Temporal response of the thermal boundary resistance in superfluid helium

J. S. Olafsen and R. P. Behringer
Department of Physics, Duke University, Durham, North Carolina 27708-0305
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We report measurements of the thermal response $\Delta T(\omega)$ across a layer (height d) of superfluid helium to an ac heat flux, $Q(t)=Q_0e^{i\omega t}$. In this case $\Delta T(\omega)$ yields the boundary resistance, R_b. Measurements show a peak in $\Delta T(\omega)$ in the same temperature regime as the anomalous dc boundary resistance. The peak frequency and amplitude depend on T and Q_0, but not d. The range of d was $0 \leq d \leq 3$ mm. The sample has a nominal ^{3}He concentration of $X=2\times10^{-7}$.

The Landau two-fluid model is thought to provide a good description of the dynamics of superfluid ^{4}He and ^{3}He-superfluid-^{4}He mixtures. This model correctly predicts that, for pure superfluid ^{4}He, moderate amounts of heat can be carried without dissipation by counterflow, and that the addition of ^{3}He leads to dissipation. A thermal conductivity measurement in a superfluid mixture yields a finite effective conductivity κ_{eff}, which is predicted[1,2] to vary inversely with the ^{3}He molar concentration X, as $X\rightarrow0$. However, recent measurements[3–7] indicate a finite and size-dependent κ_{eff} as X becomes small. These measurements present a significant challenge to what was thought to be a well established model.

One possible explanation for the unexpected experimental observations lies in the boundary resistance R_b. In typical experiments, a heat flux Q flows from one boundary, through a fluid layer of thickness d and area A, to another boundary. This results in a temperature difference ΔT with contributions from the fluid and from the serial boundary resistances. R_b for a mixture is difficult to measure, and it typically contributes a large fraction of ΔT as X becomes small. Also, R_b has recently been shown[8,6] to behave anomalously near the superfluid temperature T_λ of pure ^{4}He, where it has both a weak divergence and a nonlinear dependence on Q. The weak divergence of R_b can be explained by the suppression of superfluidity near the boundaries over a length scale comparable to the correlation length $\xi(\epsilon)$, where $\epsilon\equiv(T-T_\lambda)/T_\lambda$ is the reduced temperature. However, the Q dependence of R_b is not currently explained.

Accordingly, a method to determine R_b unambiguously is useful. Here, we show that ac heat-flow measurements can provide such a method. Using such a technique, we find that the ac response of a very dilute mixture with $X=2\times10^{-7}$ exhibits anomalous ac behavior, including an expected peak at nonzero frequencies which can be attributed to R_b.

We consider heat input of the form[9]

$$Q(t)=Q_0\exp(i\omega t)\ . \tag{1}$$

An extensive calculation[10] starting from the full equations for two-fluid dynamics predicts that for small Q_0, the temperature difference across a layer of thickness d, including boundary resistance is given by

$$\Delta T(t)=[Q_0d/\kappa_{\text{eff}}]\tan(q_0)/q_0\exp(i\omega t)+2Q_0R_b(\omega)\ . \tag{2}$$

Here, q_0 is given by

$$q_0=(\omega d^2/4\Gamma_0)^{1/2}\exp(-i\pi/4)\ . \tag{3}$$

Γ_0, the diffusion coefficient of Griffin,[11] is related for dilute mixtures, to the mass diffusion coefficient of a single ^{3}He atom:

$$\Gamma_0=D_{\text{iso}}\ . \tag{4}$$

An alternative representation of the response function is the temperature amplitude, $|\Delta T(q_0)|$, and the phase angle $\theta(q_0)$ relative to $Q(t)$. When $|q_0|\gg1$, the factor $\tan(q_0)/q_0$ falls off rapidly as $|q_0|^{-1}$.

Equation (2) is similar to the ac response of a layer of He-I: the ideal normal-fluid response[9] is identical to Eq. (2) with R_b set to zero and Γ replaced by the thermal diffusion coefficient D_T. For $R_b\neq0$, the normal-fluid response has a more complicated form than Eq. (2) because in that case there is no propagating second sound mode. An important point is that a normal fluid cannot respond to an ac flux when $\omega\gg D_T/d^2$, so that $\Delta T(\omega)$ reduces to that of the boundary resistance at the surface where the heat flux is applied. By contrast, for a superfluid, as long as $\omega d\ll u_2$, where u_2 is the superfluid second sound speed, the response function will have two factors of R_b present.

For the mixture used in these experiments, $X\simeq2\times10^{-7}$, the resistance of the fluid, $d/(A\kappa_{\text{eff}})$, is so small that the measurable temperature response should correspond only to the boundary resistance $2Q_0R_b$. In addition, the characteristic fluid frequency, $4\Gamma_0/d^2$, is typically small, so that $|q_0|\gg1$, and $|\tan(q_0)/q_0|\ll1$. As noted above, the boundary resistance is typically assumed to react instantaneously, so we expect that $\Delta T(\omega)/Q_0$ should be a real constant, independent of d, Q_0, and ω. We find that this is not always the case; the remainder of this work is devoted to showing the actual form of R_b, along with a brief description of the apparatus.

The experiments were carried out in a cryostat[12] with the unique feature that d can be changed continuously from $0\leq d\leq3$ mm without warming up the apparatus.

This is important since R_b may change with thermal cycling. We used germanium resistance thermometry with a resolution of 0.2 μK. Ac heating was achieved by providing an ac voltage $V = V_0 \sin(\omega t/2)$ to a noninductive wire-wound resistor. The resulting heat flux consists of an ac part with frequency ω superimposed on a dc part. As long as the response is linear, the dc component plays no role.

We now turn to the experimental results. Well below T_λ, for $\epsilon = 0.05$, we obtain the data of Fig. 1. As expected, $|\Delta T(\omega)|$ is independent of Q_0 and ω; similarly, $\theta = 0$. A careful investigation for height effects shows that the results well below T_λ are independent of d. This reinforces the interpretation of the response as a boundary layer effect.

On close approach to T_λ, the response is significantly different and surprising. Specifically, the response is a nonlinear function of Q_0 (not necessarily surprising) but, it shows a peak. The peak occurs at a frequency which is large compared to the bulk fluid response rate but very small compared to characteristic microscopic rates such as D_{iso}/ξ^2. Figure 2 shows results for $|\epsilon| = 10^{-3}$. There is now a heat-dependent maximum in $|\Delta T|$, and the phase is no longer 0. Note that the peak in $|\Delta T|$ vs ω first grows with increasing Q_0 until $Q_0 = 20$ μW/cm^2, and then decreases with further increase in Q_0. In addition, the peak in $|\Delta T|$ vs ω moves from low ω to a saturated

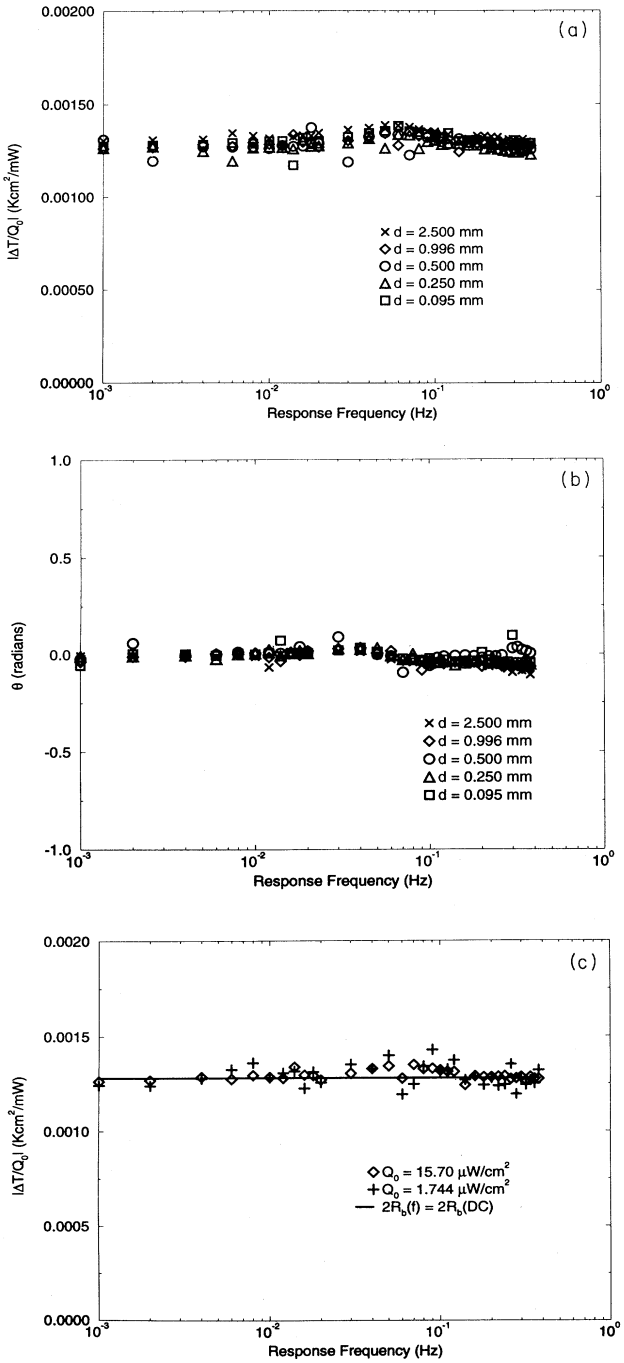

FIG. 1. Data for the thermal response at $|\epsilon| = 0.05$. In (a) and (b) we show $|\Delta T|$ and the phase θ vs frequency for a variety of d's with Q_0 fixed at 15.70 μW/cm^2. In (c) we show just $|\Delta T|$ for different values of Q_0, as labeled.

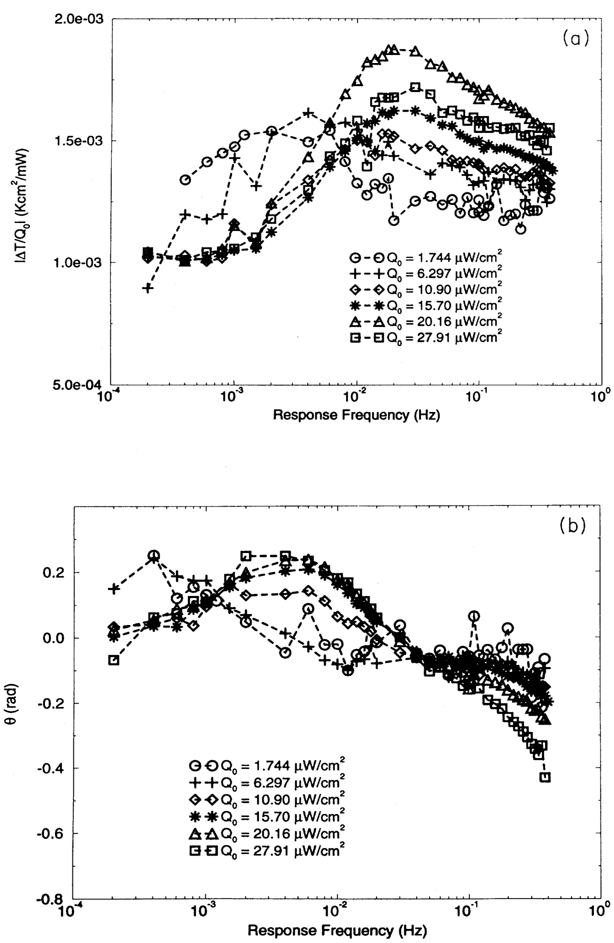

FIG. 2. Thermal response for $|\epsilon| = 0.001$ for (a) $|\Delta T|$ and (b) for θ.

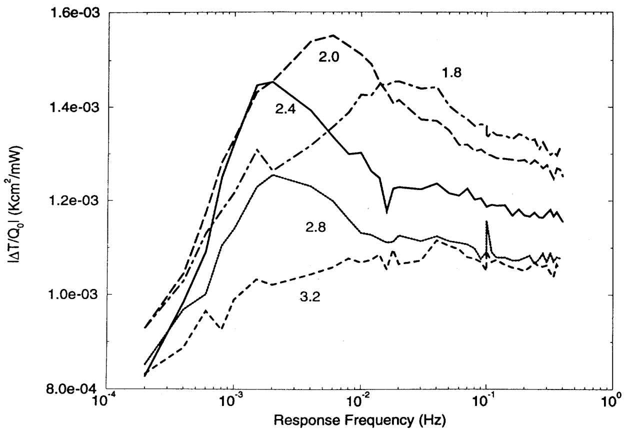

FIG. 3. ΔT vs frequency for several $|\epsilon|$ showing the onset of the peak. The value of $|\epsilon|\times 10^3$ is shown for each curve.

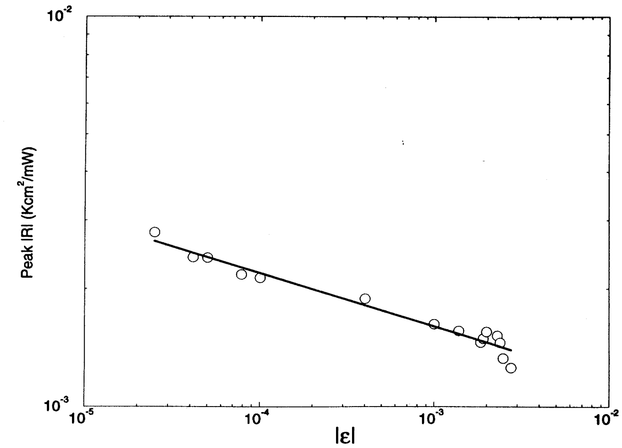

FIG. 4. Peak amplitude vs $|\epsilon|$.

value of $f=\omega/2\pi\simeq 0.02$ Hz. This frequency is fast compared to any bulk fluid relaxation rate. For instance, if we estimate $\Gamma_0\simeq 10^{-5}$ cm^2/s, then a frequency of 0.02 Hz corresponds to a fluid thickness of $(\Gamma_0/f)^{1/2}=0.02$ cm.

The onset of the anomalous boundary resistance with $\epsilon\to 0$ is sharp. In Fig. 3, we show $|\Delta T|$ vs frequency $\omega/2\pi$ for various ϵ near the onset of the anomaly. Note that the effect is well established when $\epsilon=-2.8\times 10^{-3}$ but essentially absent when $\epsilon=-3.2\times 10^{-3}$.

Recently, several authors[8,6,13,14] have described an anomalous dc boundary resistance for superfluid ^{4}He. The dc anomaly consists of two parts; a heat-independent component associated with the suppression of superfluidity near a boundary, and a heat-dependent part whose origin has not been explained. The heat-dependent ac anomaly of R_b is presumably related to the corresponding dc anomaly.

However, the ac anomaly shows several features which differ qualitatively from the dc effect. We have already noted one of these, that the response $|\Delta T(\omega)|$ has a maximum with increasing Q_0. By contrast, for the dc effect, increasing Q causes R_b to increase monotonically toward a limiting value. It is also interesting to note, Fig. 4, that the peak height appears to diverge weakly as

$$R_{\text{peak}}=A|\epsilon|^{-x}\ , \tag{5}$$

with $A=6.22\times 10^{-4}$ K cm^2/mW and $x=0.137$.

To conclude, we have identified a new aspect of the boundary resistance near the superfluid transition in ^{4}He. The effect manifests itself in the same temperature regime as the heat-dependent anomaly found in previous dc measurements with a similarly sharp onset. A future experiment will determine the boundary resistance in a more dilute mixture, $X\simeq 10^{-4}$. Here, the fluid contribution will be comparable to the thermal response.

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