Acoustic scattering of electrons in degenerate semiconductors at low lattice temperatures

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(Received 22 March 1995)

The inelastic acoustic scattering rate of electrons in a degenerate material is calculated under the condition of low temperature when the traditional approximations are not valid. The results for Si show features significantly different from those that follow from the traditional theory.

Degenerate semiconductors are used in various solid-state devices, for example, metal-insulator-semiconductor diodes, tunnel diodes, lasers, etc.[1] For device applications the theoretical study of the electron transport in a semiconductor is important. Such studies require a knowledge of the scattering rates of the free electrons for the prevalent interactions in the material at a given lattice temperature.

There is a range of low lattice temperature ($T_L \leq 20$ K) when the free electrons in a high-purity covalent semiconductor interact dominantly only with intravalley deformation acoustic phonons.[2–6] The well-developed traditional theory of the deformation acoustic-phonon scattering of the free electrons in a nondegenerate material, based on the simplifying approximations of negligible phonon energy, elastic collisions, and equipartition for the phonon distribution, is indeed valid at higher temperatures. But with the lowering of the temperature when the phonon energy compares well with the electron energy, it can neither be neglected nor can the phonon distribution be approximated by the equipartition law. This problem of inelastic acoustic scattering of the free electrons in a nondegenerate material at the low lattice temperatures has been studied by many.[6–11] The results have been rather interesting, being significantly different from what follows from the traditional theory.

At the low lattice temperatures, however, a material having a particular carrier concentration can hardly be regarded as nondegenerate when the Fermi energy $\varepsilon_F \geq k_B T_L$, k_B being the Boltzmann constant. A proper theory of the inelastic acoustic scattering of the free electrons in a degenerate material at low lattice temperatures is not yet available in the literature. The purpose of this paper is to develop the theory of the inelastic scattering of the electrons in a degenerate material due to intravalley acoustic phonons at low lattice temperatures, taking due account of the energy carried by a phonon and also of their true energy distribution. The result is then used to obtain the scattering rate and its dependence upon carrier energy, lattice temperature, degree of degeneracy, etc., for some degenerate samples of Si.

Let us consider a volume V of an isotropic, degenerate semiconductor with a single, parabolic, spherically symmetric conduction band. The scattering rate of an electron with wave vector $\mathbf{k}$ and energy $\varepsilon_{\mathbf{k}}$ for interactions with intravalley deformation acoustic phonons may be obtained directly from the perturbation theory as

$$P_{ac}(\varepsilon_{\mathbf{k}}) = \frac{2\pi}{\hbar} \sum_{\mathbf{q}} |\langle \mathbf{k}\pm\mathbf{q}|H'_{ac}|\mathbf{k}\rangle|^2 \delta(\varepsilon_{\mathbf{k}\pm\mathbf{q}} - \varepsilon_{\mathbf{k}} \mp \hbar q v_s) \times \{1 - f_0(\mathbf{k}\pm\mathbf{q})\} d\mathbf{q}\ , \tag{1}$$

where the matrix element is given by[2,3]

$$\langle \mathbf{k}\pm\mathbf{q}|H'_{ac}|\mathbf{k}\rangle = \left[\frac{E_1^2 \hbar q}{2\rho v_s V}\right]^{1/2} \left[\begin{matrix} n_q^{1/2} \\ (n_q+1)^{1/2} \end{matrix}\right],$$

where E_1 is the deformation potential constant, ρ the density of the material, $\hbar = h/2\pi$, h being Planck's constant, $\mathbf{q}$ is the phonon wave vector, $f_0(\mathbf{k}\pm\mathbf{q})$ is the probability of occupation of the final state $\mathbf{k}\pm\mathbf{q}$ and is given by the Fermi function, and n_q is the phonon population. The upper and lower sign (and n_q and n_q+1) must be taken, respectively, for the processes of absorption and emission.

The integration over the polar and azimuthal angles can be easily performed. For integration over the magnitude of the phonon wave vector one can expand $f_0(\mathbf{k}\pm\mathbf{q})$ in a Taylor's series around $\varepsilon_{\mathbf{k}}$.

In order to have a correct description of the energy balance at low temperature the true n_q is given to a good approximation by the truncated Laurent expansion,[6]

$$n_q(x) = \begin{cases} \dfrac{1}{x} - \dfrac{1}{2} + \dfrac{1}{12}x - \dfrac{1}{720}x^3 + \dfrac{1}{30\,240}x^5 & \text{for } x < \bar{x} \\ \exp(-x) & \text{for } x > \bar{x}, \end{cases} \tag{2}$$

where $x = \hbar q v_s / k_B T_L$ and $\bar{x} = 3.5$.

The integration over the normalized phonon wave vector x can be carried out after assigning the upper (x_2 or x'_2) and the lower (x_1 or x'_1) limits. The limits can be obtained from the energy and momentum conservation conditions for an electron making transitions from a state $\mathbf{k}$ to $\mathbf{k}\pm\mathbf{q}$ accompanied by either absorption or emission of a phonon. It may be mentioned here that the probability of the scattering involving more than one phonon is small at low temperatures of interest here.[2] Since the energy of

0163-1829/95/52(7)/4651(3)/$06.00

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a phonon can no longer be neglected at low temperatures, the range of the phonon wave vector involved in the process of absorption comes out to be different from that corresponding to the process of emission, though the traditional elastic approximations lead to the same range for both, viz., 0–$2\hbar k v_s/k_B T_L$. In the energy domain $0<\varepsilon_k \le \varepsilon_s$ $(=\frac{1}{2}m^* v_s^2$, m^* being the effective mass of the electron), where the carriers can only absorb phonons during the interaction, $x_1=\lambda(\sqrt{\varepsilon_s}-\sqrt{\varepsilon_k})$ and $x_2=\lambda(\sqrt{\varepsilon_s}+\sqrt{\varepsilon_k})$. It should be noted here that the maximum value of x_2 in this energy range is usually less than $\bar{x}$. In the range $\varepsilon_k>\varepsilon_s$, $x_1'=0$, $x_2=\lambda(\sqrt{\varepsilon_s}+\sqrt{\varepsilon_k})$ for the process of absorption and $x_1'=0$, $x_2'=\lambda(\sqrt{\varepsilon_k}-\sqrt{\varepsilon_s})$ for the process of emission, where $\lambda=4\sqrt{\varepsilon_s}/k_B T_L$.

Thus, performing the integrations for $T_L>8\varepsilon_s/\bar{x}k_B$, one can obtain

$$P_{ac}(\varepsilon_k)=\frac{A_{ac}}{\sqrt{\varepsilon_k}}[H_1(x_1)+H_3(x_2)-H_1(x_2)-H_3(x_1)] \quad \text{for } 0<\varepsilon_k\le\varepsilon_s , \tag{3a}$$

$$P_{ac}(\varepsilon_k)=\frac{A_{ac}}{\sqrt{\varepsilon_k}}[H_3(x_2)+H_3(x_2')-H_1(x_2)-H_2(x_2')+F_1(x_2')] \quad \text{for } \varepsilon_s<\varepsilon_k\le\varepsilon_1 , \tag{3b}$$

$$P_{ac}(\varepsilon_k)=\frac{A_{ac}}{\sqrt{\varepsilon_k}}[H_3(\bar{x})+H_3(x_2')-H_1(\bar{x})-H_2(x_2')+G_3(x_2)-G_3(\bar{x})-G_1(x_2)+G_1(\bar{x})+F_1(x_2')] \quad \text{for } \varepsilon_1<\varepsilon_k\le\varepsilon_2 , \tag{3c}$$

$$P_{ac}(\varepsilon_k)=\frac{A_{ac}}{\sqrt{\varepsilon_k}}[2H_3(\bar{x})-H_1(\bar{x})-H_2(\bar{x})-2G_3(\bar{x})+G_3(x_2)+G_3(x_2')+G_1(\bar{x})+G_2(\bar{x})-G_1(x_2)-G_2(x_2')+F_1(x_2')] \quad \text{for } \varepsilon_2<\varepsilon_k<\infty . \tag{3d}$$

Here,

$$A_{ac}=\frac{1}{16}\left[\frac{E_1^2 m^{*2}}{\hbar^4 \rho\pi v_s}\right]\left[\frac{k_B T_L}{\sqrt{\varepsilon_s}}\right]^3 ,$$

$$H_3(x)=\tfrac{1}{2}x^2-\tfrac{1}{6}x^3+\tfrac{1}{48}x^4-\tfrac{1}{4320}x^6+\tfrac{1}{241\,920}x^8 ,$$

$$F_1(x)=\tfrac{1}{3}x^3 ,$$

$$G_3(x)=-\exp(-x)(x^2+2x+2) ,$$

$$H_1(x)=\sum_{n=0}^{\infty}\left[\frac{x^{n+2}}{n+2}-\frac{x^{n+3}}{2(n+3)}+\frac{x^{n+4}}{12(n+4)}-\frac{x^{n+6}}{720(n+6)}+\frac{x^{n+8}}{30\,240(n+8)}\right]\times D^n f_0(\varepsilon_k)\frac{(k_B T_L)^n}{n!} ,$$

$$H_2(x)=\sum_{n=0}^{\infty}\left[\frac{x^{n+2}}{n+2}+\frac{x^{n+3}}{2(n+3)}+\frac{x^{n+4}}{12(n+4)}-\frac{x^{n+6}}{720(n+6)}+\frac{x^{n+8}}{30\,240(n+8)}\right]\times D^n f_0(\varepsilon_k)\frac{(-k_B T_L)^n}{n!} ,$$

$$G_1(x)=\sum_{n=0}^{\infty}\frac{(k_B T_L)^n D^n f_0(\varepsilon_k)}{n!}I_{n+2} ,$$

$$G_2(x)=\sum_{n=0}^{\infty}\frac{(-k_B T_L)^n D^n f_0(\varepsilon_k)}{n!}\left[\frac{x^{n+3}}{n+3}+I_{n+2}\right] ,$$

where

$$I_{n+2}=\int_{x_1}^{x_2}x^{n+2}\exp(-x)dx ,$$

$$\varepsilon_1=\left[\frac{\bar{x}}{\lambda}-\sqrt{\varepsilon_s}\right]^2 , \quad \varepsilon_2=\left[\frac{\bar{x}}{\lambda}+\sqrt{\varepsilon_s}\right]^2 ,$$

the operator

$$D^n=\frac{d^n}{d\varepsilon_k^n} ,$$

and the Fermi energy is related with the electron concentration N_i by

$$\varepsilon_F=\frac{\hbar^2}{2m^*}(3\pi^2 N_i)^{2/3} .$$

At high temperature when the acoustic scattering may be considered elastic and the equipartition approximation is valid one can obtain for the combined process of emission and absorption:

$$P_{ac}(\varepsilon_k)=A_{ac}[1-f_0(\varepsilon_k)]\lambda^2\sqrt{\varepsilon_k} . \tag{4}$$

It may be noted that the scattering rates for a nondegenerate ensemble at any temperature may be recovered from (3) and (4) by neglecting $D^n f_0(\varepsilon_k)$ in comparison to unity.

Thus even the traditional theory, which neglects the phonon energy, yields a distinctly different dependence of the scattering rates on the carrier energy and the lattice temperature if the degeneracy of the sample is taken into account. The scattering rate for a degenerate material is less than that for a nondegenerate material at any temperature. Moreover, both the qualitative and quantitative discrepancies between the scattering characteristics of the two materials are quite sensitive to the change of η $(=\varepsilon_F/k_B T_L)$ and increase with its value. However, for higher energies when ε_k largely exceeds ε_F, $f_0(\varepsilon_k)$ can be neglected in comparison to unity and the degeneracy hardly produces any effect on the scattering rates at any temperature.

Under the low-temperature condition the scattering characteristics turn out to be rather involved compared to what follows from the traditional theory. The scattering rates are now numerically calculated for Si with the following parameter values: $E_1=9.0$ eV, $v_s=9.037\times10^5$

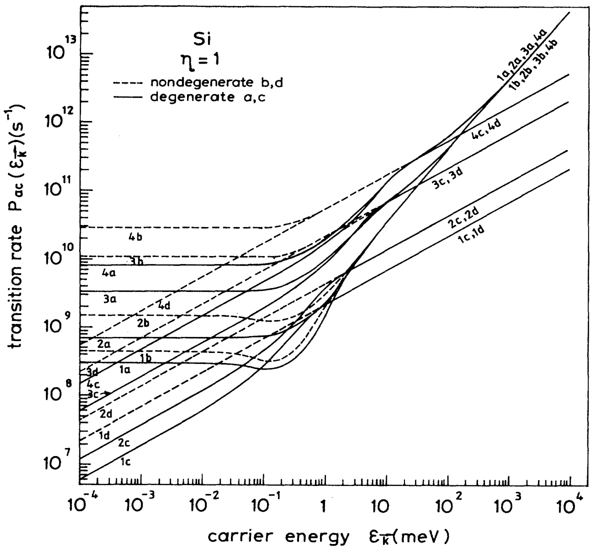

FIG. 1. Dependence of the inelastic acoustic scattering rate upon the carrier energy at different lattice temperatures for a degenerate sample of Si with $\eta=1$. Curves 1, 2, 3, and 4 are obtained for the lattice temperatures of 2, 4, 20, and 50 K, respectively, and the corresponding concentrations are 1.86×10^{15}, 5.26×10^{15}, 5.88×10^{16}, and 2.32×10^{17} cm^{-3}. Curves marked *a* and *b* are obtained for degenerate and nondegenerate semiconducting materials, respectively, taking the finite phonon energy into account. Curves marked *c* and *d* are obtained, respectively, for the same materials neglecting the phonon energy.

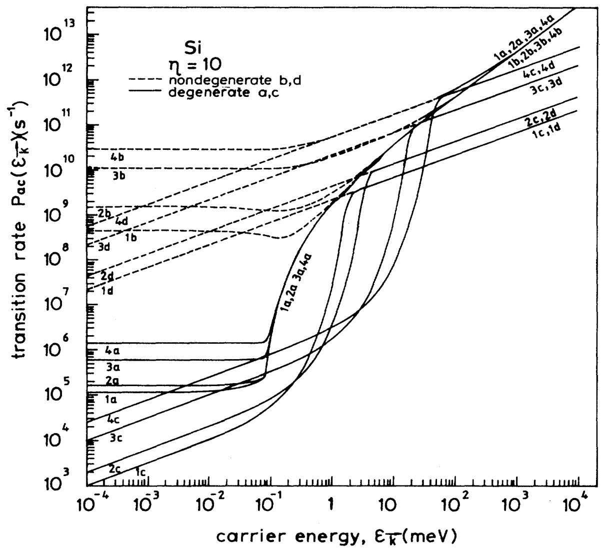

FIG. 2. Dependence of the inelastic acoustic scattering rate upon the carrier energy at different lattice temperatures for a degenerate sample of Si with $\eta=10$. Curves 1, 2, 3, and 4 are obtained for the lattice temperatures of 2, 4, 20, and 50 K, respectively, and the corresponding concentrations are 5.88×10^{16}, 1.66×10^{17}, 1.86×10^{18}, and 7.55×10^{18} cm^{-3}. Curves marked *a* and *b* are obtained for degenerate and nondegenerate semiconducting materials, respectively, taking the finite phonon energy into account. Curves marked *c* and *d* are obtained, respectively, for the same materials neglecting the phonon energy.

cm s^{-1}, $\rho=2.329$ g cm^{-3}, and $m^*=0.32m_0$. It may be further noted that the number of terms n required for convergence of the series occurring in (3) increases with the value of η. Using a logarithmic scale the dependence of the scattering rates on the carrier energy at different lattice temperatures are shown in Figs. 1 and 2 for two different values of η, viz., $\eta=1$ and $\eta=10$, respectively. The corresponding values of N_i given in the figures for these values of η obviously increase monotonically with the lattice temperature and η. Since for Si $8\varepsilon_s/\bar{x}k_B=1.97$, the characteristics could not be obtained for $T_L=1$ K. The scattering rates as obtained for the nondegenerate material and also for elastic collisions with negligible phonon energy are plotted in the same figure for a ready comparison. The figures indicate how significantly the scattering rates change if one considers the degeneracy of the material and the finite energy of phonons. The consideration of finite phonon energy produces altogether different characteristics, be it for the degenerate or the nondegenerate material. For lower energies the scattering rate for the degenerate material is lower than that of the nondegenerate material by an almost energy-independent scale factor. The value of the factor, though, hardly depends upon the phonon energy and is quite sensitive to the change in η. Just one order increase in η increases the scattering rate by several orders. Around an important energy range, $\varepsilon_s \le \varepsilon_k \le \varepsilon_F$, the discrepancy between the scattering characteristics of the two materials assumes a more and more complex form the lower the lattice temperature or higher the value of η. However, for higher energies the scattering rates become almost independent of the lattice temperature and again the effect of degeneracy can hardly be felt. The effect of degeneracy is felt at lower and lower energies the lower the lattice temperature, when again the phonon energy cannot be neglected.

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