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Electric-current transmission through the contact of two metals

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We study the current transmission through a nonidal interface between two different metals. The exact equations relating the electron distribution functions in the bulk of metals are obtained. These equations take into account the specular scattering of electrons from the contacting surface as well as the diffusive scattering from defects at the interface. A specific resistance of contact is calculated. The approach makes it possible to take into account the nonideality of the inerface in a rigorous way.

I. INTRODUCTION

The transition of electric current through a contact of two metals is a well-known problem of solid-state physics.[1] The interest in this problem is growing now thanks to recent studies of metallic superlattices. In particular, a giant magnetoresistance (GMR) has been observed in Fe/Cr and some other systems with alternating films of ferromagnetic metal and nonmagnetic spacers.[2–10]

Here we study a single interface between two metals in current perpendicular to plane (CPP) geometry. As for magnetic multilayers the CPP geometry has recently been considered theoretically[8–10] and experimentally.[11,12]

All theories based on the Boltzmann equation approach have some difficulties in a formulation of the boundary conditions at the interface. It is necessary to take into consideration the nonideality of the interface (roughness and impurity scattering of the electrons). For this purpose one normally uses some kind of Fuchs-Sondheimer boundary conditions[13,14] for electron distribution functions. A specular parameter entering these boundary conditions has been introduced phenomenologically as a constant ranging somewhere among 0 and 1. The introduction of such a parameter does not take into account the fact that the electrons can experience an interface either as specular or diffusive depending on the relation between an electron wavelength and a characteristic dimension of roughness. Thus the specular parameter should depend on the electron velocity. In a more consistent theory developed by Falkovsky for a metal surface (see the review article[15]) the boundary conditions take this feature into account quite naturally. They have been derived from a fitting condition for the electron wave functions at a nonideal interface.

In the present paper the method[15] developed for the electron reflecting from the metal surface is generalized for the case of electrons transmitting through the contact of two metals. As the interface nonideality we consider the scattering of electrons from a random two-dimensional potential which is located at the interface and has a white-noise-like correlator. In this case there is only one parameter describing the interface nonideality that equals the mean-square fluctuation of a scattering potential at the interface. All other parameters included in the boundary conditions will be derived from microscopic theory.

II. MODEL AND KINETIC EQUATIONS

Let us consider a contact of metals 1 and 2, laying in a $z=0$ plane. The dispersion of electrons in these metals is taken in the square approximation $\varepsilon_1=p^2/2m_1$; $\varepsilon_2=\varepsilon_0+p^2/2m_2$. The metals are characterized by bulk conductivities σ_1 and σ_2 and mean free paths l_1 and l_2. The external electric field $\mathbf{E}$ is applied along the z axis. Due to the different work functions of metals, a redistribution of electrons takes place resulting in a nonuniform electric field $\mathbf{E}(z)$. The characteristic inhomogeneity length near the contact is of the order of screening lengths $L_{1,2}$ and is small compared to $l_{1,2}$, $L_1\cong L_2\equiv L\ll l_{1,2}$.

In region $1(z<0)$ the Boltzmann transport equation for distribution function $f(z,\mathbf{v})$ in the weak field can be written in τ approximation as

$$\frac{eE(z)}{m_1}\frac{\partial f_0}{\partial v_z}+v_z\frac{\partial f}{\partial z}+\frac{f-f_0}{\tau_1}=0\ , \tag{1}$$

where $f_0=\{1+\exp[(\varepsilon_1-\varepsilon_{F1})/T]\}^{-1}$, ε_{F1} is the Fermi energy; $\tau_1=l_1/v_{F1}$ is the bulk relaxation time related to the scattering from impurities in the bulk of metal 1, and T is the temperature in energy units.

The boundary condition at $z\to-\infty$ reads

$$f=f_0-e\frac{\partial f_0}{\partial\varepsilon}E_1^{\infty}v_z\tau_1\ , \tag{2}$$

where $E_1^{\infty}=j_0/\sigma_1$ is the field in bulk of metal $1(|z|\gg l_1)$, and j_0 is the transport current density.

The analogous equation can be written for the distribution function $g(z,\mathbf{v})$ in region 2 $(z>0)$.

In both regions 1 and 2 we can introduce the distribution functions for electrons moving along the z axis (i.e., along $\mathbf{E}$) f^+, g^+, $(v_z>0)$, and f^-, g^-, $(v_z<0)$ for electrons moving in the opposite direction.

The solutions for these functions have the following form:

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$$\begin{aligned}
f^{+}&=f_0-e\frac{\partial f_0}{\partial \varepsilon}\int_{-\infty}^{z}E_1(z')\exp[(z'-z)/v\tau_1]dz' ,\\
f^{-}&=f_0+e\frac{\partial f_0}{\partial \varepsilon}\left[-C_1\exp(z/v\tau_1)+\int_z^0 E_1(z')\exp[-(z'-z)/v\tau_1]dz'\right] ,\\
g^{+}&=f_0-e\frac{\partial f_0}{\partial \varepsilon}\left[C_2\exp(-z/v\tau_2)+\int_0^z E_2(z')\exp[(z'-z)/v\tau_2]dz'\right] ,\\
g^{-}&=f_0+e\frac{\partial f_0}{\partial \varepsilon}\int_z^{\infty}E_2(z')\exp[-(z'-z)/v\tau_2]dz' ,
\end{aligned} \tag{3}$$

where $v\equiv|v_z|$, C_1 and C_2 are some functions of v that will be calculated below using the boundary conditions at the interface $z=0$, and $E_{1,2}(z)$ is the electric field in regions 1 and 2.

III. ELECTRIC CURRENT AND VOLTAGE DROP AT THE CONTACT

Using the expression for the current density in region 1,

$$j_1(z)=2e\int_{(v_z>0)}\frac{d^3k}{(2\pi)^3}v_z(f^{+}-f^{-}) , \tag{4}$$

and after integrating over the moment in the contact plane, from (3) we obtain

$$j_1(z)=\frac{e^2m^2}{2\pi^2\hbar^3}\int_0^{v_{F1}}v\,dv\left\{\int_{-\infty}^{z}E_1(z')\exp[(z'-z)/v\tau_1]dz'-C_1\exp(z/v\tau_1)-\int_0^z E_1(z')\exp[(z-z')/v\tau_1]dz'\right\} . \tag{5}$$

Taking $E_1(z)=E_1^{\infty}+\delta E_1(z)$, we can find from equation $j_1(z)=j_0$ that the distribution of the electric field at $z<0$ obeys

$$\int_0^{v_{F1}}v\,dv\left\{\exp(-z/v\tau_1)\int_{-\infty}^{z}\delta E_1(z')\exp(z'/v\tau_1)dz'-E_1^{\infty}v\tau_1\exp(z/v\tau_1)\right.$$
$$\left.-C_1\exp(z/v\tau_1)-\exp(z/v\tau_1)\int_z^0\delta E_1(z')\exp(-z'/v\tau_1)dz'\right\}=0 . \tag{6}$$

To estimate the integrals in Eq. (6), we assume that $\delta E_1(z)=E_1^0e^{z/L}$, where E_1^0 is the field magnitude at the interface. These integrals can be presented in the form

$$\begin{aligned}
I&=\int_0^{v_F}v\exp(-a/v\tau)dv\\
&=\frac{a^2}{\tau^2}\Gamma\left[-2,\frac{a}{v_F\tau}\right]\\
&=\begin{cases} v_F^2/2, & a/v_F\tau\ll 1\\ \dfrac{v_F^3\tau}{a}\exp\left[-\dfrac{a}{v_F\tau}\right], & a/v_F\tau\gg 1 ,\end{cases}
\end{aligned} \tag{7}$$

where $\Gamma(a,x)$ is the incomplete gamma function.[16]

Using (7) we have the following expressions for the third and fourth integrals in (6):

$$\begin{aligned}
&\int_0^{v_{F1}}v\,dv\int_{-\infty}^{z}\delta E_1(z')\exp[-(z-z')/v\tau_1]dz'\\
&\qquad\cong E_1^0v_{F1}^2L\exp(z/L) ,\\
&\int_0^{v_{F1}}v\,dv\int_z^0\delta E_1(z')\exp[-(z'-z)/v\tau_1]dz'\\
&\qquad\cong E_1^0v_{F1}^2L\exp(z/l_1)[1-\exp(z/L)] ,
\end{aligned} \tag{8}$$

where $l_1=v_{F1}\tau_1$.

Equation (6) holds for any $z<0$. The third term in (6) can be neglected at $|z|\gg L$, whereas the fourth one is a constant proportional to the voltage drop U_1 in region 1.

Thus Eq. (6) takes the form ($|z|\gg L$)

$$\int_0^{v_{F1}}v\,dv[C_1(v)+v\tau_1E_1^0-U_1]=0 , \tag{9}$$

where

$$U_1=\int_{-\infty}^{0}\delta E_1(z)dz .$$

After the integration, the final form of Eq. (9) is

$$\int_0^{v_{F1}}C_1(v)v\,dv+\tfrac{1}{3}E_1^{\infty}\tau_1v_{F1}^3+\tfrac{1}{2}v_{F1}^2U_1=0 . \tag{10}$$

Using the equation $j_2(z)=j_0$ for region 2, in an analogous way, we obtain

$$\int_0^{v_{F2}}C_2(v)v\,dv-\tfrac{1}{3}E_2^{\infty}\tau_2v_{F2}^3+\tfrac{1}{2}v_{F2}^2U_2=0 , \tag{11}$$

where

$$U_2=\int_0^{\infty}\delta E_2(z)dz$$

is the voltage drop in region 2 ($z>0$).

Equations (10) and (11) enable the determination of the total voltage drops for regions 1 and 2, provided that the coefficients C_1 and C_2 are known.

IV. BOUNDARY CONDITIONS FOR DISTRIBUTION FUNCTIONS

At the surface separating two metals, electrons are scattered from defects located at the surface. The scattering takes place either from impurities or the roughness of the boundary. In this case the scattering is supposed to be elastic.

The wave function in region 1 can be presented as a superposition of plane waves on the isoenergetic surface

$$\Psi_1(\varepsilon,z,\mathbf{r})=\int dk_z d\mathbf{k}\, a_1(k_z,\mathbf{k})\exp[i(k_z z+\mathbf{k}\cdot\mathbf{r})] \times\delta\left[\varepsilon-\frac{\hbar^2(k_z^2+k^2)}{2m}\right], \quad (12)$$

where $\mathbf{r}=(x,y)$ and $\mathbf{k}=(k_x,k_y)$.

After performing the integration over k_z, we have

$$\Psi_1(\varepsilon,z,\mathbf{r})=\int d\mathbf{k}\frac{\exp(i\mathbf{k}\cdot\mathbf{r})}{v_{1k}} \times[a_{1k}^{<}\exp(-ik_{1z}z)+a_{1k}^{>}\exp(ik_{1z}z)], \quad (13)$$

where $\hbar k_{1z}=(2m_1\varepsilon-\hbar^2k^2)^{1/2}$, $v_{1k}=\hbar k_{1z}/m_1$, and $a_{1k}^{\gtrless}\equiv a_1(k_z=\pm k_{1z},\mathbf{k})$. In region 2 we obtain a similar expression:

$$\Psi_2(\varepsilon,z,\mathbf{r})=\int d\mathbf{k}\frac{\exp(i\mathbf{k}\cdot\mathbf{r})}{v_{2k}}[a_{2k}^{<}\exp(-ik_{2z}z) +a_{2k}^{>}\exp(ik_{2z}z)], \quad (14)$$

where $\hbar k_{2z}=(2m_2\varepsilon-\hbar^2k^2)^{1/2}$, and $v_{2k}=\hbar k_{2z}/m_2$.

In order to write down Schrödinger equation at arbitrary z, it is convenient to use the Lagrangian

$$\mathcal{L}=\int dz\, d\mathbf{r}\left\{\frac{\hbar^2|\nabla\Psi_1|^2}{2m_1}[1-\Theta(z)]-\varepsilon|\Psi_1|^2[1-\Theta(z)]+\frac{\hbar^2|\nabla\Psi_2|^2}{2m_2}\Theta(z)-\varepsilon|\Psi_2|^2\Theta(z)+\delta(z)W(\mathbf{r})|\Psi(0,\mathbf{r})|^2\right\}, \quad (15)$$

where $W(\mathbf{r})\delta(z)$ is a random field located at the interface, $\Theta(z)$ is the step function, $\Psi(z<0)=\Psi_1$, $\Psi(z>0)=\Psi_2$, and $\Psi=\Psi_1=\Psi_2$ at $z=0$. Making a variation of (15) over Ψ^*, we obtain the Schrödinger equation

$$-\frac{\hbar^2}{2m_1}\nabla_i[1-\Theta(z)]\nabla_i\Psi_1-\frac{\hbar^2}{2m_2}\nabla_i\Theta(z)\nabla_i\Psi_2 +\delta(z)W(\mathbf{r})\Psi=\varepsilon\Psi_1[1-\Theta(z)]+\varepsilon\Psi_2\Theta(z). \quad (16)$$

After integrating Eq. (16) over an infinitely small neighborhood of $z=0$, we obtain the following condition for the wave functions at the interface:

$$\frac{\hbar^2}{2m_1}\frac{d\Psi_1}{dz}\bigg|_{z=-\delta}-\frac{\hbar^2}{2m_2}\frac{d\Psi_2}{dz}\bigg|_{z=\delta}+W(\mathbf{r})\Psi=0\ , \quad \delta\to+0\ . \quad (17)$$

Substituting (13) and (14) in (17), we have

$$a_{1k}^{>}-a_{1k}^{<}+a_{2k}^{<}-a_{2k}^{>} =\frac{2i}{\hbar}\int d^2q\frac{W(\mathbf{q})}{v_{1\mathbf{k}-\mathbf{q}}}(a_{1\mathbf{k}-\mathbf{q}}^{>}+a_{1\mathbf{k}-\mathbf{q}}^{<})\ , \quad (18)$$

where

$$W(\mathbf{r})=\frac{1}{2\pi}\int W(\mathbf{q})\exp(i\mathbf{q}\cdot\mathbf{r})d^2q\ .$$

The second equation follows from the continuity condition $\Psi_1(z=0,\mathbf{r})=\Psi_2(z=0,\mathbf{r})$

$$\frac{1}{v_{1k}}(a_{1k}^{>}+a_{1k}^{<})=\frac{1}{v_{2k}}(a_{2k}^{>}+a_{2k}^{<})\ . \quad (19)$$

Now, taking into account[15] the relation between distribution functions and factors $a^{\lessgtr}$, we obtain

$$|a_{1k}^{<}|^2=v_{1k}f^-,\quad |a_{1k}^{>}|^2=v_{1k}f^+\ ,$$
$$|a_{2k}^{<}|^2=v_{2k}g^-,\quad |a_{2k}^{>}|^2=v_{2k}g^+\ . \quad (20)$$

If the contacting surface is an ideal plane [i.e., $W(\mathbf{r})=0$], then from Eqs. (18) and (19) we can find

$$f^-=Rf^++Tg^-\ ,$$

where $R=(v_{1k}-v_{2k})^2/(v_{1k}+v_{2k})^2$ and $T=1-R$ are the reflection and transmission coefficients, respectively. If one sets $m_1=m_2$ and substitutes in R and T the values $k_{1,2}=(2m\varepsilon_{1,2}-\hbar^2k_F^2\sin^2\Theta)^{1/2}$, then the reflection and transmission coefficients will take the form used by Hood and Falicov[7] for the ideal interface (here Θ is the angle of incidence).

Excluding the factors $a_{1k}^{<}$ and $a_{2k}^{<}$ in turn from Eq. (19), with the help of (18) we can find the relation between other a factors at the interface. After that, formulating the expressions for distribution functions (20), and making use of the iterations in $W(\mathbf{r})$, we come to the boundary conditions for distribution functions. They should be averaged over the realizations of random fields $W(\mathbf{r})$. We consider the distribution of random fields $W(\mathbf{r})$ to be Gaussian,

$$\langle W(\mathbf{r})\rangle=0,\quad \langle W(\mathbf{r})W(\mathbf{r}')\rangle=\gamma\delta(\mathbf{r}-\mathbf{r}')\ , \quad (21)$$

so that the correlations of higher orders are absent. Hence the iterations of (18) can be cut off at terms proportional to γ.

As a result, from (18)–(20) we obtain the boundary conditions at the $z=0$ plane:

$$f_k^- = f_k^+ \left[R - \frac{16 v_{1k}(v_{1k}-v_{2k})}{(v_{1k}+v_{2k})^3} \frac{\gamma}{\hbar^2} \int \frac{d^2 q}{v_{1q}+v_{2q}} \right] + g_k^- \left[T - \frac{32 v_{1k} v_{2k}}{(v_{1k}+v_{2k})^3} \frac{\gamma}{\hbar^2} \int \frac{d^2 q}{v_{1q}+v_{2q}} \right] + \frac{16 v_{1k}}{(v_{1k}+v_{2k})^2} \frac{\gamma}{\hbar^2} \int d^2 q \frac{v_{1q}}{(v_{1q}+v_{2q})^2} f_q^+ + \frac{16 v_{1k}}{(v_{1k}+v_{2k})^2} \frac{\gamma}{\hbar^2} \int d^2 q \frac{v_{2q}}{(v_{1q}+v_{2q})^2} g_q^- , \tag{22}$$

$$g_k^+ = g_k^- \left[R - \frac{16 v_{2k}(v_{2k}-v_{1k})}{(v_{1k}+v_{2k})^3} \frac{\gamma}{\hbar^2} \int \frac{d^2 q}{v_{1q}+v_{2q}} \right] + f_k^+ \left[T - \frac{32 v_{1k} v_{2k}}{(v_{1k}+v_{2k})^3} \frac{\gamma}{\hbar^2} \int \frac{d^2 q}{v_{1q}+v_{2q}} \right] + \frac{16 v_{2k}}{(v_{1k}+v_{2k})^2} \frac{\gamma}{\hbar^2} \int d^2 q \frac{v_{2q}}{(v_{1q}+v_{2q})^2} g_q^- + \frac{16 v_{2k}}{(v_{1k}+v_{2k})^2} \frac{\gamma}{\hbar^2} \int d^2 q \frac{v_{1q}}{(v_{1q}+v_{2q})^2} f_q^+ . \tag{23}$$

Here the z components of electron velocities v_{1k} and v_{2k} are related by

$$\frac{v_{1k}}{v_{F2}} = \left[\frac{m_2}{m_1} \right]^{1/2} \left[1 - \frac{m_2}{m_1} + \frac{\varepsilon_0}{\varepsilon_{F2}} + \frac{m_2}{m_1} \left[\frac{v_{2k}}{v_{F2}} \right]^2 \right]^{1/2} ,$$
$$\frac{v_{2k}}{v_{F1}} = \left[\frac{m_1}{m_2} \right]^{1/2} \left[1 - \frac{m_1}{m_2} - \frac{\varepsilon_0}{\varepsilon_{F1}} + \frac{m_1}{m_2} \left[\frac{v_{1k}}{v_{F1}} \right]^2 \right]^{1/2} , \tag{24}$$
$$\varepsilon_{F1,2} = m_{1,2} v_{F1,2}^2 / 2 .$$

Equations (22) and (23) give us effective transmission and reflection coefficients for the nonideal interface that are exact if a random field $W(\mathbf{r})$ is Gaussian. If the field $W(\mathbf{r})$ is not Gaussian these coefficients have the same form within the second order approximation in the field $W(\mathbf{r})$. For $m_1 = m_2$ an effective transmission coefficient from the square brackets of (22) and (23) coincide with those obtained by Barnas and Fert[10] and taken within the second-order approximation in the impurity potential.

In Eqs. (22) and (23) both the potential step $\varepsilon_0 \neq 0$ and the difference of masses $m_1 \neq m_2$ were taken into account as two sources of the quantum reflection of electrons against the interface.

We can go to the one-dimensional integration in Eqs. (22) and (23) in the following way:

$$\int d^2 q \cdots \to \frac{2\pi m_1^2}{\hbar^2} \int v_1 dv_1 \cdots , \qquad \int d^2 q \cdots \to \frac{2\pi m_2^2}{\hbar^2} \int v_2 dv_2 \cdots \tag{25}$$

in regions 1 and 2, respectively. In the course of integration over the variable v_1 (v_2) the value of v_2 (v_1) should be considered as a function of v_1 (v_2) according to (24).

In our treatment the Fermi surfaces may have different sizes in regions 1 and 2. For definiteness, we take $p_{F1} > p_{F2}$. There exists an interval of velocities $0 < v_1 < v_{1\,\min}$, in which $v_2 = 0$, and

$$v_{1\,\min} = v_{F1} \left[\frac{m_2}{m_1} \right]^{1/2} \left[\frac{\varepsilon_0}{\varepsilon_{F1}} + \frac{m_1}{m_2} - 1 \right]^{1/2} . \tag{26}$$

It corresponds to the complete quantum reflection for a portion of electrons at the current transmission through the contact, which should be taken into account when integrating over v_1. In Sec. V the coefficients C_1 and C_2 determining the distribution functions (3) will be calculated from boundary conditions (22) and (23).

V. DISTRIBUTION FUNCTIONS

To determine C_1 and C_2 we write the distribution functions from Eq. (3) at the $z = 0$ plane for $L \ll l_{1,2}$:

$$f_k^+ = f_0 - e \frac{\partial f_0}{\partial \varepsilon} (E_1^\infty v_1 \tau_1 + U_1), \quad f_k^- = f_0 - e \frac{\partial f_0}{\partial \varepsilon} C_1 ,$$
$$g_k^+ = f_0 - e \frac{\partial f_0}{\partial \varepsilon} C_2, \quad g_k^- = f_0 + e \frac{\partial f_0}{\partial \varepsilon} (E_2^\infty v_2 \tau_2 + U_2) . \tag{27}$$

Substituting (27) in boundary conditions (22) and (23), we obtain the equations from which we can calculate coefficients C_1 and C_2. The result reads

$$C_1 = R(E_1^\infty v_1 \tau_1 + U_1) \left[1 - \frac{16 v_1 \gamma m_1^2 v_{F1}}{\hbar^4 (v_1^2 - v_2^2)} I_1 \right] - T(E_2^\infty v_2 \tau_2 + U_2) \left[1 - \frac{8 \gamma m_2^2 v_{F2}}{\hbar^4 (v_1 + v_2)} I_2 \right] + T \frac{8 \gamma m_1^2 v_{F1}}{\hbar^4 v_2} (E_1^\infty v_{F1} \tau_1 I_3 + U_1 I_5) - T \frac{4 \gamma m_2^2 v_{F2}}{\hbar^4 v_2} (E_2^\infty v_{F2} \tau_2 I_4 + U_2 I_6) ,$$
$$C_2 = -R(E_2^\infty v_2 \tau_2 + U_2) \left[1 - \frac{16 v_2 \gamma m_2^2 v_{F2}}{\hbar^4 (v_2^2 - v_1^2)} I_2 \right] + T(E_1^\infty v_1 \tau_1 + U_1) \left[1 - \frac{8 \gamma m_1^2 v_{F1}}{\hbar^4 (v_1 + v_2)} I_1 \right] - T \frac{4 \gamma m_2^2 v_{F2}}{\hbar^4 v_1} (E_2^\infty v_{F2} \tau_2 I_4 + U_2 I_6) + T \frac{4 \gamma m_1^2 v_{F1}}{\hbar^4 v_1} (E_1^\infty v_{F1} \tau_1 I_3 + U_1 I_5) , \tag{28}$$

where

$$I_1 = \frac{v_{1\,\min}}{v_{F1}} + \frac{1}{v_{F1}} \int_{v_{1\,\min}}^{v_{F1}} \frac{v_1 dv_1}{v_1 + v_2} ,$$
$$I_2 = \frac{1}{v_{F2}} \int_0^{v_{F2}} \frac{v_2 dv_2}{v_1 + v_2} ,$$
$$I_3 = \frac{v_{1\,\min}}{2 v_{F1}^2} + \frac{1}{v_{F1}^2} \int_{v_{1\,\min}}^{v_{F1}} \frac{v_1^3 dv_1}{(v_1 + v_2)^2} , \tag{29}$$

$$I_4=\frac{1}{v_{F2}^2}\int_0^{v_{F2}}\frac{v_2^3 dv_2}{(v_1+v_2)^2}\ ,$$

$$I_5=\frac{v_{1\,\mathrm{min}}}{v_{F1}}+\frac{1}{v_{F1}}\int_{v_{1\,\mathrm{min}}}^{v_{F1}}\frac{v_1^2 dv_1}{(v_1+v_2)^2}\ ,$$

$$I_6=\frac{1}{v_{F2}}\int_0^{v_{F2}}\frac{v_2^2 dv_2}{(v_1+v_2)^2}\ .$$

Now we shall consider some limiting cases.

(1) Nearly identical metals, i.e., $v_1=v_2=v$, $R=0$, $T=1$, and $\tau_1=\tau_2=\tau$:

$$C_1=-C_2=-(E^{\infty}v\tau+U)\left[1-\frac{2\gamma m^2 v_F}{\hbar^4 v}\right]\ . \tag{30}$$

(2) Ideal interface, $\gamma=0$:

$$\begin{aligned}C_1&=(E_1^{\infty}v_1\tau_1+U_1)R-(E_2^{\infty}v_2\tau_2+U_2)T\ ,\\ C_2&=-(E_2^{\infty}v_2\tau_2+U_2)R+(E_1^{\infty}v_1\tau_1+U_1)T\ .\end{aligned} \tag{31}$$

VI. ELECTRICAL RESISTANCE OF THE CONTACT

With the known coefficients C_1 and C_2, and using Eqs. (10) and (11) we can find the voltage drops U_1, and U_2. It can be done in the analytical form for the limiting cases considered above.

(1) For nearly identical metals,

$$U_1=U_2=U=\frac{E^{\infty}\gamma m^2 v_F\tau}{\hbar^4}\ . \tag{32}$$

The voltage drop at the contact is $2U$. The specific resistance of the contact is given by

$$R=\frac{2U}{j_0}=\frac{\gamma m^2 l}{\hbar^4\sigma}\ , \tag{33}$$

where l is the bulk mean free path of metal.

(2) For the ideal interface ($\gamma=0$),

$$\begin{aligned}U_1&=v_{F1}(E_1^{\infty}\tau_1P_1+E_2^{\infty}\tau_2\tilde{P}_1)\ ,\\ U_2&=v_{F2}(E_1^{\infty}\tau_1P_2+E_2^{\infty}\tau_2\tilde{P}_2)\ ,\end{aligned} \tag{34}$$

where $P_{1,2}$ and $\tilde{P}_{1,2}$ are the following numerical coefficients:

$$P_1=\frac{1}{D}\left[-\tfrac{1}{6}-\tfrac{1}{2}I_1^{(1)}+\tfrac{1}{3}I_2^{(2)}+I_1^{(1)}I_2^{(2)}-16\frac{v_{F2}}{v_{F1}}I_4^{(1)}I_4^{(2)}\right]\ ,$$

$$\tilde{P}_1=\frac{1}{D}\left[\frac{4}{3}\frac{v_{F2}}{v_{F1}}I_4^{(1)}+4\frac{v_{F2}}{v_{F1}}I_4^{(1)}I_1^{(2)}+2I_3^{(1)}-4I_3^{(1)}I_2^{(2)}\right]\ ,$$

$$P_2=\frac{1}{D}\left[\frac{4}{3}\frac{v_{F1}}{v_{F2}}I_4^{(2)}+4\frac{v_{F1}}{v_{F2}}I_1^{(1)}I_4^{(2)}+2I_3^{(2)}-4I_2^{(1)}I_3^{(2)}\right]\ ,$$

$$\tilde{P}_2=\frac{1}{D}\left[-\tfrac{1}{6}-\tfrac{1}{2}I_1^{(2)}+\tfrac{1}{3}I_2^{(1)}+I_2^{(1)}I_1^{(2)}-16\frac{v_{F1}}{v_{F2}}I_4^{(2)}I_3^{(1)}\right]\ ,$$

and

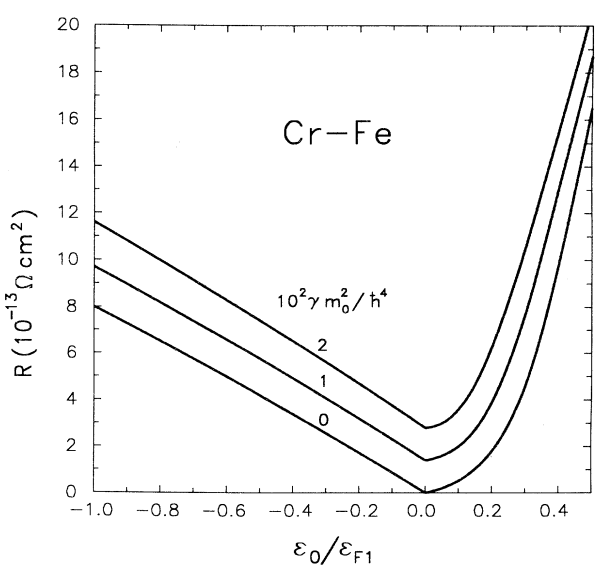

FIG. 1. Dependence of the Cr-Fe contact resistance on ε_0 for various values of γ.

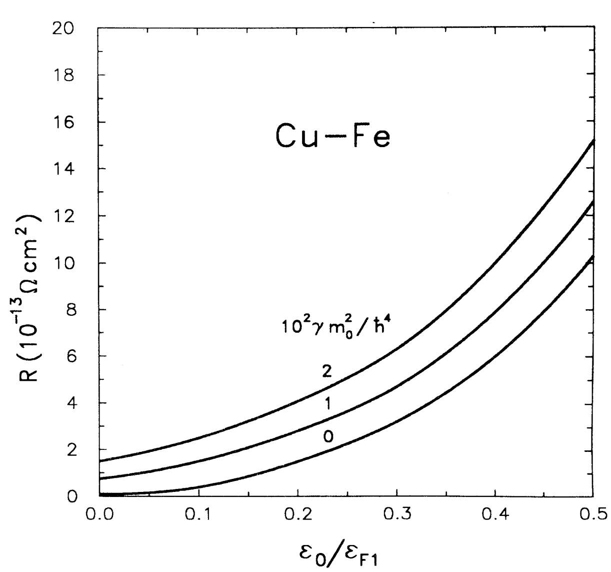

FIG. 2. The same for Cu-Fe contact resistance.

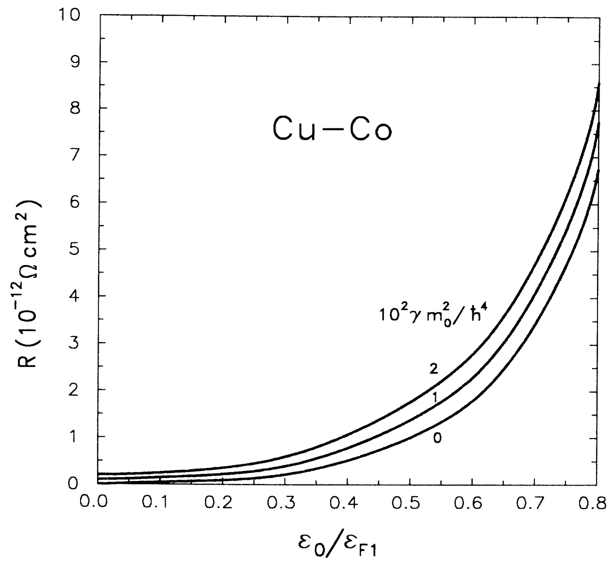

FIG. 3. The same for Cu-Co contact resistance.

$$D=16I_4^{(1)}I_4^{(2)}+\tfrac{1}{2}(I_2^{(1)}+I_2^{(2)})-I_2^{(1)}I_2^{(2)}-\tfrac{1}{4}\ ,$$

$$I_1^{(1)}=\frac{1}{v_{F1}^3}\int_0^{v_{F1}}Rv_1^2dv_1,\quad I_2^{(1)}=\frac{1}{v_{F1}^2}\int_0^{v_{F1}}Rv_1dv_1\ , \qquad (35)$$

$$I_3^{(1)}=\frac{1}{4v_{F1}^3}\int_0^{v_{F1}}Tv_1v_2dv_1\ ,I_4^{(1)}=\frac{1}{4v_{F1}^2}\int_0^{v_{F1}}Tv_1dv_1\ .$$

The integrals $I_i^{(2)}$ are given by the same expressions (35) with the replacements $v_1\leftrightarrow v_2$, $v_{F1}\leftrightarrow v_{F2}$. We should keep in mind that $v_2=0$ at $v_1<v_{1\,\min}$.

The final expression for the specific resistance in case of $\gamma=0$ is

$$R=R^{(1)}+R^{(2)}\ , \qquad (36)$$

where

$$R^{(1,2)}=\frac{2l_{1,2}}{\sigma_{1,2}}\left[P_{1,2}+\frac{\sigma_{1,2}}{\sigma_{2,1}}\frac{l_{2,1}}{l_{1,2}}\frac{v_{F1,2}}{v_{F2,1}}\tilde{P}_{1,2}\right]\ .$$

In case of different metals ($v_{F1}\neq v_{F2}$) and the nonideal contacting surface ($\gamma\neq 0$), the resistance can be obtained from the general formulas by numerical calculations. The contact resistance as a function of the potential step ε_0 is shown in Figs. 1–3. The bulk properties of metals were taken for Cr/Fe, Cu/Fe, and Co/Cu magnetic multilayers. To estimate the potential steps one can use the known Fermi energies[7] V_s, V_m, and V_M for the nonmagnetic metal, minority, and majority carriers, respectively. The sheet resistances for the minority and majority electrons can be determined from Figs. 1–3 if one takes for ε_0 the values $\varepsilon_0^{\downarrow}=(V_s-V_m)$ and $\varepsilon_0^{\uparrow}=(V_s-V_M)$, respectively. This gives $\varepsilon_0^{\uparrow}/\varepsilon_{F1}=0.036$ and $\varepsilon_0^{\downarrow}/\varepsilon_{F1}=0.33$ for Cu/Fe, $\varepsilon_0^{\uparrow}/\varepsilon_{F1}=-0.43$ and $\varepsilon_0^{\downarrow}/\varepsilon_{F1}=0.007$ for Cr/Fe, and $\varepsilon_{F1}=V_s$. For Co/Cu multilayers we have used[10] $\varepsilon_0^{\downarrow}/\varepsilon_{F1}=0.4$ and $\varepsilon_0^{\uparrow}/\varepsilon_{F1}=0.03$. We have taken the equal masses[7] $m_1=m_2=4m_0$, and the bulk conductivities[17] $\sigma_{\mathrm{Fe}}=1.16\times10^5\ \Omega^{-1}\mathrm{cm}^{-1}$, $\sigma_{\mathrm{Cr}}=7.09\times10^4\ \Omega^{-1}\mathrm{cm}^{-1}$, $\sigma_{\mathrm{Cu}}=6.45\times10^5\ \Omega^{-1}\mathrm{cm}^{-1}$, and $\sigma_{\mathrm{Co}}=1.80\times10^5\ \Omega^{-1}\mathrm{cm}^{-1}(T=300\ \mathrm{K})$.

VII. CONCLUSIONS

In this paper we have presented the solution of the problem of the contact resistance for the interface modeled by a two-dimensional random potential. Such a treatment corresponds to the sharp and flat separating surfaces. As a result, the contact resistance is determined by the macroscopic parameters of metals and the correlator of random fields. The dimensionless parameter of the interface nonideality is $\gamma m_0^2/\hbar^4$. We believe that our model can describe correctly the contacts fabricated by MBE technology.

Note that in the case of scattering from short-ranged impurity centers at the interface the roughness parameter γ can be presented as $\gamma=N_iW_0^2$, where N_i is the planar concentration of impurities and W_0 is the Fourier transform of the δ-like potential.

The explicit dependence of the specific contact resistance upon the nonideality parameter makes it possible to estimate quantitatively the effects of the interface nonideality on the contact resistance of two metals. In contrast to the Fuchs-Sondheimer approach, in which the specular parameters were introduced phenomenologically, in our treatment the nonideality is taken into account with the help of the boundary conditions for electron wave functions.

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[1] H. Y. Fan, Phys. Rev. **61**, 365 (1942); **62**, 388 (1942).
[2] M. N. Baibich, J. M. Broto, A. Fert, Nguyen Van Dau, F. Petroff, P. Eitenne, G. Creuzet, A. Friederich, and J. Chazelas, Phys. Rev. Lett. **61**, 2472 (1988).
[3] J. Barnas, A. Fuss, R. E. Camley, P. Grünberg, and W. Zinn, Phys. Rev. B **42**, 8110 (1990).
[4] J. M. George, L. G. Pereira, A. Barthelemy, F. Petroff, L. Steren, J. L. Dvail, A. Fert, R. Loloee, P. Holody, and P. A. Schoeder, Phys. Rev. Lett. **72**, 408 (1994).
[5] Y. Asano, A. Oguri, and S. Maekawa, Phys. Rev. B **48**, 6192 (1993).
[6] P. B. Visscher and Hui Zhang, Phys. Rev. B **48**, 6672 (1993).
[7] R. Hood and L. M. Falicov, Phys. Rev. B **46**, 8287 (1992); R. Q. Hood, L. M. Falicov, and D. R. Penn, *ibid.* **49**, 368 (1994).

[8]T. Valet and A. Fert, Phys. Rev. B **48**, 7099 (1993).

[9]A. Braatas and G. E. W. Bauer, Europhys. Lett. **26**, 117 (1994).

[10]J. Barnas and A. Fert, Phys. Rev. B **49**, 12 835 (1994); A. Fert, T. Valet, and J. Barnas, J. Appl. Phys. **75**, 6693 (1994).

[11]M. A. M. Gijs, S. K. J. Lenczowski, and J. B. Giesbers, Phys. Rev. Lett. **70**, 3343 (1993).

[12]W. P. Pratt, S. F. Lee, R. Loloee, P. A. Schroeder, and J. Bass, Phys. Rev. Lett. **66**, 3060 (1991).

[13]K. Fuchs, Proc. Cambridge Philos. Soc. **34**, 100 (1938).

[14]H. Sondheimer, Adv. Phys. **1**, 1 (1952).

[15]L. A. Falkovsky, Adv. Phys. **32**, 753 (1983).

[16]*Handbook of Mathematical Functions,* edited by M. Abramowitz and I. A. Stegun, Natl. Bur. Stand. (U.S.) Appl. Math. Ser. (GPO, New York, 1964).

[17]*Physical Encyclopedic Dictionary,* edited by A. M. Prokhorov (Sovietskaya, Moscow, 1983), p. 410 (in Russian).