# Theory of Shubnikov-de Haas oscillations around the $\nu = 1/2$ filling factor of the Landau level: Effect of gauge-field fluctuations

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We present a theory of magneto-oscillations around the  $\nu = 1/2$  Landau level filling factor based on a model with a fluctuating Chern-Simons field. The quasiclassical treatment of the problem is appropriate and leads to an unconventional  $\exp[-(\pi/\omega_c \tau_{1/2}^*)^4]$  behavior of the amplitude of oscillations. This result is in good qualitative agreement with available experimental data, although the experimental value of  $\tau_{1/2}^*$  exceeds considerably our theoretical estimate.

Since the discovery of the fractional quantum hall effect (FQHE),1 the physical properties of a high mobility electron gas subjected to a strong magnetic field are attracting great interest. Laughlin's theory<sup>2</sup> gives a very good description of the properties of the FQHE states with filling factors  $\nu = 1/(2m+1)$ . The subsequently proposed hierarchy scheme<sup>3</sup> explains, in principle, the existence of FQHE states with arbitrary filling factors  $\nu = p/q$  (q is odd). However, some drawbacks of this scheme were discovered later. In particular, the FQHE state with  $\nu = p/(2p\pm 1)$  appears only on the pth level of hierarchy, so that the scheme does not explain why these states are experimentally dominating. This discrepancy motivated Jain<sup>4</sup> to propose a different concept based on converting the electrons into composite fermions by attaching to them an even number of flux quanta. Following a similar approach Halperin, Lee, and Read<sup>5</sup> developed a theory for the half-filled Landau level (see also Ref. 6).

This theory gives an explanation for many experimentally observed properties of the  $\nu = 1/2$  state, such as a nonzero value of the longitudinal resistivity, an anomaly in the surface acoustic wave propagation, and a dimensional resonance of the composite fermions. 8 It predicts the formation, at half filling, of a metallic state with well-defined Fermi surface. From this point of view, the  $\nu = p/(2p\pm 1)$  series can be considered as the usual  $\nu = p$ Shubnikov-de Haas oscillations (SdHO) for the composite fermions, providing an explanation for the prominence of the above FQHE states. Indeed, the oscillating behavior of the longitudinal resistivity  $\rho_{xx}$  near  $\nu = 1/2$  is very much reminiscent of its behavior in low magnetic fields

where conventional SdHO take place.<sup>9,10</sup>

Comparison of the resistivity oscillations near  $\nu = 1/2$ and in a weak magnetic field shows, however, not only a similarity in shape, but also an important difference: the amplitude of the SdHO near half filling decreases on approaching  $\nu = 1/2$   $(p = \infty)$  much faster than it does in weak fields on approaching zero magnetic field B=0 $(\nu = \infty)$ . As a result, the  $\nu = p/(2p \pm 1)$  oscillations vanish at  $p = p_{\text{max}} = 6-9.9,10$  This reflects a difference in the physical properties between the states at  $\nu = 1/2$  and B=0, calling thus for a detailed theoretical considera-

The crucial feature that distinguishes these states is the presence of fluctuations of the fictitious, Chern-Simons (CS), gauge field,<sup>5</sup> which scatter the fermions. On the other hand, the problem of a quantum particle in a random magnetic field in two dimensions was studied in our recent papers. 11,12 We have shown that whereas the transport relaxation time  $\tau_t$  can be found within perturbation theory, the single particle properties of the model are peculiar. In particular, we found a Gaussian shape of the broadened Landau levels and unusual expressions for the single particle relaxation time  $\tau_s$  and for the amplitude of the de Haas-van Alphen oscillations. These results were obtained by using the formalism of path integrals in coordinate space. In the present report, we apply these methods to the study of magneto-oscillations of the conductivity in the FQHE regime near  $\nu = 1/2$ .

In the zero-temperature limit, the fluctuations of the CS field are determined by the randomly located impurities.<sup>5</sup> Namely, impurities produce fluctuations of fermion density and current, which are in turn coupled

to the scalar and vector components of the CS potential, respectively. This results in static spatial fluctuations of the CS field, with the correlation length determined by that of the impurity potential. Since the impurities in experiments are located in a remote layer, the fluctuations are rather smooth. These fluctuations suppress the amplitude of the SdHO. We treat this effect semiclassically, which is justified for the random magnetic field as well as for the smooth random potential. 12 We find the amplitude of the SdHO to be proportional to  $\rho_{xx}^{\rm osc} \propto \exp[-(\pi/\omega_c \tau_{1/2}^*)^4]$ , where  $\omega_c$  is the cyclotron frequency and  $\tau_{1/2}^*$  plays the role of an effective relaxation time. This result that describes well the experimental data should be contrasted with the usual behavior  $\rho_{xx}^{\rm osc} \propto \exp\left[-\pi/\omega_c \tau\right]$  that one obtains for the case of the short-range random potential.

We consider a realistic system formed by the twodimensional (2D) electron gas of density  $n_e$  and by the positively charged impurities located in a layer separated by a large distance  $d_s$  from the electron plane. The statistical transformation attaches to each electron an even number  $\tilde{\phi}$  of flux quanta of the CS gauge field. To describe the vicinity of the  $\nu=1/2$  state, we take  $\tilde{\phi}=2$ ; the same formalism with  $\tilde{\phi}=4$  can be applied to the  $\nu=1/4$  state.

In the mean field approximation, the statistical magnetic field  $B_{1/2} = 4\pi c n_e/e$  cancels exactly the externally applied field B at  $\nu=1/2$ . When the filling factor  $\nu$  is tuned away from  $\nu=1/2$ , the effective uniform magnetic field is equal to  $B_{\rm eff}=B-B_{1/2}$ . For  $\nu$  close to 1/2, the number of filled Landau levels of composite fermions  $p\gg 1$ , so that the problem can be considered quasiclassically. In the quasiclassical approximation, the quantities of interest can be expressed as a sum over classical trajectories. We will treat the random fields in the framework of quasiclassical perturbation theory, neglecting their influence on the classical trajectories. This approximation is valid provided  $\omega_c \tau_t \gg 1$ . The trajectories are then simply the cyclotron circles in the uniform field  $B_{\rm eff}$ .

In a previous paper,  $^{12}$  we used this quasiclassical approach to calculate the de Haas-van Alphen oscillations of the density of states (DOS) in the presence of a random magnetic field. The conductivity can be written as a sum over periodic orbits in a similar way.  $^{13-15}$  One starts from the Kubo formula and writes each of the two Green functions involved (retarded and advanced) as a path integral, yielding a double sum over winding numbers n, n' of the two trajectories.  $^{14,15}$  The terms with n=n' give then the nonoscillating contribution  $\sigma_{no}$ , whereas the terms with  $k=n-n'\neq 0$  represent the kth harmonic of oscillations. The resulting expression has the same structure as for the DOS,  $^{12}$ 

$$\sigma_{xx} = \sigma_{\text{no}} \left[ 1 - 2i \operatorname{Re} \sum_{k=1}^{\infty} \exp \left\{ 2\pi i p k - \frac{1}{2} \left\langle S_r^2 \right\rangle k^2 \right\} \right],$$
(1)

where  $p = \frac{1}{2} \frac{e}{c} R_c^2 B_{\text{eff}} = 2\pi \frac{cn_e}{eB_{\text{eff}}}$ ,  $R_c$  is the cyclotron radius,  $S_r$  is the contribution to the action induced by random fields along the classical path of winding number

k=1, and the angular brackets denote the average over impurity configurations. A good estimate for the amplitude of the oscillations is given by the first harmonic:

$$\sigma_{xx} = \sigma_{no} \left[ 1 - 2\cos(2\pi p) \exp\left(-\frac{1}{2} \left\langle S_r^2 \right\rangle\right) \right].$$
 (2)

At zero temperature, the gauge-field fluctuations are dominated by the randomly located impurities. Each impurity creates a scalar potential of the form

$$\int (dq)v_0(q)e^{i\boldsymbol{q}(\boldsymbol{r}-\boldsymbol{r}_i)}\;;\;v_0(q)=\frac{2\pi e^2}{\epsilon q}e^{-qd_s},\qquad(3)$$

where  $r_i$  is the projection of the impurity position to the 2D plane,  $\epsilon$  is the dielectric constant, and  $(dq) = d^2q/(2\pi)^2$ . This potential gets renormalized due to the screening by fermions and mixing with the CS field. In the random phase approximation one gets

$$A_{\mu} = \left(\delta_{\mu}^{\ \rho} - U_{\mu\nu} K^{\nu\rho}\right)^{-1} A_{\rho}^{(0)},\tag{4}$$

where we united scalar  $A_0$  and vector A potentials in a covariant vector  $A_{\mu}$ ; the vector  $A_{\rho}^{(0)}$  represents the bare impurity potentials and, therefore, has only  $\rho=0$  nonzero component. The tensors  $U_{\mu\nu}$  and  $K^{\nu\rho}$  represent the bare gauge-field propagator and the current-density response tensor of the composite fermions, respectively.

To evaluate Eq. (4), we use the Coulomb gauge div A=0, go to the momentum space, and choose the momentum q to be directed along the x axis:  $q_x=q$ ,  $q_y=0$ . Then  $A_{\mu}$  has only 2 nonzero components corresponding to  $\mu=0$ , y, and both K and U become  $2\times 2$  matrices:<sup>5</sup>

$$K^{\mu\nu}(q) = \begin{pmatrix} -m^*/2\pi & -iq\sigma_{xy} \\ iq\sigma_{xy} & \chi q^2 - 2i\omega n_e/qk_F \end{pmatrix},$$

$$U_{\mu\nu}(q) = \begin{pmatrix} v(q) & 2\pi i\tilde{\phi}/q \\ -2\pi i\tilde{\phi}/q & 0 \end{pmatrix}, \qquad (5)$$

$$A^{(0)}_{\mu}(q) = \begin{pmatrix} v_0 e^{-iq\mathbf{r}_i} \\ 0 \end{pmatrix},$$

where  $m^*$  is the effective mass of fermions,  $\chi = 1/24\pi m^*$  is the magnetic susceptibility,<sup>17</sup> and  $v(q) = 2\pi e^2/(\epsilon q)$  is the Coulomb propagator, and  $\sigma_{xy}$  is the Hall conductivity of composite fermions.

Substituting (5) in (4), we find

$$A_{\mu}(q) = \frac{v_0(q)e^{-i\mathbf{q}\mathbf{r}_i}}{\frac{m^*v(q)}{2\pi} + (\tilde{\phi}s + 1)^2 + \frac{\tilde{\phi}^2}{12}} \begin{pmatrix} \tilde{\phi}s + 1\\ i\tilde{\phi}m^*/q \end{pmatrix}, \quad (6)$$

where  $s = 2\pi\sigma_{xy} \simeq p$  in the limit  $\omega_c \tau_t \gg 1$ .

Let us now compare the first and the second term in denominator of (6). As we will see below, the typical momenta are  $q \sim (2d_s)^{-1}$ , and we get for  $\tilde{\phi} = 2$ 

$$\frac{m^*v(q)/2\pi}{(2s)^2} = \frac{m^*e^2}{4\epsilon q s^2} \sim \frac{m^*e^2}{\epsilon k_F} \frac{k_F d_s}{2p^2} \sim \frac{50}{p^2},\tag{7}$$

where  $k_F = \sqrt{4\pi n_e}$ , and we used typical experimental parameters<sup>9</sup>  $n_e = 1.1 \times 10^{11}$  cm<sup>-2</sup>,  $d_s = 80$  nm, and the ex-

perimentally estimated value for the ratio  $m^*e^2/(\epsilon k_F) \sim$  10. For the not too large p we are interested in, it is thus a reasonable approximation to neglect all but the first term in the denominator of (6). This gives

$$A_{\mu}(q) = \frac{2\pi}{m^*} \tilde{\phi} e^{-i\boldsymbol{q}\boldsymbol{r}_i} e^{-qd_s} \begin{pmatrix} p \\ im^*/q \end{pmatrix}, \tag{8}$$

The random field action  $S_r$  in Eqs. (1), (2) is given by  $S_r = -\oint A_\mu dr^\mu = -\left(\int A_0 dt - \oint A dr\right)$ , where the integration goes around a cyclotron orbit. Assuming now the impurities to be randomly distributed with concentration  $n_i$  and uncorrelated, we find

$$\langle S_r^2 \rangle = (2\pi\tilde{\phi})^2 n_i$$

$$\times \int (dq) e^{-2qd_s} \left| \frac{p}{k_F} \oint dl \ e^{-i\boldsymbol{q}\boldsymbol{r}} + \int d^2r \ e^{-i\boldsymbol{q}\boldsymbol{r}} \right|^2$$

$$= n_i (4\pi^2 \tilde{\phi} R_c)^2 \int (dq) e^{-2qd_s}$$

$$\times \left| \frac{p}{k_F} J_0(qR_c) + \frac{1}{q} J_1(qR_c) \right|^2 .$$

$$(9)$$

Here,  $\oint dl$  means integration along the cyclotron orbit and corresponds to the electric field contribution, whereas  $\int d^2r$  goes over the area surrounded by the orbit and describes the magnetic field contribution. Taking into account that  $R_c^2 = p^2/(\pi n_e)$ , we have  $R_c/2d_s = p/\sqrt{4\pi n_e d_s^2} \sim p/10 \lesssim 1$ . Thus, for relevant momenta  $q \sim 1/(2d_s)$  and level numbers  $p, qR_c \ll 1$  is a reasonable approximation. In this case Eq. (9) reduces to

$$\frac{1}{2} \left\langle S_r^2 \right\rangle = \pi^3 \tilde{\phi}^2 n_i \frac{R_c^4}{d_s^2} = \frac{n_i}{n_e} \frac{\pi \tilde{\phi}^2}{n_e d_s^2} \left( \frac{2\pi n_e}{m^* \omega_c} \right)^4. \tag{10}$$

Note that electric and magnetic field fluctuations give equal contributions in this limit. According to (2), this gives for the oscillating part of the conductivity:

$$\sigma_{xx}^{\rm osc} \propto -\cos\left(\frac{4\pi^2 n_e c}{eB_{\rm eff}}\right) \exp\left[-\left(\frac{\pi}{\omega_c \tau_{1/2}^*}\right)^4\right],$$
 (11)

where we introduced a parameter  $\tau_{1/2}^*$ , which is given according to Eq. (10) by

$$\tau_{1/2}^* \simeq \frac{m^*}{2n_e} \left(\frac{n_e d_s^2}{4\pi}\right)^{1/4} .$$
(12)

When writing Eq. (12), we made the usual assumption that concentrations of donors and charge carriers coincide:  $n_e = n_i$ .

The dependence of the amplitude of oscillations on  $\omega_c$  in Eq. (11) differs from the conventional form  $\exp(-\pi/\omega_c\tau)$ , which holds for short-range potential scattering. We have already shown in Ref. 12 that short-range magnetic field scattering leads to damping of oscillations in the DOS  $\sim \exp\left[-(\pi/\omega_c\tau)^2\right]$ . As we see now from Eq. (11), a long-range correlated magnetic field leads to the result  $\sim \exp\left[-(\pi/\omega_c\tau)^4\right]$ .

In Fig. 1 we present experimental data for the amplitude of  $\rho^{\rm osc}$  from Ref. 9 (T=0.19 K,  $B_{\rm eff}>0$ ). It

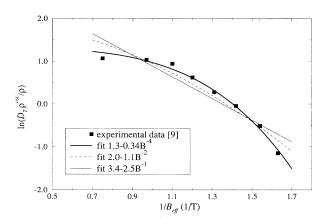


FIG. 1. Dingle plot. Logarithm of normalized amplitude of resistivity oscillations  $\ln(D_T \rho^{\rm osc}/\rho)$ , with  $D_T = \sinh(2\pi^2 T/\omega_c)/(2\pi^2 T/\omega_c)$ , as a function of inverse effective magnetic field  $B_{\rm eff}^{-1}$ . Linear, quadratic and quartic fits of experimental data from Ref. 9 are presented.

is seen that they can be fitted well by  $\exp \left[-(\pi/\omega_c \tau)^4\right]$ , whereas a  $\exp(-\pi/\omega_c \tau)$  fit is much worse.

Let us briefly consider now effect of finite temperature T. First of all, the SdHO are then suppressed by a usual factor  $D_T = (2\pi^2 T/\omega_c)/\sinh(2\pi^2 T/\omega_c)$  originating from the Fermi distribution.<sup>13</sup> In addition, the fermions are scattered by the thermal fluctuations of the gauge-field. The propagator of gauge-field fluctuations is given by

$$D_{\mu\nu}(q,\omega) = U_{\mu\rho}(q) \left[ \delta^{\rho}_{\nu} - K^{\rho\lambda}(q,\omega) U_{\lambda\nu}(q) \right]^{-1}. \quad (13)$$

In particular, for the  $D_{11}$  component determining the magnetic field fluctuations, we get

$$D_{11}(q,\omega) = (2i\omega n_e/qk_F - \tilde{\chi}q^2)^{-1};$$

$$\tilde{\chi} = \frac{1}{2\pi m^*} \left[ \frac{1}{12} + \left(s + \frac{1}{\tilde{\phi}}\right)^2 \right] + \frac{v(q)}{(2\pi\tilde{\phi})^2}.$$
(14)

In the quasistatic approximation, we find

$$\langle A_1 A_1 \rangle_q = \int rac{d\omega}{2\pi} rac{2T}{\omega} {
m Im} D_{11} = rac{T}{ ilde{\chi}q^2},$$

and consequently for the amplitude of magnetic field fluctuations  $\langle hh \rangle_q = T/\tilde{\chi}$ . If  $\omega_c \tau_t \gg 1$ , we have  $s \simeq p$  and  $\tilde{\chi} = 12p^2\chi$ . The contribution of these fluctuations to the random field action  $\langle S_r^2 \rangle$  in (2) is then  $\langle S_r^2 \rangle_T = (T/\tilde{\chi})\pi R_c^2 \approx 2\pi^2 T/p\omega_c$ , i.e., is small at  $p \gg 1$  compared to the standard term  $-\ln D_T$  and decreases with p. This result would manifest itself as an apparent decrease of the effective mass  $m^*$  (extracted in the usual way from the temperature dependence of the amplitude of SdHO) with p for moderately large p, in qualitative agreement with experimental findings.  $^{9,10,20}$  For larger p, a sharp increase in  $m^*$  was observed in Ref. 20, the origin of which is not clear to us.

For completeness, we consider now the SdHO in low fields (around B=0), where there is no CS gauge field. At low temperature the scattering is then due to screened impurity potential<sup>5</sup>  $A_0 = (2\pi/m_b)e^{-qd_s}e^{-i\mathbf{q}\mathbf{r}_i}$ , where  $m_b$ 

is the band electron mass. [It is easy to see that this follows from Eq. (6) if one puts  $\tilde{\phi}=0$ .] We find that  $\langle S_r^2 \rangle$  in Eq. (2) is given for this case by

$$\left\langle S_{r}^{2}\right\rangle =(2\pi)^{2}\frac{n_{i}}{n_{e}}R_{c}^{2}\int qdqe^{-2qd_{s}}J_{0}^{2}(qR_{c}).$$
 (15)

Assuming again  $n_i = n_e$ , we get for the amplitude of oscillations

$$\sigma_{xx}^{\rm osc} \propto -\cos\left(\frac{2\pi^2 n_e c}{eB}\right) \exp\left[-\frac{1}{\beta} \left(\frac{\pi}{\omega_c \tau_0}\right)^{\beta}\right] ,$$
 (16)

$$\tau_0 = \frac{m_b}{n_e} \left( \frac{n_e d_s^2}{2\pi} \right)^{1/2} \,, \tag{17}$$

where  $\beta=2$  for  $R_c\ll d_s$  and  $\beta=1$  for  $R_c\gg d_s$ . It is seen from Eq. (17) that the condition of week oscillations,  $\pi/\omega_c\tau_0\gg 1$ , corresponds to the latter regime, where the usual result  $\ln\sigma_{xx}^{\rm osc}\sim -(\pi/\omega_c\tau_0)$  holds.

As was already mentioned, the available experimental data around  $\nu=1/2$  (Ref. 9) apparently show the behavior  $\ln \rho^{\rm osc} \sim 1/\omega_c^4$  predicted by Eq. (11). The value of the parameter  $\tau_{1/2}^*$  which is found from such a fit (Fig. 1) is  $\tau_{1/2}^* \simeq 16 \times 10^{-12}$  s. At the same time the theoretical estimate according to Eq. (12) (with use of the parameters of Ref. 9) gives  $\tau_{1/2}^* \simeq 2.4 \times 10^{-12}$  s if one uses the experimental value of the effective mass  $m^* = 0.7m_e$ . A similar discrepancy is found for the low-field relaxation time: Eq. (17) for  $m_b = 0.07m_e$  gives  $\tau_0 \simeq 0.6 \times 10^{-12}$  s, whereas the value quoted in Ref. 9 is  $\tau_0 \simeq 9 \times 10^{-12}$  s. We note also that the theoretically estimated values for the transport relaxation rate at  $\nu = 1/2$  are typically four times greater than extracted from ex-

perimental mobilities.<sup>5,9</sup> Therefore, the theory seems to overestimate relaxation rates systematically. This situation has been discussed previously.<sup>18,19</sup> The considerable increase of relaxation times was attributed to the correlations in positions of charged impurities due to their mutual Coulomb interaction.<sup>19</sup> Note, however, that this effect, which leads to a weakening of the random potential, is probably not sufficient to remove the discrepancy of theory and experiment for the quantity  $\tau_{1/2}^*$ , namely, a factor of  $[\tau_{1/2}^*(\exp)/\tau_{1/2}^*(\operatorname{theor})]^4 \sim 2000$  in the exponent of Eq. (11).

In conclusion, we have presented a theory of magneto-oscillations around  $\nu=1/2$  Landau level filling factor based on a model with fluctuating Chern-Simons field. The quasiclassical treatment of the problem is appropriate and leads to unconventional  $\exp[-(\pi/\omega_c\tau_{1/2}^*)^4]$  behavior of the amplitude of oscillations. This result is in good agreement with available experimental data. At the same time, the experimental value of  $\tau_{1/2}^*$  considerably exceeds our theoretical estimate.

Recently, experimental data<sup>20</sup> on SdHO near  $\nu=1/2$  on a better-quality sample have been published. The obtained Dingle plot [Fig. 3(a) of Ref. 20] is again highly nonlinear, in agreement with our formula (11). On the other hand, the suggestion of the authors of Ref. 20 that this nonlinearity may be explained by the variation of the effective mass is not supported by our results, since  $m^*$  drops out from Eq. (11).

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16609 (1994).

<sup>\*</sup> Deceased.

<sup>&</sup>lt;sup>1</sup> D.C. Tsui, H.L. Stormer and A.C. Gossard, Phys. Rev. Lett. 48, 1159 (1982).

<sup>&</sup>lt;sup>2</sup> R.B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983).

F.D.M. Haldane Phys. Rev. Lett. 51, 605 (1983); B.I. Halperin, *ibid.* 52, 1583, 2390(E) (1984); R.B. Laughlin, Surf. Sci. 141, 11 (1984).

<sup>&</sup>lt;sup>4</sup> D.K. Jain, Phys. Rev. Lett. **63**, 199 (1989); Phys. Rev. B **40**, 8079 (1989); **41**, 7653 (1990).

<sup>&</sup>lt;sup>5</sup> B.I. Halperin, P.A. Lee, and N. Read, Phys. Rev. B **47**, 7312 (1993).

<sup>&</sup>lt;sup>6</sup> V. Kalmeyer and S.-C. Zhang, Phys. Rev. B **46**, 9889 (1992).

<sup>&</sup>lt;sup>7</sup> R.L. Willet, M.A. Paalanen, R.R. Ruel, K.W. West, L.N. Pfeiffer, and D.J. Bishop, Phys. Rev. Lett. **65**, 112 (1990).

<sup>&</sup>lt;sup>8</sup> R.L. Willet, R.R. Ruel, K.W. West, and L.N. Pfeiffer, Phys. Rev. Lett. **71**, 3846 (1993); W. Kang, H.L. Stormer, L.N. Pfeiffer, K.W. Baldwin, and K.W. West, *ibid.* **71**, 3850 (1993).

<sup>&</sup>lt;sup>9</sup> R.R. Du, H.L. Stormer, D.C. Tsui, L.N. Pfeiffer, and K.W. West, Solid State Commun. **90**, 71 (1994); Phys. Rev. Lett. **70**, 2944 (1993).

<sup>&</sup>lt;sup>10</sup> D.R. Leadley, R.J. Nicholas, C.T. Foxon, and J.J. Harris, Phys. Rev. Lett. **72**, 1906 (1994).

<sup>&</sup>lt;sup>11</sup> A.G. Aronov, A.D. Mirlin, and P. Wölfle, Phys. Rev. B **49**,

<sup>&</sup>lt;sup>12</sup> A.G. Aronov, E. Altshuler, A.D. Mirlin, and P. Wölfle, Europhys. Lett. **29**, 239 (1995).

<sup>&</sup>lt;sup>13</sup> A.A. Abrikosov, Introduction to the Theory of Normal Metals (Academic Press, New York, 1972).

<sup>&</sup>lt;sup>14</sup> G. Hackenbroich and F. von Oppen, Europhys. Lett. 29, 7 (1995).

<sup>&</sup>lt;sup>15</sup> K. Richter, Europhys. Lett. **29**, 151 (1995).

<sup>&</sup>lt;sup>16</sup> An analogous suppression of the gauge-field fluctuations in the external magnetic field was found in L.B. Ioffe and P. Wiegmann, Phys. Rev. B 45, 519 (1992).

<sup>&</sup>lt;sup>17</sup> In fact, both DOS and magnetic susceptibility determining the diagonal components of the response tensor  $K^{\mu\nu}$  are different from their average values  $m^*/2\pi$  and  $1/24\pi m^*$ , due to de Haas-van Alphen oscillations, which lead to a nonlinear screening as considered in A.L. Efros, Solid State Commun. **65**, 1281 (1988). However, since our primary interest is in the region of large p, where the amplitude of oscillations is small, we can neglect these nonlinear effects and use the approximation (5) for  $K^{\mu\nu}$ .

<sup>&</sup>lt;sup>18</sup> P.T. Coleridge, Phys. Rev. B **44**, 3793 (1991).

<sup>&</sup>lt;sup>19</sup> E. Buks, M. Heiblum, Y. Levinson, and H. Strikman, Semicond. Sci. Technol. 9, 2031 (1994).

<sup>&</sup>lt;sup>20</sup> R.R. Du, H.L. Stormer, D.C. Tsui, A.S. Yeh, L.N. Pfeiffer, and K.W. West, Phys. Rev. Lett. **73**, 3274 (1994).