PHYSICAL REVIEW B VOLUME 52, NUMBER 7 15 AUGUST 1995-I

# Electron-electron momentum relaxation in a two-dimensional electron gas

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(Received 17 February 1995; revised manuscript received 26 April 1995)

Evolution of sharp angular distributions in the degenerate two-dimensional gas of colliding electrons has been shown to occur essentially otherwise than in the three-dimensional gas. As early as after a few collisions the electron distribution function becomes antisymmetric in momentum, conserving, practically, its sharpness. Further spreading of this distribution in coordinate and momentum spaces occurs rather slowly, taking much more time than an electron-electron collision. Application of an extremely weak magnetic field leads to essential changes in this picture. The possibility of observing new effects in experiments on electron beam evolution in heterostructures is discussed.

In contrast to the one-dimensional case, a weak repulsion in two-dimensional conductors does not result in a rearrangement of the ground state of the electron system. However, reduction of the space dimensionality of the system from three to two affects essentially the relaxation processes associated with electron-electron collisions in a degenerate electron gas. In the case of the electron-electron energy relaxation, the reduction of the dimensionality causes only a decrease in the characteristic relaxation time by a factor $\ln(\varepsilon_F/T)$ ($\varepsilon_F$ is the Fermi energy and $T$ is the temperature).[1] The changes in the angular relaxation are, on the contrary, radical.[2,3] In the two-dimensional case a collision of an electron having a momentum $\mathbf{p}_1$ with another electron with $\mathbf{p}_2$ generally results in scattering through a small angle $\delta\varphi \simeq T/\varepsilon_F \ll 1$. The exception is the case of collisions of electrons having almost opposite momenta: $\mid \mathbf{p}_1+\mathbf{p}_2 \mid /p_F \leq T/\varepsilon_F$. The result of this collision is a pair of electrons with almost opposite momenta too; therefore, the scattering angle is arbitrary: $\delta\varphi \sim 1$. [Note that, in contrast to the three-dimensional (3D) case, in the 2D case both types of processes have equal (by the order of magnitude) probabilities.] The second type of collision is quite efficient in the relaxation of the part of the electron distribution function even in a momentum whose relaxation time is of the same order of magnitude as in the 3D case: $\tau_s \sim (\varepsilon_F/T)^2 \sim T^{-2}$. Both types of collisions affect the relaxation of the odd part of the distribution very slightly. For the first type the reason lies in the small-angle character of scattering. For the second type the reason is that a turn of an electron pair with strictly opposite momenta in the $\mathbf{p}$ space by any angle $\delta\varphi$ obviously does not affect the odd part. Thus the relaxation of the odd part in the two-dimensional case occurs notably slower than in three-dimensional case: $\tau_a \approx \tau_s(\varepsilon_F/T)^2 \sim T^{-4}$ (for details see Ref. 3).

These assertions hold for the case of a weakly anisotropic nonequilibrium distribution function. As the angle $\varphi_0$ characterizing the range of velocities of nonequilibrium electrons becomes smaller, the relaxation time of the odd distribution decreases and becomes of the order of $\tau_s$ when $\varphi_0 = \sqrt{T/\varepsilon_F}$; see Ref. 3.

It seems that the most direct and detailed information on the momentum relaxation can be provided by experiments with electron beams injected into the degenerate two-dimensional electron gas (2DEG) in heterostructures.[4,5] This paper shows the above properties of relaxation in the 2D system to be responsible for the peculiar features of the beam evolution: as a result of collisions, the primary beam soon divides into relatively narrow and long-lived electron and hole beams moving in opposite directions (together they form the odd distribution, while the even part becomes practically isotropic). By a "hole" we mean the absence of an electron under the Fermi level. Finding experimentally these secondary beams would corroborate our concept of the peculiar character of the relaxation mechanisms in 2D systems. We suggest some experiments at the end of the paper.

The effects under consideration are most pronounced at sufficiently low temperatures. Let us first assume that $T \ll |\varepsilon| \ll \varepsilon_F$, where $\varepsilon$ is the energy of an electron ($\varepsilon > 0$) or a hole ($\varepsilon < 0$) counted from the Fermi level $\varepsilon_F$. A similar situation was considered earlier by Laikhtman,[6] who, however, disregarded the facts that are essential in our consideration, viz., the difference in relaxation rate of the odd and even parts of the distribution function. Neglecting the effects of the spatial dispersion, the kinetic equation for the nonequilibrium correction $f_\mathbf{p}$ to the electron distribution can be written as

$$\frac{\partial f_\mathbf{p}}{\partial t} = I\{f_\mathbf{p}\}, \tag{1}$$

$$I\{f_\mathbf{p}\} = -\int (\nu_{\mathbf{p}'\mathbf{p}} f_\mathbf{p} - \nu_{\mathbf{p}\mathbf{p}'} f_{\mathbf{p}'}) d^2p'. \tag{2}$$

Here $I\{f_\mathbf{p}\}$ is a linearized integral of electron-electron collisions. Omitting exponentially small terms of the $\exp(-|\varepsilon|/T)$ type, we can write the kernel $\nu_{\mathbf{p}\mathbf{p}'}$ as

0163-1829/95/52(7)/4744(4)/$06.00  

$$\nu_{\mathbf{pp'}} \simeq \frac{\pi^2}{2h^5} \times \begin{cases} 2\int U_{\mathbf{pp_1p_2p'}} d^2p_1 d^2p_2, \\ \varepsilon\varepsilon' > 0, \varepsilon\varepsilon_1 > 0, \varepsilon\varepsilon_2 < 0 \\ -\int U_{\mathbf{pp'p_2p_3}} d^2p_2 d^2p_3, \\ \varepsilon\varepsilon' < 0, \varepsilon\varepsilon_2 < 0, \varepsilon\varepsilon_3 < 0, \end{cases} \tag{3}$$

$$U_{\mathbf{pp_1p_2p_3}} = W_{\mathbf{pp_1p_2p_3}} \times\delta(\varepsilon+\varepsilon_1-\varepsilon_2-\varepsilon_3)\delta(\mathbf{p}+\mathbf{p}_1-\mathbf{p}_2-\mathbf{p}_3).$$

Here $W$ is the squared modulus of the matrix element of the electron-electron interaction. Equations (1)–(3) describe the "pretemperature" relaxation stage, on a time scale shorter than the thermalization time. At this stage departures from the state $\mathbf{p}$ occur only downward in the absolute energy value, while arrivals occur only from higher $|\varepsilon|$, i.e., $\nu_{\mathbf{pp'}} = 0$ when $|\varepsilon_{\mathbf{p'}}| < |\varepsilon_{\mathbf{p}}|$. Assuming that the electron spectrum is isotropic, to within values of the order $|\varepsilon|/\varepsilon_F$, we have

$$\nu_{\mathbf{pp'}} \simeq 2C \times \begin{cases} \dfrac{|K(\varphi,\epsilon) - K(\varphi,\epsilon')|}{\eta \sin\varphi/2}, & \epsilon\,\epsilon' > 0 \\ -\arcsin \dfrac{|\epsilon+\epsilon'|}{\sqrt{(\delta\epsilon)^2 + 4\sin^2\varphi}}, & \epsilon\,\epsilon' < 0, \end{cases} \tag{4}$$

$$K(\varphi,\epsilon) = (\cos^2\varphi/2 - \epsilon\eta)^{1/2}\,\theta(\cos^2\varphi/2 - \epsilon\eta);$$
$$\eta = 1 + \left(\frac{\delta\epsilon}{4\sin\varphi/2}\right)^2;$$
$$\epsilon \equiv \varepsilon_{\mathbf{p}}/\varepsilon_F;\ \epsilon' \equiv \varepsilon_{\mathbf{p'}}/\varepsilon_F;$$
$$\delta\epsilon = \epsilon' - \epsilon;\quad |\epsilon|, |\epsilon'| \ll 1;\quad C \equiv \frac{\pi^2 W m}{2h^5}.$$

Here $\theta(x)$ is the Heaviside function, $\varphi$ is the angle between the vectors $\mathbf{p}$ and $\mathbf{p}'$, and $m$ is the electron mass. For the sake of simplicity $W$ is assumed to be a constant; a justification of this assumption will be given later. For not too small angles $\varphi$, when (4) can be expanded in $\epsilon$ and $\epsilon'$, we have

$$\nu_{\mathbf{pp'}} \simeq \frac{C}{\sin\varphi} \times \begin{cases} 2|\delta\epsilon|, \\ \epsilon\,\epsilon' > 0, \quad \varphi \gg |\delta\epsilon|\sqrt{|\epsilon'|},\ \pi - \varphi \gg \sqrt{|\epsilon'|} \\ -|\epsilon+\epsilon'|, \quad \epsilon\,\epsilon' < 0, \quad \varphi, \pi - \varphi \gg |\delta\epsilon|. \end{cases} \tag{5}$$

Note that this expression is an even function of the momenta: $\nu_{\mathbf{pp'}}$ is invariant relative to the substitution $\varphi \to \pi - \varphi$ (by definition, $0 \le \varphi \le \pi$). The odd part appears in the approximation

$$\nu^a_{\mathbf{pp'}} \simeq -C\frac{\cos\varphi}{\sin^3\varphi}\,(\epsilon'^2 - \epsilon^2)\,\theta(\epsilon\epsilon')\mathrm{sgn}\,\epsilon', \qquad \varphi, \pi - \varphi \gg \sqrt{|\epsilon'|}. \tag{6}$$

The small value and rapid decrease of the odd part of the kernel $\nu^a_{\mathbf{pp'}} \sim \varphi^{-3}$ is in agreement with the general statement of Refs. 2 and 3 on the slow rate of the odd relaxation. However, it is worth noting that the evenness of $\nu_{\mathbf{pp'}}$ in approximation (5) follows from the assumption that $W$ is constant. Indeed, the expression found in Ref. 6, which is actually similar to (5), does not reveal any evenness. However, the angular relaxation is determined by the integral of $\nu_{\mathbf{pp'}}$ over energy. A rigorous proof may be given that the odd part (relative to angle) is small for this quantity. Here we will not provide such a proof for the general case of an arbitrary dependence of $W$ on its variables. Actually, $W$ is a smooth function of the momentum that varies notably at distances of order of inverse screening length $q_{\mathrm{sc}}$. When $q_{\mathrm{sc}}{p_F}^{-1} \gg \sqrt{|\epsilon'|}$ it is possible to obtain from (3) the following estimate consistent with (6):

$$\int \nu^a_{\mathbf{pp'}} d\varepsilon \sim (\epsilon')^3 \sin^{-3}\varphi, \quad \varphi, \pi - \varphi \gg \sqrt{|\epsilon'|}. \tag{7}$$

For the even part of $\nu_{\mathbf{pp'}}$ the account of the smooth dependence $W$ is unessential.

We are going to show that the relaxation of the odd distribution does not end at the pretemperature stage because of the stated properties of the kernel $\nu_{\mathbf{pp'}}$; moreover, the relaxation practically stops after a few collisions and the distribution remains substantially anisotropic. Let us consider the stationary equation

$$\tilde{I}\{\tilde{f}\} = 0, \tag{8}$$
$$\tilde{I}\{\tilde{f}\} = -\int \nu_{\mathbf{p'p}}(\tilde{f}_{\mathbf{p}} - \tilde{f}_{\mathbf{p'}})d^2p',$$

containing the operator $\tilde{I}$ transposed with respect to the collision operator $I$. The solutions $\tilde{f}$ of this equation are known to represent the quantities conserved in the collision process. The functions const, $\mathbf{p}$, and $\varepsilon$ obviously satisfy this equation and that corresponds to the conservation of the particle number, momentum, and energy. To find highly anisotropic odd solutions of Eq. (8) (that correspond to the local, in $\mathbf{p}$ space, conservation of particles number), we consider the function

$$G(\varepsilon,\varphi) = \frac{1}{2\pi}\sum_{n=\pm1,\pm3,\ldots}^{\infty}[1 - g_n(\varepsilon)]e^{i\varphi n}. \tag{9}$$

Henceforth $0 \le \varphi \le 2\pi$, the origin of the $\varphi$ being chosen arbitrarily. We are going to show that there exist solutions of the form (9) with $g_n(\varepsilon) \to 0$ as $\varepsilon \to 0$, i.e., the function $G$ has the form $(1/2)\left[\delta(\varphi) - \delta(\varphi - \pi)\right]$ when $\varepsilon = 0$. From (8) we have the following equations for $g_n$:

$$g_n - \nu^{-1}(\varepsilon)\int \nu_{\mathbf{p'p}}\, g_n(\varepsilon')\, e^{i\varphi' n}\, d^2p' = \nu^{-1}(\varepsilon)\,\tilde{I}\{e^{i\varphi n}\}, \tag{10}$$

$$\nu(\varepsilon) = \int \nu_{\mathbf{p'p}}\, d^2p' \simeq \tau_0^{-1}\epsilon^2|\ln|\epsilon||.$$

The estimate of the right-hand side of (10) yields

$$\nu^{-1}(\varepsilon)\tilde{I}\{e^{i\varphi n}\} \simeq n^2\epsilon\frac{\ln(n^2|\epsilon|)}{\ln|\epsilon|}, \quad |\epsilon| \ll n^{-2}. \tag{11}$$

The smallness of the right-hand side of (10) is a demonstration of the slow rate of the odd relaxation. Indeed, for the odd function $\tilde{f} = \exp(i\varphi n)$, the incoming and outgoing terms in $\tilde{I}\{\tilde{f}\}$ cancel by virtue of the smoothness condition $n^2|\epsilon| \ll 1$. If we take function (11), which is substantially dependent on energy, in the capacity of $\tilde{f}$, then the terms will not be canceled. That is why Eq. (10) can be solved by an iteration method over the second (integral) term on its left-hand side. The corresponding series will converge approximately as $6^{-k}$, where $k$ is the iteration order. [We take into account that $\nu_{\mathbf{pp}'} = 0$ when $|\varepsilon'| < |\varepsilon|$ and that $g_n(\varepsilon)$ grows monotonically with $|\varepsilon|$.] In the zeroth-order iteration

$$G(\varepsilon,\varphi) \simeq \nu^{-1}(\varepsilon) \int \nu^{a}_{\mathbf{p}'\mathbf{p}}\delta(\varphi')d^2p' \simeq -\frac{\epsilon}{|\ln|\epsilon||}\frac{\cos\varphi}{\sin^3\varphi},$$

$$|\pi-\varphi|,\ \varphi \gg \sqrt{|\epsilon|}. \quad (12)$$

Thus, the existence of solutions of the form (9), with $g_n(\varepsilon) \to 0$ as $\varepsilon \to 0$, is proved [which is seen from (10)–(12)]. On the other hand, one can check that even solutions of the form (9) with this property are absent. Indeed, for an even $n$ the right-hand side of (10) depends on $\varepsilon$ weakly, like $|\ln|\epsilon||^{-1}$. Such a dependence, when integrated over energy, is almost indistinguishable from a constant. That is why the iteration method is inapplicable, and the solution does not tend to zero as $\varepsilon \to 0$.

After every collision the redundant energy of a nonequilibrium electron decreases (about three times, on the average), so that in the limit of a large number of collisions nonequilibrium electrons (holes) appear arbitrarily close to the Fermi surface. As $\varepsilon \to 0$, the relaxation terminates since the collision frequency is proportional to $(\varepsilon/\varepsilon_F)^2$. Correspondingly, Eq. (1), as $T \to 0$, has a stationary solution $\delta(\varepsilon)F(\varphi)$, where $F(\varphi)$ is an arbitrary function of the angle. Since the quantity $G(\varepsilon,\varphi)$ is conserved, it is possible to relate the odd part of the final (for the pretemperature relaxation stage) distribution in the angles $F_a(\varphi)$ with the initial distribution $f_{0\mathbf{p}} \equiv f_{\mathbf{p}}(t=0)$:

$$F_a(\varphi) = m^{-1} \int f_{0\mathbf{p}'} G(\varepsilon', \varphi - \varphi')d^2p'. \quad (13)$$

Thus, as seen from Eq. (13), the function $G(\varepsilon_0,\varphi)$ describes the odd part of the final distribution over the angles, which develops from the initial non-equilibrium distribution of the form $f_{0\mathbf{p}} \sim \delta(\mathbf{p}-\mathbf{p}_0)$, $\varphi(\mathbf{p}_0) = 0$, $\varepsilon(\mathbf{p}_0) \equiv \varepsilon_0$. Since the local even conserved quantity is absent, the even part of the final distribution is obviously isotropic.

Combining the above-obtained results with the results of the analysis of the temperature regime of the electron-electron relaxation,[3] we shall qualitatively discuss the evolution of the initial high-energy electron beam $f_{0\mathbf{p}} = A\,p_F^2\,\delta(\mathbf{p}-\mathbf{p}_0)$, $\varepsilon_0 \gg T$. For time intervals $t$ shorter than the average time $\nu^{-1}(\varepsilon_0)$ of collisions of the beam electrons with equilibrium electrons, the distribution of the scattered electrons is described by the incoming term of the collision integral

$$f_{\mathbf{p}} \simeq A\,\nu_{\mathbf{pp}_0}\,t. \quad (14)$$

Due to energy decrease of nonequilibrium electrons, each consecutive collision requires more time than the preceding one (by about six times).[7] As a result of the few first collisions, the odd part of the distribution function with a characteristic angle size $\sqrt{\varepsilon_0/\varepsilon_F}$ is shaped, while the even part becomes isotropic after about $\ln(\varepsilon_F/\varepsilon_0)$ collisions. In other words, the initial narrow beam transforms into an electron beam and a hole beam, which move in the opposite directions, each having the angular width of order $\sqrt{\varepsilon_0/\varepsilon_F}$ [together they form the odd distribution function $G(\varepsilon_0,\varphi)$]. The distribution function in the centre of these beams is of order $A\sqrt{\varepsilon_F/\varepsilon_0}$. In the $T \to 0$ approximation a distribution of this kind would be conserved infinitely long.

However, after $\ln(\varepsilon_0/T)$ collisions during the time $\nu^{-1}(T) \simeq \tau_s$ a thermalization of the electron gas occurs. The final electron temperature $T$ is determined both by the initial temperature of equilibrium electrons and by the heating of the electron system by the beam. On the time intervals longer than $\tau_s$, the odd part of the distribution broadens with time according to the law

$$\delta\varphi \simeq \delta\varphi_0 + (T/\varepsilon_F)^{1/2}\,(t/\tau_s)^{1/4}. \quad (15)$$

Thus the complete smearing of electron and hole beams requires the time $t \simeq \tau_a \sim (\varepsilon_F/T)^4$. The quantity $\delta\varphi_0$ in (15) denotes the beam smearing before the beginning of the temperature relaxation stage. Formula (15) holds for the initial beam of a low-energy $\varepsilon_0 \simeq T$ too, when the pretemperature stage of the relaxation is absent.

The above-described relatively narrow electron and hole beams can be observed in experiments similar to those reported in Ref. 4. If the distance between an injector and detector $L \le v_F\nu^{-1}(\varepsilon_0) \equiv l_0$, then a repeated collision has a small probability and, according to Eq. (14), the angular distribution of the electrons that underwent a collision is determined by $\int \nu_{\mathbf{pp}_0} d\varepsilon$. Here the maximum intensity of the scattered electrons must be observed at the distance $r \simeq L(\varepsilon_0/\varepsilon_F)^{3/2}$ from the detector normally to the beam axis. (These particles can be registered by the same detector on applying a transverse magnetic field $H \simeq rp_Fc/eL^2$.) The holes can be detected at the distance $r \simeq L\sqrt{\varepsilon_0/\varepsilon_F}$ from the injector, the width of the maximum hole beam intensity being approximately equal to $L(\varepsilon_0/\varepsilon_F)$. When $L \gg l_0$ (but $L \ll l_s$), several collisions occur on the length $L$ and the angular size of both beams become of order $\sqrt{\varepsilon_0/\varepsilon_F}$. Note that the results of Ref. 6 correspond to the background $\delta\varphi \gg \sqrt{\varepsilon_0/\varepsilon_F}$.

Another method for studying electron transport is the focusing of the beam by a transverse magnetic field,[5] when both the injector and detector are placed on the same boundary of the 2DEG. For the sake of definiteness, let the trajectory length $L$ of the primary beam equal half of the Larmour orbit. When $L \le l_0$, the scattered electrons make the same picture near the detector, as in the experiments without the magnetic field. However, the holes move along trajectories deviating far from that of the primary beam and they do not get into the region near the injector. It is interesting that in the limit $L \gg l_0$ the holes can be found near the injector: the first generated hole with the probability approximately equal to 1/2 will return to the 2DEG boundary, having deviated from the injector by approximately $l_0^2/L$ (to the opposite side from the detector).

One should bear in mind that at the temperature stage of the relaxation ($L \gg l_s$) on the mean free path $l_s$ a transition of an electron into a hole occurs (and vice versa).[3] So the beam propagation is accompanied by a one-dimensional diffusion of carriers. That is why the time $t$ in Eq. (15) is, in this case, the time of the diffusion propagation along the path $L$, $v_F t \simeq L^2/l_s$, and the beam width near the detector $r \simeq L^{3/2}(T/\varepsilon_F)^{1/2} l_s^{-1/2} \sim T^{3/2}$, $H = 0$.

We believe that the difference between the even and odd relaxations, as well as the difference-based effects, must be most clearly displayed in the following experiments. First, a beam must decay due to electron-electron collisions at distances significantly larger than the usual mean free path $l_0$. Second, the appearance of a positive charge (holes) may be expected near the injector. (This effect becomes stronger with the growth of $L$, but it also must be observed when $L \ll l_0$.) Third, when $L \gg l_s$ a considerable smearing of the beam can be observed under a extremely weak magnetic field. Under actual experimental conditions the value of $L$ is varied from the units to several tens of micrometers, while the mean free path $l_s \simeq 0.8 - 30$ $\mu$m, depending on the electron gas temperature.[4,5,8] The electron-impurity mean free path can be made to exceed these lengths. Thus the observation of the above-mentioned effects is quite realizable nowadays. If, for example, $L < l_0$ and $\varepsilon/\varepsilon_F \simeq 0.1$, then the positive charge can be observed at a distance $r \simeq 0.3L$ from the injector.

In conclusion, we have studied the mechanisms of angular relaxation of highly anisotropic distributions in a two-dimensional degenerate gas of colliding electrons. We have shown that the relaxation has a number of stages. Rather fast, during the time of order $\tau_0 \sim (\varepsilon_F/\varepsilon)^2$, the even part of the distribution becomes isotropic, while the odd part becomes wider by the angle $\sqrt{\varepsilon/\varepsilon_F}$. The further relaxation of the odd part occurs rather slowly, in $\tau_a \sim (\varepsilon_F/T)^4$, $\varepsilon \geq T$. These processes can be interpreted as the formation, from the primary beam, of secondary long-lived electron and hole beams. (We remind the reader that by a hole we mean the absence of an electron under the Fermi level.) We have shown here that electron beam experiments can provide fairly complete information on qualitatively new kinetic effects arising in a two-dimensional degenerate electron gas.

The research described herein was made possible in part by Grant No. U2D000 from the International Science Foundation.

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