Classical and quantum size effects in electron conductivity of films with rough boundaries

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(Received 20 January 1995)

We derive the classical static conductivity $\langle\sigma\rangle$ for a film with mildly sloping boundary asperities, when their root-mean-square height ζ is less than their mean length L. The formulas admit a numerical analysis of $\langle\sigma\rangle$ versus d/l and parameters ζ/L and $k_F L$ (d is the sample thickness, l is a bulk mean free path of electrons, and k_F is the Fermi wave number). The decrease of the conductivity with increasing $k_F L$ is demonstrated. At small-scale boundary defects ($k_F L < 1$), we also build the quantum theory of the electron transport. Dependencies of $\langle\sigma\rangle$ on l and d are studied. We reveal quantum dips of the conductivity versus $k_F d/\pi$ when a new conducting electron channel opens. The dips are caused by the size quantization of an electron-surface scattering frequency. The quasiclassical theory of $\langle\sigma\rangle$ at large-scale asperities ($k_F L \gg 1$) is presented. In this case the residual conductivity due to the electron-surface interaction may have both quantum and classical origins. The relation between quantum and classical effects in the film conductivity is clarified. The theoretical results are tested against recent experimental data.

I. INTRODUCTION

In investigating conducting electron properties of bounded samples, a problem of size effect is of great importance. The resistivity is known to increase rapidly as film thickness or wire diameter are reduced below the bulk mean free path of electrons. Such an increase is caused by intensive electron-surface interaction, which plays a fundamental role in nearly perfect conductors at low temperatures. In this way, the problem of size effect is mainly reduced to exploration and description of specific mechanisms of the electron scattering from surface defects. Depending on external conditions and subjects of inquiry, two approaches exist (classical and quantum) to solve the problem formulated.

Within the classical approach, the surface scattering is traditionally described via a boundary condition (BC) for the electron distribution function. The simplest form of such a BC was suggested by Fuchs.[1] In his model, surface properties are characterized by the single phenomenological parameter ρ (specularity coefficient), which is equal to a fraction of electrons specularly scattered at the sample boundary. Due to mathematical simplicity and a clear physical interpretation of results, Fuchs' model was widely used in the earlier calculations[2–9] of the static size effects and high-frequency surface impedance. Further development of the theory resulted in the transformation of local Fuchs' BC into the integral in the moment relation with the indicatrix of the electron-surface interaction. Its form contains a particular mechanism of the quasiparticle boundary scattering. A number of authors[10–14] have obtained, and the review[3] presented various microscopic BC's corresponding to the scattering of electrons by a random surface potential of some specific nature.

The quantum approach to the problem of size effect is the most actual in investigating the conductivity of ultrathin perfect samples at low temperatures, where the discrete character of quasiparticle transverse movement manifests itself (see, e.g., Ref. 15). In such microstructures, the quantization of trajectories qualitatively changes conditions of the electron interaction with surface inhomogeneities. In recent years, a number of papers were devoted to the research of this factor (see, e.g., Refs. 16–25). As far as we know, the first work in this field was Ref. 26. Its authors developed the diagram technique for the electron-surface interaction, which allowed them to analyze the ground quantum electron state, find the residual resistivity of the film, and a temperature dependence of the conductivity at low and high temperatures. In Ref. 16, a system of electrons in a thin conductor film, which experiences arbitrarily weak surface scattering, was shown to be always in localized states. This demonstrates the fundamental importance of the surface relaxation for the conductivity even at the nearly specular scattering at a boundary. The authors of Ref. 18 calculated the quasiclassical conductivity of films with statistically rough sides versus d/l (d is the sample thickness and l is the mean free path of electrons) and derived its asymptotics in the absence of bulk collisions.

The most widespread model of boundary defects is one with asperities—continuously distributed random deviations of the real surface from the ideal crystallographic plane. They are described by two microscopic parameters: root-mean-square height ζ and mean length L. Up to now, both classical[3,11–13,27] and quantum[16,18,20,22,24–26] theories of the electron-surface scattering at asperities were mainly built within the Born approximation, which was based on iterations in the small deviation ζ of the real surface from the ideal one. To

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use the Born approximation, one needs to satisfy the requirements of the smallness of asperity slope ($\zeta/L < 1$) and smallness of the Rayleigh ($k_x\zeta \ll 1$) and Fresnel ($k_F\zeta^2/L \ll 1$) parameters. Here k_x is the absolute value of normal (with respect to the average sample boundary) component of the wave vector and k_F is the Fermi wave number of electrons. Both the Rayleigh and Fresnel parameters are small within the condition $\zeta/L < 1$ in specimens with either anomalously small asperity heights ($k_F\zeta \ll 1$) or small-scale (SS) surface defects ($k_F L < 1$). In these cases, the Born approximation is automatically valid. Therefore, the majority of theoretical results were derived for, namely, those situations (see, e.g., Refs. 16, 18, 24, and 27). The simplicity of the theoretical analysis at $k_F\zeta \ll 1$ or $k_F L < 1$ is compensated by limitations on its practical realization. Actually, the inequalities $k_F\zeta \ll 1$ and $k_F L < 1$ may hold just in semiconductors and semimetals, where the electron de Broglie wavelength is much larger than the crystal lattice parameter ($k_F \lesssim 10^6$ cm^{-1}). On the contrary, $k_F L \gg 1$ in metals due to large values of $k_F \sim 10^8$ cm^{-1}. Moreover, the investigation[28] of well-treated surfaces with a scanning tunneling microscope showed that the characteristic length of asperities reached hundreds of atomic layers, i.e., $L \gtrsim 10^{-6}$ cm. This means that the case with large-scale (LS) boundary roughnesses ($k_F L \gg 1$) is evidently the most important even for semiconductors and semimetals. When the conditions $k_F L \gg 1$ and $\zeta/L < 1$ hold simultaneously, the Born approximation is, in general, justified only for the electrons with small sliding angles with respect to the surface. So, the necessity arises to build the theory of the electron-surface interaction, which would be independent of $k_x\zeta$. In Refs. 29 and 30, such a theory of wave diffraction from a statistically rough surface was developed. On the basis of this theory, the authors of Ref. 14 derived the BC for arbitrary values of the Rayleigh parameter. The only restriction is the requirement for the small obliquity of asperities ($\zeta/L < 1$). According to Ref. 28, the film boundaries were rather smooth and contained irregularities with typical atomic-scale heights ($\zeta \gtrsim 10^{-8}$ cm), so that the slope of surface defects was actually small ($\zeta/L \sim 10^{-2}$–10^{-4}). This suggests that the model of mildly sloping asperities adequately reflects a profile of a real well-treated boundary. The new BC (Ref. 14) allows us to study the statical conductivity for samples with arbitrary relationships between the electron wavelength k_F^{-1} and the average dimensions ζ and L of surface defects. In other words, one can study both cases of SS and LS boundary inhomogeneities from the generalized point of view.

Classical theoretical results are acceptably consistent with experimental data concerning quite thick specimens. As an illustration, we refer to the good coincidence of the theoretical dependencies with the experimental ones for stibium whiskers of thickness $d \sim 10^{-4}$ cm (see Refs. 3 and 4). Besides, examples[31,32] exist where the traditional classical theory can even describe kinetic properties in ultrathin samples. The investigations[31,32] demonstrate that a variety of effects in conducting microstructures may have classical as well as quantum origin. Hence, a problem arises as to the relationship between classical and quantum effects in the conductivity of thin specimens. We emphasize that earlier theoretical papers have not adequately elucidated this problem even for the simplest situation with $k_F L < 1$.

The present paper is devoted to the classical and quantum transport in films with mildly sloping surface asperities. The electron-surface scattering is included via the BC.[14]

In Sec. II, we derive the most general classical formula for the conductivity and give its brief numerical analysis. Section III deals with the full classical and quantum theories of the electron transport at SS asperities ($k_F L < 1$). The asymptotical behavior of the conductivity depending on d and l is studied therein. Limitations on the classical approach are also found out. In Sec. IV, the quasiclassical theory of the conductivity for LS roughnesses ($k_F L \gg 1$) is first constructed. The conclusions following from our investigations, as well as the test of the theoretical results against the experimental data,[33] are contained in Sec. V.

II. PROBLEM STATEMENT. PRINCIPAL EQUATIONS AND RESULTS

A plate with an electron-type conduction is bounded by randomly rough surfaces. Their asperity averages coincide with the planes $|x| = d/2$. The y axis is directed along the vectors of current density $\vec{j}$ and electric field $\vec{E}$ (see, Ref. 34). For simplicity, the dispersion law of electrons is assumed to be quadratic and isotropic, $\varepsilon = \hbar^2k^2/2m$. Our goal is to calculate the averaged conductivity,

$$\langle\sigma\rangle = \frac{1}{d}\int_{-d/2}^{d/2}\sigma(x)dx, \qquad \sigma(x) = j(x)/E, \tag{2.1}$$

as a function of the plate thickness d, the electron mean free path l, and characteristics of surface defects. Within the classical approach, the current density $j(x)$ is determined from

$$j(x) = -\frac{2e}{(2\pi)^3}\int d^3k\frac{\partial f_F}{\partial\varepsilon}v_y\chi(x,\vec{k}), \tag{2.2}$$

where e is the elementary charge, $\chi(x,\vec{k})\partial f_F/\partial\varepsilon$ is a nonequilibrium addition to the Fermi distribution function $f_F(\varepsilon)$, and $\vec{v} = \hbar\vec{k}/m$ is the velocity of electrons. The integration in Eq. (2.2) is taken over all electron wave vectors $\vec{k}$. The function χ is found from the kinetic equation

$$v_x\frac{d\chi}{dx} + \nu\chi = ev_yE, \tag{2.3}$$

where $\nu = v_F/l$ is the frequency of bulk collisions, and v_F is the Fermi velocity of electrons.

Boundary conditions for Eq. (2.3) are formulated on the asperity-averaged sides and contain information about the electron-surface scattering. They relate the function $\chi(\pm d/2; \mp k_x, \vec{k}_t)$ of the electrons flying away from the plate boundaries to the function

$\chi(\pm d/2; \pm k_x, \vec{k}_t)$ of the electrons impinging on them. For the elastic scattering, the BC's can be written as a canonical integral relationship, which ensures the absence of particle flux through the specimen surface (see, e.g., the review[3]):

$$\chi(\pm d/2; \mp k_x, \vec{k}_t) = \chi(\pm d/2; \pm k_x, \vec{k}_t) - \int_{k_t' \leq k_F} \frac{d^2 k_t'}{(2\pi k_F)^2} V(\vec{k}_t, \vec{k}_t') \times [\chi(\pm d/2; \pm k_x, \vec{k}_t) - \chi(\pm d/2; \pm k_x', \vec{k}_t')], \quad (2.4)$$

where $\vec{k}_t = \{k_y, k_z\}$ is a two-dimensional (2D) tangential wave vector. The energy conservation law specifies a dependence of the absolute value k_x on k_t: $k_x = (k_F^2 - k_t^2)^{1/2}$.

The first term in the right-hand part of Eq. (2.4) presents the specularly reflected electrons. The second one is an integral of electron-surface collisions. It accounts for partially diffuse reflection at the rough boundary and is written as a difference between outgoing and incoming terms. The integral kernel $V(\vec{k}_t, \vec{k}_t') = V(\vec{k}_t', \vec{k}_t)$ is a dimensionless probability density of a transition between states $\vec{k}_t$ and $\vec{k}_t'$ as the electron hits the surface. For a randomly rough surface, the most general expression for $V(\vec{k}_t, \vec{k}_t')$ was derived in Ref. 14:

$$V(\vec{k}_t, \vec{k}_t') = 8\pi (k_F \zeta)^2 k_x k_x' \int_0^\infty r dr |W'(r)| \frac{J_1(|\vec{k}_t - \vec{k}_t'| r)}{|\vec{k}_t - \vec{k}_t'|} \times \exp\left\{-\zeta^2 (k_x + k_x')^2 [1 - W(r)]\right\}, \quad (2.5)$$

where $\vec{r} = \{y, z\}$ is a 2D position vector, $J_n(x)$ is the Bessel function, ζ is the root-mean-square asperity height, and $W(r)$ is a binary correlator of roughness heights. The mean asperity length L is the scale of monotonous decrease of $W(r)$, and $W(0) = 1$. The prime on a function denotes a derivative with respect to the argument. For simplicity, the plate sides are assumed to be statistically identical, so BC's (2.4) contain same probability density (2.5).

Equation (2.5) adequately describes the electron scattering at mildly sloping asperities, i.e., at

$$\zeta < L, \quad (2.6)$$

and in the absence of the shadowing effect,[35] when, e.g., the Fresnel parameter is small,

$$k_F \zeta^2 / L \ll 1. \quad (2.7)$$

In general, the electron wave "illuminates" the whole rough surface at sufficiently large sliding angles, for which

$$\zeta / L < k_x / k_F \leq 1. \quad (2.8)$$

In this case there is no need to use condition (2.7). However, the left inequality in Eq. (2.8) reverses at small sliding angles, where the shadowing effect is determined by the relation between ζ and the size $(L/k_F)^{1/2}$ of the Fresnel zone. If Eq. (2.7) holds, the electrons with $k_x/k_F \lesssim \zeta/L$ due to diffraction get into the geometrical shadow region. Thus, the usage of Eq. (2.5) is limited by either single inequality (2.8) or two conditions (2.6) and (2.7).

Solving boundary-value problem (2.3) and (2.4), in line with Eqs. (2.1) and (2.2), results in the expression for the average plate conductivity $\langle\sigma\rangle$ over the bulk sample conductivity σ_0,

$$\frac{\langle\sigma\rangle}{\sigma_0} = \frac{3}{2} \int_0^1 dn_x (1 - n_x^2) \times \left\{1 - \frac{l}{d} n_x \left[\exp(d/l n_x) - 1\right] [1 - Q(n_x)]\right\}, \quad (2.9)$$

$$\sigma_0 = k_F^3 e^2 / 3\pi^2 m\nu. \quad (2.10)$$

Here we introduce the following designations:

$$n_x = |v_x|/v_F = k_x/k_F, \quad n_t = k_t/k_F = (1 - n_x^2)^{1/2}, \quad n_y = n_t \cos\varphi. \quad (2.11)$$

The dimensionless function $Q(n_x)$ is linearly connected with the nonequilibrium addition $\chi(\pm d/2; \vec{k})$ and, in accordance with BC's (2.4), satisfies an integral equation

$$\left[\exp\left(\frac{d}{l n_x}\right) - 1\right] [1 - Q(n_x)] = \int_0^{2\pi} \frac{d\varphi'}{(2\pi)^2} \int_0^1 n_x' dn_x' V(\vec{k}_t, \vec{k}_t') \left[Q(n_x) - \frac{n_y'}{n_y} Q(n_x')\right]. \quad (2.12)$$

Since the scattering surface is statistically uniform and isotropic, the probability density $V(\vec{k}_t, \vec{k}_t')$ is an even function of a difference $\varphi - \varphi'$. Therefore, the kernel Q in Eqs. (2.9) and (2.12) depends only on the variable n_x (or n_t) but not on the polar angle φ.

Due to the complicated structure of the kernel $V(\vec{k}_t, \vec{k}_t')$, the exact solution of Eq. (2.12) can be unlikely derived analytically. In spite of this, formulas (2.9), (2.12), and (2.5) are the ultimate result, in a sense. Actually, they uniquely specify $\langle\sigma\rangle/\sigma_0$ as a function of d/l and the two parameters ζ/L and $k_F L$ at a particular form of $W(r)$. We performed numerical calculations of conductivity (2.9) with the error $\delta = 10^{-3}$. Figure 1 shows plots of $\langle\sigma\rangle/\sigma_0$ versus d/l for three values of $k_F L$ and the constant asperity slope $\zeta/L = 0.1$. For the correlator $W(r)$ we used the Gaussian function, $W(r) = \exp(-r^2/L^2)$. Note that the average conductivity decreases as $k_F L$ increases. This suggests the corresponding increase of the effective frequency ν_{surf} of the electron-surface scattering. Besides, comparing curve 2 with curve 3 we see that both ν_{surf} and $\langle\sigma\rangle$ slightly depend on $k_F L$ at large values of $k_F L$ and constant slope ζ/L. This conclusion is justified by the analytical results of Sec. IV.

Often numerical methods do not allow us to realize

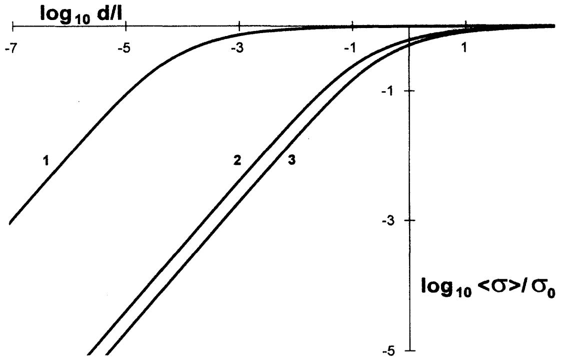

FIG. 1. Results of numerical calculations of average conductivity (2.9) (1) $k_F L = 10^{-1}$; (2) $k_F L = 1$; (3) $k_F L = 10$.

physical reasons for a particular form of curves $\langle\sigma\rangle/\sigma_0$. So, it is necessary to derive analytical solutions for $\langle\sigma\rangle$, which would be simpler than Eqs. (2.9) and (2.12). Such simplification is possible just for limiting values of some parameter. From the physical point, two types of rough surfaces seem to be the most important. Surfaces of the first type contain SS asperities of the "white noise" kind ($k_F L < 1$). For the second type, a sample boundary consists of LS inhomogeneities ($k_F L \gg 1$). Below we will analyze the electron conductivity for these two types of surfaces.

In Ref. 14, the electron reflection from mildly sloping surface asperities ($\zeta/L < 1$) was shown to be nearly specular, if the transition probability $V(\vec{k}_t, \vec{k}'_t)$ had form (2.5). This means that the frequency $\nu_{\rm surf}$ of the electron-surface relaxation is small as compared to the double frequency $|v_x|/d$ of the electron periodical motion along the x axis,

$$\nu_{\rm surf} \ll |v_x|/d. \qquad (2.13)$$

Obviously, under such a condition the surface electron scattering can compete with the bulk one only if the bulk relaxation frequency ν is also much less than $|v_x|/d$,

$$\nu \ll |v_x|/d. \qquad (2.14)$$

So, we expand expressions (2.9) and (2.12) in a small parameter $d/ln_x = \nu d/|v_x| \ll 1$, and get

$$\frac{\langle\sigma\rangle}{\sigma_0} = \frac{3}{2}\int_0^1 dn_x(1-n_x^2)Q(n_x), \qquad (2.15)$$

$$Q(n_x) + \frac{ln_x}{d}\int_0^{2\pi}\frac{d\varphi'}{(2\pi)^2}\int_0^1 n'_x dn'_x V(\vec{k}_t, \vec{k}'_t) \times \left[Q(n_x) - \frac{n'_y}{n_y}Q(n'_x)\right] = 1. \qquad (2.16)$$

The second term in the left-hand part of Eq. (2.16) is due to the surface electron relaxation and is of the order of $(\nu_{\rm surf}/\nu)Q$. Note that Eqs. (2.15) and (2.16) are remarkably valid in the linear approximation in $\nu_{\rm surf}/\nu$ even if Eq. (2.14) fails, i.e., at $\nu_{\rm surf} \ll |v_x|/d \ll \nu$. Thus, we can use them independently of Eq. (2.14) without loss of generality.

III. SMALL-SCALE MILDLY SLOPING ASPERITIES

A case of SS mildly sloping asperities is defined by two inequalities:

$$k_F L < 1 \qquad (3.1)$$

and Eq. (2.6).

It is easy to verify that both the Fresnel and Rayleigh parameters are automatically small under conditions (3.1) and (2.6), i.e., relationships (2.7) and

$$k_x\zeta \ll 1 \qquad (3.2)$$

are valid.

Within Eq. (3.2), the exponent in the integrand of Eq. (2.5) can be set to unity. Inequality (3.1) allows substitution $J_1(x) \simeq x/2$ applied at $x < 1$. As a result, the probability density $V(\vec{k}_t, \vec{k}'_t)$ is independent of the angular variables φ, φ' and takes a form

$$V(n_x, n'_x) = 6\pi\alpha(k_F\zeta)^2(k_F L)^2 n_x n'_x,$$
$$\alpha = \frac{4}{3}\int_0^\infty x dx w(x), \qquad (3.3)$$

where $w(x) = W(xL)$ is the correlation function of the dimensionless argument.

According to Eq. (3.3), the kernel $V(\vec{k}_t, \vec{k}'_t)$ is degenerate and isotropic for SS asperities (3.1) and (2.6). The isotropic property sets the incoming term in BC (2.4) to zero. Hence, the integral BC is transformed into local Fuchs' BC (Ref. 1) with the microscopic but not phenomenological specularity parameter ρ, which depends on n_x and the roughness characteristics,

$$1-\rho = \int_0^{2\pi}\frac{d\varphi'}{(2\pi)^2}\int_0^1 n'_x dn'_x V(\vec{k}_t, \vec{k}'_t). \qquad (3.4)$$

Since the diffuseness coefficient $1-\rho$ results from the only nonvanishing outgoing term in the integral of surface collisions, it is proportional to the full cross section of the surface scattering. This situation is similar to the approximation for the full relaxation time in the theory of bulk scattering. So, we can naturally introduce the surface relaxation frequency $\nu_{\rm surf}$, which represents the rate of electron transitions from the $\vec{k}$ state to any other one,

$$\nu_{\rm surf} = (1-\rho)|v_x|/d. \qquad (3.5)$$

The physical meaning of definition (3.5) is rather obvious. As $\nu_{\rm surf}$ equals a number of electron diffuse reflections in a unit time, it is written as the product of the diffuse scattering probability $1-\rho$ at a single impact with the boundary by the frequency $|v_x|/d$ of impacts.

A. Classical conductivity

All the considerations as to BC (2.4) can be directly used to simplify integral equations (2.12) and (2.16). The incoming term of the surface collision integral vanishes, and the equations become algebraic. Substituting the solution of Eq. (2.16) into integral (2.15), we get

$$\frac{\langle\sigma\rangle}{\sigma_0} = \frac{3}{2}\int_0^1 \frac{dn_x(1-n_x^2)}{1+\nu_s(n_x)/\nu}. \tag{3.6}$$

According to Eqs. (3.3) – (3.5), the frequency of the electron-surface scattering at SS asperities is $\nu_{\rm surf} \equiv \nu_s(n_x)$,

$$\nu_s(n_x) = \alpha(k_F\zeta)^2(k_FL)^2(v_F/d)n_x^2. \tag{3.7}$$

Note that inequalities (3.1) and (2.6) imply that $1-\rho \ll 1$. This confirms condition (2.13).

The integral in Eq. (3.6) is simply taken and gives the known result[27]

$$\frac{\langle\sigma\rangle}{\sigma_0} = \frac{3}{2}\left(\frac{\nu}{\nu_s^{\rm max}}\right)^{1/2} \times\left[\left(1+\frac{\nu}{\nu_s^{\rm max}}\right)\arctan\left(\frac{\nu}{\nu_s^{\rm max}}\right)^{-1/2} - \left(\frac{\nu}{\nu_s^{\rm max}}\right)^{1/2}\right]. \tag{3.8}$$

Equation (3.8) determines $\langle\sigma\rangle/\sigma_0$ as the universal function of $\nu/\nu_s^{\rm max}$ with

$$\nu_s^{\rm max} = \nu_s(n_x=1) = \alpha(k_F\zeta)^2(k_FL)^2(v_F/d). \tag{3.9}$$

At $\nu/\nu_s^{\rm max} \gg 1$ (high temperatures or thick plate, or perfect boundary) the conductivity is scarcely affected by the surface relaxation and $\langle\sigma\rangle/\sigma_0$ is described by

$$\frac{\langle\sigma\rangle}{\sigma_0} \simeq 1-\frac{1}{5}\left(\frac{\nu}{\nu_s^{\rm max}}\right)^{-1}, \qquad \nu_s^{\rm max} \ll \nu. \tag{3.10}$$

Otherwise, at $\nu/\nu_s^{\rm max} \ll 1$ (low temperatures or thin plate, or imperfect surface) we come to an asymptotics

$$\frac{\langle\sigma\rangle}{\sigma_0} \simeq \frac{3\pi}{4}\left(\frac{\nu}{\nu_s^{\rm max}}\right)^{1/2}, \qquad \nu \ll \nu_s^{\rm max}. \tag{3.11}$$

The first term in Eq. (3.10) describes the contribution to $\langle\sigma\rangle$ of the electrons that do not interact with the boundaries. The second one accounts for a small group of the electrons [of relative concentration $(1-\rho)\, l/d \sim \nu_s^{\rm max}/\nu \ll 1$] that reside close to the sample surfaces and scatter at them at all $n_x \le 1$. Conductivity (3.11) arises from a small group of sliding electrons with $n_x \lesssim (\nu/\nu_s^{\rm max})^{1/2} \ll 1$, for which $\nu_s(n_x) \sim \nu$ [see Eq. (3.6)]. In other words, at $\nu \ll \nu_s^{\rm max}$, the bulk and surface relaxations comparably contribute to the conductivity. Therefore, from Eq. (3.11) it follows that $\langle\sigma\rangle$ diverges as bulk collisions vanish ($\nu \to 0$).

B. Quantum conductivity

The above results are classical. Yet rather thin films are expected to exhibit the quantum size effect, which is due to quantization of the magnitude n_x of normal component of $\vec{k}/k_F$

$$n_x = \pi n/k_\mu d \simeq n/n_d, \quad n=1,\dots,n_d, \quad n_d = [k_\mu d/\pi], \tag{3.12}$$

where k_μ is a wave number associated with the chemical potential $\mu = \hbar^2k_\mu^2/2m$ of degenerate electron gas, and n_d is a number of electron occupied subbands of the spatial quantization (number of conducting electron channels or propagating electron modes). The square brackets denote an integer part. The quantized electron motion requires the quantum approach to find $\langle\sigma\rangle$. However, the quantum conductivity can be derived in a simple way based on the consistency principle. Replace k_F with k_μ in Eqs. (3.3)–(3.6). Applying the quantization rule (3.12) to Eqs. (3.4) and (3.6), we change the integration over n_x' and n_x for the integration over n. Then we replace the integration over n by the summation. For SS asperities, this yields

$$\frac{\langle\sigma\rangle}{\sigma_i} = \frac{3}{2}\frac{(k_\mu d/\pi)^2}{(k_Fd/\pi)^3}\sum_{n=1}^{n_d}\left[1-\frac{n^2}{(k_\mu d/\pi)^2}\right] \times\left[\frac{\nu}{\nu_s^i} + n^2n_d(n_d+1/2)(n_d+1)\left(\frac{k_Fd}{\pi}\right)^{-6}\right]^{-1}. \tag{3.13}$$

Equation (3.13) must be completed by an equation for $k_\mu = k_\mu(k_F,d)$, which follows from the conservation law for the electron concentration $N \equiv k_F^3/3\pi^2$. Allowing for Eqs. (2.13) and (2.14), we can write the equation in a case of a perfect film ($\nu_{\rm surf} = \nu = 0$) at zero temperature:

$$\frac{3}{2}\left(\frac{k_Fd}{\pi}\right)^{-3}\left(\frac{k_\mu d}{\pi}\right)^2 n_d \times\left[1-\frac{1}{3}(n_d+\tfrac{1}{2})(n_d+1)\left(\frac{k_\mu d}{\pi}\right)^{-2}\right] = 1. \tag{3.14}$$

In line with Eq. (3.14), the curve of $k_\mu d/\pi$ versus k_Fd/π is continuous and monotonously increasing but shows kinks at points $(k_Fd/\pi, k_\mu d/\pi) = (4.5^{1/3},2)$; $(19.5^{1/3},3)$; $(51^{1/3},4)$; $(105^{1/3},5)$; $(187.5^{1/3},6)$; $(304.5^{1/3},7)$; etc. Besides, Eq. (3.14) illustrates an important property of an electron subsystem: the wave number k_μ increases as the film thickness d decreases [$k_\mu/k_F \simeq (k_Fd/\pi)^{-1} \gg 1$], so that $k_\mu d/\pi = n_d = 1$ at $k_Fd/\pi = 0$. Due to this, any, however thin, sample contains at least one conducting electron channel, i.e., always $n_d \ge 1$.

Quantization (3.12) of the electron transverse movement results in the quantization of diffuseness coefficient (3.4) and frequency (3.5) of surface relaxation, $\nu_{\rm surf} \equiv \nu_{\rm sq}(n)$,

$$\nu_{sq}(n) = \alpha(k_F\zeta)^2(k_FL)^2 n_d(n_d+1/2)(n_d+1) \times (k_Fd/\pi)^{-6}(k_Fv_F/\pi)n^2. \quad (3.15)$$

Note that $\nu_{sq} \propto n_d^3$ exhibits a stepwise increase as a function of k_Fd/π (or of $k_\mu d/\pi$) since the number n_d abruptly increases by unity, when electrons start to occupy a new subband.

In contrast to the classical frequency, the quantum frequency ν_{sq} has three characteristic values. The first one, ν_s^i, is of the order of the surface relaxation frequency in a film with the single electron channel ($\nu_{sq} = 3\nu_s^i$ at $k_Fd/\pi = 1$, i.e., at $n = n_d = 1$)

$$\nu_s^i = \alpha(k_F\zeta)^2(k_FL)^2 k_Fv_F/\pi. \quad (3.16)$$

The frequency ν_s^i is completely specified by the material of a conductor and the quality of its surfaces. This frequency is an intrinsic characteristic of a sample. The second value, ν_s^{max}, is reached at $n = n_d = k_Fd/\pi \gg 1$ and coincides with its classical limit (3.9)

$$\nu_s^{max} = \nu_s^i(k_Fd/\pi)^{-1}. \quad (3.17)$$

The third characteristic value is the nonzero minimal frequency ν_s^{min} of the surface scattering. It corresponds to $n = 1$ at $n_d = k_Fd/\pi \gg 1$

$$\nu_s^{min} = \nu_s^i(k_Fd/\pi)^{-3}. \quad (3.18)$$

The introduced frequencies are inter-related in the following way:

$$\nu_s^{min} \le \nu_s^{max} \le \nu_s^i, \qquad k_Fd/\pi \ge 1. \quad (3.19)$$

Unlike classical conductivity (3.8), the quantum one (3.13) is a function of two independent parameters: ν/ν_s^i and k_Fd/π. So, it can be suitably normalized to the value σ_i,

$$\sigma_i = Ne^2/m\nu_s^i = \sigma_0\nu/\nu_s^i, \quad (3.20)$$

which depends on neither the sample thickness d nor the mean free path l of electrons.

Formula (3.13) generalizes the result[18] and has a wider region of validity (3.1) and (2.6). In general, it was proven from rigorous quantum calculations (based on Kubo's formalism) using the diagram technique from Refs. 26, 36, and 37 within a usual ladder approximation for two-particle Green's function. In this paper, we gave the elementary and physically clear derivation of Eq. (3.13) to demonstrate a relation between quantum and classical results. Both methods rely on inequalities (2.13) and (2.14). These conditions imply that broadenings $\hbar\nu_{surf}/2$ and $\hbar\nu/2$ of electron energy levels due to surface and bulk collisions are small compared to the distance $\sim \hbar|v_x|/2d$ between the neighboring levels. This allows us to regard the discrete number n as a good quantum number and quantization rule (3.12) as the acceptable leading approximation. At the same time, the randomly rough surface can be reduced to the asperity-averaged plane, which scatters electrons with the frequency $\nu_{surf} \equiv \nu_{sq}(n)$. Since spectrum (3.12) is equidistant and the electron wave function is rather simple, the application of the consistency principle gives an exact quantum relationship (3.13) but not a quasiclassical one as it usually occurs in more complex quantum systems.

C. Ultraquantum limit (2D electron gas)

In the ultraquantum limit, when there exists the single propagating electron mode in the film ($n_d = 1$), the average conductivity, according to Eqs. (3.13) and (3.14), takes the form

$$\langle\sigma\rangle = Ne^2/m[\nu + 3\nu_s^i(k_Fd/\pi)^{-6}],$$

$$0 \le (k_Fd/\pi)^3 < 9/2. \quad (3.21)$$

In the absence of bulk collisions ($\nu = 0$), this formula ensures the maximal quantum residual resistivity and coincides with that from Ref. 26.

Since wave number k_μ diverges ($k_\mu \simeq \pi/d$) at $k_Fd/\pi \to 0$, the case with $n_d = 1$ is a distinctive one: a situation is possible, when $k_Fd/\pi \ll k_\mu d/\pi$ and one needs to have $d \gg L$ to use Eqs. (3.13) and (3.21). Actually, quantum conductivity (3.13) was derived within Eq. (3.1), in which, rigorously speaking, k_F had to be replaced by k_μ. Obviously, in the regime $k_Fd/\pi \ll 1$, the condition $k_\mu L < 1$ holds only if $L \ll d$. Yet if $k_Fd/\pi \ll 1$ and $L \ll d$ holds, surface asperities are always SS, i.e., $k_FL < 1$. Note that the condition $L \ll d$ for films with SS defects is violated just at $k_Fd/\pi \ll k_FL < 1$ and holds even at $k_Fd/\pi \ge 1$.

D. Asymptotics of the quantum conductivity. Dependence on the variable ν/ν_s^i

The conductivity of thin films is usually[4] measured as a function of the temperature (of the mean free path l or the bulk scattering frequency ν). So, let us analyze $\langle\sigma\rangle$ (3.13) versus ν/ν_s^i at a fixed value of the parameter $k_Fd/\pi \ge (9/2)^{1/3}$ ($k_\mu d/\pi \ge n_d \ge 2$).

In the region of relatively high temperatures, where

$$\nu_s^{max} \ll \nu, \quad (3.22)$$

the bulk relaxation mechanism dominates and conductivity (3.13) obeys an asymptotics

$$\frac{\langle\sigma\rangle - \sigma_0}{\sigma_i} \simeq -\frac{1}{2}n_d^2(n_d+1/2)^2(n_d+1)^2\frac{(k_\mu d/\pi)^2}{(k_Fd/\pi)^9} \times \left[1 - \frac{3n_d^2+3n_d-1}{5\,(k_\mu d/\pi)^2}\right]\left(\frac{\nu}{\nu_s^i}\right)^{-2},$$

$$(k_Fd/\pi)^{-1} \ll \nu/\nu_s^i. \quad (3.23)$$

If the temperature is decreased, the frequency ν falls into an interval

$$\nu_s^{min} < \nu < \nu_s^{max}. \quad (3.24)$$

Here the bulk and surface scatterings comparably form $\langle\sigma\rangle$ and we get

$$\frac{\langle\sigma\rangle}{\sigma_i} \simeq \frac{3\pi}{4}\left[n_d(n_d+1/2)(n_d+1)\right]^{-1/2} \times \left(\frac{k_\mu d}{\pi}\right)^2 \left(\frac{\nu}{\nu_s^i}\right)^{-1/2}, \qquad (3.25)$$

$$(k_F d/\pi)^{-3} < \nu/\nu_s^i < (k_F d/\pi)^{-1}.$$

Note that the frequencies $\nu_s^{\min}$, $\nu_s^{\max}$, and ν_s^i are of the same order for "quantum" films with $n_d \simeq k_\mu d/\pi \gtrsim 2$. Therefore, range (3.24) might be too narrow for realization of Eq. (3.25).

Finally, when ν becomes the smallest of all the other frequencies,

$$\nu \ll \nu_s^{\min}, \qquad (3.26)$$

the surface scattering dominates, and the average conductivity is described by

$$\langle\sigma\rangle \simeq \sigma_{\rm res} - B(\nu/\nu_s^i)\sigma_i, \qquad \nu/\nu_s^i \ll (k_F d/\pi)^{-3}. \quad (3.27)$$

As bulk collisions vanish ($\nu \to 0$), quantum conductivity (3.27), unlike classical conductivity (3.8), sets a natural limit for $\langle\sigma\rangle$

$$\frac{\sigma_{\rm res}}{\sigma_i} = \frac{\pi^2}{4}\frac{(k_F d/\pi)^3 (k_\mu d/\pi)^2}{n_d(n_d+1/2)(n_d+1)} \times \left[1 - \frac{6}{\pi^2} n_d \left(\frac{k_\mu d}{\pi}\right)^{-2} - \frac{6}{\pi^2}\sum_{n=1}^{\infty}(n+n_d)^{-2}\right]. \qquad (3.28)$$

It is determined by the number of conducting channels (by the thickness d) and by the frequency ν_s^i (by defects of the film sides). The coefficient $B \sim (k_F d/\pi)^5$ in Eq. (3.27) is

$$B = \frac{\pi^4}{60}\frac{(k_F d/\pi)^9 (k_\mu d/\pi)^2}{n_d^2(n_d+1/2)^2(n_d+1)^2} \times \left\{1 - \frac{15}{\pi^2}\left(\frac{k_\mu d}{\pi}\right)^{-2} + \frac{90}{\pi^4}\sum_{n=1}^{\infty}(n+n_d)^{-4} \times \left[\left(\frac{n+n_d}{k_\mu d/\pi}\right)^2 - 1\right]\right\}. \qquad (3.29)$$

In the quasiclassical situation, when $k_\mu d/\pi \simeq n_d \simeq k_F d/\pi \gg 1$, Eq. (3.23) transforms into classical asymptotics (3.10). The condition $n_d \gg 1$ makes inequalities (3.19) strong ($\nu_s^{\min} \ll \nu_s^{\max} \ll \nu_s^i$) and frequency interval (3.24) reasonably wide. Here quantum asymptotics (3.25) coincides with Eq. (3.11). Thus, in the temperature range, where

$$\nu_s^{\min} \ll \nu,$$

i.e., (3.30)

$$(k_F d/\pi)^{-3} \ll \nu/\nu_s^i \quad \text{at} \quad k_F d/\pi \gg 1,$$

the film conductivity can be adequately described by the classical theory. Actually, when Eq. (3.30) holds, quantization of $\nu_{\rm surf}$ yielding the nonzero $\nu_s^{\min}$, is insignificant. Otherwise, at extremely low temperatures (3.26) the discrete spectrum of $\nu_{\rm sq}$ is important even for quasiclassical plates. At $n_d \gg 1$, quantum equation (3.27) transforms into the quasiclassical one

$$\frac{\langle\sigma\rangle}{\sigma_i} \simeq \frac{\pi^2}{4}\left(\frac{k_F d}{\pi}\right)^2 \left[1 - \frac{\pi^2}{15}\left(\frac{k_F d}{\pi}\right)^3 \frac{\nu}{\nu_s^i}\right],$$

$$\nu/\nu_s^i \ll (k_F d/\pi)^{-3} \ll 1, \quad (3.31)$$

which has no classical analog. Whereas classical asymptotics (3.11) arises from a small electron group with $n_x \lesssim (\nu/\nu_s^{\max})^{1/2} \ll 1$, quasiclassical equation (3.31) is formed by the states with $n \sim 1$, i.e., $n_x \sim n_d^{-1} \gg (\nu/\nu_s^{\max})^{1/2}$. For these states, the surface scattering dominates.

Figure 2 shows plots of Eq. (3.13) versus ν/ν_s^i for fixed values of $k_F d/\pi$. All the curves have no peculiarities and monotonously fall off as ν/ν_s^i increases. They start from the value of $\sigma_{\rm res}/\sigma_i$ (3.28). The more $k_F d/\pi$, the more the quantum residual conductivity $\sigma_{\rm res}$. Then according to Eq. (3.27), the curves change linearly with ν/ν_s^i and asymptotically approach a dashed line $\sigma_0/\sigma_i = (\nu/\nu_s^i)^{-1}$. We stress that the distinction of residual conductivity (3.28) from the first term in Eq. (3.31) is essential only for curves 1 and 2 with $k_F d/\pi = 1$ and $k_F d/\pi = 2$, respectively. For the rest of the curves, the quasiclassical condition ($k_F d/\pi \gg 1$) can be considered as met. This means that even at a small number of electron states ($n_d \geq 5$), the film conductivity is adequately described by either classical formula (3.8) or quasiclassical asymptotics (3.31) in the applicable ν/ν_s^i intervals. For example, for curve 3 such intervals are $1.5 \times 10^{-2} \lesssim \nu/\nu_s^i$ and $\nu/\nu_s^i \lesssim 7 \times 10^{-4}$, respectively. Curve 4 presents the conductivity of a sample with ten propagating electron modes. It obeys Eq. (3.31) at $\nu/\nu_s^i \lesssim 2 \times 10^{-4}$ and then, after a wide intermediate region, Eq. (3.10) at $\nu/\nu_s^i \gtrsim 4 \times 10^{-2}$. Note that intermediate asymptotics (3.25) does not exhibit on curves 1 and 2, and its classical

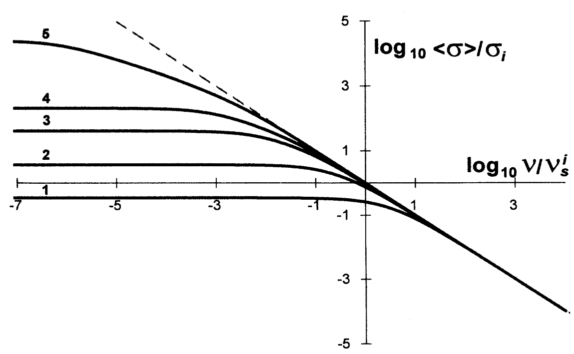

FIG. 2. Quantum conductivity (3.13) vs ν/ν_s^i at fixed $k_F d/\pi$. (1) $k_F d/\pi = 1$; (2) $k_F d/\pi = 2$; (3) $k_F d/\pi = 5$; (4) $k_F d/\pi = 10$; (5) $k_F d/\pi = 100$. The dashed line describes the bulk sample conductivity σ_0.

analog (3.11) does not exhibit on curves 3 and 4. This implies that the square-root dependence $\langle\sigma\rangle \propto \nu^{-1/2}$ can be unlikely experimentally observed in relatively thin plates with $n_d \lesssim 10$.

It is important that the interval for the existence of quasiclassical asymptotics (3.31) gets rapidly narrow as n_d increases. In addition, the maximal value of ν_s^i is about 10^{14} s^{-1} (at $k_F \sim 10^8$ cm^{-1} and $k_F\zeta \sim k_F L \lesssim 1$). Consequently, the extremely low frequencies ν needed to realize Eq. (3.31) cannot be apparently reached for any, however perfect, samples with $n_d \gtrsim 400$ at any, however low, temperatures. Therefore, in actual practice the conductivity of sufficiently thick (but still microscopically thin) plates is described by classical Eq. (3.8).

E. Dependence of the conductivity on the parameter $k_F d/\pi$

Let us examine $\langle\sigma\rangle$ (3.13) as a function of $k_F d/\pi$ at fixed ν/ν_s^i (at constant temperature). We begin from relatively high temperatures (or dirty samples), when

$$\nu_s^i \ll \nu. \tag{3.32}$$

Here the surface scattering prevails in ultraquantum films ($n_d = 1$) only, for which the inequality $(k_F d/\pi)^6 \ll (\nu/\nu_s^i)^{-1} \ll 1$ [see Eq. (3.21)] holds. Violating this inequality, the bulk scattering becomes dominating and the conductivity obeys quantum asymptotics (3.23). In a region $k_F d/\pi \gg 1$, formula (3.23) transforms into classical expression (3.10).

At sufficiently low temperatures (or in perfect conductors), when the condition

$$\nu \ll \nu_s^i \tag{3.33}$$

is met, the electron-surface scattering forms $\langle\sigma\rangle$ in relatively thin specimens, for which

$$k_F d/\pi \ll (\nu/\nu_s^i)^{-1/3}. \tag{3.34}$$

In this case, we can use quantum equation (3.27), which is reduced to quasiclassical equation (3.31) at $1 \ll k_F d/\pi \ll (\nu/\nu_s^i)^{-1/3}$. The contributions of both scattering mechanisms compete at

$$(\nu/\nu_s^i)^{-1/3} \ll k_F d/\pi \ll (\nu/\nu_s^i)^{-1}. \tag{3.35}$$

Here the average conductivity $\langle\sigma\rangle$ may be described by quantum asymptotics (3.25) [or classical one (3.11) at $k_F d/\pi \gg 1$]. Proceeding the increase of $k_F d/\pi$, we get in a region

$$1 \ll (\nu/\nu_s^i)^{-1} \ll k_F d/\pi, \tag{3.36}$$

where the bulk scattering dominates and $\langle\sigma\rangle$ obeys classical expression (3.10).

Figure 3 presents plots of Eq. (3.13) versus $k_F d/\pi$ at fixed values of ν/ν_s^i. All the curves exhibit peculiarities (sharp dips) at points where $k_\mu d/\pi$ is an integer, i.e., when a new propagating electron mode opens. Such behavior is caused by the stepwise dependence of $\nu_{\rm sq}$

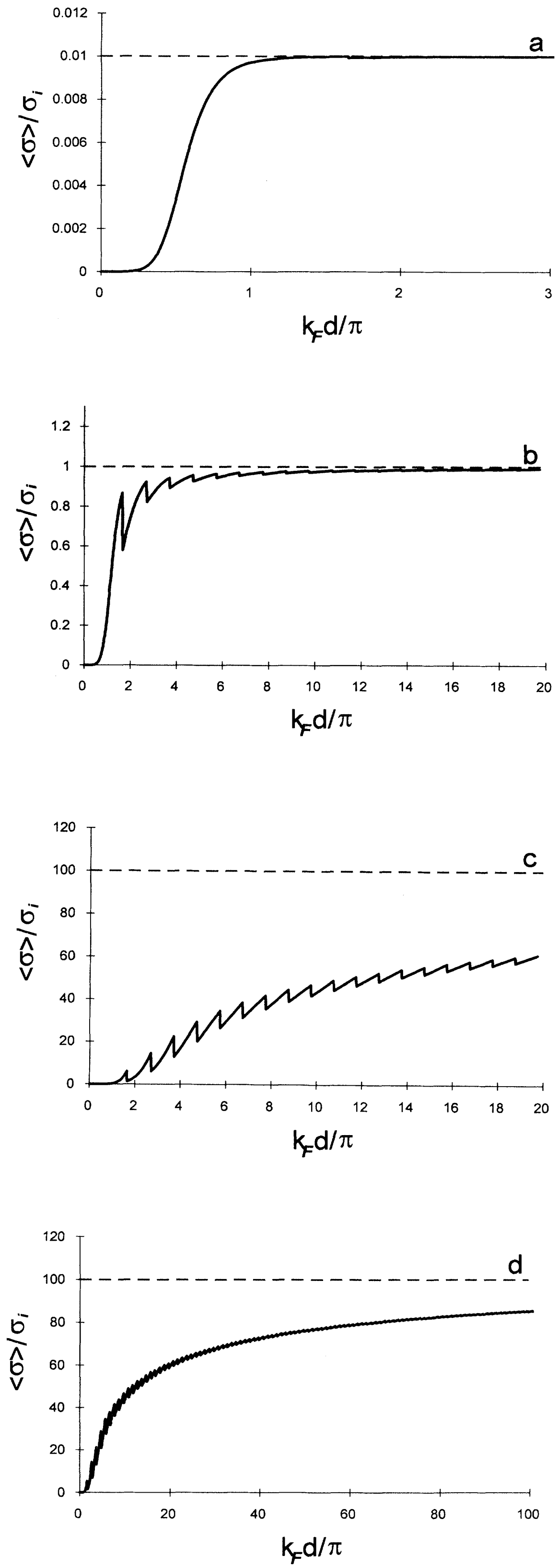

FIG. 3. Quantum conductivity (3.13) vs $k_F d/\pi$ at fixed ν/ν_s^i. (a) $\nu/\nu_s^i = 10^2$; (b) $\nu/\nu_s^i = 1$; (c), (d) $\nu/\nu_s^i = 10^{-2}$. The dashed line describes the bulk sample conductivity σ_0.

on k_Fd/π. Similar dips were observed in the numerical simulation[38] of the quantum transport through a narrow 2D strip with a randomly rough boundary. The peculiarities of the conductivity or the conductance at points of integer $k_\mu d/\pi$ have fundamental quantum origin and occur in all quantized systems. In particular, they were revealed in 2D electron waveguides[39–41] and at the quantum-Hall effect.[42,43] As increasing k_Fd/π, the relative depth of the quantum dips decreases and the curves get smoother. Figure 3(a) corresponds to a region of sufficiently high temperatures (or dirty samples) (3.32). In this region, the conductivity "saturates" ($\langle\sigma\rangle \approx \sigma_0$) even in ultraquantum films with $k_Fd/\pi \lesssim 1$. Quantum equation (3.23) describing this curve transforms into classical equation (3.10) at $k_Fd/\pi \gtrsim 2$. The dips in Fig. 3(a) are due to the right-hand side of Eq. (3.23). Their relative magnitude is $(\nu/\nu_s^i)^{-1} = 10^{-2}$. Therefore, only one dip at $k_\mu d/\pi = n_d = 2$ is seen on the plot. Decreasing the temperature (or improving samples), the competition between the frequencies ν and ν_s^i is achieved. The curve in Fig. 3(b) illustrates the case $\nu = \nu_s^i$. It becomes quite smooth and almost coincides with the dashed line $\langle\sigma\rangle = \sigma_0$ even in films with fifteen conducting channels. Such dependencies are intermediate at the crossover from high (3.32) to low (3.33) temperatures. Figures 3(c) and 3(d) present $\langle\sigma\rangle$ at low temperatures and for perfect samples (3.33). Here interval (3.34) of the surface scattering domination is $k_Fd/\pi \lesssim 6$, within which the conductivity is given by quantum formula (3.28). The interval $6 \lesssim k_Fd/\pi \lesssim 100$ in Fig. 3(d) corresponds to range (3.35), where $\langle\sigma\rangle$ is expected to obey either quantum asymptotics (3.25) or classical one (3.11). However, these formulas lead to essentially over-read values of $\langle\sigma\rangle/\sigma_i$. Thus, at $\nu/\nu_s^i = 10^{-2}$, interval (3.35) is too narrow for the square-root behavior of Eqs. (3.25) and (3.11) to exhibit. Then, at $k_Fd/\pi \gtrsim 100$, the bulk scattering becomes dominating, and the plot is described by Eq. (3.10). A portion of the curve obeying Eq. (3.10) is not presented in Fig. 3(d) because of too large k_Fd/π in the range of realization for this asymptotics.

It should be emphasized that when deriving formula (3.13), we used a constant frequency model for electron bulk collisions (ν = const). Due to this, all the peculiarities of $\langle\sigma\rangle$ versus k_Fd/π are associated with $\nu_{\rm surf}$. Within more realistic models allowing for electron-impurity, electron-phonon, and other interactions, the frequency ν depends on the density of quantized electron states, which is proportional to n_d. This results in additional peculiarities of $\langle\sigma\rangle$ caused by the size quantization of ν. However, we can state that such peculiarities produced by isotropic bulk scatterers are manifested weaker than the dips caused by $\nu_{\rm surf}$. In Ref. 44, it was shown that $\nu \propto n_d$, when electrons were elastically scattered by randomly distributed impurities with δ potential. Yet $\nu_{\rm surf} \equiv \nu_{\rm sq}$ (3.15) is proportional to n_d^3. So, allowing for the quantization of ν, we will obtain relatively weak peculiarities on the curves of Figs. 3(a) and 3(b) only, where the bulk scattering prevails. In other words, the quantum effects in the film conductivity are mainly connected with the electron scattering at boundary defects.

In conclusion, we note that the quantum peculiarities revealed herein are degenerated into the weak Shubnikov–de Haas oscillations in massive plates at high temperatures (3.30), when the leading approximation of the conductivity is given by the classical formula (3.8).

IV. LARGE-SCALE MILDLY SLOPING ASPERITIES

A case of LS mildly sloping asperities is defined by inequalities

$$k_FL \gg 1 \tag{4.1}$$

and Eq. (2.7).

Unlike the case of SS asperities (3.1) and (2.6), the smallness of the Fresnel parameter (2.7) does not follow from Eqs. (4.1) and (2.6). However, if Eqs. (4.1) and (2.7) hold, requirement (2.6) is met.

The situation with LS irregularities is more complex to analyze, since the problem of correlation between successive electron collisions with surface defects arises. This problem arises any time, when particles multiply scatter at a boundary due to periodical motion. If the distance $(n_t/n_x)d$ passed by an electron along the sample sides between neighboring impacts (in the half-period $d/|v_x|$) is less than L, the electron is subject to several collisions with the surface along the correlation radius L. Hence, the successive reflections are not statistically independent, but strongly correlated. In this case the BC (2.4) and (2.5) is not valid, because it was derived in the single-scattering approximation. The usage of Eqs. (2.4) and (2.5) in systems with periodical motion is reduced to the trivial summation over single-scattering events and gives correct results in the absence of the correlation, when

$$L \ll (n_t/n_x)d,$$

i.e., (4.2)

$$(1 + d^2/L^2)^{-1} \ll 1 - n_x^2.$$

From here on we suppose the mean length L of surface defects (a microscopic parameter) and the plate thickness d (a macroscopic parameter) to be related by a reasonable inequality

$$L \ll d. \tag{4.3}$$

Within Eq. (4.3), range (4.2) for the n_x variation is of the maximal width and correlations are strong for an anomalously small electron group with $n_t^2 \ll (L/d)^2$ only. This group will be seen below not to contribute appreciably in the film conductivity. Therefore, condition (4.2) should be regarded as a consequence of Eq. (4.3) for all electrons with $n_x \le 1$.

Relationships (4.1) and (4.3) can be combined into the double inequality

$$1 \ll k_FL \ll k_Fd/\pi \simeq n_d. \tag{4.4}$$

It is obvious from Eq. (4.4) that for LS asperities sufficient condition (4.3) for the absence of correlations results in the quasiclassical requirement. Hence, using BC (2.4) [and consequently integral equation (2.16)] in this section, we are beyond the quantum approach.

For the further analysis, we introduce the characteristic change Φ of n_x as an electron scatters at LS boundary asperities. From probability density $V(\vec{k}_t, \vec{k}_t')$ (2.5), we have

$$\Phi(n_x) \simeq \arccos[n_t - (1 + k_x\zeta)/k_F L] - \arcsin n_x \leq \Phi_{\max} = (2/k_F L)^{1/2}. \quad (4.5)$$

It is easy to verify that according to Eq. (4.4), the following inequality holds:

$$(k_F d/\pi)^{-1} \ll \Phi \ll 1. \quad (4.6)$$

The problem of correlations at the multiple surface scattering is discussed in Ref. 9 in more detail. The classical BC including the correlations was first derived in Ref. 13.

A. Flat and steep impingement of electrons. Classical conductivity

Consider integral equation (2.16). If the surface electron scattering is weaker than the bulk one, we can solve Eq. (2.16) by iterations in the small integral of surface collisions. In the opposite case, Eq. (2.16) is solved for two cases only: flat and steep impingement of electrons on the plate boundary.

The electrons are said to be flatly impinging on the surface, if n_x is less than the width Φ of the scattering indicatrix,

$$n_x \ll \Phi(n_x),$$

i.e., (4.7)

$$n_x^2 \ll (k_F L)^{-1} \ll 1.$$

According to Eqs. (2.7) and (4.7), the Rayleigh parameter $k_x\zeta$ is small. So, the exponent in the integrand of Eq. (2.5) can be set to unity and $V(\vec{k}_t, \vec{k}_t')$ is given by the Born approximation

$$V(\vec{n}_t, \vec{n}_t') = 8\pi(k_F\zeta)^2(k_F L)^2 n_x n_x' \times \int_0^\infty x\,dx\,w(x) J_0(k_F L x|\vec{n}_t' - \vec{n}_t|). \quad (4.8)$$

When inequality (4.7) holds, the factor $Q(n_x')$ in Eq. (2.16) turns out to be a sharper function of n_x' than the kernel $V(\vec{k}_t, \vec{k}_t')$. Therefore, the effective region of integration over n_x' in the incoming term is significantly less than in the outgoing one. Hence, "income" can be neglected. This allows us to reasonably introduce specularity parameter ρ (3.4) and frequency ν_{surf} (3.5) as well as for SS asperities. We substitute Eq. (4.8) into Eq. (3.4) and perform the asymptotical integration over φ' and n_x'. After this, we get the expression for the frequency $\nu_{\text{surf}} \equiv \nu_{lf}(n_x)$ of the flatly impinging electrons [cf. Eq. (3.7)]

$$\nu_{lf}(n_x) = \beta(k_F\zeta)^2(k_F L)^{-1/2}(v_F/d)n_x^2,$$
$$\beta = \frac{\Gamma^2(3/4)}{\pi^2\Gamma(3/2)} \int_0^\infty x^{3/2}\,dx\,w(x), \quad (4.9)$$

where $\Gamma(x)$ is the Euler gamma function.

Neglecting "income," we reduce integral equation (2.16) to the algebraic one,

$$Q(n_x)[1 + \nu_{lf}(n_x)/\nu] = 1, \qquad n_x^2 \ll (k_F L)^{-1} \ll 1. \quad (4.10)$$

The electrons are said to be steeply impinging on the surface, if n_x obeys the condition

$$\Phi(n_x) \ll n_x,$$

i.e., (4.11)

$$(k_F L)^{-1} \ll n_x^2 \leq 1.$$

When Eq. (4.11) is met, the indicatrix $V(\vec{k}_t, \vec{k}_t')$ in Eq. (2.16) is a sharper function of n_x' than $Q(n_x')$. So, the outgoing and incoming terms almost compensate each other. In other words, Eq. (4.11) implies small wave vector transfer at scattering, and the integral of surface collisions can be written within the Fokker-Planck approximation[45] by expanding it in powers of $\vec{n}_t' - \vec{n}_t$ up to quadratic terms. Integral equation (2.16) takes the differential form[14]

$$Q(n_x) - 2|w''(0)|(\zeta/L)^2 \frac{l}{d}\frac{n_x}{n_y}[n_x^2\nabla^2 - 2\vec{n}_t\nabla] \times n_y Q(n_x) = 1. \quad (4.12)$$

Here ∇ and ∇^2 are a two-dimensional gradient and Laplacian in $\vec{n}_t$ space.

Using relationships (2.11), let us rewrite Eq. (4.12) in the variables n_x, φ. Since Q depends only on n_x but not on φ, partial differential equation (4.12) becomes ordinary,

$$Q - 2|w''(0)|(\zeta/L)^2 \frac{l}{d} n_x \times \left[(1 - n_x^2)\frac{d^2Q}{dn_x^2} + \frac{1 - 5n_x^2}{n_x}\frac{dQ}{dn_x} - 2Q\right] = 1. \quad (4.13)$$

This equation must be completed by two BC's. They are obtained from original integral relationship (2.16): the function $Q(n_x)$ should be regular and equal to unity at $n_x = 0$.

The general solution of Eq. (4.13) is expressed via not completely studied Heun's functions.[46] Fortunately, Eq. (4.13) allows a simple asymptotical analysis at the weak and strong surface scattering. If the bulk electron relaxation dominates, the second term in the left-hand part is small and can be included as a perturbation. In the opposite situation, the first term can be neglected and Q is presented as $[2|w''(0)|(\zeta/L)^2(l/d)]^{-1}$ multiplied by the unity-order function. In the subsequent discussion, we shall also use a qualitative solution of Eq. (4.13) based on reasonable physical considerations. For this purpose, replace the differential operator in Eq. (4.13) with a multiplication one of the same order of magnitude, i.e., represent the second term in the left-hand part as $(\nu_{\rm surf}/\nu)Q$. In doing so, the effective frequency $\nu_{\rm surf} \equiv \nu_{ls}(n_x)$ for the steeply impinging electrons takes the form

$$\nu_{ls}(n_x) = 2|w''(0)|(\zeta/L)^2(v_F/d)n_x^{-1}. \qquad (4.14)$$

After the replacing procedure, Eq. (4.13) becomes algebraic,

$$Q(n_x)[1+\nu_{ls}(n_x)/\nu] = 1, \qquad (k_FL)^{-1} \ll n_x^2 \le 1. \qquad (4.15)$$

The solution of this equation does not satisfy a BC $Q(n_x = 0) = 1$, but it is of the same order as the exact solution of Eq. (4.13) at $n_x \gtrsim (k_FL)^{-1/2}$. Besides, in the intermediate region $n_x \sim (k_FL)^{-1/2}$ the frequency $\nu_{ls}(n_x)$ is of the order of $\nu_{lf}(n_x)$ (4.9). This ensures the "joint" of the solutions of Eqs. (4.15) and (4.10) for the steep and flat impingement.

Equations (4.10) and (4.15) allow us to write $\langle\sigma\rangle$ (2.15) in the usual form [cf. Eq. (3.6)]

$$\frac{\langle\sigma\rangle}{\sigma_0} = \frac{3}{2}\int_0^1 \frac{dn_x(1-n_x^2)}{1+\nu_l(n_x)/\nu}. \qquad (4.16)$$

Here $\nu_l(n_x)$ is the frequency of surface relaxation as electrons scatter at LS asperities,

$$\nu_l(n_x) = \begin{cases} \nu_{lf}(n_x), & n_x < (k_FL)^{-1/2} \\ \nu_{ls}(n_x), & (k_FL)^{-1/2} < n_x \le 1. \end{cases} \qquad (4.17)$$

Unlike $\nu_s(n_x)$ (3.7) for SS irregularities, the frequency $\nu_l(n_x)$ is a nonmonotonous function of n_x. It starts from the minimal frequency $\nu_{lf}^{\rm min}$ for the flatly impinging electrons, which corresponds to the nonzero lower quantum limit $n_x^{\rm min} = (k_Fd/\pi)^{-1}$,

$$\nu_{lf}^{\rm min} = \beta(k_F\zeta)^2(k_FL)^{-1/2}(k_Fd/\pi)^{-2}(v_F/d). \qquad (4.18)$$

Then the value of ν_l increases as n_x^2 according to Eq. (4.9). At $n_x \sim (k_FL)^{-1/2}$, this increase terminates and ν_l reaches the maximum, which is of the order of

$$\nu_l^{\rm max} = (k_F\zeta)^2(k_FL)^{-3/2}(v_F/d). \qquad (4.19)$$

After the crossover from the flat impingement to the steep one, ν_l decreases as n_x^{-1} [see Eq. (4.14)]. Finally, when the electrons fall onto the boundary vertically ($n_x = 1$), the value of $\nu_l(n_x)$ coincides with the minimal frequency $\nu_{ls}^{\rm min}$ for the steeply impinging electrons,

$$\nu_{ls}^{\rm min} = 2|w''(0)|(\zeta/L)^2(v_F/d). \qquad (4.20)$$

Within condition (4.4) the characteristic values $\nu_{lf}^{\rm min}$, $\nu_{ls}^{\rm min}$, and $\nu_l^{\rm max}$ are interrelated as

$$\nu_{lf}^{\rm min} \ll \nu_{ls}^{\rm min} \ll \nu_l^{\rm max}. \qquad (4.21)$$

B. Asymptotics of the conductivity. Quasiclassical and classical residual conductivity

If the bulk mechanism of electron relaxation prevails over the surface one, i.e.,

$$\nu_{ls}^{\rm min} \ll \nu, \qquad (4.22)$$

Eq. (2.16) can be solved through successive approximations in the small integral of surface collisions. According to Eqs. (2.5), (2.15), (4.1), and (2.7), we get

$$\frac{\langle\sigma\rangle}{\sigma_0} \simeq 1 - \frac{3}{4}\left(\frac{\nu}{\nu_{ls}^{\rm min}}\right)^{-1}. \qquad (4.23)$$

Conductivity (4.23) is formed by steeply impinging electrons (4.11), for which the surface scattering is relatively weak. Hence, this result can be also derived from differential equation (4.13). The same order diffuse correction to σ_0 follows from model formula (4.16) as well.

Unexpected results are obtained, when investigating $\langle\sigma\rangle$ at low temperatures, where

$$\nu \ll \nu_{ls}^{\rm min}. \qquad (4.24)$$

Here a new characteristic frequency $\nu_l^{\rm eq}$ arises,

$$\nu_l^{\rm eq} = 16\pi^2|w''(0)|^2(k_F\zeta)^2(k_FL)^{-7/2}(v_F/d) \ll \nu_{ls}^{\rm min}. \qquad (4.25)$$

Depending on the relation between frequencies $\nu_l^{\rm eq}$ and $\nu_{lf}^{\rm min}$ (4.18), the conductivity $\langle\sigma\rangle$ as a function of d/l exhibits various types of behavior.

For sufficiently thick (nearly classical) plates,

$$1 \ll (k_FL)^{3/2} \ll k_Fd/\pi,$$

i.e., (4.26)

$$\nu_{lf}^{\rm min} \ll \nu_l^{\rm eq} \ll \nu_{ls}^{\rm min}.$$

In this case within the frequency interval $\nu_l^{\rm eq} \ll \nu \ll \nu_{ls}^{\rm min}$, the conductivity is mainly formed by the steeply impinging electrons with $n_x \sim 1$, for which the surface scattering dominates. The corresponding asymptotics of Eq. (4.16) is

$$\frac{\langle\sigma\rangle}{\sigma_0} \simeq \frac{3}{8}\frac{\nu}{\nu_{ls}^{\rm min}}, \qquad \nu_l^{\rm eq} \ll \nu \ll \nu_{ls}^{\rm min}. \qquad (4.27)$$

The same formula can be also derived from differential equation (4.13).

At $\nu \sim \nu_l^{\rm eq}$, the contributions to $\langle\sigma\rangle$ of both electron groups compete, and in the region $\nu \ll \nu_l^{\rm eq}$ the flatly impinging electrons play the decisive role in the conductivity. When the temperature is not too low, the square-root asymptotics is applicable [cf. Eq. (3.11)],

$$\frac{\langle\sigma\rangle}{\sigma_0} \simeq \frac{3\pi}{4}(k_F L)^{-1/2}\left(\frac{\nu}{\nu_l^{\max}}\right)^{1/2}, \qquad \nu_{lf}^{\min} \ll \nu \ll \nu_l^{\rm eq}. \tag{4.28}$$

At extremely low temperatures, where $\nu \ll \nu_{lf}^{\min}$, the quantization of ν_{lf} is essential and results in the nonzero $\nu_{lf}^{\min}$ (see Sec. III). Therefore, $\langle\sigma\rangle$ is described by the first term of quasiclassical formula (3.31), in which $\nu_s^{\min}$ should be replaced by $\nu_{lf}^{\min}$ [i.e., ν_s^i should be replaced by $(k_F d/\pi)^3 \nu_{lf}^{\min}$]:

$$\frac{\langle\sigma\rangle}{\sigma_0} \simeq \frac{\pi^2}{4}\left(\frac{k_F d}{\pi}\right)^{-1}\frac{\nu}{\nu_{lf}^{\min}}, \qquad \nu \ll \nu_{lf}^{\min}. \tag{4.29}$$

Thus, in sufficiently thick samples (4.26) at $\nu = 0$, the residual conductivity is given by Eq. (4.29) and associated with the surface scattering of the flatly impinging electrons.

The inequality from Eq. (4.29) sets an upper limit on $k_F d/\pi$ and Eq. (4.26) sets its lower limit. To satisfy Eqs. (4.26) and (4.29) simultaneously, the condition must hold

$$(k_F L)^3(\zeta/L)^{-2} \ll k_F l/\pi. \tag{4.30}$$

This requirement to the parameter $k_F l/\pi$ is rather stringent because the left-hand part of Eq. (4.30) is a product of the two large parameters. This suggests that residual conductivity (4.29) can be apparently observed in films with not too large values of $(\zeta/L)^{-1}$ and $k_F L$. For example, for specimens with $k_F \sim 10^8$ cm^{-1}, $k_F L \sim 10$, $\zeta/L \sim 10^{-2}$, asymptotics (4.29) can be realized in perfect plates with $l \gtrsim 10^{-1}$ cm.

Note that $\langle\sigma\rangle$ from Eqs. (4.27) and (4.29) is independent of ν. Corrections to them, which would specify such dependence, can be due to either the flatly or steeply impinging electrons.

For relatively thin (quasiclassical) samples

$$k_F L \ll k_F d/\pi \ll (k_F L)^{3/2}, \tag{4.31}$$

i.e.,

$$\nu_l^{\rm eq} \ll \nu_{lf}^{\min}.$$

In such plates at any frequency ν of bulk scattering, $\langle\sigma\rangle$ is mainly formed by the steeply impinging electrons. So, within whole region (4.24), conductivity is described by Eq. (4.27).

It should be emphasized that unlike Eq. (4.29), residual conductivity (4.27) has the classical origin. Actually, steeply impinging electrons (4.11) scatter at surface at any n_x. This ensures the minimal frequency $\nu_{ls}^{\min}$ even in the classical approach. At the same time, under the classical consideration, there always exist such flatly impinging electrons (4.7) that do not scatter at the plate sides ($\nu_{lf} = 0$). They give the infinite conductivity in the absence of bulk relaxation. Just the quantum (or quasiclassical) approach forbids such electron states and sets up the lower limit $n_x^{\min} = (k_F d/\pi)^{-1}$ thus causing nonzero $\nu_{lf}^{\min}$.

C. Quasiclassical model. Graphical study of $\langle\sigma\rangle$ vs d/l

Figure 4 presents plots of

$$\frac{\langle\sigma\rangle}{\sigma_0} = \frac{3}{2}\left(\frac{k_F d}{\pi}\right)^{-1}\sum_{n=1}^{n_d}\left[1 - n^2\left(\frac{k_F d}{\pi}\right)^{-2}\right] \times[1 + \nu_l(n_x)/\nu]^{-1} \tag{4.32}$$

versus $\nu/\nu_{ls}^{\min} \sim (\zeta/L)^{-2}(d/l)$ for two characteristic values of $k_F d/\pi$. Formula (4.32) combines classical (4.16) and quasiclassical (4.29) asymptotics. It is derived similarly to Eq. (3.13), but the region of its validity is restricted by quasiclassical approach (4.4) with neglecting a distinction between k_F and k_μ. The value of n_x in Eq. (4.32) has discrete spectrum (3.12).

Both curves in Fig. 4 monotonously rise as $\nu/\nu_{ls}^{\min}$ increases. Curve 1 shows $\langle\sigma\rangle$ of thin samples (4.31), in which the steeply impinging electrons dominate. It can be approximated by Eq. (4.27) at $\nu/\nu_{ls}^{\min} \lesssim 10^{-2}$. Residual conductivity (4.27) is independent of $k_F d/\pi$, so curve 1 is universal at $\nu/\nu_{ls}^{\min} \ll 1$. For $\zeta/L \sim 10^{-1}$ in perfect plates with $l \sim 10^{-1}$ cm, Eq. (4.27) can be realized at $d \ll 10^{-3}$ cm. As increasing $\nu/\nu_{ls}^{\min}$, curve 1 asymptotically approaches the abscissa $\langle\sigma\rangle/\sigma_0 = 1$. At high temperatures, where $3 \lesssim \nu/\nu_{ls}^{\min}$, curve 1 is described by Eq. (4.23). Curve 2 reflects a behavior of the conductivity for thick samples (4.26), in which the contributions of both electron groups compete. This curve obeys quasiclassical asymptotics (4.29) within the interval $\nu/\nu_{ls}^{\min} \lesssim 10^{-4}$, and classical expression (4.28) at

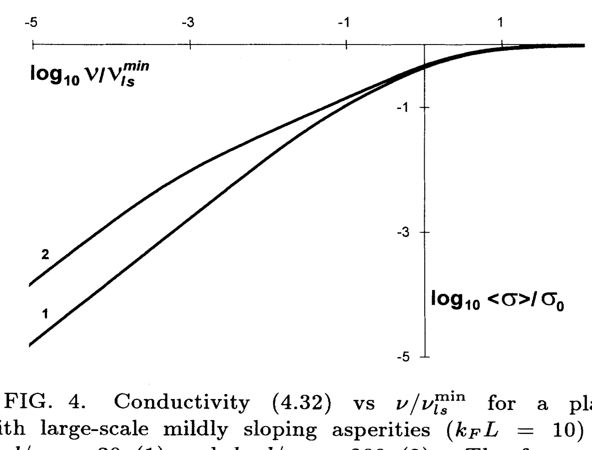

FIG. 4. Conductivity (4.32) vs $\nu/\nu_{ls}^{\min}$ for a plate with large-scale mildly sloping asperities ($k_F L = 10$) at $k_F d/\pi = 20$ (1) and $k_F d/\pi = 200$ (2). The frequency $\nu_l(n_x)$ is found from Eqs. (4.17), (4.9), and (4.14) with $\beta = 2|w''(0)| = 1$.

$3\times10^{-3} \lesssim \nu/\nu_{ls}^{\rm min} \lesssim 3\times10^{-1}$. Then, when the inequality $3\times10^{-1} \lesssim \nu/\nu_{ls}^{\rm min}$ holds, curve 2, as would be expected, almost coincides with curve 1. However, curve 2 does not have a portion described by Eq. (4.27) because $\nu_l^{\rm eq}$ is of the same order as $\nu_{ls}^{\rm min}$ at $k_FL \sim 10$.

V. SUMMARY AND DISCUSSION

We derived the most general for the present time closed system of Eqs. (2.9)–(2.12), and (2.5) for the classical static conductivity of a plate with randomly rough boundaries. The region of its validity is only restricted by the small slope of surface irregularities (2.6). This system can be fully analyzed numerically, which allows us to reveal a number of peculiarities in the behavior of $\langle\sigma\rangle$. For example, when increasing k_FL, the conductivity decreases. In plates with LS boundary defects, where $k_FL \gtrsim 10$, both the electron-surface scattering frequency $\nu_{\rm surf}$ and the average conductivity are slightly dependent on k_FL at the constant asperity slope ζ/L. We performed not only numerical but also detailed analytical study of $\langle\sigma\rangle$ at SS ($k_FL < 1$) and LS ($k_FL \gg 1$) roughnesses.

For films with SS asperities ($k_FL < 1$), we built not only classical but also the most general quantum theory of the conductivity. Comparing the classical results with the quantum ones, we can conclude that the quantum or classical nature of the electron transport in films is, in addition, determined by a method of measuring $\langle\sigma\rangle$, i.e., either the conductivity is measured versus the plate thickness d or the frequency ν of bulk collisions. So, $\langle\sigma\rangle$ as a function of ν has the essentially quantum origin only in samples with a small number of conducting electron channels ($n_d < 5$). Beginning from $n_d = 5$, the static conductivity is adequately described by the quasiclassical theory, and at $n_d \gtrsim 400$ one can use the classical theory. When bulk collisions vanish ($\nu \to 0$), quantum conductivity (3.13), unlike the classical one (3.8), approaches finite residual value (3.28) caused by the electron-surface scattering.

The study of $\langle\sigma\rangle$ versus k_Fd/π revealed the peculiarities (dips) appearing at points where a new propagating electron mode opens. Those dips are mainly due to the spatial quantization of the frequency $\nu_{\rm surf}$. This confirms the necessity to allow for the electron-surface interaction at the quantum size effect. We emphasize that classical conductivity (3.8) does not coincide with quantum one (3.13) averaged over the dips. The quantum conductivity turns out to be less than the classical one for the same values of $k_F\zeta$, k_FL, d/l. This distinction is particularly noticeable in thin perfect films (cf. curves 1 and 2 in Fig. 5).

The situation with LS asperities ($k_FL \gg 1$) is more complex, since the problem of correlations between neighboring collisions of electrons with the surface arises. We found $\langle\sigma\rangle$ at additional (but quite reasonable) restriction (4.3), within which both requirements for the absence of correlations (4.2) and the quasiclassical approach (4.4) are met. Strong correlations are actual in essentially quantum films. We revealed the nonmonotonous dependence of the effective surface scattering frequency on an angle of electron impact with the boundary. This gives rise to the competition between the quasiparticles flatly and steeply impinging on the sample surface. If the flatly impinging electrons dominate, residual conductivity (4.29) has the quantum origin. However, in sufficiently thin films (4.31), the conductivity mainly arises from the steeply impinging particles, for which the size quantization is insignificant. In this case residual conductivity (4.27) has the exclusively classical origin.

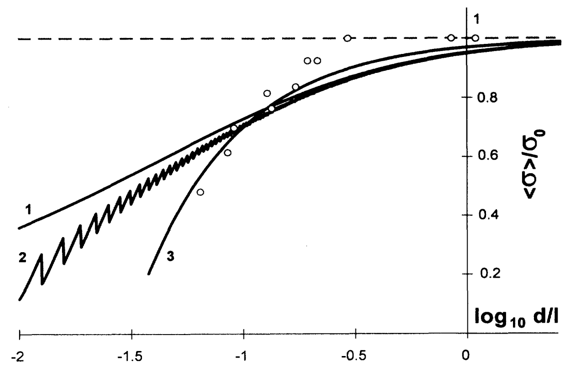

FIG. 5. Test of theoretical results against the experimental data (circles) of Hensel *et al.* (Ref. 33) (1) conductivity (3.8); (2) quantum formula (3.13); (3) classical asymptotics (4.23).

A. Comparison with the experiment

Let us analyze experimental data from Ref. 33. This research dealt with measurements of the average resistivity $\langle\rho\rangle = \langle\sigma\rangle^{-1}$ for epitaxially grown single-crystal films $CoSi_2$ versus their thickness d (60 Å$\lesssim d \lesssim$ 1100 Å, $l \approx$ 1000 Å). In Fig. 5, we tested theoretical dependencies (curves 1–3) against experimental points. All the curves were plotted by the same scheme: their location corresponds to the least root-mean-square deviation from the experimental values. For the correlator $W(r)$, we used the Gaussian function. Note that three experimental points falling in the interval $0.2 \lesssim d/l \lesssim 0.3$ reside at a substantial distance from all the theoretical curves. However, those points scarcely affect the locations of curves 1–3.

Curve 1 describes classical conductivity (3.8) at SS asperities ($k_F\zeta \approx 0.7$ and $k_FL \approx 0.9$). We call attention to a good deal of discrepancy between curve 1 and experimental points for ultrathin films with $d/l \lesssim 0.1$. Suppose this to be associated with the quantum origin of $\langle\sigma\rangle$. Curve 2 presents quantum conductivity (3.13). It was plotted at $k_F \approx 10^8$ cm^{-1} and nearly the same values of $k_F\zeta$ and k_FL as curve 1. These values correspond to the case when the frequency ν_s^i far exceeds ν (3.33) and the surface mechanism of electron relaxation dominates over the bulk one at $d/l \lesssim 0.1$. Therefore, the quantum dips are well distinct and curve 2 passes noticeably closer to the experimental points. Nevertheless, such improvement is not satisfactory, because the quantum curve as well as the classical one gives considerably overread values of $\langle\sigma\rangle$ for ultrathin films with $d/l \lesssim 0.1$. Besides, the values of $k_F\zeta$ and k_FL, which give the best fit of the theory to the experiment, are close to the limits of validity for corresponding Eqs. (3.8) and (3.13) [see Eqs. (3.1) and (2.6)]. This allows us to conclude that the films used in Ref. 33 did not actually contain SS surface asperities.

The best agreement with the experiment is given by curve 3 ($\zeta/L \approx 0.1$), which describes classical conductivity (4.23) of a plate with LS irregularities at the weak surface scattering. It correlates excellently with the experimental behavior of $\langle\sigma\rangle$ even at $d/l \lesssim 0.1$. Thus, one can conclude that the specimens[33] contained LS surface defects. According to our estimation at $\zeta/L \approx 0.1$, the range of surface scattering domination is $d/l \lesssim 3(\zeta/L)^2 \approx 0.03$. This allows us to suggest that the electron scattering at the sample boundary was relatively weak in the films.[33] For the last experimental point only ($d/l \approx 0.065$), both relaxation mechanisms might comparably contribute to the conductivity.

The above conclusions are also confirmed by additional investigations provided by the authors of Ref. 33. They recognized the cubic symmetry for single-crystal films of $CoSi_2$, metallic type of the conduction and determined the electron concentration $N \approx 2 \times 10^{22}$ cm^{-3} (i.e., $k_F \approx 10^8$ cm^{-1}). Besides, using the transmission electron microscopy, they revealed surface irregularities to be extremely smooth with $\zeta \approx 10^{-8}$ cm. All this validates our deduction as to the, namely, LS mildly sloping asperities realized in the experiments.[33]

The weak surface scattering results in rather monotonous (with no significant dips) falling of the experimental points in Fig. 5 as d/l decreases. This suggests that the quantum peculiarities of $\langle\sigma\rangle$ do not exhibit even for ultrathin metallic films with $d \lesssim 100$ Å. Hence, in accordance with our research, to observe the quantum nature of the conductivity in metallic films with LS boundary asperities, one needs to fabricate extremely thin samples with $d/l \lesssim 10^{-2}$. Such specimens will be probably obtained in the near future, since the development of the modern experimental technology is closely associated with the production of nanostructures with perfect spatial quantization. For those films, plots of $\langle\sigma\rangle/\sigma_0$ versus d/l are expected to show sharp dips at points where a new conducting electron channel opens. To describe this adequately, the quantum theory of the conductivity at $k_FL \gg 1$ must be built.

ACKNOWLEDGMENTS

This work was supported by Project No. 2.3/19 "Metal" of Ukrainian Committee in Science and Technology. One of the authors (M.A.V.) was also financially supported by a Soros Foundation grant awarded by the American Physical Society.

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