Second-sound wave in photoinjected plasma in semiconductors

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We consider the question of propagation of thermal perturbations in the carrier system in photoinjected plasma in semiconductors. A hydrodynamiclike approach is introduced in the spirit of extended irreversible thermodynamics (EIT) that incorporates the energy flux as a basic thermodynamic variable. Its equation of evolution is derived at the microscopic level using the nonequilibrium statistical operator method which provides the foundations for informational statistical thermodynamics. This equation is of the Maxwell-Cattaneo type of EIT, which replaces the Fourier constitutive equation of classical irreversible thermodynamics. Used in conjunction with the equation of conservation for the energy density, together with the definition of a nonequilibrium temperature, it leads to a wave equation with damping, implying the propagation of second sound.

I. INTRODUCTION

In the preceding paper in this issue, hereafter referred to as (I), we presented an analysis of the hydrodynamics of the carrier system in a highly photoexcited plasma in direct-gap polar semiconductors (HEPS).[1] The hydrodynamic modes of the system are associated with material and thermal motion (plasma waves and heat waves, respectively). Both types of modes are coupled together through cross-kinetic terms in the equations of evolution. For the purpose of a clearer presentation and discussion of the relevant physical properties associated with these phenomena, we have separated the analyses of both types of motion. Each one is quite relevant by itself, and, as discussed in (I), the coupling terms do not introduce any new or fundamental physical fact—at least within the conditions we are considering here—but only minor numerical corrections to the frequency and lifetime in the dispersion relations of the hydrodynamical modes. In (I), after presentation of the general theory, we considered the material motion in the carrier system in HEPS (damped plasma waves), as decoupled from the thermal motion. Here we consider thermal motion (leading to second-sound waves) as decoupled from the material motion.

Thermal effects are, of course, also expected to occur in the lattice: in fact, Guyer and Krumhansl, by means of a calculation based on a Boltzmann-like transport theory for the pure phonon field, obtained a set of macroscopic equations which describe a Poiseuille-like flow and propagation of second sound with damping.[2] It has been shown that these results can also be obtained within the framework provided by extended irreversible thermodynamics.[3]

This theory,[4] which supersedes classical irreversible thermodynamics, needs to be introduced because, depending on experimental conditions, thermal waves are not compatible with the local equilibrium assumption since they imply heat transport from cold to hot regions during short-time intervals, and, further, an infinite speed of propagation (as implied by Fourier's law) is an inadmissible feature, even though it is usually not a dangerous paradox in the context of classical irreversible thermodynamics in certain limiting conditions, as discussed below in this paper. The question of transmission of heat has been reviewed and interpreted in a relevant article by Joseph and Preziosi,[5] which presented a relatively complete chronology of thought about the subject up to the late 1980s. To look for thermal effects in the carrier system in HEPS, we resort to the methods described in (I); that is, an informational statistical thermodynamics[6] (sometimes referred to an informational-theoretic thermodynamics). We derive microscopic mechanostatistical level equations of evolution of the Maxwell-Cattaneo type of extended irreversible thermodynamics[4] for the energy flux and for the energy density. Introducing a space- and time-dependent nonequilibrium carrier temperature, we find it to satisfy an evolution equation of the form of a damped wave, corresponding to a second sound, that in the charged carrier system is tempting to call *second plasma wave.*

II. SECOND-SOUND WAVE IN HEPS

We again consider the system of photoinjected carriers in HEPS, as described in (I). To deal with thermal effects in the system we introduce as basic dynamical variables the energy density and energy flux, as given by Eqs. (3) and (4), respectively, in (I). Consequently, the auxiliary maximum entropy formalism nonequilibrium statistical operator method (MAXENT-NSOM) is given for this case by the generalized Gibbsian-like nonequilibrium distribution

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$$\bar{\rho}(t,0)=\exp\left\{-\phi(t)-\beta(t)[\hat{H}_0-\mu_e(t)\hat{N}_e-\mu_h(t)\hat{N}_h]-\sum_{\mathbf{q}\gamma}F_{\mathbf{q}\gamma}(t)\hat{\nu}_{\mathbf{q}\gamma}-\sum_{\alpha}\mathbf{V}_\alpha(t)\cdot\hat{\mathbf{I}}_\alpha\right.$$
$$\left.-\sum_{\alpha}\sum_{\mathbf{Q}\neq 0}[\beta_\alpha(\mathbf{Q},t)\hat{\epsilon}_\alpha(\mathbf{Q})-\beta(t)\mu_\alpha(\mathbf{r},t)\hat{n}_\alpha(\mathbf{Q})-\mathbf{V}_\alpha(\mathbf{Q},t)\cdot\hat{\mathbf{I}}_\alpha(\mathbf{Q})]\right\}. \quad (1)$$

In Eq. (1), β, μ, $\mathbf{V}$, and F are the Lagrange multipliers (intensive variables thermodynmically conjugated to the basic ones) that the method introduces [see (I)].

Resorting to the method described in (I), we find two variables of interest, namely

$$\epsilon_\alpha(\mathbf{Q},t)=\mathrm{Tr}\{\hat{\epsilon}_\alpha(\mathbf{Q})\bar{\rho}(t,0)\}\ , \quad (2a)$$

$$\mathbf{I}_\alpha(\mathbf{Q},t)=\mathrm{Tr}\{\hat{\mathbf{I}}_\alpha(\mathbf{Q})\bar{\rho}(t,0)\}\ , \quad (2b)$$

where $\hat{\epsilon}_\alpha(\mathbf{Q})$ and $\hat{\mathbf{I}}_\alpha(\mathbf{Q})$ are the dynamical variables of Eqs. (3) and (4) in (I), and their equations of evolution are

$$\frac{\partial}{\partial t}\epsilon_\alpha(\mathbf{Q},t)-i\mathbf{Q}\cdot\mathbf{I}(\mathbf{Q},t)=\sigma_\alpha(\mathbf{Q},t)\ , \quad (3a)$$

$$\frac{\partial}{\partial t}\mathbf{I}_\alpha(\mathbf{Q},t)-i\underset{\sim}{\psi}_\alpha(\mathbf{Q},t)\mathbf{Q}=\boldsymbol{\sigma}_\alpha(\mathbf{Q},t)+i\mathcal{N}_\alpha(\mathbf{Q},t)\ , \quad (3b)$$

where, we recall, $Q\neq 0$. Transformed to direct space, Eqs. (3) are continuity equations with sources. In Eq. (3b) $\underset{\sim}{\psi}$ is the flux of the flux of energy, or the second flux of the energy density, a tensorial quantity given by

$$\underset{\sim}{\psi}_\alpha(\mathbf{Q},t)=\sum_{\mathbf{k}}\frac{\hbar^2}{2m_\alpha^*}[\mathbf{k}\cdot(\mathbf{k}+\tfrac{1}{2}\mathbf{Q})+\tfrac{1}{2}Q^2]$$
$$\times\frac{\hbar^2}{2m_\alpha^*}[(\mathbf{k}+\tfrac{1}{2}\mathbf{Q})\otimes(\mathbf{k}+\tfrac{1}{2}\mathbf{Q})]n_{\mathbf{kQ}}^\alpha(t)\ , \quad (4)$$

where $\otimes$ stands for the tensorial product of vectors, namely, $\tau=\mathbf{a}\otimes\mathbf{b}$ is the tensor of components $\tau_{ij}=a_ib_j$, and

$$n_{\mathbf{kQ}}^\alpha(t)=\mathrm{Tr}\{\hat{n}_{\mathbf{kQ}}^\alpha\bar{\rho}(t,0)\}\ . \quad (5)$$

Furthermore,

$$\mathcal{N}_\alpha(\mathbf{Q},t)=V(Q)(\hbar^2/2m_\alpha^2)$$
$$\times\sum_{\mathbf{k}}f_{\mathbf{k}}^\alpha[2(\mathbf{k}\cdot\mathbf{Q})\mathbf{k}-\mathbf{Q}(\mathbf{k}\cdot\mathbf{Q}+\tfrac{1}{2}Q^2)]n(\mathbf{Q},t)\ , \quad (6)$$

where $V(Q)=4\pi e^2/\epsilon_0 Q^2$ is the matrix element of the Coulomb interaction, and $n(\mathbf{Q},t)=n^e(\mathbf{Q},t)+n^h(\mathbf{Q},t)$. The last term in Eq. (3b), given in Eq. (6), is a result of Coulomb interaction dealt with in the random-phase approximation. It couples the thermal motion we are analyzing with the material motion (plasma waves). It plays the role of a source in Eq. (3b), and, as already noted, for our purpose here of analyzing the pure thermal modes it is ignored in what follows [we recall the arguments advanced in (I) that the coupling of both types of movement is only redundant in minor corrections to frequencies and lifetimes in the dispersion relations of the polar modes]. The relaxation terms are

$$\sigma_\alpha(\mathbf{Q},t)=-(\pi\hbar/2m_\alpha^*)\sum_{\mathbf{kq}\gamma}A^\alpha(\mathbf{kq}\gamma;t)[\mathbf{q}\cdot(2\mathbf{k}-\mathbf{q})][n_{\mathbf{kQ}}^\alpha(t)+n_{\mathbf{k},-\mathbf{Q}}^\alpha(t)]\ . \quad (7a)$$

$$\boldsymbol{\sigma}_\alpha(\mathbf{Q},t)=-(\pi/\hbar)\sum_{\mathbf{kq}\gamma}A^\alpha(\mathbf{kq}\gamma;t)\{\xi_{0\alpha}(\mathbf{k},\mathbf{q})+\underset{\sim}{\xi}_{1\alpha}(\mathbf{k},\mathbf{q})\mathbf{Q}+\xi_{2\alpha}(\mathbf{k},\mathbf{Q})\mathbf{Q}+(\hbar^3/2m_\alpha^2)[Q^2+\tfrac{1}{2}\mathbf{Q}\otimes\mathbf{Q}]\mathbf{q}\}$$
$$\times[n_{\mathbf{kQ}}^\alpha(t)+n_{\mathbf{k},-\mathbf{Q}}^\alpha(t)]\ , \quad (7b)$$

where $A^\alpha(\mathbf{kq}\gamma;t)$ and $f_{\mathbf{k}}^{e(h)}$ are given in Eqs. (13g) and (13h) in (I):

$$\xi_{0\alpha}(\mathbf{k},\mathbf{q})=(\pi\hbar^3/2m_\alpha^2)[(2\mathbf{k}\cdot\mathbf{q}-q^2)\mathbf{k}+(\mathbf{k}-\mathbf{q})^2\mathbf{q}]\ , \quad (8a)$$

$$\underset{\sim}{\xi}_{1\alpha}(\mathbf{kq})=(\pi\hbar^3/2m_\alpha^2)[2(\mathbf{k}{:}\mathbf{q})-(\mathbf{q}\otimes\mathbf{q})]\ , \quad (8b)$$

$$\xi_{2\alpha}(\mathbf{k},\mathbf{q})=(\pi\hbar^3/2m_\alpha^2)(2\mathbf{k}\cdot\mathbf{q}-q^2)\ . \quad (8c)$$

It must be stressed that the relaxation terms σ_α and $\boldsymbol{\sigma}_\alpha$ of Eqs. (7a) and (7b) are a result of carrier-phonon collisions. In addition to these contributions, contributions associated with the interaction of carriers with radiation (laser and recombination) fields are also present. The latter involve interband transitions and are of relevance to determine the homogeneous state,[7] but in the dipolar approximation for the radiation fields they do not affect the inhomogeneous variables. Furthermore we call attention to the fact that Eqs. (3) for the inhomogeneous variables ($\mathbf{Q}\neq 0$), are coupled to the equations of evolution for the remaining basic variables, namely the homogeneous variables consisting of the energy of electrons and holes, the particle numbers (or concentration), the homogeneous flux, and the phonon populations. They are given elsewhere,[7] and are not of direct interest for us in what follows; we need only keep in mind that Eqs. (3), and therefore their solutions, are dependent on the homogeneous variables, or, better to say, on the variables conjugated to the latter, which are $\beta(t)$, $\mu_\alpha(t)$, and $F_{\mathbf{q}\gamma}(t)$.

Inspection of Eqs. (3) tells us that it requires a closure procedure. As shown elsewhere,[8] a complete description of the system requires the introduction of an infinite set of variables, composed of all fluxes of tensorial rank $r=2,3,\ldots,$ which are disregarded in our calculation.

Such a truncation procedure has been discussed in (I). Consequently, the closure of the equations of evolution requires us to express quantities ψ, σ, and σ in terms of the basic variables. Since all of them are linear combinations of $n_{\mathbf{kQ}}(t)$ of Eq. (5), we need to express the latter in terms of the chosen basic variables. For that purpose we resort to a Heims-Jaynes perturbation expansion for expectation values,[9] in a quite similar way as done in (I).

We assume the inhomogeneities to be weak, and then we take only the first term in the Heims-Jaynes expansion; i.e., we introduce a *linear approximation* in deviations from the homogeneous state. After some algebra, we find that

$$n_{\mathbf{kQ}}^{\alpha}(t)=-a_{\alpha}(\mathbf{k},t)\beta_{\alpha}(\mathbf{Q},t)+\mathbf{b}_{\alpha}(\mathbf{Q},t)\cdot\mathbf{V}_{\alpha}(\mathbf{Q},t)+c_{\alpha}(\mathbf{k},t)\beta(t)\mu_{\alpha}(\mathbf{Q},t)\ , \tag{9}$$

where

$$a_{\alpha}(\mathbf{k},t)=(\hbar^2k^2/2m_{\alpha}^{*})f_{\mathbf{k}}^{\alpha}(t)[1-f_{\mathbf{k}}^{\alpha}(t)]\ , \tag{10a}$$

$$\mathbf{b}_{\alpha}(\mathbf{k},t)=(\hbar^2k^2/2m_{\alpha}^{*})(\hbar\mathbf{k}/m_{\alpha}^{*})f_{\mathbf{k}}^{\alpha}(t)[1-f_{\mathbf{k}}^{\alpha}(t)]\ , \tag{10b}$$

$$c_{\alpha}(\mathbf{k},t)=f_{\mathbf{k}}^{\alpha}(t)[1-f_{\mathbf{k}}^{\alpha}(t)]\ , \tag{10c}$$

$$f_{\mathbf{k}}^{e}(t)=\mathrm{Tr}\{c_{\mathbf{k}}^{\dagger}c_{\mathbf{k}}\bar{\rho}_0(t,0)\}\ , \tag{10d}$$

$$f_{\mathbf{k}}^{h}(t)=\mathrm{Tr}\{h_{-\mathbf{k}}^{\dagger}h_{-\mathbf{k}}\bar{\rho}_0(t,0)\}\ , \tag{10e}$$

with $\bar{\rho}_0(t,0)$ being the homogeneous part of the auxiliary NSO, namely

$$\bar{\rho}_0(t,0)=\exp\{-\phi_0(t)-\beta(t)[\hat{H}_c-\mu_e(t)\hat{N}_e-\mu_h(t)\hat{N}_h]+\sum_{\alpha}\mathbf{V}_{\alpha}(t)\cdot\hat{\mathbf{I}}_{\alpha}-\sum_{\mathbf{q}\gamma}F_{\mathbf{q}\gamma}(t)\hat{\nu}_{\mathbf{q}\gamma}\}\ , \tag{10f}$$

where ϕ_0 ensures its normalization. Moreover, we have neglected the dependence on $\mathbf{Q}$ of the coefficients a, b, and c, that is, spatial correlations are ignored.

The expression of Eq. (9) depends on the intensive variables β and $\mathbf{V}$, which we can replace in terms of the extensive variables ϵ and $\mathbf{I}$, resorting again to the Heims-Jaynes expansion in first order to obtain

$$n_{\alpha}(\mathbf{Q},t)=-\lambda_{12}^{\alpha}(t)\beta_{\alpha}(\mathbf{Q},t)+\lambda_{11}^{\alpha}\beta(t)\mu_{\alpha}(\mathbf{Q},t)\ , \tag{11a}$$

$$\epsilon_{\alpha}(\mathbf{Q},t)=-\lambda_{22}^{\alpha}(t)\beta_{\alpha}(\mathbf{Q},t)+\lambda_{21}^{\alpha}(t)\beta(t)\mu_{\alpha}(\mathbf{Q},t)\ , \tag{11b}$$

$$\mathbf{I}_{\alpha}(\mathbf{Q},t)=\lambda_{33}^{\alpha}(t)\mathbf{V}_{\alpha}(\mathbf{Q},t)\ , \tag{11c}$$

where

$$\lambda_{11}^{\alpha}(t)=\sum_{\mathbf{k}}c_{\alpha}(\mathbf{k},t)\ , \tag{12a}$$

$$\lambda_{12}^{\alpha}(t)=a_{21}^{\alpha}(t)=\sum_{\mathbf{k}}a_{\alpha}(\mathbf{k},t)\ , \tag{12b}$$

$$\lambda_{22}^{\alpha}(t)=\sum_{\mathbf{k}}(\hbar^2k^2/2m_{\alpha}^{*})^2c_{\alpha}(\mathbf{k},t)\ , \tag{12c}$$

$$\lambda_{33}^{\alpha}(t)=\sum_{\mathbf{k}}(2/3m_{\alpha}^{*})(\hbar^2k^2/2m_{\alpha}^{*})^3c_{\alpha}(\mathbf{k},t)\ . \tag{12d}$$

Finally, using Eqs. (9) and (11), we find that

$$n_{\mathbf{kQ}}^{\alpha}(t)=g_1^{\alpha}(\mathbf{k},t)n_{\alpha}(\mathbf{Q},t)+g_2^{\alpha}(\mathbf{k},t)\epsilon_{\alpha}(\mathbf{Q},t)+\mathbf{g}_3^{\alpha}(\mathbf{k},t)\cdot\mathbf{I}_{\alpha}(\mathbf{Q},t)\ , \tag{13}$$

where

$$g_1^{\alpha}(\mathbf{k},t)=[c_{\alpha}(\mathbf{k},t)\lambda_{22}^{\alpha}(t)-a_{\alpha}(\mathbf{k},t)\lambda_{12}^{\alpha}(t)]D_{\alpha}^{-1}(t)\ , \tag{14a}$$

$$g_2^{\alpha}(\mathbf{k},t)=[a_{\alpha}(\mathbf{k},t)\lambda_{11}^{\alpha}(t)-c_{\alpha}(\mathbf{k},t)\lambda_{12}^{\alpha}(t)]D_{\alpha}^{-1}(t)\ , \tag{14b}$$

$$\mathbf{g}_3^{\alpha}(\mathbf{k},t)=\mathbf{b}_{\alpha}(\mathbf{k},t)/a_{33}^{\alpha}(t)\ , \tag{14c}$$

$$D_{\alpha}(\mathbf{k},t)=a_{11}^{\alpha}(t)a_{22}^{\alpha}(t)-a_{12}^{\alpha}(t)a_{21}^{\alpha}(t)\ . \tag{14d}$$

Using Eq. (13) to express ψ and σ in Eq. (8b), we obtain an equation of evolution for the fluxes of energy of the form

$$\frac{\partial}{\partial t}\mathbf{I}_{\alpha}(\mathbf{Q},t)=L_{\alpha}^{\epsilon}(t)i\mathbf{Q}\epsilon_{\alpha}(\mathbf{Q},t)+L_{\alpha}^{n}(t)i\mathbf{Q}n(\mathbf{Q},t)-\Theta_{I_{\alpha}}^{-1}(t)\mathbf{I}_{\alpha}(\mathbf{Q},t)-\Lambda_{\alpha}^{2}(t)(\mathbf{Q}\otimes\mathbf{Q}+Q^2\underset{\sim}{1})\mathbf{I}_{\alpha}(\mathbf{Q},t)\ , \tag{15}$$

where $\underset{\sim}{1}$ is the unit tensor, and

$$\theta_{I_{\alpha}}^{-1}(t)=(\pi\hbar^2/3m_{\alpha}^{*2})[a_{33}^{\alpha}(t)]^{-1}\times\sum_{\mathbf{kq}\gamma}A^{\alpha}(\mathbf{kq}\gamma;t)\frac{\hbar^2k^2}{2m_{\alpha}^{*}}\frac{\hbar}{m_{\alpha}^{*}}k^2(\mathbf{k}\cdot\mathbf{q})\times f_{\mathbf{k}}^{\alpha}(t)[1-f_{\mathbf{k}}^{\alpha}(t)]\ , \tag{16a}$$

$$L_{\alpha}^{\epsilon}(t)=(2/3m_{\alpha}^{*})\sum_{\mathbf{k}}(\hbar^2k^2/2m_{\alpha}^{*})^2g_2^{\alpha}(\mathbf{k},t)\ , \tag{16b}$$

$$L_{\alpha}^{n}(t)=(2/3m_{\alpha}^{*})\sum_{\mathbf{k}}(\hbar^2k^2/2m_{\alpha}^{*})^2g_1^{\alpha}(\mathbf{k},t)\ , \tag{16c}$$

$$\Lambda_{\alpha}^{2}(t)=(\pi\hbar^2/3m_{\alpha}^{*2})[a_{33}^{\alpha}(t)]^{-1}\times\sum_{\mathbf{kq}\gamma}A^{\alpha}(\mathbf{kq}\gamma;t)(\hbar^2k^2/2m_{\alpha}^{*})(\hbar/m_{\alpha}^{*})(\mathbf{k}\cdot\mathbf{q})\times f_{\mathbf{k}}^{\alpha}(t)[1-f_{\mathbf{k}}^{\alpha}(t)]\ . \tag{16d}$$

We recall that the time dependence of the kinetic coefficients is a result of evolution in time of the homogeneous state of reference.

In direct space, Eq. (15) becomes

$$-\frac{\partial}{\partial t}\mathbf{I}_{\alpha}(\mathbf{r},t)=L_{\alpha}^{\epsilon}(t)\nabla\epsilon_{\alpha}(\mathbf{r},t)+\Theta_{I_{\alpha}}^{-1}(t)\mathbf{I}_{\alpha}(\mathbf{r},t)-\Lambda_{\alpha}^{2}(t)[\nabla\otimes\nabla+\nabla^2\underset{\sim}{1}]\mathbf{I}_{\alpha}(\mathbf{r},t)\ , \tag{17}$$

where, consistently, we have ignored the source term with the cross-kinetic coefficient L_{α}^{n}.

Equation (17), which replaces the Fourier constitutive equation of classical irreversible thermodynamics, is of the Maxwell-Cattaneo type of extended irreversible thermodynamics (EIT):[4] θ plays the role of a relaxation time (of Maxwell's conjecture[10]), and on the right-hand side the first term is a driving thermodynamic force created by a gradient of energy, while the last term is a driving force created by the rate of change in space of the flux itself. Also, Eq. (17) is of the same form as the one derived for the heat flow in a lattice.[2]

Consider now the equation of evolution for the energy density, viz. Eq. (3a). Proceeding in a similar way as in the case for the derivation of the equation for the flux, after some algebra we obtain, in direct space, that

$$\frac{\partial}{\partial t}\epsilon_{\alpha}(\mathbf{r},t)+\operatorname{div}\mathbf{I}_{\alpha}(\mathbf{r},t)=-\Theta_{\epsilon_{\alpha}}^{-1}(t)\epsilon_{\alpha}(\mathbf{r},t)+\varphi_{\alpha}(t)\ , \tag{18}$$

where $\varphi_{\alpha}(t)$ stands for the contributions provided by the pumping external source, and recombination in the homogeneous state.[7] Present in Eq. (18) is the reciprocal relaxation time

$$\Theta_{\epsilon_{\alpha}}^{-1}(t)=(2\pi\hbar/m_{\alpha}^{*})\sum_{\mathbf{k}\mathbf{q}\gamma}A^{\alpha}(\mathbf{k}\mathbf{q}\gamma;t)(2\mathbf{k}\cdot\mathbf{q}-q^{2})g_{2}^{\alpha}(\mathbf{k},t)\ . \tag{19}$$

Let us next introduce a space- and time-dependent quasitemperature for carriers through the identification

$$\beta_{\alpha}(\mathbf{r},t)=1/k_{B}T^{*}(\mathbf{r},t)\ , \tag{20}$$

where the same quasitemperature is taken for both kinds of carriers, taking into account the very rapid (subpicosecond) thermalization of the carriers brought in by Coulomb interaction.[11] The relation of T^{*} with energy can be obtained from Eqs. (11): after some algebra we find that

$$\epsilon_{\alpha}(\mathbf{Q},t)=-l_{\alpha}(t)\beta(\mathbf{Q},t)+k_{\alpha}(t)n_{\alpha}(\mathbf{Q},t)\ , \tag{21}$$

where

$$l_{\alpha}(t)=\lambda_{22}^{\alpha}(t)-[\lambda_{12}^{\alpha}(t)\lambda_{21}^{\alpha}(t)/\lambda_{11}^{\alpha}(t)]\ , \tag{22a}$$

$$k_{\alpha}(t)=\lambda_{12}^{\alpha}(t)/\lambda_{11}^{\alpha}(t)\ . \tag{22b}$$

Transforming back to direct space, using Eq. (20) together with the fact that the linear approximation that has been used implies a weak local departure of the local quantities from their homogeneous values, and then $T^{*}(\mathbf{r},t)=T^{*}(t)+\Delta T^{*}(\mathbf{r},t)$, with $\Delta T^{*}(\mathbf{r},t)\ll T^{*}(t)$, and also $n_{\alpha}(\mathbf{r},t)=n_{\alpha}(t)+\Delta n_{\alpha}(\mathbf{r},t)$, we have

$$\nabla\epsilon_{\alpha}(\mathbf{r},t)=[l_{\alpha}(t)/k_{B}^{2}T^{*2}(t)]\nabla T^{*}(\mathbf{r},t)+k_{\alpha}(t)\nabla n_{\alpha}(\mathbf{r},t)\ . \tag{23}$$

Once again neglecting the coupling with the material motion ($k_{\alpha}=0$), the role of $T^{*}(\mathbf{r},t)$ as a nonequilibrium temperature for the carriers can be better shown when in Eq. (17) we take the limit of a near-stationary flux, $\partial\mathbf{I}/\partial t\simeq 0$. After neglecting the last term ($\Lambda^{2}=0$), we obtain that

$$\mathbf{I}(\mathbf{r},t)=-\mathcal{H}(t)\nabla T^{*}(\mathbf{r},t)\ , \tag{24}$$

where $\mathbf{I}=\mathbf{I}_{e}+\mathbf{I}_{h}$ is the total energy flux, and $\mathcal{H}=\mathcal{H}_{e}+\mathcal{H}_{h}$, with $\mathcal{H}_{\alpha}=L_{\alpha}^{\epsilon}l_{\alpha}\theta_{I_{\alpha}}/k_{B}T^{*2}$. Equation (24) is the Fourier constitutive equation of classical irreversible thermodynamics, with $\mathcal{H}$ being the thermal conductivity.

Let us return to the equations of evolution in extended thermodynamics, but considering now Eqs. (17) and (18) in the case of the presence of a *reference homogeneous state in stationary conditions* (which follows when the sample is under the action of continuous laser light illumination), so that all kinetic coefficients are constant in time. Taking the time derivative of Eq. (18), and next replacing in it Eq. (17), we find a hyperbolic-type equation of propagation for the energy density, namely

$$\frac{\partial^{2}}{\partial t^{2}}\epsilon_{\alpha}(\mathbf{r},t)+\theta_{\epsilon_{\alpha}}^{-1}\frac{\partial}{\partial t}\epsilon_{\alpha}(\mathbf{r},t)-L_{\alpha}^{\epsilon}\nabla^{2}\epsilon(\mathbf{r},t)=[\theta_{I_{\alpha}}^{-1}-\Lambda_{\alpha}^{2}(\nabla\otimes\nabla+\nabla^{2}\underset{\sim}{1})]\operatorname{div}\mathbf{I}_{\alpha}(\mathbf{r},t)\ , \tag{25}$$

where in the stationary state the pumping term φ in Eq. (18) is compensated for by the loss of energy to the thermal bath, i.e., $\varphi_{\alpha}-\theta_{\epsilon_{\alpha}}^{-1}\epsilon_{\alpha}^{0}=0$. Introducing the quasitemperature $T^{*}(\mathbf{r},t)$, using Eq. (21) and taking into account that ϵ_{α}^{0} is a constant, after some algebra we obtain an equation of propagation for this local quasitemperature, namely

$$\frac{1}{c_{T}^{2}}\frac{\partial^{2}}{\partial t^{2}}T^{*}(\mathbf{r},t)+\frac{1}{D_{T}}\frac{\partial}{\partial t}T^{*}(\mathbf{r},t)-\nabla^{2}T^{*}(\mathbf{r},t)=\sum_{\alpha}[\chi_{\alpha}^{-1}-\tilde{\Lambda}_{\alpha}^{2}(\nabla\otimes\nabla+\nabla^{2}\underset{\sim}{1})]\operatorname{div}\mathbf{I}_{\alpha}(\mathbf{r},t)\ , \tag{26}$$

where

$$c_{T}^{-2}=c_{e}^{-2}+c_{h}^{-2},\quad D_{T}^{-1}=D_{e}^{-1}+D_{h}^{-1}\ , \tag{27a}$$

$$c_{e(h)}^{2}=L_{e(h)}^{\epsilon},\quad D_{e(h)}=L_{e(h)}^{\epsilon}\theta_{e(h)}\ , \tag{27b}$$

$$\tilde{\Lambda}_{e(h)}^{2}=\Lambda_{e(h)}^{2}k_{B}T_{0}^{2}/L_{e(h)}^{2}l_{e(h)}=\Lambda_{e(h)}^{2}\Theta_{I_{e(h)}}/\mathcal{H}_{e(h)}\ , \tag{27c}$$

where T_{0} is the quasitemperature in the homogeneous and stationary states, and

$$\mathcal{H}_{e(h)}=L_{e(h)}^{\epsilon}l_{e(h)}\theta_{I_{e(h)}}/k_{B}T_{0}^{2}\ . \tag{27d}$$

Equation (26) is of the form of the telegrapher's equation with a source.[12] This source is associated with space derivatives of the energy flux, and then relevant in the presence of steep local variations (even though its amplitude is small). Equation (26) implies a damped propagation of the local nonequilibrium quasitemperature of the carrier system. If in this equation we take the velocity c as going to infinity, while D is kept finite, and the source on the right-hand side is neglected, we recover Fourier's law for heat propagation of classical irreversible thermodynamics, with D_T playing the role of a diffusion coefficient. Furthermore, from the left side of Eq. (26), after Fourier transforming in space and time, we obtain a dispersion relation for the propagation of the damped thermal waves, namely

$$\frac{\omega^2}{c_T^2}+i\frac{\omega}{D_T}-Q^2=0\ , \tag{28}$$

i.e., a propagation with velocity c_T and a lifetime approximately given by $2D_T/c_T^2$ when $c_TQ > c_T^2/2D_T \equiv 1/2\Theta_T$, where we have defined the characteristic time Θ_T, or relaxation time for quasitemperature. In this limit of the group velocity c_T going to infinity, while the thermal diffusivity D_T is kept finite, from Eq. (28) we find that there follows an approximate solution corresponding to a frequency with a real part (reciprocal period) null and whose imaginary part is D_TQ^2. The system then goes from a regime of damped propagation of thermal waves to a diffusive regime. In this limit, as already noted, the telegrapher's equation goes over Fourier's diffusion equation. This limit is equivalent, i.e., follows Fourier's law, if one considers a near-stationary flux; that is, when in Eq. (17) it is taken that $\partial \mathbf{I}(\mathbf{r},t)/\partial t \simeq 0$.

We made an explicit application of the hydrodynamic theory above to the case of GaAs samples under cw laser illumination, as reported elsewhere.[13] Here we reproduce two figures corresponding to a carrier concentration of $n=1.4\times10^{17}$ cm^{-3} (laser power ~ 0.2 kW cm^{-2}), when the carrier system can be considered nondegenerate. Under such conditions analytic expressions can be obtained for the relaxation time Θ_T, resulting from collisions arising out of a predominant Fröhlich interaction. Figure 1 shows the dependence on the carrier quasitemperature of the relaxation time (dashed line), and the limiting condition $c_TQ=1/2\Theta_T$. Figure 2 shows the Raman lines expected to arise from scattering by these thermal waves, for the indicated value of the wave number and the three different states of the system indicated by cross points in Fig. 1. These results are discussed in Sec. III.

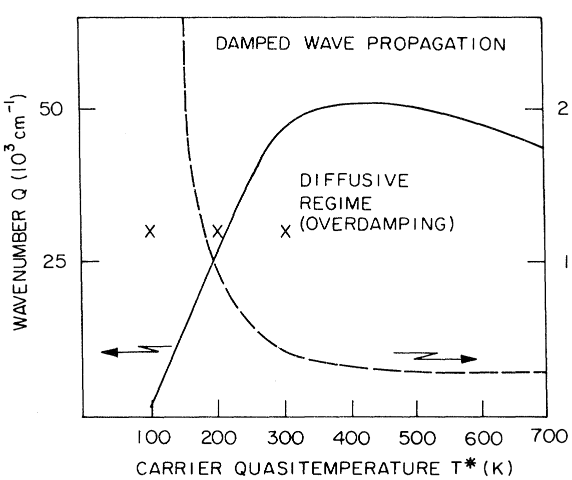

FIG. 1. The dashed line is the relaxation time (right ordinate) for a range of values of the carrier quasitemperature. The full line separates the regions of values of the wave number (left ordinate), which for each quasitemperature corresponds to either damped wave propagation (upper part) or to diffusive movement (lower and right parts). Parameters characteristic of GaAs and a carrier concentration $n=1.4\times10^{17}$ cm^{-3} were used.

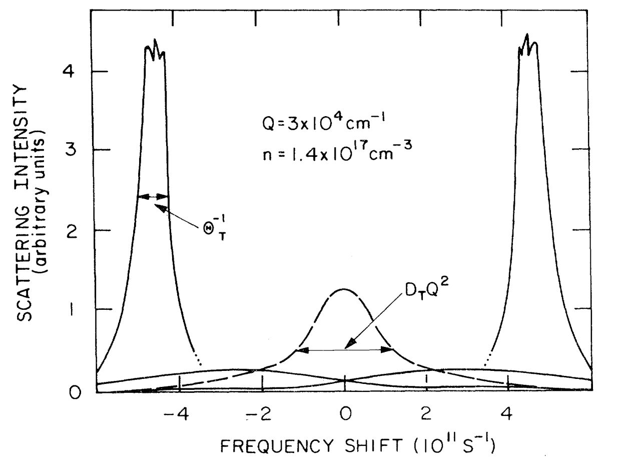

FIG. 2. A description of the Raman intensity, upper full lines for $T^*=100$ K, lower full lines for $T^*=200$ K, and dashed line for $T^*=300$ K. The transition from the damped to overdamped regimes is clearly shown. The crosses in Fig. 1 indicate the corresponding three situations considered here.

III. CONCLUSIONS

The results presented in Sec. II have shown that carriers in HEPS, in interaction with the thermal bath of phonons, behave like a Poiseuille-like fluid that can sustain thermal waves. We have already stressed the formal similarity of our Eq. (17) with the one derived by Guyer and Krumhansl for heat propagation in lattices in insulating crystals,[2] which describes the propagation of second sound.[14]

We call attention to the approximations introduced, namely (i) we have neglected coupling between thermal and material (plasma) waves, i.e., neglected cross-kinetic terms that would couple Eqs. (3) with the equations for the density and momentum density; (ii) correlations in space and time have been disregarded [i.e., we took space-independent kinetic coefficients and an instantaneous (or memoryless) approximation]; and (iii) we have considered small deviations from the homogeneous state of reference (linear theory).

A detailed study of the complete generalized extended hydrodynamics of HEPS is underway. It includes coupling effects of thermal and mass motion, and with it the polarization effects of Coulomb interaction [disregarded here because of the approximation indicated in item (i) above] together with a detailed calculation of the kinetic coefficients. This complete treatment will allow us to obtain the hydrodynamic correlation functions, and from them the expression for the Raman-scattering cross sections, which permits us to characterize the different hydrodynamic modes, including the second-sound wave. The Raman spectrum is to be composed of Stokes and anti-Stokes lines associated with density oscillations, that is the optical and acoustical plasma waves,[1] as well as the Stokes and anti-Stokes lines, centered at $\pm c_T Q$, associated with the thermal oscillations we described in Sec. II, having a linewidth roughly given by $2D_T/c_T^2 = 2\Theta_T$. As noted in Sec. II, in the diffusive limit (we recall that c_T is large and D_T is finite, and refer to a near-stationary flux) the two lines due to scattering by the damped thermal waves collapse into a unique line at a zero-frequency shift and with linewidth $D_T Q^2$ that is characteristic of the Rayleigh scattering.[15] This is clearly shown in Fig. 2. Experimental observation of the peaks due to scattering by these second-sound waves should be greatly impaired by the background radiation (single-quasiparticle scattering) at these low-frequency shifts. We stress that the carrier system goes from the situation of damped wave propagation to that of diffusive movement (and thus the hyperbolic equation of evolution in extended thermodynamics goes over Fourier's parabolic one of heat diffusion in classical irreversible thermodynamics) when the wave motion goes from a damped to an overdamped regime (i.e., from $c_T Q > c_T^2/2D_T = 1/2\Theta_T$ to $c_T Q < c_T^2/2D_T = 1/2\Theta_T$). This is shown in Fig. 1. The extended hydrodynamic theory presented in this paper should then replace the usual one based on classical irreversible thermodynamics when the limit of validity of the latter is not satisfied. We recall that this limit corresponds to the case of phenomena characterized mainly by large wavelengths and small frequencies (as shown in the calculations of previous sections, these limiting conditions mean a negligible divergence and time derivative of the energy flux). It ought to be stressed that when wavelengths are shorter (and consequently frequencies of the waves are higher), according to the criterion for truncation in informational statistical thermodynamics,[8] higher- and higher-order fluxes of the energy density are required to be included in order to obtain a more accurate description of the system, and a better agreement with experiment.

The implications of the propagation of thermal waves in nonequilibrium solids on the physical meaning of nonequilibrium entropy and nonequilibrium temperature in EIT are discussed elsewhere.[3,16] The question of a quasitemperature and its measurement in HEPS is being addressed in a forthcoming paper.[17]

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