Coherence effects in a normal-metal–insulator–superconductor junction

Fei Zhou and B. Spivak
Physics Department, University of Washington, Seattle, Washington 98195

A. Zyuzin
Physics Department, University of Cincinnati, Cincinnati, Ohio 45221-0011

(Received 2 November 1994)

Coherence effects arising due to Andreev reflections in a superconductor–insulator–normal-metal junction are considered. It is shown that in the absence of electron-electron interaction in the metal at low temperatures, the electrical potential drop on the insulator I_0R_I, caused by the current density I_0 through the junction, vanishes, although the resistance of the device R_∞ measured by the two-probe method can be large. The electron-electron interaction determines the zero-temperature value of R_I. The implications of these effects for the theory of the superconductor–normal-metal–superconductor junction are discussed.

Recent experiments on superconductor–insulator–normal-metal (SIN) junctions with high transmission coefficient of the insulator[1] revived interest in this area. The resistance of a SIN junction at high temperature is determined by one-particle tunneling through the insulator.[2,3] In this paper, we consider the resistance of the SIN junction at low temperatures $T \ll |\Delta|$, when quasiparticles with energies $\epsilon \ll |\Delta|$ cannot tunnel from the metal into the superconductor. Here $|\Delta|$ is the superconducting gap. In this case, the resistance of the system is determined by the tunneling of electron pairs, which is known as the Andreev reflection.[4] This reflection gives rise to a coherence between electrons and holes inside the metal. As a result, at low T, in the absence of electron-electron interaction in the metal, the resistance of the SIN junction R_∞ turns out to be proportional to t^{-1}[5,6] rather than t^{-2}, which naively might be expected. Here $t=t_0v_F$, t_0, and v_F are the insulator transmission coefficient, the dimensionless transmission coefficient, and Fermi velocity, respectively.

At small temperatures T, when $l \ll \xi$, L_T, the coherence between electrons and holes, arising in the metal due to the Andreev reflection, extends over distances of the order of $L_T \gg l$. Here $L_T=\sqrt{D/T}$ and ξ are the coherence lengths of the normal metal and the superconductor, respectively, $D=\frac{1}{3}v_Fl$ is the electron diffusion coefficient, and l is the elastic mean free path. In this case, electron wave packets which carry current in the metal are coherent superpositions of electron and hole wave functions. The minimum size of the packets is of the order of $L_T \gg l$. The effects considered below are related to the fact that the electrochemical potential cannot change significantly over distances smaller than the typical size of the packets L_T. It is interesting that at small T, the effective charge of the packets turns out to be much smaller than the electron charge [see the discussion after Eq. (19)]. This is very different from the situation in the bulk of the metal, where the minimum size of the packets is less than l and their effective charge equals the electron charge.

The question arises: How does the nonequilibrium electric charge distribution induced by the current through the junction reflect the fact that the minimum size of the electron packets is large and their effective charge is small? We show below that the electrical potential drop across the junction is inversely proportional to t but takes place in the metal far from the SN boundary. This is different from the case of the normal-metal–insulator–normal-metal junction (NIN), where the electrical potential drop takes place right on the insulator. As a consequence, the resistance of the NIS junction crucially depends on the way the resistance is measured. At low temperature, in the absence of the electron-electron interaction in the metal, the quantity $R_I=[\Phi(+0)]/I_0$ goes to zero although the resistance of the system measured by the two-probe method $R_\infty \sim t^{-1}$ can be large. Here I_0 is the current through the junction and $\Phi(+0)$ is the electrical potential drop across the junction.

It turns out that in this situation the electron-electron repulsive interaction in the metal determines the low-temperature value of R_I.

I. KINEMATIC SCHEME FOR DISORDERED SUPERCONDUCTORS

The conventional Boltzmann equation[7] cannot be used for the description of the effects mentioned above because they arise at $l \ll L_T$; ξ. The set of equations describing kinetic phenomena in superconductors in this case were derived in.[8–11] We will use the Keldysh diagram technique[12] for the matrix form of Green's functions

$$\check{G}=\begin{bmatrix}\hat{G}_A(\epsilon,\mathbf{r},\mathbf{r}') & \hat{G}_K(\epsilon,\mathbf{r},\mathbf{r}') \\ 0 & \hat{G}_R(\epsilon,\mathbf{r},\mathbf{r}')\end{bmatrix} \tag{1}$$

Here $\hat{G}_A;\hat{G}_R;\hat{G}_K$ are advanced, retarded, and Keldysh Green's function matrices in the Numbu space,[9,12,13] the index i stands for A,R, or K, G and F stand for normal and anomalous Green's functions,

$$\hat{G}_i=\begin{bmatrix} G_i(\epsilon,\mathbf{r},\mathbf{r}') & F_i(\epsilon,\mathbf{r},\mathbf{r}') \\ F_i^+(\epsilon,\mathbf{r},\mathbf{r}') & \tilde{G}_i(\epsilon,\mathbf{r},\mathbf{r}') \end{bmatrix}, \quad (2)$$

$$G^R=-i\theta(t_1-t_2)\langle[\Psi(1),\Psi^+(2)]_+\rangle ,$$

$$G^A=i\theta(t_2-t_1)\langle[\Psi(1),\Psi^+(2)]_+\rangle ,$$

$$F_1^R=-i\theta(t_1-t_2)\langle[\Psi(1),\Psi(2)]_+\rangle ,$$

$$G^K=-i\langle[\Psi(1),\Psi^+(2)]_-\rangle ,$$

$$F^K=i\langle[\Psi^+(1),\Psi^+(2)]_-\rangle ,$$

and $\Psi(t,\mathbf{r}')$ is the electron field operator. Here the brackets $\langle\ \rangle$ stand for both quantum mechanical averaging and for averaging over the random potential realizations, and the brackets $[\ ]_{+,-}$ stand for commutators and anticommutators, respectively.

The equation for $\check{G}$ averaged over realizations of the elastic scattering potential is given by the diagrams shown in Fig. 1, where thin solid lines correspond to unperturbed electron Green's functions, thick solid lines correspond to averaged over realizations of random potential Green's functions in the presence of electron-electron interaction, the vertex corresponds to the electron-electron interaction constant α, dashed lines correspond to the correlation function of the elastic scattering potential $\langle u(\mathbf{r})u(\mathbf{r}')\rangle=(1/\pi\tau\nu_0)\delta(\mathbf{r}-\mathbf{r}')$, ν_0 is the density of states in the normal metal on the Fermi surface, and τ is the electron elastic mean free time. We use the standard technique of averaging over the random scattering potential.[13]

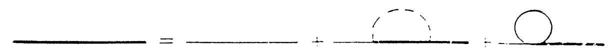

FIG. 1. The diagram equation for Green's functions.

In the diffusion approximation, when all the quantities change on the spatial scale much larger than l, it is useful to introduce Green's functions $\check{g}$ integrated over ξ_p and averaged over $\mathbf{n}_p$, where $\mathbf{n}_p$ is the unit vector parallel to $\mathbf{p}$,

$$\check{g}=i/v\nu_0\int d\xi_p d\mathbf{r}\,d\mathbf{n}_p\check{G}(\epsilon;\mathbf{R}+\mathbf{r}/2;\mathbf{R}-\mathbf{r}/2)\times\exp(-i\mathbf{pr}) . \quad (3)$$

Here v is the volume of the sample, μ is the chemical potential in the metal, and $\xi_p=\mathbf{p}^2/2m-\mu$. We use the symbol $\hat{g}_i$ to represent the elements of $\check{g}$.

In this approximation one can find a solution of the equations in Fig. 1 in the form[9]

$$\hat{g}_K(\epsilon,\mathbf{R})=\hat{g}_R(\epsilon,\mathbf{R})\tilde{f}(\epsilon,\mathbf{R})-\tilde{f}(\epsilon,\mathbf{R})\hat{g}_A(\epsilon,\mathbf{R}) . \quad (4)$$

Here $\tilde{f}(\epsilon,\mathbf{R})=f(\epsilon,\mathbf{R})I+f_1(\epsilon,\mathbf{R})\tau_z$ is the generalized quasiparticle distribution function of the superconductor, τ_z is the Pauli matrix, and I is the unit matrix.

And, finally, the complete set of equations for the stationary situation consists of the Usadel equations

$$D/2[g^R(\epsilon,\mathbf{R})(\partial_\mathbf{R}-2ie\,\mathbf{A})^2F_1^R(\epsilon,\mathbf{R})-F_1^R(\epsilon,\mathbf{R})\partial_\mathbf{R}^2g^R(\epsilon,\mathbf{R})]=\left[-i\epsilon+\frac{1}{\tau_{\text{in}}}\right]F_1^R+i\Delta g^R(\epsilon,\mathbf{R}) ,$$
$$D/2[g^R(\epsilon,\mathbf{R})(\partial_\mathbf{R}+2ie\,\mathbf{A})^2F_2^R(\epsilon,\mathbf{R})-F_2^R(\epsilon,\mathbf{R})\partial_\mathbf{R}^2g^R(\epsilon,\mathbf{R})]=\left[-i\epsilon+\frac{1}{\tau_{\text{in}}}\right]F_2^R+i\Delta^*g^R(\epsilon,\mathbf{R}) , \quad (5)$$

and the equations for the two distribution functions f and f_1,

$$D/4\partial_\mathbf{R}[\Pi_1(\epsilon,\mathbf{R})\partial_\mathbf{R}f(\epsilon,\mathbf{R})+\mathbf{J}(\epsilon,\mathbf{R})f_1(\epsilon,\mathbf{R})]=I_1^{\text{PH}}(f) ,$$
$$D/4\partial_\mathbf{R}[\Pi_2(\epsilon,\mathbf{R})\partial_\mathbf{R}f_1(\epsilon,\mathbf{R})+\mathbf{J}(\epsilon,\mathbf{R})f(\epsilon,\mathbf{R})]=I_2^{\text{PH}}(f_1) . \quad (6)$$

Here

$$\Pi_1(\epsilon,\mathbf{R})=2+2|g^R(\epsilon,\mathbf{R})|^2-|F_1^R(\epsilon,\mathbf{R})|^2-|F_2^R(\epsilon,\mathbf{R})|^2 ,$$
$$\Pi_2(\epsilon,\mathbf{R})=2+|g^R(\epsilon,\mathbf{R})|^2+|F_1^R(\epsilon,\mathbf{R})|^2+|F_2^R(\epsilon,\mathbf{R})|^2 , \quad (7)$$
$$\mathbf{J}(\epsilon,\mathbf{R})=-2\,\text{Im}[F_1^R(\partial_\mathbf{R}+2ie\,\mathbf{A})F_2^R-F_2^R(\partial_\mathbf{R}-2ie\,\mathbf{A})F_1^R] .$$

g^R and F^R are defined as elements of the matrix

$$g^{\hat{R}}=\begin{bmatrix} g^R(\epsilon,R) & F_1^R(\epsilon,\mathbf{R}) \\ -F_2^R(\epsilon,\mathbf{R}) & -g^R(\epsilon,\mathbf{R}) \end{bmatrix} . \quad (8)$$

τ_{in} is the inelastic electron mean free time, $\mathbf{A}$ is the vector potential of the magnetic field, and I_1^{PH} and $I_2^{\text{PH}}(f_1)\sim f_1/\tau_{\text{in}}$ are electron-phonon collision integrals.

The self-consistency equation for the order parameter Δ, the electroneutrality condition, and the expression for current $j=j_n+j_s$ have the forms[9]

$$\Delta=\alpha\int_{-\infty}^{+\infty}d\epsilon g_{(1,2)}^{\text{K}}(\epsilon,\mathbf{R})=\int d\epsilon(F_1^R-F_2^{R*})f-(F_1^R+F_2^{R*})f_1 , \quad (9)$$

$$\Phi=\int_{-\infty}^{+\infty}d\epsilon\,\text{Tr}\hat{g}^K(\mathbf{R},\epsilon)=\int_{-\infty}^{+\infty}d\epsilon f_1(\mathbf{R},\epsilon)\nu(\mathbf{R},\epsilon) , \quad (10)$$

$$\mathbf{j}=D\nu_0\int_{-\infty}^{+\infty}d\epsilon\text{Tr}\{\tau_Z(\hat{g}^R\partial_\mathbf{R}\hat{g}^K+\hat{g}^K\partial_\mathbf{R}\hat{g}^A)\} , \quad (11)$$

$$\mathbf{j}_n=D\nu_0\int_{-\infty}^{+\infty}d\epsilon\Pi_2(\epsilon,\mathbf{R})\partial_\mathbf{R}f_1(\epsilon,\mathbf{R}) ,$$
$$\mathbf{j}_s=D\nu_0\int_{-\infty}^{+\infty}d\epsilon\mathbf{J}(\epsilon,\mathbf{R})f(\mathbf{R},\epsilon) . \quad (12)$$

Here j_n and j_s are normal and superconducting components of the current, $\nu(\mathbf{R},\epsilon)=\nu_0\text{Re}g^R(\mathbf{R},\epsilon)$ is the one-particle density of states, $\Phi(\mathbf{R})=e\phi(\mathbf{R})+\partial_t\chi(\mathbf{R},t)$ is the gauge invariant potential, $\phi(x)$ is electrical potential, and χ is the phase of the order parameter, and the anomalous Green's function.

The boundary condition for Eqs. (5) and (6) are of the form[14]

$$D\check{g}_k \mathbf{n}\partial_{\mathbf{R}}\check{g}_k(\epsilon,\mathbf{R})=t[\check{g}_1(\epsilon,\mathbf{R})\check{g}_2(\epsilon,\mathbf{R})-\check{g}_2(\epsilon,\mathbf{R})\check{g}_1(\epsilon,\mathbf{R})] \ . \tag{13}$$

Here $\mathbf{n}$ is a unit vector normal to the NS surface and $k=1,2$ stands for the superconductor and normal-metal sides of the junction, respectively.

In equilibrium, $f_1=0$ and $f=\tan(-\epsilon/2T)$. In the pure case $l \gg L_T$, ξ one can use the conventional Boltzmann equation for quasiparticle distribution function $n(\mathbf{p})$. In this case, the relation between $n(\mathbf{p})$, the two functions f and f_1, and Φ are $f_1=\xi_{\mathbf{p}}/\epsilon_{\mathbf{p}}[n(\xi_{\mathbf{p}})-n(-\xi_{\mathbf{p}})]$, $f=1-n(-\xi_{\mathbf{p}})-n(\xi_{\mathbf{p}})$; $\Phi=\sum_{\mathbf{p}}(\xi_{\mathbf{p}}/\epsilon_{\mathbf{p}})n(\mathbf{p})$, where $\epsilon_{\mathbf{p}}=\sqrt{\xi_{\mathbf{p}}^2+\Delta^2}$. The last equation means that at $\Delta\neq 0$ the effective charge, which connects Φ and $n(\mathbf{p})$, vanishes at $\xi_{\mathbf{p}}=0$. It implies that at $\xi=0$, quasiparticle wave functions are superpositions of electron and hole wave functions with equal weights. We will see below that due to the Andreev reflection the same situation arises in normal metal near the NS boundary.

II. RESISTANCE OF SIN JUNCTION

A. The case of the noninteracting electrons in normal metal

In the linear approximation in the external electric field, $j_n(\mathbf{R})$ and $\Phi(\mathbf{R})$ in metal are determined only by the distribution function f_1, which is related to the imbalance in the population of the electron and the hole branches of the spectrum.[15]

Let us consider the case when the electron-electron interaction constant in the metal $\alpha_N=0$. In this case, supercurrent j_s and superfluid velocity $\mathbf{v}_s=(2e/c)\mathbf{A}+\partial_{\mathbf{R}}\chi$ inside the metal are zero. We also neglect the phonon collision integrals in Eq. (6) using parameters L_t/L_T and L_T/L_t at low and high temperatures, respectively.

Taking into account the normalization condition $(g^R(\epsilon,\mathbf{R}))^2=F_2^R(\epsilon,\mathbf{R})F_1^R(\epsilon,\mathbf{R})+1$, one can express g and F in the form

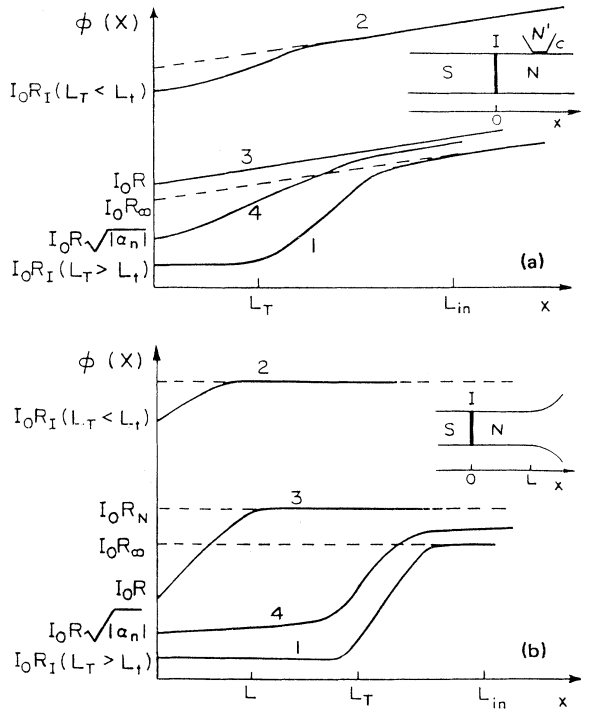

FIG. 2. (a) and (b) Spatial distributions of $\Phi(x)$. Curve 1: The case of NIS junctions at $L_T \gg L_t$. Curve 2: The case of NIS junctions at $L_T \ll L_t$. Curve 3: The case of NIN junctions. Curve 4: The case of NIS junctions at low T and in the presence of electron-electron interaction.

$$\cos\theta(\epsilon,\mathbf{R})=g^R(\epsilon,\mathbf{R}) \ ,$$
$$\sin\theta(\epsilon,\mathbf{R})=iF_1^R(\epsilon,\mathbf{R}) \ , \tag{14}$$

where $\theta(\epsilon,\mathbf{R})=\theta_1(\epsilon,\mathbf{R})+i\theta_2(\epsilon,\mathbf{R})$ is a complex number. In this case, the Usadel equations and the equation for f_1 take the simple form

$$\frac{D}{2}\partial_{\mathbf{R}}^2\theta(\epsilon,\mathbf{R})+\left[i\epsilon-\frac{1}{\tau_{\text{in}}}\right]\sin\theta(\epsilon,\mathbf{R})=0 \ , \tag{15}$$

$$D\partial_{\mathbf{R}}\{\cosh^2\theta_2(\epsilon,\mathbf{R})\partial_{\mathbf{R}}f_1(\epsilon,\mathbf{R})\}=0 \ , \tag{16}$$

with boundary conditions

$$D\cosh^2\theta_2(\epsilon,0^+)\mathbf{n}\partial_{\mathbf{R}}f_1(\epsilon,0^+)=t\{f_1(\epsilon,0^+)-f_1(\epsilon,0^-)\}$$
$$\times\cosh\theta_2(\epsilon,0^+)\cosh\theta_2(\epsilon,0^-)\cos[\theta_1(\epsilon,0^+)-\theta_1(\epsilon,0^-)]D\mathbf{n}\partial_{\mathbf{R}}\theta(\epsilon,x)$$
$$=t\{\theta(\epsilon,0^+)-\theta(\epsilon,0^-)\} \tag{17}$$

and the expressions for j_n and Φ are of the form

$$j_n=D\nu_0\int_{-\infty}^{+\infty}\cosh^2\theta_2(\epsilon,\mathbf{R})\partial_{\mathbf{R}}f_1(\epsilon,x)d\epsilon \ ,$$
$$\Phi(\mathbf{R})=-\int_{-\infty}^{+\infty}\text{Re}[\cos\theta_1(\epsilon,\mathbf{R})\cosh\theta_2(\epsilon,\mathbf{R})]f_1(\epsilon,\mathbf{R})d\epsilon \ . \tag{18}$$

Here we use $0^+,0^-$ to represent the normal-metal side and superconductor side of the interface.

Let us consider one-dimensional geometry of the junction shown in Fig. 2(a). At low temperatures $T\ll|\Delta|$ the solution of Eq. (15) in the metal $(L_{\text{in}}>L_T>x>0)$ with the boundary conditions of Eq. (17) is

$$\theta(\epsilon,x)=\begin{cases}\frac{\pi}{2}-\sqrt{2}(1-i)\frac{x+L_t}{L_\epsilon} & \text{if } x+L_t \ll L_\epsilon\\ \frac{\pi L_\epsilon}{4L_t}(1+i)\exp\left[-(1-i)\frac{x}{L_\epsilon}\right] & \text{if } L_\epsilon \ll L_t\\ 4\left[\frac{\sqrt{2}-1}{\sqrt{2}+1}\right]^{1/2}\exp\left[-(1-i)\frac{x}{L_\epsilon}\right] & \text{if } L_\epsilon \ll x\ .\end{cases} \tag{19}$$

Here, $L_t=D/t$ is assumed to be much smaller than $L_{\rm in}=\sqrt{D\tau_{\rm in}}$, $L_\epsilon=\sqrt{D/\epsilon}$. At $x \gg L_{\rm in}$, Eq. (19) acquires the additional factor $\exp(-x/L_{\rm in})$. The main feature of Eq. (19) is that in the normal metal, at small ϵ and small distances from the boundary, $\theta(\epsilon,x)$ and, consequently, $g^R(\epsilon,x)$ and $F_1^R(\epsilon,x)$, have the same values as in the bulk of the superconductor. In particular, it means that at small energy, $\nu(\epsilon,x)$ is close to zero, provided $x<L_\epsilon$. As has been mentioned above, it follows from Eq. (10) that the effective charge which connects Φ and $f_1(\epsilon)$ is zero at $\epsilon=0$. The value of $\theta(\epsilon,x)$ approaches zero at large ϵ and x, which means that the values of $g^R(\epsilon,x)$ and $F_1^R(\epsilon,x)$ are close to their normal-metal value.

Substituting the solution of this equation into Eq. (6) and using the boundary conditions of Eq. (17), we get the expression for the spatial distribution of $\Phi(x)$ [we assume that inside superconductor $\Phi(x<0)=0$],

$$\Phi(x>0)=\frac{I_0}{\sigma_D}\int_{-\infty}^{+\infty}\cos\theta_1(\epsilon,x)\cosh\theta_2(\epsilon,x)\partial_\epsilon\tan(\epsilon/2kT)\times\left\{\frac{L_t}{\cosh\theta_2(\epsilon,x=0^+)\cosh\theta_2(\epsilon,x=0)-\cos[\theta_1(\epsilon,x=0^+)-\theta_1(\epsilon,x=0^-)]}+\int_0^x\frac{1}{\cosh^2\theta_2(\epsilon,x')}dx'\right\}. \tag{20}$$

Here I_0 is a current density through the junction and σ_D is the Drude conductivity.

Using Eqs. (19) and (20), we get the expression for the resistance of the insulator per unit area of the junction $R_I=\Phi(x=+0)/I_0$,

$$R_I=\gamma R\frac{L_t}{L_T}\ . \tag{21}$$

Here $R=(t\nu_0)^{-1}$ is the resistance of the insulator per unit area of the junction in the case of the metal-insulator-metal (NIN) junction; $\zeta(x)$ is the ζ function, and γ is a coefficient of the order of unity, which is equal to $4(1-\sqrt{2})\pi^{-1/2}\zeta(\frac{1}{2})$ at $L_T \gg L_t$ and to $2^{7/2}(1-\sqrt{2})\pi^{-3/2}\zeta(\frac{1}{2})$ at $L_T \ll L_t$.

Depending on the ratio between L_t and L_T, we have different regimes. Let us start with the case of low temperatures, when $L_t \ll L_T \ll L_{\rm in}$. In this case, following Eq. (21), $R_I \ll R$ and R_I approaches zero at small T.

In this case, following Eqs. (19) and (20), the spatial distribution of $\Phi(x)$ has the form

$$\frac{\Phi(x>0)}{I_0}=\begin{cases}R_I+\frac{a}{\sigma_D}\frac{(2L_t+x)x}{L_T} & \text{if } x \ll L_T\\ R_\infty+\frac{x}{\sigma_D}\left[1-\beta\frac{L_T^2}{x^2}\right] & \text{if } L_{\rm in} \gg x \gg L_T\ .\end{cases} \tag{22}$$

Here $a=4(1-\sqrt{2})\pi^{-1/2}\zeta(\frac{1}{2})$, β is a constant of order of unity, and R_∞ is the value of the insulator resistance which can be extracted from the asymptotic behavior of $\Phi(x)$ at large x. The curves 1 and 3 in Fig. 2(a) show the qualitative picture of the spatial distribution $\Phi(x)$ in the SIN junction at $L_T \gg L_t$ and the NIN junction, respectively. Using Eq. (18) and the fact that the effective diffusion coefficient in Eq. (16) for f_1 differs from D by the factor $\cosh^2\theta_2$, which is of the order of unity at $0<x<L_T$, we get $R-R_\infty \sim L_T/\sigma_D$. In the case of the SN junction, such an estimate has been made in Ref. 16. The above-considered distribution $\Phi(x)$ can be measured using four-probe measurements. We assume that the probes have a short phase memory time and electron coherence effects in the probes can be neglected.

It is interesting that the electric field is expelled out of the region of the size L_T near the NIS boundary. This is in qualitative agreement with Ref. 17, where it was shown that the local conductivity near the NS boundary diverges at small T. On large distances from the boundary $x \gg L_T$, the electrical field $E(x)=\partial_x\Phi(x)$ approaches the bulk value $E_0=I_0/\sigma_D$. The corrections to the electric field in the metal $E_0-E(x)\sim E_0L_T^2/x^2$, arising due to the NIS boundary, survive on distances up to $L_{\rm in} \gg x \gg L_T$.

At high temperatures $L_t \gg L_T$ (but still $T \ll \Delta$), it follows from Eq. (20) that $R_I \gg R$. In this case, the electric field $E(x)$ in the metal region is close to E_0 while the corrections to E_0 in the metal due to the presence of the superconductor survive over distances which are smaller than $L_{\rm in}$ (but can be much larger than L_T). Curve 2 in Fig. 2(a) qualitatively shows $\Phi(x)$ in this case.

Using Eqs. (19) and (20) in the case $L_{\rm in}>L_t>L_T$, we get

$$\frac{E_0-E(x)}{E_0}=\begin{cases}\dfrac{\pi^2}{8}\dfrac{L_T^2}{L_t^2}\mathrm{Ln}\dfrac{L_t}{x} & \text{if } L_T \ll x \ll L_t \\ \sim\dfrac{L_T^2}{x^2} & \text{if } L_{\rm in} > x \gg L_T \\ \sim\exp\left[-\dfrac{x}{L_{\rm in}}\right] & \text{if } x \gg L_{\rm in}\ .\end{cases} \tag{23}$$

In the case $L_t \gg L_{\rm in} \gg L_T$, we have

$$\frac{E_0-E(x)}{E_0}=\begin{cases}\dfrac{\pi^2}{8}\dfrac{L_T^2}{L_t^2}\mathrm{Ln}\left[\dfrac{x}{L_{\rm in}}\right] & \text{if } L_T \ll x \ll L_{\rm in} \\ \dfrac{L_T^2}{L_t^2}\exp\left[-\dfrac{x}{L_{\rm in}}\right] & \text{if } x \gg L_{\rm in}\ .\end{cases} \tag{24}$$

In the case considered above, one can calculate both the two-probe resistance of the junction[18] and the distribution of the electric field in the metal using the perturbation theory with respect to t.

Let us now turn to the case of the junction geometry shown in Fig. 2(b). We will assume that $L_t \gg L$ and at $T>T_c$ the resistance of the normal-metal–insulator–normal-metal junction $R_N=\Phi^N(x=\infty)/I_0$ is determined by the resistance of the insulator $R\sim(t\nu_0)^{-1}$. Here T_c is the critical temperature of the superconductor. In this case, at low temperature $L_T \gg L_t$ the solution of Eqs. (15) and (16) reproduces the result obtained in Refs. 5, 6, and 19, namely, the resistance of the SIN junction $R_\infty=\Phi(x=\infty)/I_0$ is of the order of $R(R-R_\infty \sim R_N-R_\infty \sim L/\sigma_D)$. One can see it from the fact that at low temperatures, Eq. (16) for f_1 and the boundary conditions Eq. (17) differ from the corresponding equations for the NIN junction by the factors $\cosh^2\theta_2$, which are of the order of unity. Following Eq. (10), this means that at $x \gg L_T$ the value of $\Phi(x)$ is of the same order as in the case of the NIN junction. On the other hand, the "resistance of the insulator" $R_I=\Phi(+0)/I_0$ is given by the same formula Eq. (21) as in the above-considered one-dimensional case. In particular, it vanishes at small T. The spatial distribution of $\Phi(x>0)$ in this case is shown qualitatively in Fig. 2(b), curve 1. The nontrivial feature of the spatial distribution $\Phi(x)$ in this case is that the voltage drop takes place far away from the insulator on the distance that is even larger than L. This is very different from the NIN case where the main voltage drop takes place on the insulator and there is no $\Phi(x)$ dependence at $x \gg L$. Curve 3 in this figure corresponds to $\Phi(x)$ in the case of the NIN junction.

Following Eq. (21), at $L_t \gg L_T$ the resistance of the system exceeds significantly the resistance R of the corresponding NIN junction.

B. The case of repulsive electron-electron interaction in the metal

The repulsive electron-electron interaction in the metal changes the above-presented results at small $T \ll |\Delta_N(x)|$: R_I does not approach zero at small temperatures. In this case, Eq. (15) has the form

$$\frac{D}{2}\partial_x^2\theta(\epsilon,x)+\left[i\epsilon-\frac{1}{\tau_{\rm in}}\right]\sin\theta(\epsilon,x)=\Delta_N\cos\theta(\epsilon,x)\ . \tag{25}$$

Here $\Delta_N(x)=\Delta(x>0)$ is the order parameter in the metallic region. Using Eqs. (9) and (15) we can estimate it $|\Delta_N(x)|\sim|\alpha_N|D/L_t x$ at $x \ll L_t$. Here $0>\alpha_N>-1$ is the interaction constant in the metal. It follows from Eqs. (25) and (17) that

$$\theta(\epsilon=0,x=0)-\frac{\pi}{2}\sim\frac{L_t}{L_\Delta}\sim\sqrt{|\alpha_N|} \tag{26}$$

and $\nu(\epsilon=0,x=0)\sim\nu_0 L_t/L_\Delta$. This is different from the case without interaction when $\theta(0,0)=\pi/2$ and $\nu(0,0)=0$.

Using Eqs. (18) and (26) in the limit $L_T \gg L_t$, $L_\Delta=\sqrt{D/|\Delta_N(L_t)|}$, we have

$$R_I(T=0)\sim R\frac{L_t}{L_\Delta}\sim R\sqrt{|\alpha_N|}\ . \tag{27}$$

Electric field is expelled from the metal region of the size of the order of L_Δ near the NS surface. Since $R_I\sim\sqrt{|\alpha_N|}$, Eq. (27) provides a way to measure interaction constants in metals. Curves 4 in Figs. 2(a) and 2(b) represent $\Phi(x)$ at small T in the presence of the electron-electron interaction.

It is also worth mentioning that, according to Eq. (6), in the presence of repulsive interaction, the supercurrent penetrates into the normal metal over the region L_Δ. The penetration of the supercurrent into normal metal does not significantly affect our results at high temperatures $T \gg |\Delta_N(L_t)|$. At $T \ll |\Delta_N(L_t)|$, it can change the results for R_I by a factor of the order of unity. However, since $|\alpha_N|<1$, in the mean-field approximation the electron-electron interaction cannot change results for R_∞ significantly.

We would also like to mention that the mean-field description of electron-electron interaction in the metal may not capture all essential features of the effect. The effect of macroscopic quantum tunneling similar to that considered in the SNS junction[20] can contribute significantly in the destruction of the electron-hole coherence in the metal and thereby can change the results for SIN junction resistance.

The above-considered dependence $\Phi(x)$can be measured, for example, with the help of an additional tunneling junction "c" [see Fig. 2(a)]. We assume that the tunneling transmission coefficient t' of the NIN′ junction is small and $L_{t'} \gg L_T$, but still $L_t \ll L_t$. In this case, the normal metal N' is in the high-temperature regime and we can neglect proximity effects in N' metal. Using the high-temperature expansion, similar to what we have done above, we arrive at the conclusion that the "x"

dependence of the current I_c through the "c" junction follows the corresponding dependence $\Phi(x)$,

$$I_c = t'\nu_0 \Phi(x) \ . \tag{28}$$

On the other hand, at high enough temperatures $L_{t'} \gg L_T \gg L_t$, the conductance of the c junction G_c is significantly suppressed due to small density of states in the "N" normal metal. As a result, the four-probe measurements (the measurements of the voltage which has to be applied to the c contact to make the total current through the contact be zero) will not show the significant suppression of $\Phi(x)$ near the SIN boundary.

III. NONLINEAR EFFECTS AND MAGNETIC-FIELD DEPENDENCE OF THE RESISTANCE

Nonlinearity in external electric field destroys the electron-hole coherence and increases the resistance of the system considered above. As a result, $R_I(V)$ and $R_\infty(V)$ have sharp minimums at $V=0$. If $L_V \ll L_T$ we have

$$R_I \sim R \frac{L_t}{L_V} \ , \tag{29}$$

Magnetic field destroys the electron-hole coherence and increases the resistance of the system considered above. This is in agreement with the theoretical results obtained in Refs. 5 and 6 as well as with the experiment in Ref. 1, where such a minimum in $R_\infty(V)$ and its magnetic-field sensitivity were observed. To get the expressions for the magnetoresistance in the case $L_T \gg L_H$ one can substitute L_T for L_H in the formulas presented above. Here L_H is the magnetic length.

Let us consider now the following question: Can we measure the minimum in ϵ dependence of the density of states $\nu(\epsilon)$ at $\epsilon=0$ [see Eqs. (14), (19), and (26) using an additional tunneling NIN′ junction C in Fig. 2(a)? It has been noted that in the case $L_V \ll L_{t'}$ we can use the perturbation theory with respect to t' and I-V characteristics of the NIN′ junction directly reflect the minimum in $\nu(\epsilon)$. In particular, dI/dV goes to zero at small V. Here t' is the transmission coefficient of the NIN′ junction. We assume that N and N' metals have the same diffusion constants. On the other hand, as has been shown above, in the case of low temperatures $L_t, L_{t'} \ll L_T$, the resistance of the NIN′ junction is relatively small and of the order of $(t'\nu_0)^{-1}$.

ACKNOWLEDGMENTS

We would like to thank H. Courtois, A. Kastalski, Y. Nazarov, B. Pannetier, and A. Shelankov for useful discussions. This work was partially supported by the Division of Material Sciences, U.S. National Science Foundation, under Contract No. DMR-9205144.

[1]A. Kastalsky, A. W. Kleinsasser, L. H. Greene, R. Bhat, F. P. Milliken, and J. P. Harbison, Phys. Rev. Lett. **67**, 3026 (1991).

[2]G. E. Blonder, M. Tinkham, and T. M. Klapwijk, Phys. Rev. B **25**, 4515 (1982).

[3]S. N. Artemenko and A. F. Volkov, Sov. Phys. Usp. **22**, 295 (1979) [Usp. Fiz. Nauk. **128**, 3 (1979)].

[4]A. F. Andreev, Zh. Eksp. Teor. Fiz. **46**, 1832 (1964); **51**, 1510 (1966) [Sov. Phys. JETP **19**, 1228 (1964); **24**, 1019 (1964)].

[5]C. W. J. Beenakker, Phys. Rev. B **46**, 12841 (1992); I. K. Marmorkos, C. W. J. Beenakker, and R. A. Jalabert (unpublished).

[6]B. J. Van Wees, P. De Vries, P. Magnee, and T. M. Klapwijk, Phys. Rev. Lett. **69**, 510 (1992).

[7]A. G. Aronov, Yu. M. Galperin, V. L. Gurevich, and V. I. Kozub, Adv. Phys. **30**, 539 (1981).

[8]V. Galaiko and V. Shumeiko, Zh. Eksp. Teor. Fiz. **71**, 671 (1976) [Sov. Phys. JETP **44**, 353 (1976)].

[9]A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **68**, 1915 (1975); **73**, 299 (1977) [Sov. Phys. JETP **41**, 960 (1977); **46**, 155 (1977)].

[10]A. L. Shelankov, Zh. Eksp. Teor. Fiz. **78**, 2359 (1980) [Sov. Phys. JETP **51**, 1186 (1980)].

[11]A. Schmidt and Gerd Schon, J. Low Temp. Phys. **20**, 207 (1975).

[12]L. Keldysh, Zh. Eksp. Teor. Fiz. **47**, 1515 (1964) [Sov. Phys. JETP **20**, 1018 (1965)].

[13]A. A. Abrikosov, L. P. Gorkov, and I. E. Dzialoshinskii, *Quantum Field Theoretical Methods in Statistical Physics* (Pergamon, New York, 1985).

[14]M. Y. U. Kuprianov and V. F. Lukichev, Zh. Eksp. Theor. Fiz. **94**, 139 (1988) [Sov. Phys. JETP **67**, 1163 (1988)].

[15]M. Tinkham and J. Clark, Phys. Rev. Lett. **28**, 1366 (1972).

[16]A. Shelankov (unpublished).

[17]Y. Takakane and H. Ebisawa, J. Phys. Soc. Jpn. **60**, 3130 (1991); **61**, 3466 (1992).

[18]F. W. J. Hekking and Yu. V. Nasarov, Phys. Rev. Lett. **71**, 1625 (1993).

[19]A. V. Zaitsev, Pis'ma Zh. Eksp. Teor. Fiz. **51**, 35 (1990) [JETP Lett. **51**, 41 (1990)]; A. F. Volkov and T. M. Klapwijk, Phys. Lett. A **168**, 217 (1992).

[20]A. O. Caldeira and A. J. Legett, Phys. Rev. Lett. **46**, 221 (1981); A. J. Legett, S. Chakravarty, A. T. Dorsey, M. P. Fisher, A. Garg, and W. Zweger, Rev. Mod. Phys. **59**, 1 (1987).