_0=1/[1-\tilde{V}F_\sigma(0)]$, the Stoner exchange-enhancement factor.

Starting from (2.17) we can similarly pursue how the attenuation constant of sound wave depends on the (additional) sin splitting of the electron energy bands due to an external magnetic field. If we put

$$\alpha_\omega(\eta)=\alpha_\omega+\Delta\alpha_\omega(\eta) , \tag{2.36}$$

we obtain

$$\frac{\Delta\alpha_\omega(\eta)}{\alpha_\omega}=g_1\frac{\eta}{W}+g_2\left[\frac{\eta}{W}\right]^2+\cdots , \tag{2.37}$$

$$g_1=2\left[\left[\frac{s}{s_0}\right]^2\frac{\tilde{P}}{N(0)}-1\right]f_1+2\sum_\sigma D_\sigma\tilde{V}F_\sigma(0)b_{1\sigma} . \tag{2.38}$$

As for g_2, since its expression is too complicated, we give it only for the paramagnetic state where $g_1=0$,

$$g_2=2\left[\left[\frac{s}{s_0}\right]^2\frac{\tilde{P}}{N(0)}-1\right]f_2+2\tilde{V}\tilde{F}(0)\sum_\sigma\{b_{2\sigma}-\tilde{V}\tilde{F}(0)b_{1\sigma}^2\} . \tag{2.39}$$

Based on the above results, in the next section we proceed to discuss the effect of an external magnetic field on the velocity and attenuation of sound wave separately for the paramagnetic and ferromagnetic states of a metal.

III. MODEL CALCULATION OF THE MAGNETIC-FIELD EFFECTS

A. Magnetic-field effect on sound propagation in the paramagnetic state of a metal

1. Magnetic-field effect on sound velocity for $T>T_C$

In the paramagnetic state, from Eqs. (2.33) and (2.30) we find $f_1=0$. Thus, we have

$$\frac{\Delta s(\eta)}{s}\equiv-\frac{1}{2}\left[\frac{s_0}{s}\right]^2K\left[\frac{\eta}{W}\right]^2 , \tag{3.1}$$

$$K=\left[\frac{1}{2}\left\{\frac{F''(0)}{F(0)}-\left[\frac{F'(0)}{F(0)}\right]^2\right\}+\tilde{V}F(0)D_0\left[\frac{F'(0)}{F(0)}\right]^2\right]W^2 . \tag{3.2}$$

If we note the relation,

$$\eta=\left[1+\frac{\tilde{V}}{2\mu_B^2}\chi_S\right]\mu_BH=D_0\mu_BH , \tag{3.3}$$

$\chi_S=2\mu_B^2F(0)/[1-\tilde{V}F(0)]$ being the Stoner magnetic susceptibility, we arrive at[2]

$$\frac{\Delta s(H)}{s}=k\left[\frac{\mu_BH}{W}\right]^2 , \quad k=-\frac{1}{2}\left[\frac{s_0}{s}\right]^2D_0^2K . \tag{3.4}$$

In carrying out a numerical calculation on k, and other magnetic-field-effect coefficients, we use the simple model electronic density of states given by

$$N(\varepsilon)=\frac{6N}{W^3}\varepsilon(W-\varepsilon) , \tag{3.5}$$

which is illustrated in Fig. 2. This density of states can accommodate one electron per spin, per atom. We give the result of our numerical calculation in Fig. 3, where we assumed $W=1$ eV, $\tilde{V}N(0)\equiv\bar{V}=1.3$, and $\xi=2$, and chose the locations of Fermi energy at $T=0$ of the (hypothetical) spin unsplit state at $\varepsilon_F/W=0.1$, 0.3, and 0.5. Together with k, we also showed k/D_0^3, to demonstrate that $k\propto D_0^3$. Being proportional to D_0^3, k varies in a quite wide range. The characteristic features of the temperature dependence are remarkably different for cases with different values of ε_F/W. Thus, it is not difficult to have $|k|\sim10^2$ as in Ni, and $k\sim-10^6$ as in FeNi, with the same common value of $\tilde{V}N(0)$. Within the present simple model electronic density of states of Fig. 2, Ni is represented by the case of $\varepsilon_F/W\cong0.5$, and FeNi is represented by the case of $\varepsilon_F/W\cong0.3-0.4$. For comparison, in Fig. 4 we show the temperature dependence of sound velocity without the effect of an external magnetic field which is calculated from Eq. (2.16) for the same electronic density of states of Fig. 2 both for $T>T_C$ and

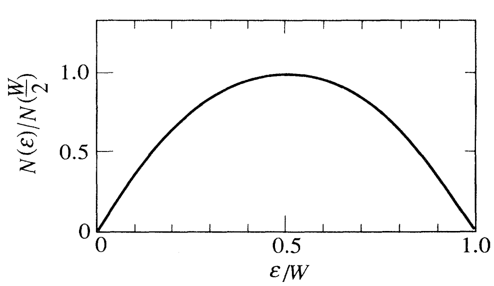

FIG. 2. The model electronic density of states given by Eq. (3.5). W is the width of the band.

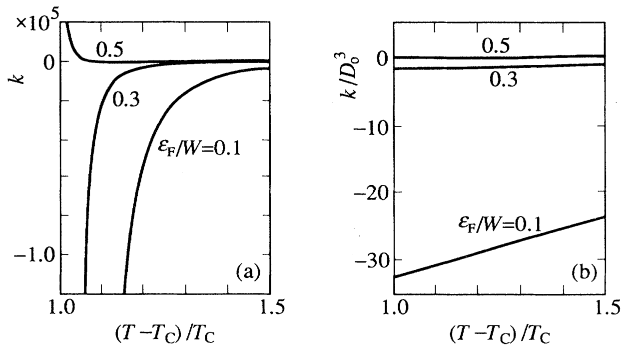

FIG. 3. The temperature dependence of the coefficient, k, of the magnetic-field dependence of sound velocity for $T > T_C$ as defined by Eq. (3.4), for different locations of the Fermi energy in the model electronic density of states of Fig. 2. We assumed $W = 1$ eV, $\overline{V} = \tilde{V}N(0) = 1.3$, and $\xi = 2$. To illustrate that k is proportional to D_0^3, we show k/D_0^3, (b), in addition to k itself, (a).

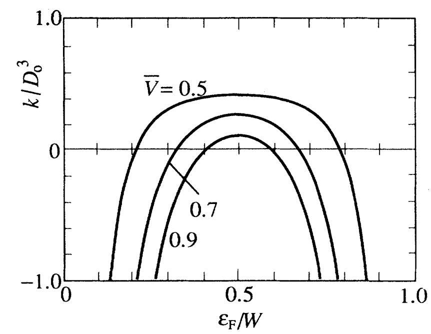

FIG. 5. The magnetic-field dependence of sound velocity in nonferromagnetic metals, for which $\overline{V} = \tilde{V}N(0) < 1$, calculated from Eq. (3.4) with the model electronic density of states of Fig. 2 at $T = 0$, for different values of $\overline{V}$ and ε_F/W. To illustrate that k is proportional to D_0^3, we show k/D_0^3.

$T < T_C$. For $T > T_C$, the effect of the difference between electronic structures near the Fermi surface, which is represented by the different values of ε_F/W, is very small, and actually indistinguishable.

Since $k \propto D_0^3$, even in a paramagnetic metal where $\overline{V} = \tilde{V}N(0) < 1$, k can be significantly exchange enhanced to make $|k| \gg 1$. There, however, the temperature dependence of k is expected to be much less important than in a ferromagnetic metal. Corresponding to such situations, in Fig. 5 we calculated k for $T = 0$ by changing the values of $\overline{V}$ and ε_F/W in the model electronic density of states of Fig. 2 with $W = 1$ eV. Unlike in Figs. 3 and 4, here we used the approximations $F'(0) \cong -N'(0)$, $F''(0) \cong N''(0)$, which are valid for $(k_B T/\varepsilon_F)^2 \ll 1$, in Eq. (3.2).

A paramagnetic system in which the magnetic-field effect on sound velocity was measured is the $A15$ compounds. In V_3Si, for instance, it was observed that $\Delta s(H)/s \cong -1\%$ for $H \cong 80$ kG.[7] To account for such an observation from Eq. (3.4) with $|k| = O(1)$, we require $\varepsilon_F \cong W < 10^{-2}$ eV.[8] This fact was actually used to corroborate the linear chain model for the $A15$ compounds.[9] However, detailed band calculations have shown that a conduction electron band with such a small width is unrealistic; see, for instance, Klein *et al.*[10] If $W > 0.1$ eV, as various band calculations suggest, we need to have $|k| \cong 10^2$. According to the result of Fig. 5, k can easily take such a value with a moderate exchange enhancement.

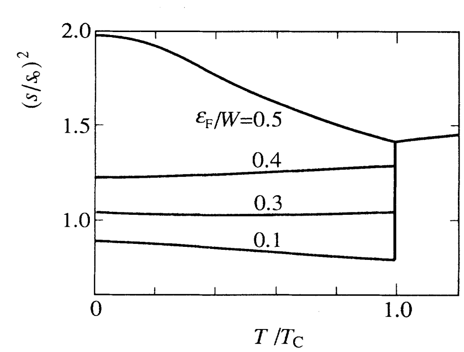

FIG. 4. The temperature dependence of the sound velocity calculated from Eq. (2.16) for different locations of the Fermi energy in the spin unsplit state of the model electronic density of states of Fig. 2. The same values of W, $\overline{V}$, and ξ are used as in Fig. 3. Note that for $T > T_C$, the differences between the behaviors for different values of ε_F/W become indistinguishable.

2. *Magnetic-field effect on sound attenuation for $T > T_C$*

If we write the attenuation constant in the presence of an external magnetic field **H** as given in Eq. (2.36), from Eqs. (2.37)–(2.39) we obtain

$$\Delta\alpha_\omega(H)/\alpha_\omega = \nu(\mu_B H/W)^2 ,$$

$$\nu = D_0^2 K \left[\left[\frac{s_0}{s} \right]^2 - 2D_0 \right] + G , \tag{3.6}$$

where K is given in Eq. (3.2), and

$$G = \left[\overline{V} D_0^3 \left\{ \frac{F''(0)}{F(0)} - \left[\frac{F'(0)}{F(0)} \right]^2 \right\} + 3\overline{V}^2 D_0^4 \left[\frac{F'(0)}{F(0)} \right]^2 \right] W^2 . \tag{3.7}$$

Note that the magnetic-field effect is exchange enhanced by D_0^4. [In Refs. 2 and 5, there is an error in the expression for ν; it should be corrected as in Eq. (3.6).]

In Fig. 6(a) we show our numerical result on the temperature dependence of ν for $T > T_C$ of a ferromagnetic metal which is obtained similarly to Fig. 3. To demonstrate that $\nu \propto D_0^4$, we also show ν/D_0^4 in Fig. 6(b). Besides the large exchange enhancement, we note the sensi-

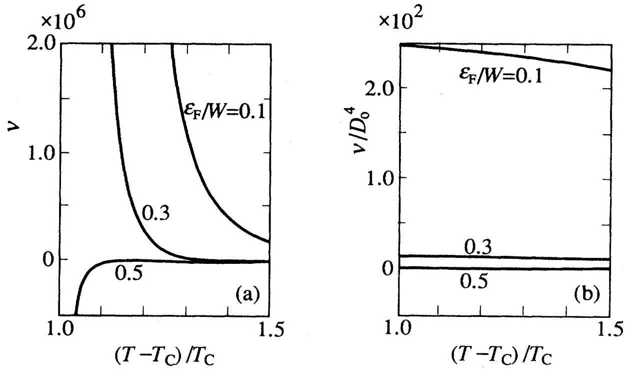

FIG. 6. The temperature dependence of the magnetic-field effect on the attenuation constant of sound propagation for $T > T_C$ as calculated from Eqs. (3.6) and (3.7) with the model electronic density of Fig. 2 similar to Fig. 3. To illustrate that ν is proportional to D_0^4, we show in addition to ν, (a), ν/D_0^4, (b).

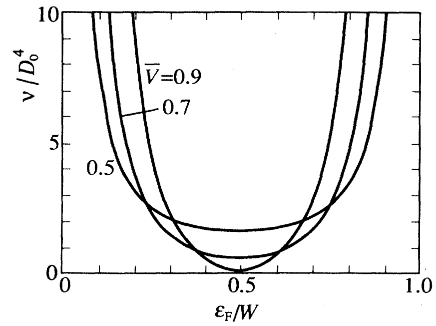

FIG. 8. The magnetic-field dependence of sound attenuation in nonferromagnetic metals, for which $\bar{V} = \tilde{V}N(0) < 1$, as calculated from Eqs. (3.6) and (3.7) with the model electronic density of states of Fig. 2 for different values of $\bar{V}$ and ε_F/W at $T=0$. ν being proportional to D_0^4, we show ν/D_0^4.

tive dependence on the value of ε_F/W, similar to the case of k given in Fig. 3. For comparison, in Fig. 7 we show the temperature dependence of the sound attenuation without the effect of a magnetic field both for $T > T_C$ and $T < T_C$. We again notice that the effect of the difference in the electronic structure near the Fermi surface is drastically amplified in the behavior of ν compared in that of $\alpha_\omega/\alpha_{\omega 0}$ itself.

In Fig. 8, considering the case of nonferromagnetic metals, we show how ν at $T=0$ changes with the values of $\bar{V}$ (<1) and ε_F/W in the model electronic density of states. We see that since $\nu \propto D_0^4$, ν can take a value much larger than $O(1)$ even with a moderate exchange enhancement and reflect the difference in the electronic structure in a drastically amplified form.

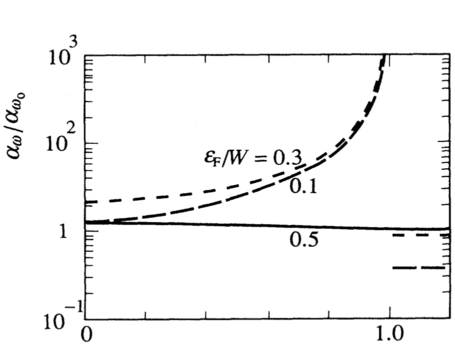

FIG. 7. The temperature dependence of the attenuation constant of sound propagation without the effect of magnetic field calculated from Eq. (2.17) with the model electronic density of states of Fig. 2 for different values of ε_F/W. Note the large jumps at T_C in the attenuation constant except for the case of $\varepsilon_F/W=0.5$.

B. Magnetic-field effect on sound propagation in the ferromagnetic state of a metal

1. Magnetic-field effect on sound velocity for $T < T_C$

Differing from the case of the paramagnetic state, in the ferromagnetic state the magnetic-field effect begins from the first order in η or H. Thus, here we retain only up to the first-order terms. From Eqs. (2.32) and (2.33) we obtain

$$\frac{\Delta s(\eta)}{s} = f_1\left[\frac{\eta}{W}\right] = \left[\frac{s_0}{s}\right]^2 Y(\varepsilon_F,\bar{V})\frac{4F_+(0)F_-(0)}{F_+(0)+F_-(0)}\eta\ , \tag{3.8}$$

where $(s/s_0)^2$ is given in (2.16), and we put

$$Y(\varepsilon_F,\bar{V}) = \frac{1}{2}N(0) \times \sum_\sigma \sigma\frac{F'_\sigma(0)/F_\sigma(0)}{\{1-\tilde{V}F_\sigma(0)\}^2} \Bigg/ \left[\sum_\sigma \frac{F_\sigma(0)}{1-\tilde{V}F_\sigma(0)}\right]^2 . \tag{3.9}$$

In the ferromagnetic state, η is related to the external magnetic field as

$$\eta = (1+\tfrac{1}{2}\tilde{V}\bar{\chi}_{\rm hf})\mu_B H = \frac{F_+(0)+F_-(0)}{4F_+(0)F_-(0)}\bar{\chi}_{\rm hf}\mu_B H\ , \tag{3.10}$$

where $\chi_{\rm hf} = \mu_B^2\bar{\chi}_{\rm hf}$ is the Stoner high-field magnetic susceptibility,

$$\frac{1}{\bar{\chi}_{\rm hf}} = \frac{1}{4}\left[\sum_\sigma \frac{1}{F_\sigma(0)} - 2\tilde{V}\right] . \tag{3.11}$$

Thus, we obtain

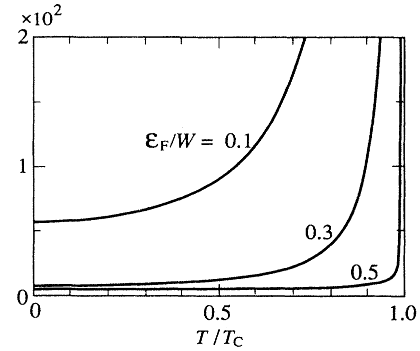

FIG. 9. The temperature dependence of the magnetic-field effect on sound velocity for $T < T_C$ of ferromagnetic metals calculated from Eq. (3.12) with the model electronic density of states of Fig. 2 for different values of ε_F/W. We assumed $W=1$ eV, $\bar{V}=1.3$, and $\xi=2$, the same values as in Figs. 3, 4, and 6.

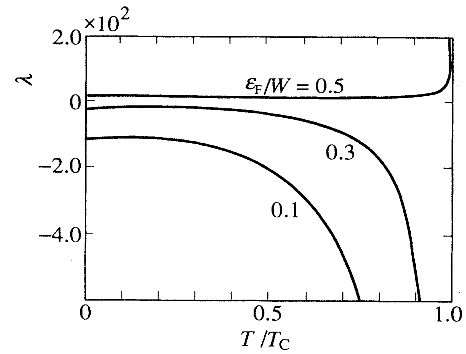

FIG. 10. The temperature dependence of the magnetic-field effect on the attenuation constant of sound propagation for $T < T_C$ of ferromagnetic metals calculated from Eq. (3.13) with the model electronic density of states of Fig. 2 similar to Fig. 9.

$$\frac{\Delta s(H)}{s} = A\left[\frac{\mu_B H}{W}\right],$$
$$\frac{A}{W} = \frac{F_+(0)+F_-(0)}{4F_+(0)F_-(0)}\bar{\chi}_{\rm hf}\frac{f_1}{W} = (s_0/s)^2 Y\bar{\chi}_{\rm hf}\,. \tag{3.12}$$

In Fig. 9 we show the temperature dependence of A which is calculated similarly to Figs. 3 and 6 with the model electronic density of states of Fig. 2; we use the same values, $W=1$ eV, $\bar{V}=1.3$, and $\xi=2$, as in Fig. 3. We find the magnitude of A and its temperature dependence very sensitively depend upon the value of ε_F/W; A can take the value of $O(10^2)$ as observed in FeNi,[1] and $\sim O(1)$ as observed in Ni,[1] if we assume, respectively, ~ 0.3 and ~ 0.5 for ε_F/W within the model electronic density of states of Fig. 2. Of course, the real situations would be quite different; the electronic densities of FeNi and Ni are not as simple as that of Fig. 2.

2. Magnetic-field effect on sound attenuation for $T < T_C$

Retaining up to first-order terms in Eq. (2.37) we have

$$\frac{\Delta\alpha_\omega(H)}{\alpha_\omega} = \lambda\frac{\mu_B H}{W},$$
$$\frac{\lambda}{W} = 2\left[\left[\frac{s}{s_0}\right]^2\frac{\tilde{P}(0)}{N(0)} - 1\right]\frac{A}{W} + \frac{\tilde{V}\bar{\chi}_{\rm hf}}{\sum_\sigma D_\sigma^2}\sum_\sigma \sigma D_\sigma^3\frac{F'_\sigma(0)}{F\sigma(0)}\,. \tag{3.13}$$

In Fig. 10 we present the result of our numerical calculation on Eq. (3.13) which was carried out similarly to that of Fig. 9 for A. We again observe that the magnetic-field effect on the sound attenuation for $T < T_C$ very strongly depends on the value of ε_F/W and varies in wide range.

IV. CONCLUDING REMARKS

In this paper, first we systematically derived general expressions for the effect of a magnetic field on the velocity and attenuation of longitudinal acoustic sound for $T > T_C$ and $T < T_C$. Then, as an illustration of our results, we carried out a numerical calculation on the temperature dependence of those magnetic-field effects by using a simple model electronic density of states. Our numerical results shows how sensitively the electronic structure and electron interaction of a metal, particularly that of a ferromagnetic one, would be reflected in those magnetic-field effects. This implies the possibility of using the magnetic-field effect as a sensitive probe of the electronic structure and electron interaction. Such importance of studying those magnetic-field effects on sound propagation is not yet widely known.

The essence of the origin of those magnetic-field effects is the spin splitting of the conduction electron bands produced by a magnetic field. In this paper we studied it with the Stoner model. We now know that we have to go beyond the Stoner model, by considering, for instance, the spin fluctuation effect. Note, then, that our result in this paper can be useful even there, since the spin fluctuation effects are described in terms of Stoner-type magnetic susceptibility.

A study of the magnetic-field effect on sound can also contribute to the exploration of the fundamental mechanism of itinerant-electron ferromagnetism. It is still controversial whether the electron-phonon interaction is involved in it in any important way.[11] The possible effect of the electron-phonon interaction on magnetism derives its origin in the magnetization dependence of phonon frequency. How a phonon frequency depends on magnetization can be inferred from how the phonon frequency changes with a magnetic field.[5] A study of the magnetic-field effect on sound is also very important in this respect.

ACKNOWLEDGMENT

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[1]G. A. Alers, J. R. Neighbors, and H. Sato, J. Phys. Chem. Solids **13**, 401 (1960).
[2]D. J. Kim, Solid State Commun. **36**, 373 (1980).
[3]D. J. Kim and I. Yoshida, in *Proceedings of the International Conference on the Physics of Transition Metals,* edited by P. M. Oppeneer and J. Kübler (World Scientific, Singapore, 1993), p. 605.
[4]D. J. Kim, Solid State Commun. **38**, 441 (1981).
[5]D. J. Kim, Phys. Rev. B **25**, 6919 (1982); Phys. Rep. **171**, 129 (1988).
[6]See, for instance, D. Pines, *Elementary Excitations in Solids* (Benjamin, New York, 1964).
[7]W. Rehwald, M. Rayl, R. W. Cohen, and G. D. Cody, Phys. Rev. B **6**, 363 (1972).
[8]W. Dieterich and P. Fulde, Z. Phys. **248**, 154 (1971).
[9]M. Weger, Rev. Mod. Phys. **36**, 175 (1964); J. Labbe and J. Friedel, J. Phys. (Paris) **27**, 153 (1966).
[10]B. M. Klein, L. L. Boyer, D. A. Papaconstantopoulos, and L. F. Mattheiss, Phys. Rev. B **18**, 6411 (1978).
[11]C. Herring, in *Magnetism,* edited by G. T. Rado and H. Suhl (Academic, New York, 1966), Vol. 4.