High-frequency pumping of Josephson soliton oscillators

M. Cirillo
Dipartimento di Fisica, Universitá di Roma "Tor Vergata", I-00133 Roma, Italy

G. Rotoli
Dipartimento di Energetica, Universitá di L'Aquila, I-67040 L'Aquila, Italy

A. R. Bishop, N. Grønbech-Jensen, and P. S. Lomdahl
Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545
(Received 14 November 1994)

We study the phase locking of a long Josephson junction operating in a fluxon oscillator regime to external rf signals whose frequency is an even harmonic of the oscillator frequency. The phenomenon is investigated for different values of the dc bias current in the junction, and around each value the current intervals that allow phase locking are evaluated. These intervals can be well identified even when the drive frequency is sixteen times the fluxon oscillator frequency. The explanation of the numerical data that we obtain is given in terms of analytical approximations for a long Josephson junction model. An equation containing relevant experimental parameters is derived for the current-locking ranges generated by the different harmonics. This equation establishes that the amplitude of these intervals decreases exponentially with the harmonic number and we show how this result can be related to a property of the Fourier spectrum of fluxon oscillations. In all the cases that we analyze very good agreement is found between the numerical evidence and theoretical analysis.

I. INTRODUCTION

The study of coherence and phase locking phenomena in extended Josephson structures is a very interesting issue in solid state physics because it deals with the response of a system containing an internal (spatial) degree of freedom to external excitations. In the past much attention has been devoted to the phase locking of one-dimensional junctions biased on zero-field steps of the current-voltage characteristics.[1–5] These singularities are generated by magnetic-flux-quanta shuttling motion in the junction and in several cases the problem of the phase locking has been reduced to the study of the motion of a particle (whose dynamics is governed by an effective Hamiltonian) in an external oscillating field.[1,2,4,5] Very recently, the study of phase locking in cases where the phase oscillations inside a long junction cannot be treated as a point particle has begun.[6,7] Results have also been reported concerning the phase locking of long junctions operated in the flux-flow regime[8] and interesting experiments have been performed on phase-locked inductively coupled junction oscillators.[9,10]

The wide interest that the problem of phase locking may have in nonlinear dynamics has recently been recognized even for the discretized versions of wave equations. In fact, the problem of phase locking in discrete Josephson transmission lines was investigated[11] and it has been shown by Cai *et al.*[12] that the phenomenon of the locking of nonlinear coherent structures to oscillating fields can produce interesting effects even for other discretized wave equations.

In spite of the growing interest that phase locking has received, a systematic study giving direct information about ranges of locking, parameter spaces, and other experimental quantities is still lacking. A study of the dependence of parameters of the current-voltage characteristics of long Josephson junctions in the presence of uniform rf driving currents has been reported recently by Cirillo *et al.*[13] These authors found that, in the case of a junction driven uniformly by a rf external current, there exists a simple and straightforward relation between the intervals of locking and the dynamical resistance of the zero-field steps. In particular, the current ranges of locking are proportional to the applied rf current through a coefficient in which the dynamical resistance enters linearly. The case investigated represents perhaps the simplest possible physical configuration, very suitable for experimental investigations in which the Josephson junction can be considered uniformly irradiated by rf fields.[14,15]

The results mentioned in the previous paragraph, however, were obtained for the case in which the fluxon voltage is equal to the Josephson voltage, i.e., when the frequency of the applied rf signal equals twice the fluxon oscillator frequency. The case in which the drive frequency is a higher multiple of the fluxon oscillation frequency was not analyzed in detail. In particular, no attempts were made to give phenomenological or analytical explanations for the few numerical data. Subharmonic locking is relevant for practical applications as it is not trivial to find a nonlinear system that can generate it in a stable and controlled manner. In the present work we consider the subharmonic locking in long Josephson junctions.[5] We show that the subharmonic locking is a well controllable mechanism that can be obtained even for relatively high harmonic numbers due to the extremely sharp nature of the voltage peaks associated with a moving fluxon. We find the parameter ranges for the phase-locking intervals and analyze the details of the locking process in terms of fluxon dynamics.

In this paper we will concentrate on phase-locking phenomena generating major effects in the observable current-voltage characteristics of Josephson junctions. For this reason we will consider only phase-locking phenomena for drive frequencies equal to even multiples of the fluxon oscillation frequency. The odd multiple frequencies can be shown to generate phase locking over much more limited intervals of parameter values.[5]

The paper is organized as follows. In the next section we briefly review the results obtained for the phase locking of the long overlap junction to a drive signal whose frequency is twice the fluxon oscillator frequency. In Sec. III we show the results obtained numerically for the subharmonic locking. In Sec. IV we compare the obtained results with analytical approximations and present physical explanations for the observed phenomena. In Sec. V we conclude the paper and briefly discuss the impact of the results on Josephson junction physics and devices.

II. UPPER LIMIT FOR THE LOCKING INTERVALS

In the following we briefly review previous work as it is relevant for the present purposes. In Refs. 5 and 13 large parameter excursions were considered in the case of a long overlap Josephson junction uniformly driven in space by a rf signal whose frequency is twice the fluxon oscillation frequency. We recall that when the external drive frequency is twice the fluxon oscillation frequency the dc voltage due to the ac Josephson effect and the "fluxon" voltage are equal.[16]

The model equation under analysis is the sine Gordon with dc and rf driving forces[13]

$$\phi_{tt} - \phi_{xx} + \sin\phi + \alpha\phi_t = \rho_0 + \rho_1 \sin(\omega t) \tag{1}$$

with open circuit boundary conditions, i.e., $\phi_x(0,t) = \phi_x(l,t) = 0$ over the finite spatial interval $[0,l]$. In Eq. (1) and everywhere else in this paper time and space scales are normalized to Josephson characteristic time and lengths[16] and the current terms are normalized to the maximum Josephson supercurrent in the junction. The numerical results presented in this paper have been obtained for a long junction model of length $l = 4$ and a loss parameter $\alpha = 0.1$. The investigations we performed were made for three different values of the dc bias current ρ_0: 0.26, 0.3, and 0.35. For these values of the dc bias current and for a fixed value of the dissipative parameter the corresponding fluxon oscillation frequencies were, respectively, 0.678, 0.704, and 0.726. We recall that these frequencies correspond to the inverse of the time spent by the fluxon to cover a distance $2l$.

For the sake of reproducibility of our results we specify here the most relevant parameters of our numerical integration procedures. We obtained current-voltage characteristics by sweeping the dc bias current in increments of 10^{-5}. At every increment a transient of 2000 rf cycles was allowed to develop, and after this transient average values of voltages from Eq. (1), i.e., $\langle\phi_t\rangle$, were obtained over 1000 rf cycles. We found that these transient and averaging times were safe margins for our investigations. The CPU time required for these simulations is such they could be performed on desktop machines such as SUN workstations or 486 PC's.

In Fig. 1 we show the results for the phase-locking intervals ΔI as a function of ρ_1 for the three different values of the dc bias current investigated. According to the results of Ref. 13 the slope of the straight lines in Fig. 1 (least squares fits to the data) can be predicted by the equation

$$\Delta I_2/\rho_1 = (1/\sqrt{2})\alpha R_d \,, \tag{2}$$

where R_d is the normalized dynamical resistance evaluated at the corresponding dc bias point on the zero-field step.[13] We recall that $1/\alpha$ in Eq. (2) represents the normalized Ohmic resistance of the long junction and, therefore, on the right hand side of the equation we have the ratio of two resistances. From the data of Fig. 1 we have found that the discrepancy between the numerical result and Eq. (2) is within a few percent. In Eq. (2) the subscript in ΔI_2 indicates that we are considering the case $N = 2$, i.e., the drive frequency is twice the fluxon oscillation frequency.

When comparing Eq. (2) with numerical results attention must be paid to the evaluation of the dynamical resistance because an inaccurate estimate of this parameter may give rise to larger discrepancies than the ones we obtained. We evaluated the dynamical resistance by differentiating an eighth-order polynomial curve fitting the current-voltage characteristic points spaced by a 10^{-5} current step. Fitting by lower-order polynomials gave results that were identical. The uncertainty of our fittings was always within 3%. We point out that Eq. (2) uses a linear approximation for the portion of zero-field step centered on the dc bias point of interests.[13] In general, we have found that the values of the rf drive levels for which this can be a good approximation depend on junction length, loss parameter, and dc bias level.

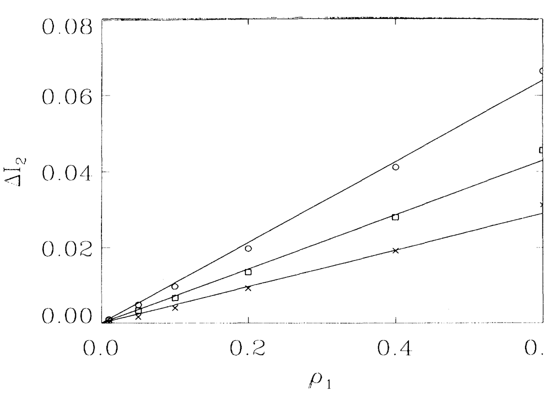

FIG. 1. Current intervals of locking as a function of the applied rf current (ρ_1) for the three dc bias points that we analyze. The uppermost data correspond to $\rho_0 = 0.26$, the middle data to $\rho_0 = 0.3$, and the lowermost to $\rho_0 = 0.35$.

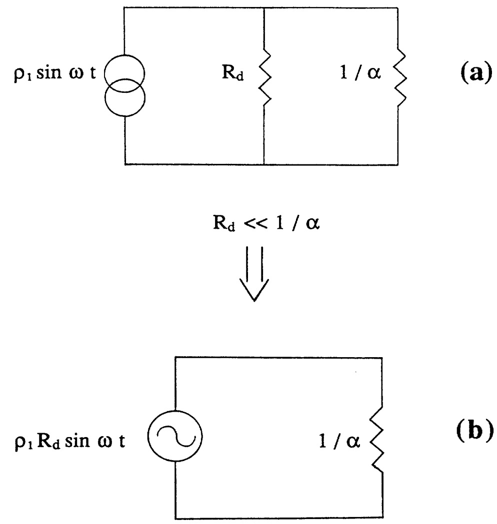

FIG. 2. Circuit modeling of the driven long Josephson junction. Since the dynamical resistance of the junction biased on a zero-field step is always small with respect to the Ohmic impedance of the tunnel junction, we can step from current bias to voltage bias.

The basic idea beyond Eq. (2) is the fact that the long junction is viewed (follow Fig. 2) as a parallel combination of a resistance R_d, the dynamical resistance of the junction when biased on the zero-field step, and the resistance $1/\alpha$. Since generally $R_d \ll \alpha^{-1}$, the parallel combination of these two resistances and the current generator $\rho_1 \sin \omega t$ can be approximated by a voltage generator of amplitude $\rho_1 R_d \sin \omega t$ applied to the resistance $1/\alpha$. The rms current across this resistance gives the maximum current available for the locking process at the given frequency. We recall that the functional relationship leading to Eq. (2) can also be obtained on the basis of a perturbation model for the sine-Gordon Hamiltonian. In this case, however, we get a different value for the slope of the straight lines of Fig. 1.[13] Both the circuit analysis and the perturbation model rely on the low-amplitude rf drive limit, but, as we can see from Fig. 1, both the functional dependencies and the slope prediction apply very well up to values as high as half the critical current of the junctions [currents in Eq. (1) are normalized to the maximum Josephson supercurrent in the junction].

In Sec. IV we will see how it is possible to extend the approximate analysis performed for the $N = 2$ case to higher even harmonics. At this point, however, we can say that, for a fixed value of the rf drive amplitude, the locking intervals at higher harmonics are always smaller than those evaluated for $N = 2$ and therefore an upper limit has been established on the basis of readily evaluable experimental parameters. We will next analyze the higher-harmonic case, focusing our attention on possible experimental evidence.

For the higher harmonics we have considered the eventuality of adding to the model of Fig. 2 a capacitance in parallel with the resistance $1/\alpha$. This element (modeling the total capacitance of the tunnel junction) at higher frequencies can short the current flowing through $1/\alpha$. However, we found that the predicted effects were absolutely negligible with respect to the ones observed in the simulations.

III. NUMERICAL RESULTS

As for the $N = 2$ case, for the three given dc bias points we evaluated the response to the different harmonic excitations. The results for the bias point $\rho_0 = 0.26$ are shown in Fig. 3(a). In this figure we see, from top to bottom, the locking ranges corresponding, respectively, to $N = 2, 4, 6, 8, 10$. Even in this case the dependence of

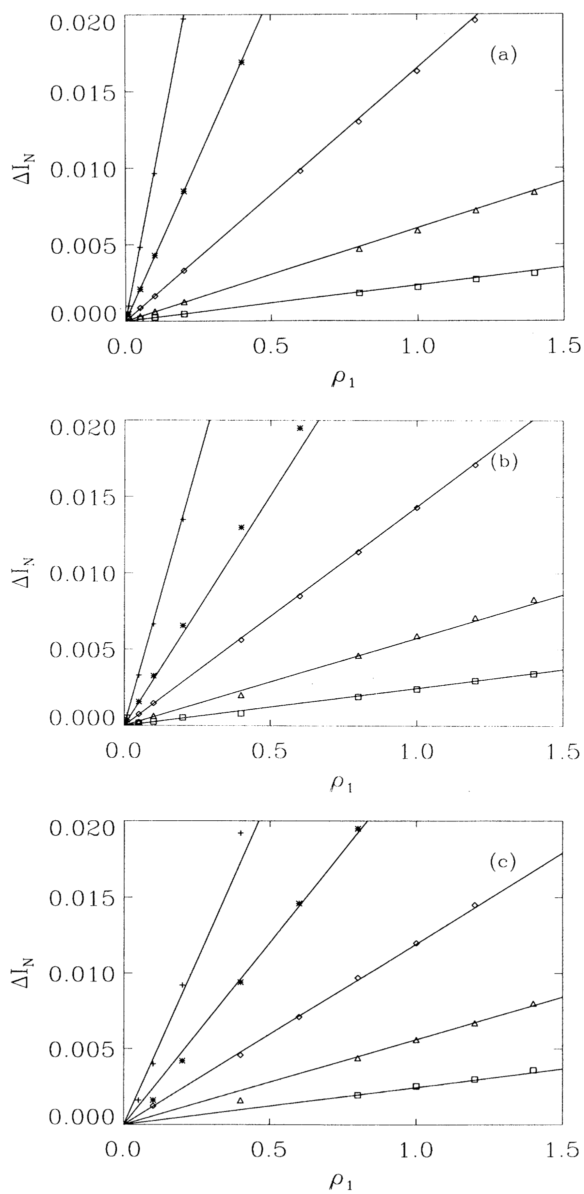

FIG. 3. Current ranges of locking for $\rho_0 = 0.26$ (a), $\rho_0 = 0.3$ (b), and $\rho_0 = 0.35$ (c). From the top to the bottom we report, respectively, the harmonic locking at $N = 2, 4, 6, 8, 10$.

the locking intervals upon the external rf field is linear. Indeed, as a general feature we have seen that the dependence remains linear even for higher values of the rf field (including $\rho_1 \gg 1$), while the same does not happen for the $N = 2$ case. However, the slope of the straight lines in this case cannot be related in a simple manner to the locking intervals and it is clear that an explanation of the observed locking ranges can only be obtained by considering the full problem of harmonic locking for higher values of N.[5]

The difficulty concerning slope prediction anticipated in the last paragraph is more evident from Fig. 3(b) where we show the results for the bias point $\rho_0 = 0.3$. Comparing Figs. 3(a) and 3(b) we see that on increasing N the difference between the slopes of the linear fit to the data corresponding to the same harmonic for different bias points decreases. This tendency is confirmed by the data relative to the $\rho_0 = 0.35$ bias point: for increasing values of N there is very little difference between the slopes of the straight lines obtained for the three different dc bias values [see Fig. 3(c)].

It is somewhat surprising that the linear dependence of the locking ranges upon the rf amplitude persists for high harmonics even for relatively high amplitudes. Increasing the harmonic number further, it becomes difficult to observe phase locking for low values of the applied rf current. However, increasing the value of this parameter one can still observe a linear dependence of the locking intervals upon the rf current amplitude. This observation is demonstrated by Fig. 4 in which we plot the dependence of the locking intervals upon ρ_1 for $N = 12$ and $N = 16$ for the three different values of the dc bias current. We see that even in this case there is very little dependence of the slopes upon the dc bias.

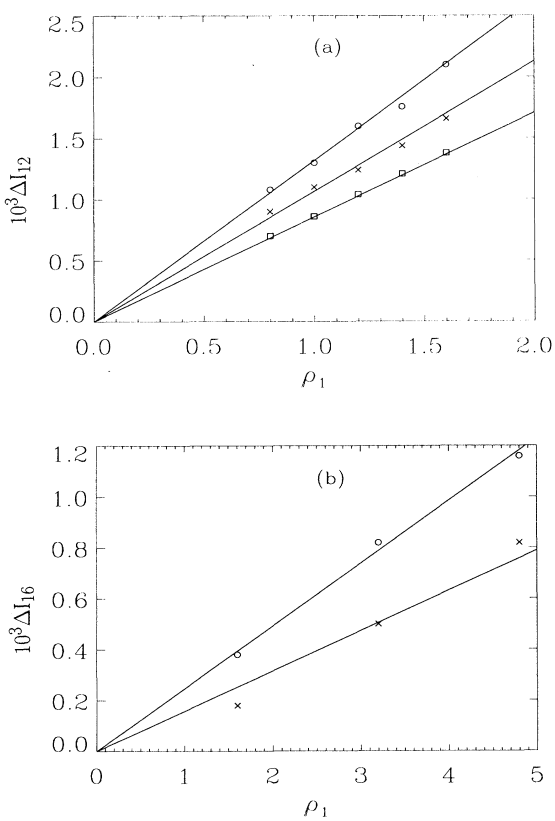

FIG. 4. Current-locking ranges for $N = 12$ (a) and $N = 16$ (b). In (a) the circles, crosses, and squares refer, respectively, to the bias points 0.35, 0.3, and 0.26. Note that in this case the slope of the straight lines increases for higher values of the dc bias. In (b) we observe the same tendency, as the circles refer to the bias point 0.35 and the crosses to 0.3.

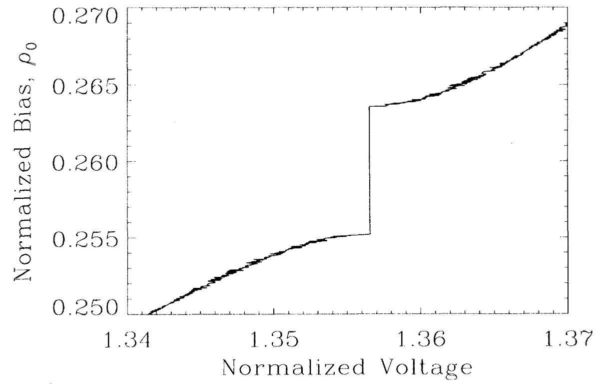

FIG. 5. A current-voltage characteristic for the long junction model when pumped with a rf current having amplitude 10 (in normalized units) and a frequency corresponding to the 12th harmonic of the soliton oscillation frequency. The phase-locked step appears at the subharmonic frequency on the zero-field step.

In Fig. 5 we show a current-voltage characteristic which exhibits a phase-locked current step on a zero-field step obtained for the value $\rho_1 = 10$ and $N = 12$ (a); the range of locking for this value of the rf current is equal to ten times the one obtained for $\rho_1 = 1.0$, meaning that we are still in a linear regime. We note that the current amplitude of the phase-locked step for $N = 12$ is 8.3×10^{-3} which represents 3.2% of the dc bias point value (0.26 in this case).

In order to gain more insight into the functional dependences of the locking intervals for the different harmonics we have plotted the dependence of the slopes $\Delta I_N/\rho_1$ for all the harmonics corresponding to a given dc bias point. In Fig. 6 the triangles represent the numerical data for $\rho_0 = 0.26$. As emphasized by the dashed line interpolating the triangles we see that there exists an exponential dependence of the intervals upon the harmonic number N. Since the first point ($N = 2$) corresponds to the locking at twice the soliton frequency [Eq. (2)] our conclusion is that the following extension of Eq. (2), as a first rough approximation, can account for the dependence of the locking ranges upon the harmonic number:

$$\Delta I_N = \Delta I_2 e^{-k(N-2)} , \tag{3}$$

where $N = 2, 4, 6 \ldots, k$ is a constant, and ΔI_2 is given by Eq. (2).

In Fig. 6 we have also plotted the dependence of the locking intervals upon the harmonic number for the other two bias points. In the figure the squares correspond to

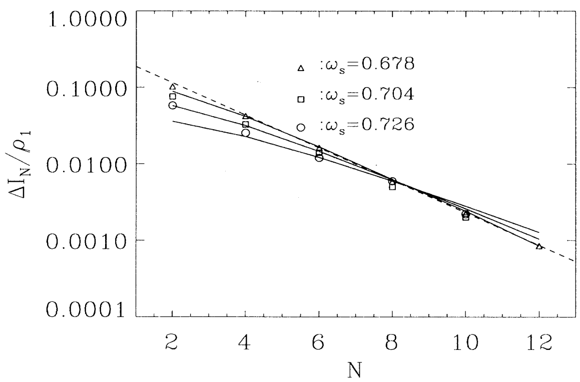

FIG. 6. Dependencies of the slopes $\Delta I/\rho_1$ upon the harmonic number N for $\rho_0 = 0.26$ (triangles), $\rho_0 = 0.3$ (squares), and $\rho_0 = 0.35$ (circles); the straight-dashed line represents a least-squares fit to the data for $\rho_0 = 0.26$ and the full lines represent the predictions of the theory [Eq. (5)] for the three cases.

$\rho_0 = 0.3$ and the circles to $\rho_0 = 0.35$. We see that the basic features described above are again present. From the data of Fig. 6 it seems that there is weak dependence of the slope of the straight line upon the dc bias point. However, the exponential tendency is present even for the two other bias points. Fitting the data of Fig. 6 in the higher-N region with straight lines we have found that the value of the constant k in the exponent is equal to 0.48 for the data corresponding to $\rho_0 = 0.26$, to 0.45 for $\rho_0 = 0.3$, and to 0.4 for $\rho_0 = 0.35$.

Equation (3) contains most of the information needed for evaluating the phase-locking intervals for the uniform dc and rf bias case. In a given experimental condition it is simple, given the profile of a zero-field step, to evaluate the dynamical resistance. Also, from the measurement of the subgap resistance one can derive the McCumber parameter $\beta_c = 1/\alpha^2$. These are all the parameters needed to check our results from an experimental point of view.

The voltage ranges of locking that may be measurable in direct Josephson radiation experiments can be evaluated from Eq. (3) following the Taylor-expansion arguments employed in Ref. 5 for the locking ranges at $N = 2$, i.e., $(\Delta V_N/\Delta I_N) = R_d$. Therefore we expect

$$\Delta V_N = \Delta V_2 e^{-k(N-2)} \,, \tag{4}$$

where $\Delta V_2 = (1/\sqrt{2})\alpha R_d^2 \rho_1$.

It is worth noting at this point that Eqs. (3) and (4) represent only an ansatz that fits the data reasonably well. However, the same ansatz, applied to the results obtained for a completely different parameter set,[5] gives a very good explanation of the numerical data. For the set of data presented in Ref. 5 the value of the constant k (the slope) is equal to 0.25. The parameter set used in these simulations was $\rho_0 = 0.0336, \alpha = 0.025$, and $l = 8$.

IV. DISCUSSION AND MODELING

We have seen at the end of the last section that our ansatz is consistent with a set of data reported in Ref. 5 that, in turn, are consistent with a theoretical, perturbation analysis of the phase-locking phenomenon in long, uniformly driven long junctions. Thus it is natural to check whether the proposed theoretical model can be applied even for moderately long junctions like the one that we used, having a normalized length equal to 4. Therefore, in Fig. 6 we have reported the prediction of the theory [Eqs. (11) and (12) of Ref. 5] superimposed on our numerical data. We note that for the three cases we have analyzed there is little difference between the ansatz Eq. (3) and the perturbation analysis results. The difference becomes more evident for lower values of N. Indeed, one can derive analytically an expression showing that in the large N limit our ansatz and the theoretical analysis basically coincide. Specifically, in the limit $u \to 1$ and large N the following analytical expression can be derived from Eqs. (12)–(14) of Ref. 5:

$$\Delta I_N/\rho_1 = 2\gamma^{-4}(u)\pi^2 \frac{N}{L^2} \exp\left(-\frac{N\pi^2}{2L\gamma(u)}\right) \,. \tag{5}$$

In these expressions $\gamma(u) = 1/\sqrt{1-u^2}$ and u is the soliton power balance velocity.[17] Equation (5) gives a functional dependence for the locking intervals as a function of the harmonic number which is not very different from Eq. (4) because the dominating exponential determines the shape of the function. For large N the functional dependencies of Eqs. (4) and (5) are basically the same. Equation (5) gives the correct values for the slopes of the straight lines interpolating the data of Fig. 5 for large N. For the three values of the bias current $0.26, 0.3$, and 0.35 we get, respectively, for the coefficient of the exponential of Eq. (5) $0.54, 0.48$, and 0.42. These coefficients agree very well with those that we have found numerically $(0.48, 0.45, 0.4)$. The discrepancy decreases on increasing the dc bias current as we expect because of the high-velocity approximation made in deriving Eq. (5). Better agreement is expected if the original, but more complicated, expressions from Ref. 5 were used.

The good agreement between the theoretical prediction and the data ensures us that the boundary-pumping model proposed by one of the authors[5] can be applied to predict the ranges of phase locking for long junctions whose length is relatively short (4 in our case). Going through important features of the theoretical analysis employed in Ref. 5 enables us to understand the higher-harmonics results within the same equivalent-circuit analysis reviewed in Sec. II.

The necessary condition for obtaining the phase-locking current intervals in the framework of the perturbation model is derived by requiring the time integral (evaluated over one period of motion) of the time derivative of the total energy of the system to be zero. An inspection of the form of this integral [Eqs. (7) and (10) of Ref. 5] indicates that it consists essentially of the Fourier transform of the soliton profile. Thus we conclude that the "phase-locking" currents are related to the Fourier transform of the soliton wave form. Bearing in mind that ϕ_t is related to the voltage generated by the fluxon in the long junction, we have translated this observation in terms of circuit analysis as follows.

In the circuit model discussed in Sec. II one proceeds from the current-source model to the voltage-source model knowing the voltage generated across R_d and $1/\alpha$.

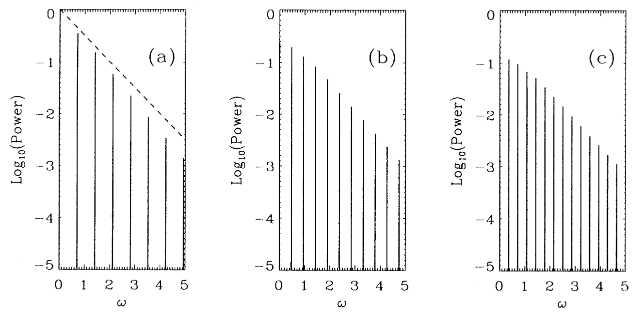

FIG. 7. Fourier spectrum of the voltage wave form generated by fluxon oscillations at one end of the junction for three different lengths, $l = 4$ (a), $l = 6$ (b), and $l = 8$ (c). We see that the amplitude is a linearly decreasing function of the harmonic number N. The dashed line in the leftmost spectrum represents the result expected from our model.

One implicitly assumes that the amplitude of the component at the frequency $\omega_2 = 2\omega_s$ of the voltage present across the two resistors must be equal. This is very reasonable because the average voltages at this frequency in the phase-locked state must be equal. It is clear that the same kind of arguments can be used if we have information about higher components of the voltages present across R_d.

We have performed fast Fourier transforms (FFT's) of the voltage wave form generated by the oscillating fluxon and the result, for three different lengths, is shown in Fig. 7. These wave forms have been obtained for zero external rf current, with a dc bias point equal to 0.3 and for the three lengths $l = 4, 6, 8$. In the figure we see the amplitude of the components decreasing linearly, indicating that, considered the logarithmic scale of the power axis, the amplitude of the Fourier components is an exponentially decreasing function of the harmonic number. This result tells us that an approximation for amplitude of the fluxon-voltage Nth Fourier component is

$$V_N = V_2 e^{-s(N-2)} , \tag{6}$$

where s is a constant. We know that, as far as phase locking is concerned, a good approximation for V_2 is $\rho_1 R_d$. If we apply the voltage $V_N \sin \omega_N t$ across the resistor $1/\alpha$, from the Kirchhoff voltage law we obtain an expression for the current-locking ranges identical to Eq. (3). In Fig. 7(a) the dashed line indicates the slope that we obtain for $s = 0.48$, i.e., for the coefficient obtained from the data of Fig. 6. We see in Fig. 7 that the slope accounts very well for the observed exponential decay of the Fourier component's amplitudes.

V. CONCLUSIONS

In the present work we have shown how it is possible to quantify the most physically relevant phase-locking phenomena in uniformly driven long Josephson junctions. The uniformly driven Josephson junction is described by a model equation that has been successfully employed in the past in several physical circumstances to describe experimentally observed phenomena[15] and physics of Josephson devices. Complementary to Ref. 5, we have demonstrated that, due to the large harmonic content of the wave forms associated with shuttling fluxons, it is possible to phase-lock the long junctions to rf signals whose frequency is much higher than the fundamental frequency associated with the fluxon oscillations. We have reported here on phase-locking intervals when the frequency of the external drive was up to 16 times the fundamental fluxon oscillation frequency.

This phenomenon could be very useful for devices in which a low-frequency reading of high-frequency signals is necessary. A typical application for example would be a readout of the pump signal for voltage standard devices.[18] The Josephson voltage standard is usually pumped with a 70–90 GHz signal. An integration of the standard including a superconducting (or semiconductor) oscillator would certainly be simplified if the readout of the frequency of the signal can be made at lower frequencies. Our simulations have shown that the long junction fluxon oscillator could provide the basic ingredients for this kind of property. We note that we have observed dc locking ranges at the 16th harmonic of the soliton oscillation frequency representing up to 10% of the bias current. If we consider that a typical value for a current bias point on a zero-field step can be of the order of several hundred microamperes, we conclude that the amplitude of the phase-locked step can be of the order of several tens of microamperes.

An important feature of the operation described in the last paragraph is the fact that the fluxon oscillator, according to our simulations, requires moderate amplitudes of the rf pumping current for the ranges of operation that can be interesting in practice. For example, a down conversion of a factor 10, which means going down for example from 90 GHz to 9 GHz, would require a rf current less than ten times the Josephson critical current. This critical current for a long junction fluxon oscillator is typically of the order of 1 mA which means coupling to the oscillator ten times as much rf current. These current levels are nowadays available even from integrated superconducting millimeter-wave oscillators.

We have given expressions for the voltage and current ranges of locking that are versatile and contain readily evaluable experimental parameters. The agreement between an analytical perturbation treatment of the sine-Gordon equation containing rf and dc driving terms and the numerical results has given us the key to interpret the numerical data in terms of a circuit model of a driven long Josephson junction. The exponentially decreasing amplitude of the locking ranges as a function of the harmonic number is directly related to the exponentially decreasing amplitude of the Fourier components of the voltage wave form of fluxon oscillations.[5] This physical phenomenon has given us the key to establish a link between numerical data, analytical approximations, and circuit modeling.

ACKNOWLEDGMENTS

This work was performed under the auspices of the U.S. Department of Energy. Partial financial support by the Superconductive and Cryogenic Technologies project of the CNR (Italy) is also acknowledged.

[1]M. Salerno, M. R. Samuelsen, G. Filatrella, S. Pagano, and R. D. Parmentier, Phys. Lett. A **137**, 75 (1989); Phys. Rev. B **41**, 6641 (1990).
[2]B. A. Malomed, Phys. Rev. B **41**, 2037 (1990).
[3]N. F. Pedersen and A. Davidson, Phys. Rev. B **41**, 178 (1990).
[4]N. Grønbech-Jensen, Phys. Rev. B **47**, 5504 (1993).
[5]N. Grønbech-Jensen, Phys. Lett. A **169**, 31 (1992).
[6]N. Grønbech-Jensen and M. Cirillo, Phys. Rev. B **50**, 12 851 (1994).
[7]N. Grønbech-Jensen, P. S. Lomdahl, and M. Cirillo, Phys. Rev. B **51**, 11 690 (1995).
[8]A. V. Ustinov, J. Mygind, and V. A. Oboznov, J. Appl. Phys. **72**, 1203 (1992).
[9]T. Holst, J. Bindslev Hansen, N. Grønbech-Jensen, and J. A. Blackburn, Phys. Rev. B **42**, 127 (1990).
[10]A. V. Ustinov, H. Kohlstedt, and C. Heiden, Appl. Phys. Lett. **65**, 1457 (1994).
[11]A. V. Ustinov, M. Cirillo, and B. A. Malomed, Phys. Rev. B **47**, 8357 (1993).
[12]D. Cai, A. R. Bishop, N. Grønbech-Jensen, and B. A. Malomed, Phys. Rev. E **50**, 694 (1994).
[13]M. Cirillo, P. Cocciolo, V. Merlo, N. Grønbech-Jensen, and R. D. Parmentier, J. Appl. Phys. **75**, 2125 (1994).
[14]M. Cirillo and F. L. Lloyd, J. Appl. Phys. **61**, 2581 (1987).
[15]R. L. Kautz, in *Structure, Coherence and Chaos in Dynamical Systems*, edited by P. L. Christiansen and R. D. Parmentier (Manchester University Press, Manchester, 1989), pp. 207–226; *Metrology at the Frontiers of Physics and Technology*, edited by L. Crovini and T. J. Quinn (North-Holland, Amsterdam, 1992), pp. 259–296.
[16]R. D. Parmentier, in *The New Superconducting Electronics*, edited by H. Weinstock and R. W. Ralston (Kluwer, Dordrecht, 1993), pp. 221–248.
[17]D. W. McLaughlin and A. C. Scott, Phys. Rev. A **18**, 1672 (1978).
[18]M. Cirillo, Italian Patent No. RM94A000418 (24 June 1994).