# Electron localization in a two-dimensional system with random magnetic flux

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Using a finite-size scaling method, we calculate the localization properties of a disordered two-dimensional electron system in the presence of a random magnetic field. Below a critical energy $E_c$ all states are localized and the localization length $\xi$ diverges when the Fermi energy approaches the critical energy, i.e., $\xi(E) \propto |E - E_c|^{-\nu}$. We find that $E_c$ shifts with the strength of the disorder and the amplitude of the random magnetic field while the critical exponent ($\nu \approx 4.5$) remains unchanged, indicating universality in this system. Implications on the experiment in the half-filling fractional quantum Hall system are also discussed.

The fractional quantum Hall (FQH) system[1] is an ideal candidate to study localization properties in strongly correlated electron systems. In a noninteracting picture, according to the scaling theory of localization,[2] all electrons in a two-dimensional (2D) system are localized in the absence of a magnetic field. When the two-dimensional electron system is subject to a strong perpendicular magnetic field, the energy spectrum becomes a series of impurity broadened Landau levels. Extended states appear at the center of each Landau band, while states at other energies are localized. This gives rise to the integer quantum Hall effect. Finite-size scaling techniques[3,4] have been used extensively to study the integer quantum Hall localization transition with the important finding that the critical transition at the center of each Landau band is universal in the sense that it is independent of the disorder strength and range and also of the Landau level index. In the FQH regime, electron-electron interaction plays an important role,[1] however, recent experiments and theories indicate that the critical properties of the plateau transition might also be in the same universality class.[5,6]

Recently, Halperin, Lee, and Read[7] and Kalmeyer and Zhang[8] developed an effective Chern-Simons field theory to understand electronic properties of the FQH systems. In their theory the quasiparticles are weakly interacting composite fermions,[9] which can be constructed by attaching an even number of flux quanta to electrons under a Chern-Simons transformation. In this simple picture, the fractional quantum Hall effect can be mapped into the *integer* quantum Hall effect for the composite fermion system subject to an effective magnetic field.[9] At the filling factor $\nu_f = \frac{1}{2}$, although the effective magnetic field $B^*$ vanishes, composite fermions are subject to the random fluctuations of the gauge field, induced by the ordinary impurities.[7,8] Thus it is important to study the localization properties of noninteracting charged particles in a random background magnetic field to understand the half-filling FQH system. The problem of charged particles moving in a random magnetic field is also relevant to the theoretical studies of high-$T_c$ models where the gauge field fluctuations play an important role.[10]

According to the conventional scaling theory of localization, the random flux system belongs to the unitary ensemble, which is described by a nonlinear $\sigma$ model with unitary symmetry.[11] Since there is no net magnetic field, the topological term of the uniform magnetic field case is absent. Perturbative renormalization-group calculations show that all states are localized.[11] However, it has been argued recently by Zhang and Arovas[12] that, although the constant topological term is absent, there is a term describing the long-ranged interaction between the topological densities, and they conjectured that this new term could lead to a phase transition from localized to extended states. There have been a number of conflicting numerical investigations on the localization properties of the 2D random magnetic field system. The conclusions in these studies range from all states localized[13,14] to extended states around band center.[8,15] In Ref. 16 the authors found evidence for a mobility edge, but the system was neither large enough to see good scaling nor close enough to the critical regime to obtain a conclusive critical exponent.

In this paper we systematically investigate the localization properties of a disordered two-dimensional electron system in the presence of a random magnetic field. The localization length is calculated using a transfer matrix technique and finite-size scaling analysis. An important strength of our calculation is using system widths (up to 128) that are substantially (by a factor 4) larger than those[13,15,16] existing in the literature. We find the following results: (i) A mobility edge $E_c$ is observed and the localization length $\xi$ diverges when the Fermi energy approaches the critical energy; (ii) the critical energy $E_c$ shifts with increasing disorder strength; (iii) $E_c$ shifts with changing randomness in the magnetic field; (iv) the critical exponent ($\nu \approx 4.5$) remains unchanged while varying the disorder strength and the randomness of the magnetic field, indicating universality in the metal-insulator transition; and (v) the mobility edge survives in the presence of a weak but nonzero average random magnetic field.

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We model our two-dimensional system in a very long strip geometry with a finite width ($M$) square lattice with nearest-neighbor hopping. The disorder potential is modeled by the on-site white-noise potential $V_{im}$ ($i$ denotes the column index, $m$ denotes the chain index) ranging from $-W/2$ to $W/2$. A random magnetic field is introduced by varying the flux in each lattice plaquette uniformly between $-\phi_r/2$ and $\phi_r/2$ (in this case the average field is zero; we also discuss the situation of weak but nonzero average random magnetic field in the later part of this paper). The Hamiltonian of this system can be written as

$$\mathcal{H} = \sum_i \sum_{m=1}^{M} V_{im}|im\rangle\langle im| + \sum_{\langle im;jn\rangle} \left[ t_{im;jn}|im\rangle\langle jn| + t^{\dagger}_{im;jn}|jn\rangle\langle im| \right], \quad (1)$$

where $\langle im;jn\rangle$ indicates nearest neighbors on the lattice. The amplitude of the hopping term is chosen as the unit of energy. A specific gauge is chosen so that the intercolumn hopping does not carry a complex phase factor (i.e., $t_{im;i+1,m} = -1$). The only effect of random magnetic field shows up on the phase factor of the intracolumn (interchain) hopping term. If the random flux in a plaquette cornered by $(im)$, $(i+1,m)$, $(i+1,m+1)$, and $(i,m+1)$ is $\phi_{im}$, then

$$\frac{t_{i+1,m;i+1,m+1}}{t_{im;i,m+1}} = \exp\left[ i2\pi \frac{\phi_{im}}{\phi_0} \right], \quad (2)$$

where $\phi_0 = hc/e$ is the magnetic flux quantum. For a specific energy $E$, a transfer matrix $T_i$ can be easily set up by mapping the wave-function amplitudes at column $i-1$ and $i$ to those at column $i+1$, i.e.,

$$\begin{pmatrix} \psi_{i+1} \\ \psi_i \end{pmatrix} = T_i \begin{pmatrix} \psi_i \\ \psi_{i-1} \end{pmatrix} = \begin{pmatrix} H_i - E & -I \\ I & 0 \end{pmatrix} \begin{pmatrix} \psi_i \\ \psi_{i-1} \end{pmatrix}, \quad (3)$$

where $H_i$ is the Hamiltonian for the $i$th column and $I$ is a $M \times M$ unit matrix. Using a standard iteration algorithm,[3] we can calculate the Lyapunov exponents for the transfer matrix $T_i$. The localization length $\lambda_M(E)$ for energy $E$ at finite width $M$ is then given by the inverse of the smallest Lyapunov exponent. In our numerical calculation, we choose the sample length to be over $10^4$ so that the self-averaging effect automatically takes care of the ensemble statistical fluctuations. A sample of our calculated finite width localization length for various energies is shown in Fig. 1(a).

We use the standard one-parameter finite-size scaling analysis[3] to obtain the thermodynamic localization length $\xi$. According to the one-parameter scaling theory, the renormalized finite-size localization length $\lambda_M/M$ can be expressed in terms of a universal function of $M/\xi$, i.e.,

$$\frac{\lambda_M(E)}{M} = f\left( \frac{M}{\xi(E)} \right), \quad (4)$$

where $f(x) \propto 1/x$ in thermodynamic limit $M \to \infty$ while approaching a constant ($\sim 1$) at the mobility edge where the thermodynamic localization length diverges. Numerically we shift the data in Fig. 1(a) onto a smooth function with a least-squares fit. Note that we have to select data for a large enough sample in the scaling analysis so as to avoid severe finite-size effect. In our calculation, we choose the data for sample width greater than 16 (inclusive). The thermodynamic localization length is given by the amount of shifts on a log-log plot. A sample of the scaling function and corresponding thermodynamic localization length is shown in Fig. 1(b). Because of the symmetry in the problem, we only study the branch with negative energy $E < 0$.

We first study the case with random magnetic field characterized by fluctuation amplitude $\phi_r = 1.0$. We find that if the Fermi energy is below $E_c = -3.0$, the finite-size localization length is well converged and always smaller than the sample width, indicating that all states are localized below $-3.0$. On the contrary, for the electronic states with energy higher than $-3.0$ the inverse of the Lyapunov exponent is always larger than the sample width, which is the feature of extended states.

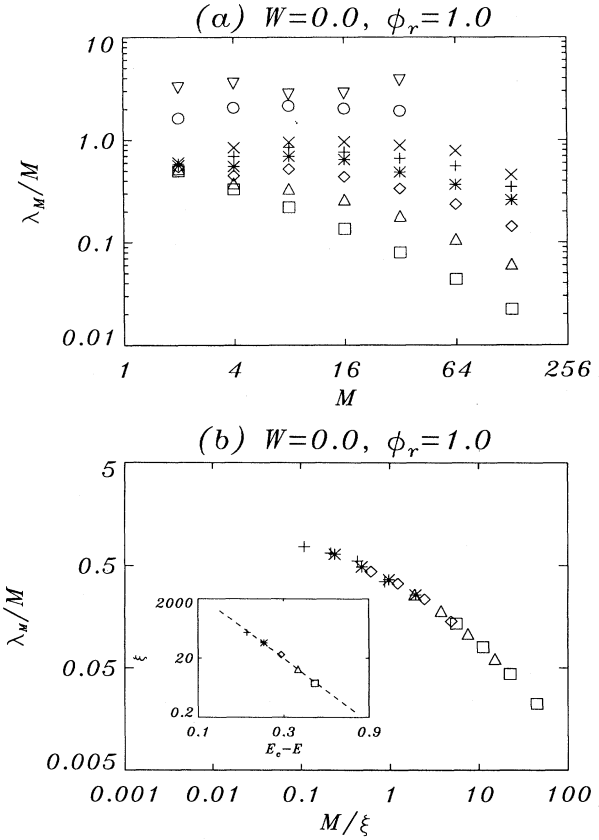


FIG. 1. (a) Renormalized finite-size localization length ($\lambda_M/M$) for different sample width $M$ in a random magnetic field ($\phi_r = 1.0$) without on-site disorder ($W = 0$). (b) Scaling function and localization length in the thermodynamic limit (inset) with $\nu = 4.52 \pm 0.08$ and $E_c = -3.00$. Here different symbols represent different energies: $\nabla$, $-2.1$; $\circ$, $-2.7$; $\times$, $-3.15$; $+$, $-3.19$; $*$, $-3.23$; $\diamond$, $-3.29$; $\triangle$, $-3.36$; $\square$, $-3.45$.

We also find that the thermodynamic localization length $\xi$ diverges while approaching $E_c = -3.0$, indicating the existence of a mobility edge around $-3.0$. Our best fit analysis indeed gives a critical energy $E_c = -3.00$ with a critical exponent $\nu = 4.52$ appearing in $\xi = |E - E_c|^{-\nu}$ for $E < E_c$ [the critical exponent is given by the slope of the straight line in the inset log-log plot of Fig. 1(b)]. Note that in the extended state regime above $E_c$, we could not obtain a second branch (for states above the mobility edge) in the scaling function from our finite-size data. This is consistent with the scenario that the extended state regime is a line of critical points[16] due to the two-dimensional nature of the system[2] where localization is "marginal." Our conclusion for the existence of a mobility edge in this problem is based on the following two facts: (i) The finite-size scaling evidence for the mobility edge is as strong as in the integer quantum Hall problem where the existence of the extended states is well established, and (ii) in the 2D Anderson Hamiltonian with random on-site energies where all states are known to be localized, all energy states can be fit into one scaling curve,[18] whereas in our random flux model only states with $E < E_c$ can be fit into the scaling curve [see Fig. 1(b)]. Here we should comment on the two recent numerical studies[13,14] where all states are found to be localized. Although the raw data in Ref. 13, obtained with sample sizes smaller than ours, seem to be consistent with ours, the authors came to a different conclusion based on the absence of the second branch. Reference 14 uses the network model where the basic assumption is that the impurity potential is smooth enough such that its correlation length is much larger than the cyclotron radius $(e\hbar/cB)^{1/2}$, where $B$ is the root-mean-square value of the field. However, in our model the impurity correlation length is roughly one lattice constant, which is of the same order as the cyclotron radius. Thus, in some sense our calculations are for finite short-range disorder and those of Ref. 14 are for long-range disorder.

We expect that the mobility edge should shift to lower energy if the magnetic field is less random so that extended states are more favorable. This is exactly what we observe in our calculation. In Fig. 2(a) we show the typical scaling function and the localization length for a random magnetic field with $\phi_r = 0.9$. In Table I we present the critical exponents and critical energies for different randomness in the magnetic field. We find that the critical energy increases almost linearly with the random flux amplitude, which is proportional to the energy fluctuation created by the random magnetic field.

TABLE I. The critical exponent ($\nu$) and critical energy ($E_c$) for different representations of the random magnetic field ($\phi_r$) and on-site disorder ($W$). The last row is for the correlated disorder model while the other data are for the independent disorder model.

| $\phi_r$ | $W$ | $\nu$ | $E_c$ |
|---|---|---|---|
| 0.7 | 0.0 | $4.53 \pm 0.11$ | -3.40 |
| 0.8 | 0.0 | $4.60 \pm 0.06$ | -3.28 |
| 0.9 | 0.0 | $4.98 \pm 0.10$ | -3.13 |
| 1.0 | 0.0 | $4.52 \pm 0.08$ | -3.00 |
| 1.0 | 1.0 | $4.79 \pm 0.17$ | -2.85 |
| 1.0 | 2.0 | $4.86 \pm 0.02$ | -2.73 |
| 1.0 | 3.0 | $5.29 \pm 0.60$ | -2.13 |
| 0.8 | 2.0 | $4.45 \pm 0.10$ | -3.35 |

We now discuss the situation with both a random magnetic field and an on-site disorder potential. We consider two types of disorder potential. (i) First is the independent model, where the distribution of disorder potential is completely independent of the random magnetic field. This model is relevant for the case with random distributed nonmagnetic impurities in the sample. (ii) Second is the correlated model in which the strength of disorder is associated with the local random magnetic field (numerically we select each on-site disorder so that it is proportional to the random flux in the neighboring plaquette). This model is relevant for the case with random distributed magnetic impurities, for example, disorder-pinned random flux lines in the sample.

In the independent model, since nonmagnetic random disorder potential tends to localize all the electronic states, we expect that the mobility edge in a random magnetic field should shift to higher energy with increasing strength of the disorder potential because the local-

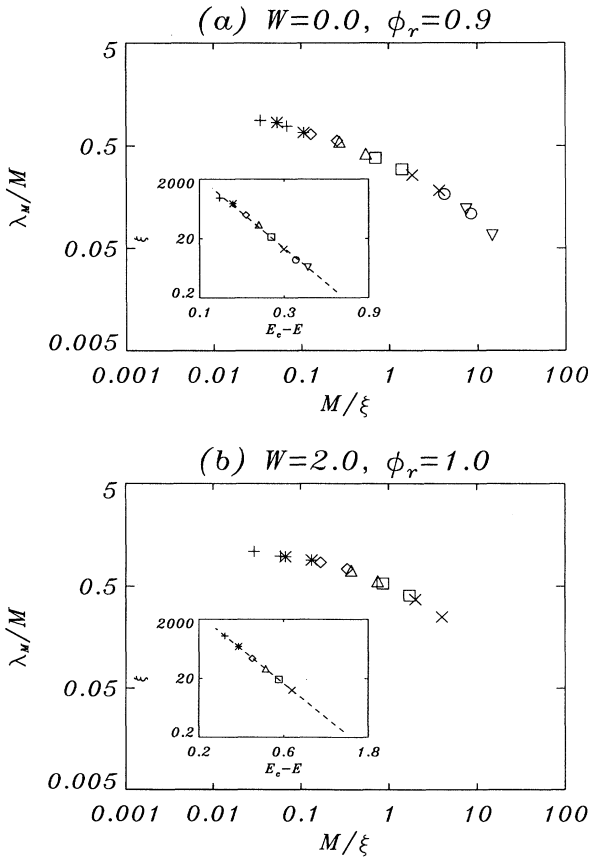


FIG. 2. Scaling functions and thermodynamic localization length (inset) in random magnetic fields (b) with or (a) without on-site disorder. The corresponding critical exponents and critical energies are (a) $\nu = 4.98 \pm 0.10, E_c = -3.13$ and (b) $\nu = 4.86 \pm 0.02, E_c = -2.73$.

ized states are more favorable in this situation. In Fig. 2(b) we show the typical scaling function and localization length for a strongly disordered sample with $W = 2.0$ in a random magnetic field with $\phi_r = 1.0$. As presented in Table I, the critical energy moves to higher energy as the disorder strength $W$ increases for a fixed random magnetic field ($\phi_r = 1.0$), just as we expect. Presented in the last row of Table I are the data for a correlated disorder model—the behavior of the mobility edge is quite similar to that for the independent disorder model.

It is interesting to notice that even though the mobility edge is shifting for various random magnetic fields and disorder strength and correlation, the critical exponent for the metal-insulator transition is more or less unchanged, which indicates universality for this critical transition. Our calculated exponent ($\nu \approx 4.5$) is quite different from the value obtained in Ref. 16. However, our system size is larger (4 times larger) than the system studied in Ref. 16, our scaling is better, and our data are closer to the critical regime; thus our calculated exponent should better represent the true critical exponent.

In Fig. 3 we present the localization length for a two-dimensional system subject to a random magnetic field when the average field is weak but nonzero (i.e., $\langle\phi\rangle \neq 0$). Clearly there exists a mobility edge at $E_c \approx -3.0$ for $\langle\phi\rangle = 0.01$. It is consistent with the argument that in the vicinity of $\langle B^*\rangle = 0$, the composite fermion system behaves as a Fermi liquid.

As mentioned earlier, there is currently considerable disagreement in the literature about the nature of two-dimensional one-electron eigenstates in a random flux environment. An earlier finite-size scaling analysis[13] concluded that all states are localized, but the localization lengths are exponentially large near the band center. Our largest system widths (=128) are four times larger than those (=32) used in Ref. 13 and our results are consistent with the existence of mobility edges separating localized and delocalized states with a localization exponent $\nu \approx 4.5$. We mention that our calculated critical exponent $\nu \approx 4.5$ is the best one can do with the currently available computer sizes. Our demonstration of the existence of a mobility edge should, however, be quite

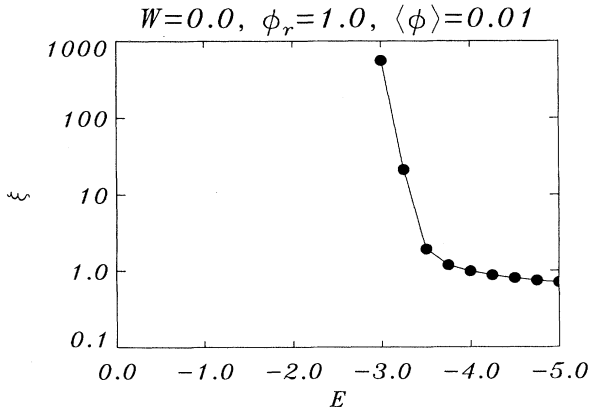


FIG. 3. Thermodynamic localization length in the presence of a weak but nonzero average random magnetic field.

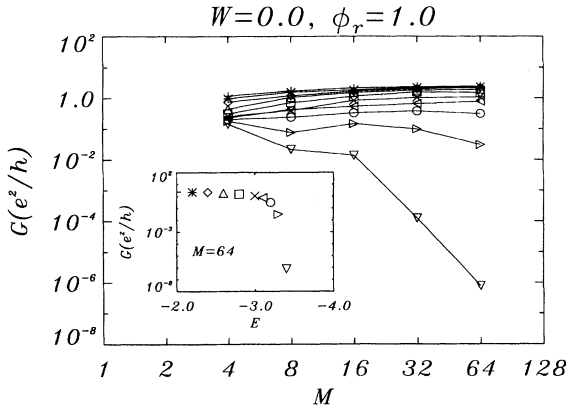


FIG. 4. Conductance for finite-size samples at different energies (different symbols). Inset: conductance for sample size $M = 64$.

robust. We emphasize[19] that no finite-size scaling analysis can distinguish between extended states and states with extremely large (orders of magnitude larger than the system length) localization lengths.[13,17] We contend that experiments in real samples cannot distinguish between these two scenarios either, i.e., the situation with extremely large localization lengths would behave very much like a weakly dirty metal. In this context, it is particularly significant that our finite-size scaling analysis (which presumably enables one to estimate the localization length in the thermodynamic limit from finite-size localization length data) clearly indicates the existence of a mobility edge $E_c$ whose position depends on the strength of randomness. Our finite-size scaling data for the 2D random flux problem looks very similar to the integer quantum Hall scaling results[3,4] rather than the 2D zero-field situation,[18] which is known not to have a mobility edge. The important point to realize is that the finite-size scaling evidence in favor of a delocalization transition presented in this paper is as strong as it is in the corresponding quantum Hall plateau transitions,[3] where the existence of extended states is not in doubt. We have also calculated the conductance $g$ as a function of the system width for various (Fermi) energies (shown in Fig. 4) using a direct transfer matrix Landauer formula type approach, finding results completely consistent with the existence of mobility edge (i.e., an insulator for $E < E_c$ and a metal for $E > E_c$). In particular, on the extended side, the conductance is insensitive to the sample width, consistent with the scenario that this regime is a line of critical points. Our efforts to construct a beta function $\beta(g)$ have not, however, yielded unambiguous results because of inherent fluctuations in the calculated conductance. Systems much larger than those used here ($M \gg 128$), which are not accessible with the currently available computers, will be needed to obtain an unambiguous $\beta(g)$; in that sense, we have a disagreement here with the conclusion of Ref. 13.

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