# Pseudo-marginal-Fermi-liquid behavior in antiferromagnetic metals

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Effects of interactions of conduction electrons with spin waves on electronic properties of a metallic antiferromagnet are investigated. At the lowest temperatures $T < T^* \sim (\Delta/E_F)T_N$ ($\Delta$ is the antiferromagnetic splitting of the electron spectrum) Fermi-liquid behavior takes place, and nonanalytic $T^3 \ln T$ contributions to specific heat are present for three dimensions (3D). At the same time, for $T > T^*$, in 2D and "nested" 3D systems the picture corresponds to a marginal Fermi liquid ($T \ln T$ contributions to specific heat and nearly $T$-linear dependence of resistivity). A comparison with the case of a ferromagnet is performed.

Recently, a possible formation of states which differ from the usual Fermi liquid was extensively discussed for highly correlated electron systems. The non-Fermi-liquid behavior of the excitation spectrum down to the lowest energies is now reliably established in the one-dimensional case (the "Luttinger liquid"[1]). However, such a situation can also be assumed in some two-dimensional (2D) and even 3D systems with strong electron correlations.[2] To describe properties of high-$T_c$ superconductors (HTSC's), Varma *et al.*[3] proposed a phenomenological "marginal Fermi liquid" (MFL) theory where electron damping is linear in energy $E$ (referred to the Fermi level) and the effective mass is logarithmically divergent at $E \to 0$. According to Ref. 3, such behavior is the result of interaction with local Bose excitations which possess a peculiar (linear in their energy and weakly $q$-dependent) spectral density. The MFL theory was developed further in a number of papers (see, e.g., Refs. 4–6). In the simplest way, the MFL electron spectrum may be reproduced in some crossover energy region for interacting electron systems under the requirement of almost perfect nesting in 2D case,[7–10] which seems to be too strict for real systems. Similar results were obtained with account of antiferromagnetic (AFM) spin fluctuations in the vicinity of the AFM instability.[11] Almost $T$-linear behavior of the resistivity in paramagnetic 2D metals with strong AFM fluctuations was obtained by Moriya *et al.*[12] in a broad temperature region.

A behavior corresponding to the MFL ($T \ln T$ term in the electronic specific heat, unusual power-law $T$ dependences of resistivity and magnetic susceptibility, etc.) in some temperature intervals was found experimentally in a number of uranium and cerium systems [$U_xY_{1-x}Pd_3$,[13] $UPt_{3-x}Pd_x$,[14] $UCu_{5-x}Pd_x$,[15] $CeCu_{6-x}Au_x$,[16] $U_xTh_{1-x}Be_{13}$, $Th_{1-x}U_xRu_2Si_2$, and $Ce_xLa_{1-x}Cu_2Si_2$ (Ref. 17)]. This behavior is as a rule interpreted within a two-channel Kondo scattering mechanism.[18] At the same time, in a number of systems ($UCu_{5-x}Pd_x$, $CeCu_{6-x}Au_x$, $U_xY_{1-x}Pd_3$) the non-Fermi-liquid behavior correlates apparently with the onset of antiferromagnetic (AFM) ordering,[16,19] i.e., is connected with multicenter effects.

In the present work we propose a description of the MFL state formation in AFM metals with a 2D spectrum (HTSC's) or nesting features of the Fermi surface for $D = 3$ (anomalous Ce- and U-based systems) owing to interaction with the usual spin waves. Treatment of this mechanism seems to be justified in that almost all the systems under consideration are characterized by pronounced local magnetic moments and spin fluctuations.

The peculiarities of the spectrum and damping of the quasiparticles near the Fermi level are due to the interaction with low-energy collective excitations, either well defined or of dissipative nature (phonons, zero sound, paramagnons, etc.). Migdal[20] proved for $D = 3$ in a general form that the corresponding nonanalytic contributions to the self-energy $\Sigma(E)$ are of the order of $E^3 \ln E$, which results in $T^3 \ln T$ terms in the electronic specific heat.[21] Fermi-liquid behavior might seem to take place for AFM metals since in the long-wavelength limit ($q \to 0$) the electron-magnon interaction is equivalent to the interaction with acoustical phonons (the spectrum of the Bose excitations is linear and the scattering amplitude is proportional to $q^{1/2}$). However, in the case of AFM spin waves there exists one more "dangerous" region $\mathbf{q} \to \mathbf{Q}$ ($\mathbf{Q}$ is the wave vector of the AFM structure), where the magnon frequency $\omega_{\mathbf{q}}$ tends to zero and the scattering amplitude diverges as $\omega_{\mathbf{q}}^{-1/2}$. At very small $E$ such processes are forbidden because of the presence of the AFM splitting in the electron spectrum. At the same time, at not too small $E$ one may expect that these processes lead to stronger singularities. Thus the Fermi-liquid picture may be violated in this energy region. We shall demonstrate that a number of physical properties of 2D and "nested" 3D antiferromagnets exhibit MFL behavior in some temperature interval, although the collective excitation spectrum is quite different from that in the theory.[3] In this sense we use the term "pseudo-marginal-Fermi-liquid."

To investigate the effects of interaction of current carriers with local moments we use the $s$-$d(f)$ exchange model

$$H = \sum_{\mathbf{k}\sigma} t_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} - I \sum_{\mathbf{qk}} \sum_{\sigma\sigma'} \mathbf{S}_{\mathbf{q}} c^\dagger_{\mathbf{k}\sigma} \sigma_{\sigma\sigma'} c_{\mathbf{k}+\mathbf{q}\sigma'} + \sum_{\mathbf{q}} J_{\mathbf{q}} \mathbf{S}_{\mathbf{q}} \mathbf{S}_{-\mathbf{q}}, \quad (1)$$

where $c^\dagger_{\mathbf{k}\sigma}$, $c_{\mathbf{k}\sigma}$, and $\mathbf{S}_{\mathbf{q}}$ are operators for conduction elec-

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trons and localized spins in the quasimomentum representation, $I$ is the $s$-$d(f)$ exchange parameter, and $\sigma$ are the Pauli matrices. Introducing spinor operators $\Psi_{\mathbf{k}}^\dagger = (c_{\mathbf{k}\uparrow}^\dagger, c_{\mathbf{k+Q}\downarrow}^\dagger)$ ($2\mathbf{Q}$ is taken for simplicity equal to a reciprocal lattice vector) and passing to the magnon representation for spin operators, we calculate the matrix electron Green's function $G(\mathbf{k},E)$ to second order in the electron-magnon interaction (this approximation corresponds to first order in the quasiclassical small parameter $1/2S$; see Ref. 22). The AFM splitting of the electron spectrum with the subbands

$$E_{\mathbf{k}1,2} = \frac{1}{2}(t_{\mathbf{k}} + t_{\mathbf{k+Q}}) \pm E_{\mathbf{k}}\,, \tag{2}$$

$$E_{\mathbf{k}} = (\tau_{\mathbf{k}}^2 + I^2\overline{S}^2)^{1/2}, \quad \tau_{\mathbf{k}} = \frac{1}{2}(t_{\mathbf{k}} - t_{\mathbf{k+Q}})$$

($\overline{S}$ is the sublattice magnetization) is included in the zero-order approximation.

One obtains for the correction to the density of states

$$\delta N(E) = -\frac{1}{\pi}\sum_{j\mathbf{k}}[\mathrm{Im}\Sigma_j(\mathbf{k},E)/(E-E_{\mathbf{k}j})^2 - \mathrm{Re}\Sigma_j(\mathbf{k},E)\delta'(E-E_{\mathbf{k}j})]. \tag{3}$$

The first term in (3) corresponds to the incoherent (nonquasiparticle) contribution, and the second one describes the renormalization of the quasiparticle spectrum. The self-energies are given by

$$\Sigma_i(\mathbf{k},E) = \frac{1}{2}I^2\overline{S}\sum_{\mathbf{q}}\sum_{j,l=1,2}\{L_{\mathbf{kq}}[(-1)^{i+j+1}] - (-1)^{i+l}M_{\mathbf{kq}}[(-1)^{i+j+1}]\}A_{\mathbf{kq}}^l\{E_{\mathbf{p}j}\}, \tag{4}$$

where

$$L_{\mathbf{kq}}(\pm) = (u_{\mathbf{q}}^2 + v_{\mathbf{q}}^2)(1 \pm I^2\overline{S}^2/E_{\mathbf{k}}E_{\mathbf{k+q}}) \pm 2u_{\mathbf{q}}v_{\mathbf{q}}\tau_{\mathbf{k}}\tau_{\mathbf{k+q}}/E_{\mathbf{k}}E_{\mathbf{k+q}}, \tag{5}$$

$$M_{\mathbf{kq}}(\pm) = I\overline{S}(1/E_{\mathbf{k}} \pm 1/E_{\mathbf{k+q}}), \tag{6}$$

$$A_{\mathbf{kq}}^{1,2}\{E_{\mathbf{p}j}\} = \frac{f(\pm E_{\mathbf{k+q}j}) + N_{\mathbf{q}}}{E - E_{\mathbf{k+q}j} \pm \omega_{\mathbf{q}}}, \tag{7}$$

$f(E)$ and $N_{\mathbf{q}} = N_B(\omega_{\mathbf{q}})$ are the Fermi and Bose distribution functions. The Bogoliubov transformation coefficients and the magnon frequency are given by

$$u_{\mathbf{q}}^2 = 1 + v_{\mathbf{q}}^2 = \frac{1}{2}[1 + \overline{S}(J_{\mathbf{q+Q}} + J_{\mathbf{q}} - 2J_{\mathbf{Q}})/\omega_{\mathbf{q}}],$$

$$\omega_{\mathbf{q}} = 2\overline{S}(J_{\mathbf{Q-q}} - J_{\mathbf{Q}})^{1/2}(J_{\mathbf{q}} - J_{\mathbf{Q}})^{1/2}.$$

It should be noted that, despite the absence of long-range order at finite temperatures, the result (4) is valid also in the 2D case up to $T \sim J$, $\overline{S}$ being replaced by the square root of the Ornstein-Cernike peak intensity in the pair correlation function.[23]

Nonanalytic contributions to $N(E)$ at $E \to 0$, $T = 0$ originate from spin waves with $\mathbf{q} \to 0$, $\mathbf{q} \to \mathbf{Q}$. Because of the $q$ dependence of the interaction matrix elements $[(u_{\mathbf{q}} - v_{\mathbf{q}})^2 \propto q$ and $(u_{\mathbf{q}} + v_{\mathbf{q}})^2 \propto q^{-1}$at $q \to 0]$, the intersubband contributions (say, to $\Sigma_1$ which originate from $A^l\{E_{\mathbf{p}2}\}$) are, generally speaking, more singular than the intrasubband ones. However, because of the quasimomentum and energy conservation laws, intersubband transitions are possible at $q, |\mathbf{q}-\mathbf{Q}| > q_0 \sim \Delta/v_F$ ($\Delta = 2I\overline{S}$ is the antiferromagnetic splitting and $v_F$ is the electron velocity at the Fermi level), so that the corresponding divergences are cut at $T^* = cq_0 \sim T_N\Delta/v_F$ (in the 2D case, $T_N \to J$).

Averaging $\Sigma_i(\mathbf{k},E)$ over the Fermi surface $E_{\mathbf{k}i} = 0$, we obtain for the intrasubband contribution in the 3D case at $T \ll |E|$

$$\mathrm{Im}\Sigma_i(E) = -\frac{I^2\overline{S}}{6\pi c^3}(|E|^3 + \pi^2|E|T^2)\langle[L_{\mathbf{kq}}(-) - (-1)^i\mathrm{sgn}E\, M_{\mathbf{kq}}(-)]/\omega_{\mathbf{q}}\rangle_{E_{\mathbf{k}i}=E_{\mathbf{k+q}i}=0}[\omega_{\mathbf{q}}\langle\delta(E_{\mathbf{k+q}i})\rangle_{E_{\mathbf{k}i}=0}]_{\mathbf{q}=0} \tag{8}$$

where $c$ is the magnon velocity. After analytical continuation, the contributions to $\mathrm{Im}\Sigma(E)$, proportional to $E^2|E|$, result in corrections of the form $\delta\mathrm{Re}\Sigma(E) \propto E^3\ln|E|$, which agrees with the microscopic Fermi-liquid theory.[20] Then the second term in (3) yields contributions of the form $\delta N(E) \propto E^2\ln|E|$. The corresponding contribution to the electronic specific heat,

$$C(T) = d\overline{H}/dT = \frac{d}{dT}\int dE f(E)EN(E),$$

is proportional to $T^3\ln(T/J)$ (cf. Ref. 21).

For $D = 2$ the symmetric (even in $E$) part of $\mathrm{Im}\Sigma(E)$ is proportional to $E^2$ and does not result in the occurrence of nonanalytic terms in $\mathrm{Re}\Sigma(E)$ and $N(E)$. In a 2D paramagnet, electron-electron scattering results in the contributions $\mathrm{Im}\Sigma(E) \propto E^2\ln|E|$ and in $T^2\ln T$ terms in the resistivity.[24] In our case the non-Fermi-liquid terms in (4), which contain $M$ and are proportional to $I^3$ (they describe the Kondo effect in the AFM state[23,25]), result in asymmetrical damping $\mathrm{Im}\Sigma(E) \propto E|E|$. The latter yields after analytical continuation $\mathrm{Re}\Sigma(E) \propto E^2\ln|E|$ and $\delta N(E) \propto E\ln|E|$. However, being odd in $E$, such terms in $N(E)$ do not lead to nonanalytic corrections to $C(T)$ after integrating over $E$.

At $|E| > T^*$ we may use instead of (4) the simple second-order perturbation expression

$$\Sigma_{\mathbf{k}}(E) = I^2\overline{S}\sum_{\mathbf{q}l}(u_{\mathbf{q}} - v_{\mathbf{q}})^2A_{\mathbf{kq}}^l\{t_{\mathbf{p}}\}, \tag{9}$$

the intersubband contributions corresponding to processes with small $|\mathbf{q}-\mathbf{Q}|$. Averaging in $\mathbf{k}$ over the Fermi surface we obtain

$$\mathrm{Im}\Sigma(E) = -2\rho^{-1}\sum_{\mathbf{q}\simeq\mathbf{Q},\ T^*\le\omega_{\mathbf{q}}<|E|}\frac{\lambda_{\mathbf{q}}}{\omega_{\mathbf{q}}} \tag{10}$$

with

$$\lambda_{\mathbf{q}} = 2\pi I^2\overline{S}^2(J_0 - J_{\mathbf{Q}})\sum_{\mathbf{k}}\delta(t_{\mathbf{k}})\delta(t_{\mathbf{k+q}}),\quad \rho = \sum_{\mathbf{k}}\delta(t_{\mathbf{k}})$$

($t_{\mathbf{k}}$ is referred to the Fermi level). In the general 3D case we have $\mathrm{Im}\Sigma(E) \propto E^2$. For $D = 2$ we derive

$$\mathrm{Im}\Sigma(E) = -\frac{1}{\pi\rho c^2}\lambda_{\mathbf{Q}}|E| \tag{11}$$

so that $\mathrm{Im}\Sigma(E) \propto |E|$. Then, by virtue of the analytical properties of $\Sigma(E)$, $\mathrm{Re}\Sigma(E) \propto E\ln|E|$ and

$$\delta N(E) = -\frac{4}{\pi^2 c^2}\lambda_{\mathbf{Q}}\ln|E|. \tag{12}$$

A similar picture occurs in the peculiar 3D case where the electron spectrum satisfies the "nesting" condition $t_{\mathbf{k}} = -t_{\mathbf{k}+\mathbf{Q}}$ in a significant part of the Brillouin zone, so that $\lambda_{\mathbf{q}} \propto 1/|\mathbf{q}-\mathbf{Q}|$. Such a situation is typical for itinerant-electron AFM systems since the onset of AFM ordering is connected with the nesting. Besides that, for localized-moment metallic magnets, which are described by the $s$-$f$ model, the value of $\mathbf{Q}$ is also often determined by the nesting condition.[26]

It may be shown that the incoherent contribution to $N(E)$ [see (3)], which is owing to intersubband transitions, is linear in $|E|$ up to $E = 0$ for $D = 2$ (but not in the 3D nesting situation). This contibution may be observed in tunneling experiments.

For the 2D "nested" antiferromagnet, the perturbation theory damping is very large, $\mathrm{Im}\Sigma(E) \propto \ln|E|$. Estimating the damping in the second-order self-consistent approximation [i.e., replacing the denominator in (9) and (7) by the exact electron Green's function] we derive $\mathrm{Im}\Sigma(E) \propto |E|^{1/2}$. Thus one has to expect in this case a strongly non-Fermi-liquid behavior at not too small $|E|$. Note that the situation is different from the power-law nonanalyticity in the Anderson model at very small $|E|$, which is an artifact of the noncrossing approximation (NCA).[27]

The intersubband contribution to $C(T)$ reads

$$\delta C_{\mathrm{inter}}(T) = \frac{8}{3}\pi T \sum_{\mathbf{q}\simeq\mathbf{Q},T<\omega_{\mathbf{q}}} \lambda_{\mathbf{q}}/\omega_{\mathbf{q}}^2. \tag{13}$$

In the 2D or "nesting" 3D situation the integral is logarithmically divergent at $\mathbf{q}\to\mathbf{Q}$, and the divergence is cut at $\omega_{\mathbf{q}} \simeq T$, so that we obtain the $T\ln T$ dependence of the specific heat. In the model accepted, the nonanalytic contributions to the magnetic susceptibility should mutually cancel, as well as for electron-phonon interaction.[28] However, such contributions may occur in the presence of relativistic interactions (heavy atoms).

Now we discuss transport properties. To second order in $I$, using the Kubo formula,[29] we obtain for the inverse transport relaxation time at $T > T^*$

$$1/\tau = \langle(\mathbf{v}_{\mathbf{k}+\mathbf{Q}} - \mathbf{v}_{\mathbf{k}})^2\rangle_{t_{\mathbf{k}}=0} \times \sum_{\mathbf{q}\simeq\mathbf{Q}} \lambda_{\mathbf{q}}\left(-\frac{\partial N_{\mathbf{q}}}{\partial\omega_{\mathbf{q}}}\right) \Bigg/ \sum_{\mathbf{k}} \mathbf{v}_{\mathbf{k}}^2\delta(t_{\mathbf{k}}) \tag{14}$$

with $\mathbf{v}_{\mathbf{k}} = \partial t_{\mathbf{k}}/\partial\mathbf{k}$. For $D = 3$ we have a quadratic temperature dependence of the spin-wave resistivity, $R(T) \propto (T/T_N)^2$. Note that the intersubband contribution (14) dominates over the intrasubband contribution [the latter is analogous to the electron-phonon scattering one and is proportional to $T^5$ (Ref. 32)]. In the 2D or "nesting" 3D situation one obtains

$$R(T) \propto T\ln[1-\exp(-T^*/T)] \simeq T\ln(T/T^*).$$

Thus in our model, unlike Ref. 3, the linear dependence of $\mathrm{Im}\Sigma(E)$ results in $T\ln T$ rather than $T$-linear behavior of the resistivity because of the lower-limit divergence of the integral with the Bose function. However, the deviation from the linear law is hardly important from the experimental point of view.

The impurity contribution to transport properties is determined by the energy dependence of $N(E)$ near the Fermi level,

$$\delta R_{\mathrm{imp}}(T)/R^2 \propto -V^2 \int dE[-\partial f(E)/\partial E]\delta N(E),$$

where $V$ is the impurity potential. The quasiparticle renormalization effects owing to $1 - d\mathrm{Re}\Sigma(E)/dE = 1/Z$ do not contribute to impurity scattering since $\tau \to \tau/Z$ and $v_F\to v_F Z$, so that the mean free path is unrenormalized.[1] At the same time, incoherent terms in $N(E)$ yield in the 2D case $\delta R_{\mathrm{imp}}(T) \propto T$ down to the lowest temperatures.

The asymmetric "Kondo" contribution to $N(E)$ in the 2D case, which is proportional to $E|E|$, should result in a large temperature-independent contribution to the thermoelectric power (cf. Ref. 30):

$$\delta Q(T) \propto \frac{1}{T}\int dE[-\partial f(E)/\partial E]E\delta N(E).$$

For 3D metallic ferromagnets with the quadratic dispersion law of magnons ($\mathbf{Q} = 0$), we have for a given spin projection

$$\Sigma_{\mathbf{k}\pm}(E) = 2I^2\overline{S}\sum_{\mathbf{q}} A_{\mathbf{kq}}^{1,2}\{t_{\mathbf{p}} \mp I\overline{S}\}. \tag{15}$$

Then we obtain the one-sided singular contributions $\mathrm{Im}\Sigma_\sigma(E) \propto \theta(\sigma E)|E|$ at $|E| > T^* \sim (\Delta/v_F)^2 T_C$, so that the crossover energy scale is considerably smaller than in the AFM case (see Refs. 33 and 34); these papers treat also the quasiparticle damping at small $E$ due to electron-magnon scattering, which occurs in the second order in $1/2S$). Then we obtain $\delta N_\sigma(E) \propto -\ln|E|$ for $|E| > T^*$. At the same time, the incoherent contribution, which survives up to $E = 0$, has the form $\delta N_\sigma(E) \propto \theta(\sigma E)|E|^{3/2}$.[35,34]

For $T > T^*$ the $T\ln T$ term in the specific heat of the ferromagnet is present.[31,33] On the other hand, the spin-wave resistivity at $T > T^*$ is proportional to $T^2$ for $D = 3$ (and $T^{3/2}$ for $D = 2$) because of the factor $(\mathbf{v}_{\mathbf{k}} - \mathbf{v}_{\mathbf{k}+\mathbf{q}})^2$ [cf. (14)]. However, this factor is absent for the scattering between spin subbands, which yields the $T\ln T$ term in the resistivity of ferromagnetic alloys.[36]

The situation in a 2D ferromagnet is similar to that in the above-discussed "nested" 2D antiferromagnet: the damping in the perturbation theory is large, $\mathrm{Im}\Sigma(E) \propto |E|^{1/2}$, this result being valid in the self-consistent approximation too.

To conclude, electronic properties of 2D and "nested" 3D metallic antiferromagnets agree on the whole with the MFL picture[3] in a rather wide interval $T^* < T < J$, the value of the crossover temperature being determined by the $s$-$f$ exchange parameter. At $T < T^*$ this behavior is changed to the usual Fermi-liquid one. At the same time, in contrast to Ref. 3, no special assumptions about the spectrum of the Bose excitations are used: in our model they are just spin waves with a linear dispersion law. Unlike Refs. 11 and 12 we need not consider the special case of the vicinity to the AFM instability, but can treat the AFM metal with well-defined local moments. Thus AFM ordering itself, together with the rather natural assumption about the "nesting," may explain violations of the Fermi-liquid picture which are observed in some anomalous rare-earth and actinide systems. The use of perturbation theory in the electron-magnon interaction within the $s$-$d(f)$ exchange model seems to be a reasonable phenomenological approach for highly correlated electron systems, which takes into account the SU(2) symmetry of exchange interactions. The electron spectrum $t_{\mathbf{k}}$ and parameter $I$ may be considered as effective ones (including many-electron renormalizations). Note that similar results for the electron-magnon interaction effects may be obtained in the Hubbard model ($I \to U$, cf. Refs. 33 and 22).

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