Spin-density-wave transition in a two-dimensional spin liquid

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Strongly correlated two-dimensional electrons are believed to form a spin liquid in some regimes of density and temperature. As the density is varied, one expects a transition from this spin-liquid state to a spin-density-wave antiferromagnetic state. In this paper we show that it is self-consistent to assume that this transition is second order and, on this assumption, determine the critical behavior of the $2p_F$ susceptibility, the NMR rates T_1 and T_2 and the uniform susceptibility. We compare our results to data on high-T_c materials.

I. INTRODUCTION

High-temperature superconductors may be created by adding carriers to magnetic insulators. At low dopings the compounds have long-range magnetic order at $T = 0$; at high doping they do not. Therefore, a $T = 0$ magnetic–nonmagnetic transition must occur when the carrier density exceeds a critical value. The idea that some of the anomalous properties of high-T_c superconductors are due to their proximity to this quantum phase transition has attracted substantial recent interest.

The properties of the transition depend on the ordering wave vector and on the nature of the disordered phase. Several different possibilities have been studied in some detail including antiferromagnet-singlet transitions in insulating magnets[1] and ferromagnetic and antiferromagnetic transitions in Fermi liquids.[2,3] Here we consider an important case which has not so far been discussed in the literature, namely, that the disordered phase is a "spin liquid" and the ordering occurs at the wave vector $|\mathbf{Q}| = 2p_F$.

By "spin liquid" we mean a liquid of charge zero spin-1/2 fermion excitations coupled by a singular gauge-field interaction; in the ground state the fermions fill a large Fermi sea which occupies a substantial part of the Brillouin zone.[4] The spin-liquid model has been argued to describe the normal phase of high-temperature superconductors.[5] By "$2p_F$" we mean a wave vector which connects two points on the Fermi surface with parallel tangents (see Fig. 1).[6] For a circular Fermi surface any vector $\mathbf{Q}$ of magnitude $2p_F$ connects two such points. One important motivation for studying the $2p_F$ case is the high-T_c superconducting material $La_{2-x}Sr_xCuO_4$, in which strong magnetic fluctuations have been observed;[7] the fluctuations are peaked at an x-dependent wave vector $\mathbf{Q}(x)$ which is claimed to be a "$2p_F$" wave vector of

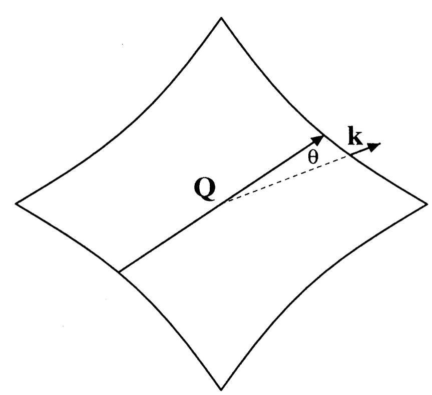


FIG. 1. Sketch of Fermi surface and important wave vectors. The Fermi surface shown here is similar to that claimed to be appropriate to $La_{1.8}Sr_{0.14}CuO_4$. The vector $\mathbf{Q}$ connects two points on the Fermi surface. It is assumed that the tangent to the Fermi surface at one end of the vector $\mathbf{Q}$ is parallel to the tangent to the Fermi surface at the other end. We parametrize the vector $\mathbf{k}$ by the angle θ shown on the figure and the magnitude $k_\parallel$ shown on the figure as the solid line part of $\mathbf{k}$.

the Fermi surface calculated by standard band-structure techniques for this material.[8]

The $2p_F$ spin-density-wave transition in a spin liquid is different from the $2p_F$ transition in a conventional Fermi liquid because the gauge-field interaction leads to divergences in the fermion response functions at wave vectors $|\mathbf{Q}|$ near $2p_F$. In this paper we calculate the exponents and the scaling functions characterizing the $2p_F$ transition in the spin liquid. Our starting point is a Hamiltonian, $H = H_{\mathrm{FL}} + H_{\mathrm{gauge}}$, where H_{FL} describes fermions moving on a lattice and interacting with each other via a short-range four-fermion interaction W:

$$H_{\mathrm{FL}} = \sum_{p\alpha} \epsilon(p) c^\dagger_{p,\alpha} c_{p,\alpha} + W \sum_{p,p',q,\alpha,\beta} c^\dagger_{p,\alpha} c_{p+q,\alpha} c^\dagger_{p',\beta} c_{p'-q,\beta} \tag{1.1}$$

and H_{gauge} describes the gauge field and its coupling to the fermions; it is discussed in detail in the literature[4,9] and below in Sec. II.

It has recently been shown that two possibilities arise, depending on the strength of the fermion-gauge-field interaction.[9] If this interaction is weak, a critical value W_c of the short-range four-fermion interaction separates a disordered phase with nondivergent spin fluctuations at $W < W_c$ from an ordered phase at $W > W_c$. In this paper we evaluate the exponents characterizing the transition at $W = W_c$ in the weak gauge-field coupling case and show how they depend on the strength of the fermion-gauge-field interaction. If, however, the fermion-gauge-field interaction is sufficiently strong, then the $2p_F$ spin susceptibility diverges as $T \to 0$ for arbitrary $W < W_c$, although there is no long-range order at $T = 0$.[9] We have not studied the transition at $W = W_c$ in the strong gauge-field interaction case, but we give the exponents characterizing the $W < W_c$ phase and discuss the physical consequences.

We assume that the transition at $W = W_c$ leads to a spin-density-wave state with long-range order with wave vector $\mathbf{Q}$ and that this transition is second order. If $|\mathbf{Q}| \neq 2p_F$, the gauge field does not modify the fermion susceptibility,[9] so we expect previously developed theories[2,3] to apply. We now outline our approach to the $|\mathbf{Q}| = 2p_F$ case. Because we expect the physics in this region to be determined by the exchange of spin-density fluctuations we use a Hubbard-Stratonovich transformation to recast Eq. (1.1) as a theory of fermions coupled to spin fluctuations $\mathbf{S}_{\mathbf{q}}$:

$$H' = \sum_{p\alpha} \epsilon(p) c^\dagger_{p,\alpha} c_{p,\alpha} + g \sum_{p,k,\alpha,\beta} c^\dagger_{p,\alpha} \vec{\sigma}_{\alpha\beta} c_{p+k,\beta} \vec{S}_{-k} + H_{\mathrm{gauge}} + \sum_k \mathbf{S}_k^2 \ . \tag{1.2}$$

Here g is a fermion-spin-fluctuation coupling constant derived from W; in weak coupling $g^2 = W$. One may study H' as it stands or one may integrate out the fermions completely, obtaining a theory of interacting spin fluctuations which is described by the action

$$\mathcal{A}[S] = \sum_{k,\omega} \vec{S}_{\omega,k} \chi_0^{-1}(k,\omega) \vec{S}_{-\omega,k} + \frac{1}{4} \sum_{\omega_i,k_i} U^{k_1,\dots k_4}_{\omega_1,\dots\omega_4} (\vec{S}_{\omega_1,k_1} \cdot \vec{S}_{\omega_2,k_2}) \times (\vec{S}_{\omega_3,k_3} \cdot \vec{S}_{\omega_4,k_4}) \delta\left(\sum_i \omega_i\right) \delta(\Sigma_i k_i) + \cdots \ . \tag{1.3}$$

Here $\vec{S}_{\omega,k}$ represents a spin fluctuation of Matsubara frequency ω and wave vector $\mathbf{k}$. χ_0 is the susceptibility and U is a four-spin-fluctuation interaction proportional to g^4. We shall derive and interpret this action in more detail below. This action is difficult to treat even for a Fermi liquid without the gauge-field interaction because χ_0 and U diverge as $T, \omega \to 0$ and $\mathbf{k} \to \mathbf{Q}$. The gauge field causes additional singular renormalizations of χ_0, U, g, and the fermion propagator.[9,10]

In this paper we present and justify a self-consistent one-loop approximation method for extracting physical results from the formally divergent theory. We supplement this treatment with an analysis based on Eq. (1.2) of the effect of the spin fluctuations on the fermions. We find that the susceptibility is less singular than the susceptibility obtained for transitions with $Q \neq 2p_F$. This is because the $2p_F$ singularity of the fermions leads to long-range Ruderman-Kittel-Kasuya-Yosida interactions; this weakens the singularities associated with the transition for the same reason that long-range dipolar interactions lower the critical dimension of classical models of ferroelectric transition.[11]

The self-consistent one-loop approximation succeeds in the spin-liquid case because the fermion-gauge-field interaction makes the singular part of the fermion response symmetric in the variable $|\mathbf{k}| - 2p_F$. This approach however fails for the conventional Fermi liquid because in this case the singularities in the fermion polarizability are not symmetric in the variable $|\mathbf{k}| - 2p_F$ as discussed below. We have found that this asymmetry implies that fluctuation effects do not permit a second-order transition at $|\mathbf{Q}| = 2p_F$ in a Fermi liquid. We will present a detailed treatment elsewhere.[12]

The outline of this paper is as follows. Section II reviews the relevant theory of the spin liquid. Section III is devoted to the transition in the spin liquid for weak fermion gauge-field interaction. Section IV discusses the properties of the small-W critical phase occurring when the gauge interaction is strong enough that the $2p_F$ susceptibility diverges as $T \to 0$. Readers uninterested in the details of the derivations may proceed directly to Sec. V which contains a summary of the results and a conclusion.

II. GAUGE THEORY OF SPIN LIQUID

In this section we review the properties of the "spin-liquid" regime of the t-J model, which has been argued to describe the normal state of the high-T_c superconductors.[5] In the spin-liquid regime the elementary excitations are charge e, spin 0 "holons," charge 0, spin-1/2 fermionic "spinons," and a gauge field which mediates a long-range, singular interaction. For temperatures above the Bose condensation temperature of the holons, the holons have a negligible effect on the magnetic properties, which are determined by the properties of the degenerate Fermi gas of spinons (which we call a spin liquid), coupled by the gauge interaction and by a short-range four-fermion interaction W.

The properties of the spin liquid have been studied by many authors.[4,5,9] The fermion propagator has been found to have the form

$$G = \frac{1}{i\omega_0^{1/3}\omega^{2/3} - v(p_\parallel + p_\perp^2/p_0)}, \tag{2.1}$$

instead of the Fermi-liquid form $G = [i\omega - v(p_\parallel + p_\perp^2/p_0)]^{-1}$. Here v_F is the Fermi velocity, p_0 is the radius of curvature of the Fermi line and $p_\parallel (p_\perp)$ are momentum components normal (tangential) to the Fermi line as measured from the points $\pm\mathbf{Q}/2$ and ω_0 is an energy scale of order $p_F v_F$ which is defined more precisely in Ref. 9.

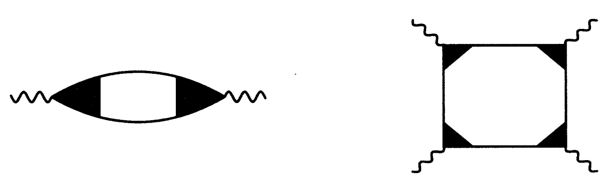
Recently, we have shown[9] that the fermion-gauge-field interaction causes a power-law divergence in the vertex Γ_{2p_F} which is shown as a black triangle in Fig. 2(a). Specifically we found

$$\Gamma_{2p_F} \simeq \left[\frac{1}{\max\left[\left(\frac{\omega}{\omega_0}\right), \left(\frac{v_F k_\parallel}{\omega_0}\right)^{3/2}\right]}\right]^\sigma. \tag{2.2}$$

We were able to calculate σ in the limits $N \to \infty$ and $N \to 0$, where N is the spin degeneracy of the fermions. We found

$$\sigma \sim 1/2N, \quad N \to \infty$$
$$\sigma \sim 16\sqrt{2}/9\pi\sqrt{N}, \quad N \to 0.$$

Extrapolation of these results to the physical case $N = 2$ gives the estimates $1/4 \lesssim \sigma \lesssim 3/4$. The combination of the change in form of the Green function and the divergence of Γ_{2p_F} has profound effects on the polarization



a b

FIG. 2. (a) Diagram yielding the nonanalytic momentum and frequency dependence of the susceptibility χ_0. The solid lines are the fermion propagators. The shaded triangles are vertices which have power-law divergences. (b) Diagram yielding the nonanalytic momentum and frequency dependence of the four-spin fluctuation interaction U.

operator $\Pi(\omega, k)$ [which is shown in Fig. 2(a)] and on the four-spin-fluctuation interaction U [which is shown in Fig. 2(b), cf. Eq. (3.2)].

If $1/6 < \sigma < 1/3$, we found[9]

$$\Pi(\omega, k_\parallel) = \Pi_0 - \sqrt{\frac{\omega_0 p_0}{v_F^3}}\left[c_\omega\left(\frac{|\omega|}{\omega_0}\right)^{\frac{2}{3}-2\sigma} + c_k \mathrm{Re}\left(\frac{k_\parallel v_F}{\omega_0}\right)^{1-3\sigma}\right]. \tag{2.3}$$

Here c_ω and c_k are constants of order unity. For $\sigma < 1/6$, the term involving $k_\parallel$ has the same sign as $k_\parallel$. Thus the maximum of Π and therefore the ordering wave vector must occur at a $|\mathbf{Q}|$ different from $2p_F$ and we expect previous treatments[2,3] of the critical behavior to apply. For $\sigma > 1/6$ the $k_\parallel$ term is always positive and a second-order transition at $|\mathbf{Q}| = 2p_F$ is possible. We restrict our attention to $\sigma > 1/6$ in the rest of the paper. In this case the nonanalyticity of $\mathrm{Re}k_\parallel^{1-3\sigma}$ affects only the magnitude of c_k, that is not important for the scaling arguments which we will present. We henceforth write the $k_\parallel$ term as

$$c_k\left(\frac{k_\parallel v_F}{\omega_0}\right)^{1-3\sigma}.$$

On the other hand, if $\sigma > 1/3$, Π diverges as $\omega, k_\parallel \to 0$. We found[9]

$$\Pi(\omega, k_\parallel) = \sqrt{\frac{\omega_0 p_0}{v_F^3}} \frac{1}{\left[c_\omega\left(\frac{|\omega|}{\omega_0}\right)^{2\sigma-2/3} + c_k\left(\frac{|k_\parallel| v_F}{\omega_0}\right)^{3\sigma-1}\right]}. \tag{2.4}$$

The full susceptibility χ is obtained by combining the irreducible bubble Π with the short-range four-fermion vertex W. We have shown[9] that the gauge-field interaction renormalizes a sufficiently weak initial W_i to zero. Therefore we expect a $T = 0$ transition as W is varied through a critical value W_c. In the weak-coupling case $\sigma < 1/3$, the fact that $\Pi(\omega = 0, k_\parallel = 0)$ is nondivergent implies that the usual random-phase approximation (RPA) formula

$$\chi(\omega, k) = \frac{\Pi(\omega, k)}{1 - W\Pi(\omega, k)} \tag{2.5}$$

is the correct starting point of a theory of the transition. We discuss the transition occurring when $W = W_c = \Pi(0,0)^{-1}$ in more detail below in the next section.

In the strong coupling, $\sigma > 1/3$, case we still expect a transition when W exceeds a critical value. However the divergence of $\Pi(0,0)$ implies that the RPA formula (2.5) is not correct. For $W < W_c$, $\chi(\omega, k) = \Pi(\omega, k)$ with corrections of order the product of W and a positive power of frequency or $k_\parallel$. The $T = 0$ critical point separates a $W < W_c$ phase which has power-law spin correlations from a $W > W_c$ phase which has long-range order.

III. WEAK FERMION-GAUGE-FIELD COUPLING

There are two possible approaches to the spin-density-wave transition at $W = W_c$. One is to integrate out the fermions, obtaining an effective action $\mathcal{A}[S]$, describing the spin fluctuations, $S_{\omega,k}$, and then to analyze this action. If $Q \neq 2p_F$ all terms in the effective action are finite in both the Fermi liquid and in the spin liquid, and the action may be treated by standard renormalization-group (RG) techniques.[3] As $Q \to 2p_F$, however, the coefficients of the quartic and higher-order terms diverge, implying that standard RG techniques cannot be used and that the effect of higher-order terms must be investigated. Another approach is to investigate the effect of the spin fluctuations on the fermion propagator. Here we consider both approaches.

Taking the first approach we proceed in four steps. First, we integrate out the fermions obtaining the action $\mathcal{A}[S]$ given in Eq. (1.3). Second, we truncate the action, retaining only the quadratic and quartic terms. Third, we solve the truncated action in the self-consistent one-loop approximation. This approximation has been extensively used to study three-dimensional magnets[13] and gives the same results as the renormalization group above the upper critical dimension.[3] Fourth, we show using the formalism of the effective action $\mathcal{A}[S]$ that corrections to the self-consistent one-loop approximation lead at most to logarithmic corrections to the self-consistent one-loop results and that logarithms were in fact not present in any diagram we investigated. Fifth, we confirm this result using the formalism of the spin fluctuations interacting with fermions. Finally, we discuss the physical consequences of the theory.

A. Derivation of truncated action

We begin by evaluating the coefficients χ_0 and U in action (1.3). We shall be interested in momenta close to the momentum $\mathbf{Q}$ at which χ_0 is maximal. For wave vectors near $\mathbf{Q}$ the momentum and frequency dependence of χ and U are nonanalytic and controlled by Fermi-surface singularities. It will be convenient to parametrize the momentum $\mathbf{k}$ in terms of a magnitude $k_{\|}$ and an angle θ as shown in Fig. 1. For each angle θ there is a momentum $\mathbf{p}_F(\theta)$ such that $2\mathbf{p}_F(\theta)$ spans the Fermi surface. Note $2\mathbf{p}_F(\theta = 0) = \mathbf{Q}$. We define $k_{\|}$ to be the difference $|\mathbf{k}| - 2p_F(\theta)$. These definitions are generalizations for the noncircular Fermi surface of coordinates which are the convenient choice for a circular Fermi surface.

The fermion contribution $\Pi(\omega, k_{\|})$ to the inverse susceptibility χ_0^{-1} can be calculated by summing all diagrams which are irreducible with respect to the fermion-fermion interaction and have two external $S_{\omega,k}$ legs. This sum has contributions from short length scale processes which give $\Pi(\omega, k_{\|})$ an analytic dependence on $k_{\|}$ and θ and also contributions from Fermi-surface singularities, which lead to a nonanalytic dependence of $\Pi(\omega, k_{\|})$ on $k_{\|}$ and ω. The Fermi-surface singularities come from the diagram shown in Fig. 2(a), which leads to the expression (2.3). Thus we get the susceptibility

$$\chi_0(\omega, k) \simeq \frac{\sqrt{\frac{v_F^3}{\omega_0 p_0}}}{g^4 \left[c_\omega \left(\frac{|\omega|}{\omega_0}\right)^{2/3-2\sigma} + c_k \left(\frac{|k_{\|}| v_F}{\omega_0}\right)^{1-3\sigma} + \theta^2 + \left(\frac{\kappa_0}{p_F}\right)^2\right]} , \tag{3.1}$$

where g is the fermion-spin-fluctuation coupling. The coefficients c_ω and c_k are of the order of unity, they are sensitive to the details of the band structure and the momentum dependence of the interaction. The coefficient κ_0^2 is determined by the difference of the interaction from its critical value. Note that we are using Matsubara frequencies so that χ is purely real.

The most singular contribution, U_{sing}, to U is given by the diagram shown in Fig. 2(b). It diverges if the reduced momenta $(k_{\|}, \theta)$ and frequencies of all four legs are zero. For our subsequent calculations we shall need to estimate the asymptotic behavior when the momenta and frequencies on the external legs satisfy

$$k_{\|} \approx k_{\|a} \approx k_{\|b} \gg k_{\|c} \approx k_{\|d} \; ,$$
$$\omega \approx \omega_a \approx \omega_b \gg \omega_c \approx \omega_d \; ,$$

and $\theta_a \approx \theta_b$; $\theta_c \approx \theta_d$ but with $\theta_a - \theta_c$ arbitrary. Here a, b, c, d are any permutation of the legs shown in Fig. 2(b). By evaluating the diagram in Fig. 2(b) we find that in this limit the vertex is

$$U(k_{\|}, \omega; \theta_1, \theta_2) \simeq \frac{g^4}{\omega_0^2} \sqrt{\frac{\omega_0 p_0}{v_F^3}} \frac{1}{\left(\frac{|\omega|}{\omega_0}\right)^{4\sigma+2/3} + \left(\frac{v_F |k_{\|}|}{\omega_0}\right)^{6\sigma+1} + \left(\frac{v_F p_F^2 (\theta_1 - \theta_2)^2}{p_0 \omega_0}\right)^{6\sigma+1}} . \tag{3.2}$$

Equation (3.2) is not strictly correct at arbitrary momentum and frequency but gives the correct asymptotics of the vertex, which are all we need. The result may be motivated by the following arguments: the scaling $\omega_0^{1/3}\omega^{2/3} \sim v_F k_{\|} \sim v_F (p_F \theta)^2 / p_0$ comes from the structure of the expanded fermion Green function (2.1); we believe that this is the correct generic scaling in this problem. In the case of the circular Fermi surface the

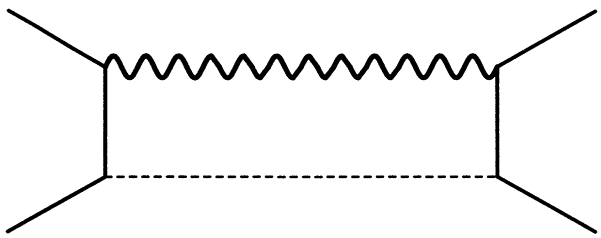


FIG. 3. Diagram yielding renormalization of the spin-fluctuation contribution to the four-fermion vertex W by gauge field. Here the solid lines are fermions, the dashed line represents the gauge field, and the wavy line represents the spin.

dependence on the large angular separation $(\theta_a - \theta_c)$ follows from rotational invariance; a noncircular Fermi surface can still be mapped locally to a circle, proving this result in the general case. In writing these equations we have included the renormalization of Γ_{2p_F} due to the fermion-gauge-field interaction. The renormalization of the fermion vertex g^2 due to the gauge field does not enter our discussion of the critical phenomena. To see this, note that the renormalization is due to the logarithmic divergence of the diagram shown in Fig. 3. In the present case the interaction is sharply peaked, so the logarithm is confined to momenta and frequencies less than the scale set by κ [see Eq. (3.1)], and therefore does not affect our treatment of the critical point. The problem is thus to investigate the self-consistency of the theory defined by Eqs. (1.3), (3.1), and (3.2).

B. Self-consistent one-loop approximation

We treat the theory defined by Eqs. (1.3), (3.1), and (3.2) in the self-consistent one-loop approximation shown diagrammatically in Fig. 4, i.e., we require that the full susceptibility obeys the equation

$$\chi(k,\omega)^{-1} = \chi_0^{-1}(k,\omega) + g^4 \sum_{\eta,q} \chi(\eta,q) U^{k,q}_{\eta,\omega} \,, \tag{3.3}$$

where χ_0 is given by (3.1). Using (3.1) and (3.2) we see that the frequency or momentum derivative of the integral (3.3) is infrared divergent. Estimating this integral we get

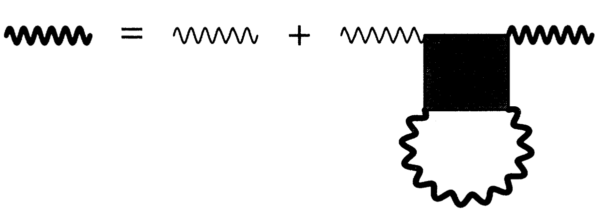


FIG. 4. Diagrams defining the self-consistent one-loop approximation for the susceptibility χ (thick wavy line) in terms of the interaction U (shaded box) and the bare susceptibility χ_0 (thin wavy line).

$$\chi^{-1}(k,\omega) - \chi_0^{-1}(k,\omega) = g^4 \sqrt{\frac{\omega_0 p_0}{v_F^3}} \left[\left(\frac{|\omega|}{\omega_0}\right)^{2/3-2\sigma} + \left(\frac{v_F |k_{||}|}{\omega_0}\right)^{1-3\sigma} \right] , \tag{3.4}$$

which has the same order of magnitude and the same frequency and momentum dependence as the bare χ_0^{-1}. Here as in Eq. (3.2) the formula gives only the correct asymptotics when one of the variables $[\omega_0^{1/3}\omega^{2/3}, v_F k_{||}, v_F (p_F\theta)^2/p_0, v_F \kappa^2/p_0]$ is much larger than the others.

Due to the four-spin interaction U the cutoff, $\kappa_R^2(T)$, is renormalized as compared to κ_0^2. At a finite temperature the sum over η in (3.3) is taken over Matsubara frequencies $\eta_n = 2\pi n T$. The temperature-dependent part of the correction is dominated by the first term in this sum. Estimating it at zero external momenta and frequency we get the leading contribution to

$$\frac{\kappa^2(T)}{p_F^2} = \left(\frac{T}{\omega_0}\right)^{\frac{2}{3}-2\sigma} + \frac{\kappa^2(0)}{p_F^2} \,. \tag{3.5}$$

Note that $\chi^{-1}(\theta) \sim \theta^2$; the singular interaction does not change the power law in the angle dependence. This may be most easily seen via a *reductio ad absurdum*. Suppose the four-spin-wave interaction had led to an exponent less than two for θ. Then the angular dependence of χ_0 in Eq. (3.3) could have been neglected. However, for a circular Fermi surface, Eqs. (3.2) and (3.3) are rotationally invariant (apart from terms due to χ_0), and can therefore lead to no angular dependence at all, in contradiction to the hypothesis of angular dependence with an anomalous exponent. The same argument applies to a nonspherical Fermi surface, because it may be mapped into a spherical one, with errors of order θ^2. We conclude that in general the self-consistent one-loop equations cannot produce a θ dependence different from θ^2.

C. Marginality of higher-order corrections

We now argue that corrections to the self-consistent solution (3.1) do not change the momentum, frequency, and temperature dependences of Eqs. (3.1) and (3.5). There are two kinds of corrections: those arising from higher-orders of perturbation theory in the S^4 coupling using the truncated action, Eq. (2.1), and those arising from higher-order $S^6, S^8, \ldots$ nonlinearities omitted from Eq. (1.3). We first consider the renormalization of the vertex U at the second order in U. The corresponding diagram is shown in Fig. 5. This diagram is infrared

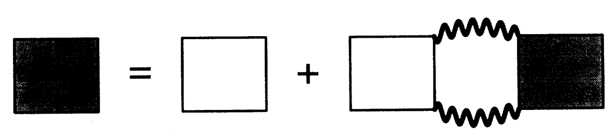


FIG. 5. Diagram giving leading renormalization of full four-spin interaction U (shaded box) in terms of bare U (open box) and spin fluctuations (wavy lines).

divergent and may therefore be estimated by writing the integrals with zero external momenta and frequencies and then cutting off the resulting divergence by the largest of the external momenta or frequency. This gives

$$\begin{aligned}\delta U(\Omega,q,\theta_1,\theta_2) &\sim \frac{g^4 p_F}{\omega_0^4}\int_{\omega>\Omega}\int_{k_\parallel>q}\frac{d\omega\, d\theta\, dk_\parallel}{\left[\left(\frac{\omega}{\omega_0}\right)^{2/3-2\sigma}+\left(\frac{v_F k_\parallel}{\omega_0}\right)^{1-3\sigma}\right]^2}\\ &\quad\times\frac{1}{\left[\left(\frac{|\omega|}{\omega_0}\right)^{4\sigma+2/3}+\left(\frac{v_F k_\parallel}{\omega_0}\right)^{6\sigma+1}+\left(\frac{v_F p_F^2(\theta-\theta_2)^2}{p_0\omega_0}\right)^{6\sigma+1}\right]\left[\left(\frac{|\omega|}{\omega_0}\right)^{4\sigma+2/3}+\left(\frac{v_F k_\parallel}{\omega_0}\right)^{6\sigma+1}+\left(\frac{v_F p_F^2(\theta_1-\theta)^2}{p_0\omega_0}\right)^{6\sigma+1}\right]}\\ &\sim\frac{g^4}{\omega_0^2}\sqrt{\frac{\omega_0 p_0}{v_F}}\frac{1}{\left(\frac{|\omega|}{\omega_0}\right)^{4\sigma+2/3}+\left(\frac{v_F k_\parallel}{\omega_0}\right)^{6\sigma+1}+\left(\frac{v_F p_F^2(\theta_1-\theta_2)^2}{p_0\omega_0}\right)^{6\sigma+1}}\,. \end{aligned}\tag{3.6}$$

This estimate shows that higher-order corrections in the S_q^4 interaction do not change the power-law dependence of the mode coupling interaction, and therefore cannot change the powers coming from the solution of the self-consistent equation. This argument, of course, does not rule out logarithmic corrections to χ^{-1}.

The effect of the higher-order nonlinearities (S_q^6, S_q^8, etc.) may be analyzed similarly. Let us compare the contribution to the susceptibility from one bare S_q^{2n+2} vertex to the contribution from one bare S_q^{2n} vertex. The former contains one more integral over $(\omega, k_{||}, \theta)$ and one more factor of the susceptibility χ. The diagram for the bare vertex S_q^{2n+2} contains also two more fermion Green functions. Performing the extra integration we see that the additional singularity coming from the extra fermion Green functions in the bare vertex and extra factor of susceptibility is precisely cancelled by the phase volume.

These results imply that the theory is marginal, in the sense that higher-order interactions give the same infrared behavior as lower-order interactions. The marginality may also be demonstrated by a scaling argument. If we rescale momenta and frequencies via $\omega' = b\omega$, $k'_\parallel = b^{2/3}k_\parallel$, $\theta' = b^{1/3}\theta$, then we must scale the field $S'_q = b^{-4/3+\sigma}S_q$ to keep the quadratic term in the action invariant. The scaling dimension of the S_q^{2n} vertex is then $b^{(-8/3+2\sigma)n}$ (from the fields) times $b^{-4n/3+2-2n\sigma}$ (from the vertex) times $b^{2(2n-1)}$ (from the integrals). Adding the powers we see that the total scaling dimension of the vertex is zero, so all interactions are marginal in the renormalization-group sense.

Thus, we conclude that higher-order effects of the spin-wave interaction do not change the exponents characterizing the divergence of the susceptibility (3.1) or the interaction between spin fluctuations (3.2).

D. Effect of spin fluctuations on fermions

We now consider the alternative approach of estimating the feedback of the interaction mediated by the spin fluctuations (3.1) on the fermions. The lowest-order contribution to the fermion self-energy Σ is

$$\begin{aligned}\Sigma(\epsilon,p) = g^2\int G(\epsilon',p')\chi(\epsilon-\epsilon',p-p')\\ \times[\Gamma_{(p+p')/2,(\epsilon+\epsilon')/2}(\epsilon-\epsilon',p-p')]^2(dp'd\epsilon')\,.\end{aligned}\tag{3.7}$$

We are interested in the leading frequency dependence of Σ; the main contribution to this comes when p' is on the Fermi line. We then find the angle at which χ is peaked; this turns out to be the point which is located symmetrically opposite to p: $|\theta'-\theta+\pi| \sim (\kappa^2+4\theta^2)^{\frac{1}{2(1-3\sigma)}}$ where θ' and θ are the polar coordinates of the points p' and p. Estimating the contribution of this region we get

$$\Sigma(\epsilon,\theta) = \frac{-ic_\Sigma\epsilon}{\left[\left(\frac{\kappa}{p_F}\right)^2+4\theta^2\right]^{\frac{1}{2(1-3\sigma)}}}\,,\tag{3.8}$$

where $c_\Sigma \sim 1$. Over most of the Fermi line, this self-energy is small compared to the fermion self-energy $\omega_0^{1/3}\epsilon^{2/3}$ due to the gauge-field interaction. The spin-fluctuation contribution becomes important only in a small region $|\theta| \leq \theta^*$ near the points connected by the wave vector Q, with θ^* given by

$$\theta^* = (\epsilon/\omega_0)^{\frac{(1-3\sigma)}{3}}\,.\tag{3.9}$$

At a finite temperature T the typical energy of the fermion is T, and it should replace ϵ in (3.9):

$$\theta^*(T) = (T/\omega_0)^{\frac{(1-3\sigma)}{3}}\,.\tag{3.10}$$

Since $\theta^*(T) \sim \kappa/p_F$ [cf. Eq. (3.5)], in the region $\theta \lesssim \theta^*(T)$ the self-energy (3.8) remains of the order of $\omega_0^{1/3}\epsilon^{2/3}$.

We do not have a reliable expression for the self-energy $\Sigma(\epsilon,\theta)$ in the region $|\theta| < |\theta^*|$ at $\epsilon \gg T$. We can, however, argue that fermions in this region make only a marginal contribution to physical quantities such as polarization operator $\Pi(\omega,k)$. The analytic expression corresponding to Fig. 2(a) for Π is

$$\Pi(\omega, k_\parallel, \theta) = \int dp\, d\epsilon\, \Gamma^2_{2p_F} G(\mathbf{p}+\mathbf{k}, \epsilon+\omega) G(\mathbf{p}, \epsilon) \ . \tag{3.11}$$

Here $\mathbf{k}$ is a vector parametrized as shown in Fig. 1 by variables $k_\parallel, \theta$. As a function of angular variable θ, $\Pi(\omega, \theta)$ has a smooth maximum of the width $\theta'(\omega) \sim (\omega/\omega_0)^{1/3-\sigma} \sim \theta^*(\omega)$. Thus all physical quantities are controlled by spin fluctuations with $\theta \gtrsim \theta'(\omega)$. Using Eq. (2.1) for G and (2.2) for Γ_{2p_F} we see that the integral over momenta is dominated by fermions with momenta near the Fermi surface which have angles very close to θ [the dominant contribution comes from the range of fermion angles θ': $|\theta'-\theta| \sim (\omega/\omega_0)^{1/3} \ll \theta$]. Thus for $k_\parallel$ and θ which are important for the susceptibility, for the self-energy $\Sigma(\epsilon)$ or for the T dependence of κ, the polarization bubble is controlled by the fermions at $\theta \gtrsim \theta^*$. Thus, we believe that the fermion-spin-wave interaction leads only to corrections of the order of unity to physical quantities. Certainly, these arguments are based on power counting and can easily miss logarithmically large contributions; however, we have verified that the leading vertex and self-energy corrections do not contain logarithms.

E. Physical consequences

We now discuss the physical content of the results. We first note that the fermion-gauge-field interaction has two effects on the polarizability near $Q = 2p_F$: it changes the form of the nonanalyticity at $\omega = 0$ and $Q = 2p_F$ (introducing the exponent σ) and it washes out the nonanalyticity associated with the lower boundary of the particle-hole continuum at $\omega = 2v_F k_\parallel$. Therefore the scaling form, Eq. (3.1), gives the correct result for the imaginary part of the susceptibility, χ'', which may be measured in neutron scattering. Using this and Eq. (2.5) gives

$$\chi''(q,\omega) = \frac{\Pi''(q,\omega)}{[1-g^2\Pi(q,\omega)]^2} \ . \tag{3.12}$$

The NMR T_1 relaxation rate is therefore

$$\frac{1}{T_1 T} = A^2 g^4 \int dk_\parallel p_F d\theta \lim_{\omega\to 0} \frac{\mathrm{Im}\Pi(k_\parallel,\omega)}{\omega} [\chi(0,k_\parallel,\theta)]^2 \ . \tag{3.13}$$

Here A is a constant proportional to the hyperfine coupling. We can neglect the weak dependence of Π on θ since the singular dependence on θ comes only via χ. For $T > 0$ and $k_\parallel$ small [$k_\parallel \lesssim p_F(T/J)^{2/3}$] we have

$$\begin{aligned}\lim_{\omega\to 0} \frac{\mathrm{Im}\Pi(k_\parallel = 0,\omega)}{\omega} &= \int G(\epsilon, p+Q/2) G(\epsilon, p-Q/2) \\ &\quad\times [\Gamma^{(R)}_{\epsilon,p}(0,Q)]^2 \frac{(dp\, d\epsilon)}{2T\cosh^2(\epsilon/2T)} \\ &= c_T \sqrt{\frac{p_0}{\omega_0 v_F^3}} \left(\frac{\omega_0}{T}\right)^{\frac{1}{3}+2\sigma} ,\end{aligned} \tag{3.14}$$

while for larger $k_\parallel$ it decreases as

$$\lim_{\omega\to 0} \mathrm{Im}\Pi(k_\parallel,\omega) \sim \mathrm{Im}\Pi(0,\omega) \left(\frac{T^{2/3}\omega_0^{1/3}}{v_F|k_\parallel|}\right)^{(1+6\sigma)/2} . \tag{3.15}$$

The divergence of $\mathrm{Im}\Pi(k_\parallel = 0,\omega)/\omega$ is the usual Kohn anomaly modified by the fermion-gauge-field interaction. This interaction has two effects. First, the increased fermion damping proportional to $\epsilon^{2/3}$ weakens the T dependence from $T^{-1/2}$ to $T^{-1/3}$ but does not change the q dependence. Second, the extra vertex correction then strengthens the T dependence to $T^{-(2\sigma+1/3)}$ and the q dependence to $|q-2p_F|^{-(3\sigma+1/2)}$. If $\sigma > 1/6$ as we assume throughout, the strengthened q dependence leads to a divergence of $1/T_1T \sim T^{(1-6\sigma)/3}$ even far from the critical point.

If the system is tuned to the critical point, then the T dependence of χ becomes important and we obtain

$$\frac{1}{T_1 T} = A^2 \frac{1}{g^4} \sqrt{\frac{v_F p_F}{\omega_0}} \sqrt{\frac{p_F}{p_0}} \left(\frac{\omega_0}{T}\right)^{\frac{2}{3}-\sigma} . \tag{3.16}$$

If $\sigma < 1/3$, the critical contribution is more singular than the background contribution. However as σ is increased to $1/3$, the difference between the critical contribution and the background contribution disappears.

One may calculate the T_2 rate similarly. The electronic contribution to the NMR T_2^{-1} rate is related to the real part χ' of the susceptibility[14]

$$T_2^{-1} \sim \frac{A^2}{a} \sqrt{\sum_k [\chi'(\omega=0,k)]^2} \ ,$$

where a is the lattice constant. If $\sigma < 1/9$, the critical contribution to T_2^{-1} is

$$T_2^{-1} \sim A^2 \frac{v_F}{a g^4} \sqrt{\frac{p_F}{p_0}} \left(\frac{\omega_0}{T}\right)^{1/6-3\sigma/2} .$$

Since we have assumed $\sigma > 1/9$, T_2^{-1} is not divergent as $T \to 0$.

Proximity to the antiferromagnetic transition has also an effect on the uniform susceptibility. We have shown elsewhere[15] that the leading low-T behavior of $\chi(\mathbf{q},0)$ in the limit $\mathbf{q}\to 0$ is given by

$$\chi_U = \lim_{\mathbf{q}\to 0} \chi(\mathbf{q},0) = \sum_{k,\nu} D(k,\nu)\chi(k,\nu) \ , \tag{3.17}$$

where the coefficient D is given by the diagram in Fig. 2(b) with two of the spin-fluctuation vertices replaced by vertices coupling the fermions to the external magnetic field. We denote this coupling to the external magnetic field by g_e. In particular, terms arising from triangular vertices coupling two spin fluctuations to one small-q external field lead only to terms proportional to integer powers of T, which are much smaller than the terms we keep. The calculation of D is very similar to the calculation of U [Eq. (3.2)] except that the coupling D has two

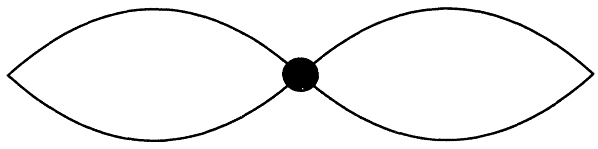


FIG. 6. Diagram yielding leading T dependence of the uniform susceptibility for the case of strong gauge-field-fermion coupling. The solid lines are fermion propagators and the heavy dot is the short-range four-fermion interaction dressed by the gauge field.

small-q vertices and only two large Q vertices Γ_{2p_F}; the degree of singularity therefore differs from that of U and the dependence on $\theta_1 - \theta_2$ is absent. We find

$$D(k_\parallel, \nu) = \frac{p_0^{1/2}}{(v_F\omega_0)^{3/2}} \frac{g^2 g_e^2}{\left(\frac{\nu}{\omega_0}\right)^{2/3+2\sigma} + \left(\frac{k_\parallel v_F}{\omega_0}\right)^{3\sigma+1}} \,. \tag{3.18}$$

Substituting this expression for D and Eq. (3.1) for χ into Eq. (2.13), we find at $g = g_c$ that

$$\chi_U = \text{const} + D_0 (T/\omega_0)^{\frac{2}{3}-\sigma} \,, \tag{3.19}$$

where D_0 is a constant of order $g_e^2 p_F/(g^2 v_F)$; D_0 is positive if $\kappa/p_F > (T/\omega_0)^{1/3-\sigma}$, and D_0 is negative if $\kappa/p_F < (T/\omega_0)^{1/3-\sigma}$. The relation between the sign of D_0 and the magnitude of κ comes because there are two sources of T dependence in Eq. (3.17): one is the T dependence of the cutoff κ; the other is the discreteness of the frequency variable.

If $\kappa(T=0) > 0$ then a crossover occurs. For $T^{\frac{1}{3}-\sigma} > \kappa$, the result (3.19) holds. For lower T, the renormalization of the $2p_F$ interaction by the gauge field shown in Fig. 6 becomes important and the T dependence becomes weaker. We discuss the T dependence in the $g < g_c$ regime in the next section because it is important for $\sigma > 1/3$.

IV. STRONG GAUGE-FIELD INTERACTION $\sigma > 1/3$

If the exponent σ arising from the fermion-gauge-field interaction is greater than the critical value 1/3, then the fermion spin susceptibility diverges at $q = 2p_F$ and $\omega = 0$ for all $g < g_c$, leading to a phase with power-law spin correlations at $T = 0$. At $g = g_c$ a transition will occur to a phase with long-range order. We have not yet formulated a theory of this transition. In this section we outline the physical consequences expected for the $g < g_c$ phase.

For $\sigma > 1/3$ the Kohn singularity at $q = 2p_F$ becomes nonintegrable, so χ' and χ''/ω diverge at $T = 0$ as shown in Eq. (2.4). The power law describing the divergence is θ independent. The NMR relaxation rates are therefore given by summing the appropriate combinations of susceptibilities over $k_\parallel$. We find

$$\frac{1}{T_1 T} \sim A^2 \frac{p_F T_0^{1/2} \omega_0^{1/2}}{v_F^{5/2}} \left(\frac{T}{\omega_0}\right)^{\frac{1}{3}-2\sigma} \,. \tag{4.1}$$

If $\sigma < 1/2$, the rate T_2^{-1} is nondivergent and if $\sigma > 1/2$,

$$\frac{1}{T_2} \sim A^2 \frac{\sqrt{p_F p_0}\,\omega_0}{v_F^2 a} \left(\frac{T}{\omega_0}\right)^{1-2\sigma} \,. \tag{4.2}$$

Here a is the lattice constant. We now consider the uniform susceptibility $\chi(q \to 0, 0)$. It has contributions from diagrams involving the four-fermion vertex W. The leading diagram is shown in Fig. 6. Because, as we have shown in a previous paper,[9] W is renormalized by the gauge field, χ_U acquires an anomalous temperature dependence. The renormalization of W is cut off by the largest of the temperature, the energy of any fermion line, the scaled momentum $(vp_\parallel)^{3/2}$ of any fermion line, or the difference from π of the angle $\theta_1 - \theta_2$ between the momentum of the incoming particles, leading to[9]

$$W \sim W_0 \max\left[\frac{\omega}{\omega_0}, \left(\frac{k_\parallel v_F}{\omega_0}\right)^{3/2}, (\theta_1 - \theta_2 - \pi)^3\right]^\beta \,, \tag{4.3}$$

where the exponent β is N dependent and is not simply related to σ. As $N \to \infty$, $\beta(N) = \frac{4}{3} - \frac{1}{N}$; as $N \to 0$ $\beta(N) \sim c_1\sqrt{N}$ with $c_1 > 0$.

To calculate the temperature dependence of $\chi(q \to 0,0)$ we insert the vertex given in Eq. (4.3) into Fig. 6, and then determine the phase volume in the (ϵ, p), (ϵ', p') integrals in which the T dependence of W is important. We find

$$\chi_U = \text{const} + D_0''(T/\omega_0)^{1+\beta} \,, \tag{4.4}$$

where D_0'' is a constant of order $\left(\frac{p_F g_e}{v_F}\right)^2 W$, whose sign is positive for repulsive W and negative for attractive W.

V. CONCLUSION

We have determined the scaling behavior of the spin susceptibility near an antiferromagnetic critical point at a "$2p_F$" wave vector of a spin liquid. By a "$2p_F$" wave vector we mean one which connects two points on the Fermi surface with parallel tangents. An example is shown in Fig. 1. We distinguish between very weak, weak, and strong fermion-gauge-field coupling. The cases are defined by the value of the exponent σ appearing in Eq. (2.2). This exponent depends only on the fermion spin degeneracy, N; we do not know how to calculate it analytically for the physically relevant case $N = 2$. For estimates of σ, see Ref. 9. In the very weak coupling case, $\sigma < 1/6$, a second-order transition at $|\mathbf{Q}| = 2p_F$ is impossible because the maximum of χ is at $|\mathbf{Q}| \neq 2p_F$. In the weak-coupling case $1/6 < \sigma < 1/3$ the critical point occurs at $T = 0$ when a short-range interaction W equals a critical value W_c. In the strong-coupling case $\sigma > 1/3$ there is a critical phase with power-law correlations at

$T = 0$ for $0 \leq W < W_c$ and an additional transition, which we did not study, at $W = W_c$.

In weak- and strong-coupling cases the $2p_F$ singularities of the fermions lead to physically important effects. First, the scaling is anisotropic. The dependence of the susceptibility on wave vectors $k_\parallel$ which are parallel to the ordering wave vector $\mathbf{Q}$ involves a different exponent than does the dependence on wave vectors $k_\perp$ perpendicular to $\mathbf{Q}$. The $k_\perp$ exponent takes the conventional Ornstein-Zernike value 2, while the $k_\parallel$ exponent is always between 0 and 1 [cf. Eq. (3.1)]. The small value of the $k_\parallel$ exponent comes from a long-range (in position space) interaction due to the $2p_F$ singularity of the fermions. It drastically weakens the singularities associated with the transition. For example, in the weakly coupled spin-liquid case the NMR T_2 relaxation rate, which is given by $T_2^{-2} \sim \sum_q (\chi'_q)^2$, does not diverge as $T \to 0$, in contrast to transitions in two-dimensional insulating magnets, where $T_2^{-1} \sim 1/T$. The T_1^{-1} rate is weakly divergent, see Eq. (3.16). The uniform susceptibility is nonvanishing as $T \to 0$, but the leading temperature dependence is a power of T between 0 and 1; see Eq. (3.19).

In the strongly coupled spin liquid the scaling is again anisotropic. There is no singular dependence of $\chi(\mathbf{q}, 0)$ on the direction of $\mathbf{q}$. The only singular dependence is on the difference between $\mathbf{q}$ and "$2\mathbf{p}_F(\theta)$", the vector spanning the Fermi surface and parallel to q. For $\sigma > 1/3$ the NMR relaxation rate $(T_1T)^{-1}$ is always divergent as $T \to 0$ [cf. Eq. (4.1)]; T_2^{-1} diverges if $\sigma > 1/2$ [cf. Eq. (4.2)]. Both quantities diverge more strongly as σ is increased. We also found that the leading temperature dependence of χ is $\chi \sim \text{const} + T^{1+\beta}$, where β is an independent exponent which tends to 0 as σ becomes large and to $4/3 - 2\sigma$ as $\sigma \to 0$ [cf. Eq. (3.19)]. Our results for T_1, T_2 and the uniform susceptibility are summarized in Table I.

One reason for studying spin fluctuations in two-dimensional systems is that the high-T_c superconductors have been shown to have strong antiferromagnetic spin fluctuations. The two best studied high-T_c materials are $La_{2-x}Sr_xCuO_4$ and $YBa_2Cu_3O_{7-\delta}$. In $La_{2-x}Sr_xCuO_4$, neutron scattering has observed peaks centered at incommensurate x-dependent wave vectors.[7] At $x = 0.14$ the incommensurability was shown to correspond to a "$2p_F$" vector of the LDA band structure,[8] suggesting that the results of the present paper should be relevant. NMR experiments have shown that the copper T_1 rate ${}^{Cu}T_1^{-1}$ has the temperature dependence

$$ {}^{Cu}(T_1T)^{-1} \sim 1/T \qquad (5.1)$$

for $100 < T < 500$ K.[16] The T_2 rate has not been measured in this material, but in other high-T_c materials with

TABLE I. Relaxation rates and susceptibility in different cases.

	Weakly coupled Spin liquid ($1/6<\sigma<1/3$)	Strongly coupled Spin liquid ($\sigma>1/3$)
$\frac{1}{T_1T}$	$\frac{1}{T^{2/3-\sigma}}$	$\frac{1}{T^{2\sigma-1/3}}$
$\frac{1}{T_2}$	Nondivergent	$\frac{1}{T^{2\sigma-1}}$
$\chi(q\to 0)$	$C+T^{2/3-\sigma}$	$C+T^{1+\beta}$

$({}^{Cu}T_1T)^{-1} \sim 1/T$, $T_2 \sim T^{-x}$ with $\frac{1}{2} \lesssim x \lesssim 1$.[17] The uniform susceptibility is given by $\chi \sim \text{const} + AT$ at least for $150 < T < 400$ K.[18] None of these properties are consistent with the weakly coupled spin-liquid results of Sec. II. The strongly coupled spin-liquid results with $\sigma \approx 2/3$ are in rough agreement with the data.

In the $YBa_2Cu_3O_{7-\delta}$ materials, ${}^{Cu}(T_1T)^{-1} \sim 1/T$ and $T_2^{-1} \sim T^{-x}$ for T greater than a δ dependent "spin-gap" temperature and above this temperature $\chi \sim \text{const}+A'T$ with A' δ dependent. The neutron scattering indicates broad and flat-topped peaks centered at the commensurate wave vector (π, π). It is possible that the observed structure is due to several overlapping and unresolved singularities at a $2p_F$ wave vectors. However, it has also been argued[19] that in $YBa_2Cu_3O_{7-\delta}$ the magnetism is not driven by a $2p_F$ instability, and is peaked at a commensurate wave vector because of a strongly peaked interaction. If the latter point of view is correct, the theory developed here is irrelevant. If the former point of view is correct, then we are forced to conclude that the relevant fixed point is the strongly coupled spin liquid with $\sigma \approx 2/3$.

One other aspect of the strongly coupled spin liquid requires further comment. We noted already that in this case χ is singular at any Q spanning the Fermi surface. In a translationally invariant system this would predict peaks in the neutron-scattering cross section on a ring of radius $2p_F$ centered at the origin. As noted by Littlewood *et al.*,[8] for fermions on a lattice one obtains instead one or more curves traced out by the vectors Q connecting points with parallel tangents, and also one obtains additional families of curves displaced by reciprocal-lattice vectors, $\mathbf{G}$. One gets further peaking when members of different families of curves intersect. The resulting structure is very sensitive to the details of the Fermi surface and may resemble the data in $YBa_2Cu_3O_{7-\delta}$ as well as the data on $La_{2-x}Sr_xCuO_4$.

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