Magnetic properties of a superlattice of amorphous multilayered films

Abdelilah Benyoussef and Hamid Ez-Zahraouy
Laboratoire de Magnetisme et de Physique des Hautes Energies, Département de Physique, B.P. 1014, Faculté des Sciences, Rabat, Morocco

(Received 8 June 1994; revised manuscript received 6 February 1995)

Using a finite-cluster approximation, the magnetic properties of a multilayer spin-$\frac{1}{2}$ superlattice with antiferromagnetic exchange coupling between amorphous magnetic layers across a nonmagnetic spacer are investigated. For reasonable values of the parameters of the proposed model, the magnetic properties of such a system are in agreement with the results observed experimentally on (Fe-Si)/Pd multilayered films. The magnetization exhibits three different kinds of behavior as a function of the amorphization.

I. INTRODUCTION

In recent years the study of multilayer structures has been of increasing interest for experiments and theoretical works. This gives the potential for many technological advances in the synthesis of new magnets for a variety of applications.[1-9] Much attention has been paid to the properties of layered structures consisting of alternating magnetic and nonmagnetic materials. The most commonly studied magnetic multilayers are those of a ferromagnetic transition metal such as iron, cobalt, or nickel and a nonmagnetic transition or noble metal. Many experiments have shown that magnetization enhancement exists in multilayered films consisting of magnetic layers and Pd-metal layers.[10-12] This enhancement is induced by the polarization effect of Pd atoms. A free Pd atom is nonmagnetic. When the nonmagnetic layers in multilayered films are sufficiently thin, magnetic coupling between magnetic layers begins to occur. It was found that ferromagnetic or antiferromagnetic coupling can exist between magnetic layers separated by Cu, Cr, Pd, or Ru layers, and an oscillatory interlayer coupling was found with a variation in the thickness of nonmagnetic layers.[13-15] When the interlayer coupling changes, the magnetization orientation in adjacent magnetic layers will change from parallel to antiparallel, or reverse. From the theoretical point of view, great interest has been paid to spin-wave excitations as well as critical phenomena in two-component layered magnetic superlattices. These researches are usually on Heisenberg or Ising multilayered systems consisting of only spin-$\frac{1}{2}$ ions with coupling exchange constants of different magnitudes within each layer.[16-20] Many experiments show that the interface region between the two-component layered superlattices looks amorphous.[21] The existence of amorphous interfaces plays an important role for the magnetic properties of multilayered thin films. The influences of disordered interfaces on the phase diagram in a bilayer system consisting of two spin-$\frac{1}{2}$ Ising ferromagnetic layers with different bulk properties have been investigated.[22] The exchange coupling in magnetic multilayers between magnetic layers across a nonmagnetic spacer is calculated.[15] The magnetic properties of a three-layer system consisting of two spin-$\frac{1}{2}$ Ising ferromagnetic layers with different bulk properties and amorphous interfaces are investigated within an effective-field theory.[6] Our work is motivated partly by the experiment done by Liu, Ma, and Mei[23] on (Fe-Si)/Pd multilayered films and by study of the magnetic properties of a multilayer spin-$\frac{1}{2}$ superlattice system with different amorphous thickness layers, within a finite-cluster approximation.[24,25]

II. MODEL AND METHOD

A. Model

We consider a three-dimensional spin-$\frac{1}{2}$ Ising superlattice consisting of n layers of type A and m layers of type B (see Fig. 1). In this model we assume that the layers of type A have magnetic amorphous structure, while type B are nonmagnetic and have crystalline structure. If we consider only nearest-neighbor interactions, the Hamiltonian of the system is given by

$$H=-\sum_{\langle ij\rangle} J_{ij}\sigma_i\sigma_j - h\sum_i \sigma_i \ , \qquad (1)$$

where σ_i is the spin variable which takes two values ± 1 and the summation runs over all pairs of nearest neighbors. J_{ij} is the exchange interaction, taking the value J_a if both spins are in A, J_b if both spins are in B, and J_{ab} if one spin is in A and its nearest neighbor is in B. h is the external magnetic field applied on each site i of the system. In this model J_{ab} is negative and J_a is assumed to be randomly distributed according to the probability distribution law

$$P(J_{ij})=\tfrac{1}{2}[\delta(J_{ij}-J_a-\Delta J_a)+\delta(J_{ij}-J_a+\Delta J_a)] \ . \qquad (2)$$

The parameter $\delta=\Delta J_a/J_a$ is a measure of the fluctuation in the exchange interactions and is called the amorphization.

Distribution (2) has been extensively used by experimentalists to fit their data. Especially for Fe-based amorphous alloys, values of δ as high as 0.5 have been used to

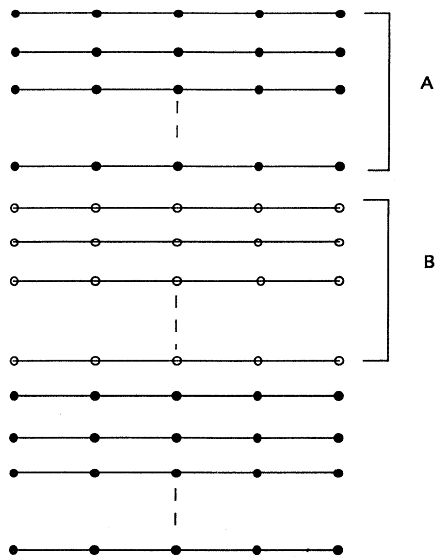

FIG. 1. Cross section of the superlattice used (A, amorphous structures; B, crystalline structures). Cyclic boundary conditions are used in the perpendicular direction (the last layer of B is connected with the first one of A) in order to have an infinite system.

give a qualitative agreement between experiments and the Handrich formula.[26] Other results were obtained by Prasad, Bhtnager, and Jagannathan,[27] which indicate that the agreement with the experimental result is improved even in the Handrich formula, when δ is made temperature dependent in such a way that it decreases with temperature.[27] In particular, for a better choice of this dependence[27] $\delta(T)=\delta(0)[(1-(T/T_c)^2]$, it gives a good fit to the experimental points for a-$Fe_{74}Co_{10}B_{16}$, a-$Fe_{80}B_{20}$, a-$Fe_{40}Ni_{20}B_{20}$, and a-$Fe_{78}B_{13}Si_9$.

For disordered models the mean-field approximation which neglects all spin correlations is not satisfactory. But to compute the average over all spin configurations $\langle\langle \sigma_i \rangle\rangle_D$, where $\langle \cdots \rangle$ indicates the thermal average and $\langle \cdots \rangle_D$ means the random configurational average, we will use the finite-cluster approximation[24,25] within an expansion technique for spin-$\frac{1}{2}$ cluster identities,[28] which still neglects correlations between different spins, but takes into account relations such as $\langle (\sigma_i)^2 \rangle = 1$ exactly.

B. Method

Using a single-site cluster approximation[24,25] in which attention is focused on a cluster comprising just a single selected spin labeled 0 and the neighboring spins with which it directly interacts, we get the Hamiltonian containing 0, namely,

$$H_0=(A-h)\sigma_0 , \tag{3}$$

where

$$A=-\sum_{j=1}^{N} J_{0j}\sigma_j ,$$

with J_{0j} the exchange couplings between spins at site 0 and the nearest-neighbor spins σ_j and N is the coordination number.

The starting point for the single-site cluster approximation is a set of formal identities of the type

$$\langle\langle \sigma_0^z \rangle_C\rangle = \left\langle \frac{\mathrm{tr}_0 \sigma_0^z \exp(-\beta H_0)}{\mathrm{tr}_0 \exp(-\beta H_0)} \right\rangle , \tag{4}$$

where $\langle \sigma_0^z \rangle_C$ denotes the mean value of the spin 0 of the z layer for a given configuration c of all other spins, i.e., when all other spins σ_j $(j\neq 0)$ have fixed values. $\langle \cdots \rangle$ denotes the average over all spin configurations. tr_0 means the trace performed over σ_0^z only. $\beta=1/K_B T$, T is the absolute temperature, and K_B is the Boltzmann constant.

To calculate $\langle \sigma_0^z \rangle_C$, one has to effect the inner traces in Eq. (4) over the states of spin 0. In this way it follows that

$$\langle \sigma_0^z \rangle_C = \tanh\left\{ \frac{1}{K_B T} \left[\sum_{j=1}^{N-2} J_{0j}\sigma_j^z + J_{0z-1}\sigma_0^{z-1} + J_{0z+1}\sigma_0^{z+1} + h \right] \right\} , \tag{5}$$

where J_{0z-1} and J_{0z+1} are, respectively, the exchange couplings between spins in site 0 of the z layer, spins in site 0 of the $z-1$ layer and spins in site 0 of the z layer, spins in site 0 of the $z+1$ layer, while J_{0j} is the exchange coupling between spin 0 of the z layer and spins in site j of the z layer.

The magnetization m_z of the z layer is given by

$$m_z = \langle\langle\langle \sigma_0^z \rangle_C\rangle_D\rangle , \tag{6}$$

where $\langle \cdots \rangle$ denotes the average over all configurations of the other spins $\sigma_j (j\neq 0)$ and $\langle \cdots \rangle_D$ denotes the average over all configurations of the disorder of the exchange interactions J_{0j}. Using the distribution of J_{0j} mentioned above, the average over the disorder of J_{0j} (for the amorphous layers) of $\langle \sigma_0^z \rangle_c$ is given by

$$\langle\langle \sigma_0^z \rangle_C\rangle_D = \int \langle \sigma_0^z \rangle_C \prod_{j=1}^{N-2} P(J_{0j}) P(J_{0z-1}) P(J_{0z+1})\, dJ_{0j}\, dJ_{0z-1}\, dJ_{0z+1} . \tag{7}$$

Then, for $z \in A$ $(z=2,\ldots,n-1)$,

$$\langle\langle\sigma_0^z\rangle_C\rangle_D=\frac{1}{2^N}\sum_{k_1=0}^{1}\sum_{k_2=0}^{1}\sum_{k_3=0}^{N-2}C_{k_3}^{N-2}\tanh\left\{\frac{J_a}{K_BT}\left[(1-\delta)\left[k_1\sum_{j=1}^{k_1}\sigma_{j-1}^{z-1}+k_2\sum_{j=1}^{k_2}\sigma_{j-1}^{z+1}+(1-\delta_{k_3,0})\sum_{j=1}^{k_3}\sigma_j^z\right]\right.\right.$$
$$+(1+\delta)\left[(1-k_1)\sum_{j=1}^{1-k_1}\sigma_{j-1}^{z-1}+(1-k_2)\sum_{j=1}^{1-k_2}\sigma_{j-1}^{z+1}\right.$$
$$\left.\left.\left.+(1-\delta_{k_3,N-2})\sum_{j=1}^{N-2-k_3}\sigma_{j+k_3}^z\right]+\frac{h}{J_a}\right]\right\}, \tag{8}$$

$$\langle\langle\sigma_0^n\rangle_C\rangle_D=\frac{1}{2^{N-1}}\sum_{k_1=0}^{1}\sum_{k_3=0}^{N-2}C_{k_3}^{N-2}\tanh\left\{\frac{J_a}{K_BT}\left[(1-\delta)\left[k_1\sum_{j=1}^{k_1}\sigma_{j-1}^n+(1-\delta_{k_3,0})\sum_{j=1}^{k_3}\sigma_j^n\right]\right.\right.$$
$$\left.\left.+(1+\delta)\left[(1-k_1)\sum_{j=1}^{1-k_1}\sigma_{j-1}^{n-1}+(1-\delta_{k_3,N-2})\sum_{j=1}^{N-2-k_3}\sigma_{j+k_3}\right]+\frac{J_{ab}}{J_a}(\sigma_0^{n+1})+\frac{h}{J_a}\right]\right\}, \tag{9}$$

where

$$\delta_{i,j}=\begin{cases}1 & \text{for } i=j\\ 0 & \text{for } i\neq j\,,\end{cases}$$

and $\delta=\Delta J_a/J_a$ is the amorphization.

Since only the layers of type A have an amorphous structure, for each layer $z\in B(z=n+1,\ldots,n+m)$ we have $\langle\langle\sigma_0^z\rangle_C\rangle_D=\langle\sigma_0^z\rangle_C$; then,

$$\langle\sigma_0^{n+1}\rangle_C=\tanh\left\{\frac{J_a}{K_BT}\left[\frac{J_{ab}}{J_a}\sigma_0^n+\frac{J_b}{J_a}\left[\sum_{i=1}^{N-2}\sigma_i^{n+1}+\sigma_0^{n+2}\right]+\frac{h}{J_a}\right]\right\}, \tag{10}$$

$$\langle\sigma_0^z\rangle_C=\tanh\left\{\frac{J_a}{K_BT}\left[\frac{J_b}{J_a}\left[\sigma_0^{z-1}+\sum_{i=1}^{N-2}\sigma_i^z+\sigma_0^{z+1}\right]+\frac{h}{J_a}\right]\right\}. \tag{11}$$

To calculate $\langle\langle\langle\sigma_0^z\rangle_C\rangle_D\rangle$ we have used the expansion technique for cluster identities of spin-$\frac{1}{2}$ Ising systems[26] as follows.

Suppose one considers the general product

$$\prod_{j=1}^{N}\left\{\sum_{k=0}^{1}(\sigma_j)^k\right\},$$

which contains 2^N terms. From these terms one may collect together all those terms containing p_1 factors of σ_j^z, p_2 factors of σ_j^{z+1}, and p_3 factors of σ_j^z. Such a group is denoted by $\{\sigma\}_{p_1,p_2,p_3}$.

Our aim is to expand the function $\langle\langle\sigma_0^z\rangle_C\rangle_D$ of Eqs. (8)–(11) in terms of these $\{\sigma\}_{p_1,p_2,p_3}$. Thus, one can write, for $z\in A\,(z=2,\ldots,n-1)$,

$$\langle\langle\sigma_0^z\rangle_C\rangle_D=\frac{1}{2^N}\sum_{k_1=0}^{1}\sum_{k_2=0}^{1}\sum_{k_3=0}^{N-2}C_{k_3}^{N-2}\sum_{p_1=0}^{1}\sum_{p_2=0}^{1}\sum_{p_3=0}^{N-2}A_{p_1,p_2,p_3,k_1,k_2,k_3}\{\sigma\}_{p_1,p_2,p_3}, \tag{12}$$

$$\langle\langle\sigma_0^n\rangle_C\rangle_D=\frac{1}{2^{N-1}}\sum_{k_1=0}^{1}\sum_{k_3=0}^{N-2}C_{k_3}^{N-2}\sum_{p_1=0}^{1}\sum_{p_2=0}^{1}\sum_{p_3=0}^{N-2}U_{p_1,p_2,p_3,k_1,k_3}\{\sigma\}_{p_1,p_2,p_3}, \tag{13}$$

$$\langle\langle\sigma_0^1\rangle_C\rangle_D=\langle\langle\sigma_0^n\rangle_C\rangle_D,$$

and, for $z\in B\,(z=n+2,\ldots,n+m-1)$,

$$\langle\langle\sigma_0^z\rangle_C\rangle_D=\sum_{p_1=0}^{1}\sum_{p_2=0}^{1}\sum_{p_3=0}^{N-2}B_{P_1,P_2,P_3}\{\sigma\}_{P_1,P_2,P_3}, \tag{14}$$

$$\langle\langle\sigma_0^{n+1}\rangle_C\rangle_D=\sum_{p_1=0}^{1}\sum_{p_2=0}^{1}\sum_{p_3=0}^{N-2}V_{P_1,P_2,P_3}\{\sigma\}_{P_1,P_2,P_3}. \tag{15}$$

To calculate the coefficients $A_{p_1,p_2,p_3,k_1,k_2,k_3}$, U_{p_1,p_2,p_3,k_1,k_3}, B_{p_1,p_2,p_3}, and V_{p_1,p_2,p_3}, we use the expansion technique for the cluster identifies of the spin-$\frac{1}{2}$ Ising systems,[26] namely,

$$A_{p_1,p_2,p_3,k_1,k_2,k_3}=\frac{1}{2^N C_{p_3}^{N-2}}\sum_{i_1=0}^{k_1}\sum_{i_2=0}^{k_2}\sum_{i_3=0}^{k_3}C_{i_3}^{k_3}\sum_{j_1=0}^{1-k_1}\sum_{j_2=0}^{1-k_2}\sum_{j_3=0}^{N-2-k_3}C_{j_3}^{N-2-k_3}\sum_{\mu_1=0}^{i_1}\sum_{\mu_2=0}^{i_2}\sum_{\mu_3=0}^{i_3}$$
$$\times\sum_{\nu_1=0}^{j_1}\sum_{\nu_2=0}^{j_2}\sum_{\nu_3=0}^{j_3}(-1)^{\mu+\nu}C_{\mu_3}^{i_3}C_{\nu_3}^{j_3}C_{p_1-\mu_1-\nu_1}^{1-i_1-j_1}C_{p_2-\mu_2-\nu_2}^{1-i_2-j_2}C_{p_3-\mu_3-\nu_3}^{N-2-i_3-j_3}f_{ijk}(\delta)\,, \tag{16}$$

with

$$i=i_1+i_2+i_3\,,\quad j=j_1+j_2+j_3\,,\quad k=k_1+k_2+k_3\,,$$
$$\mu=\mu_1+\mu_2+\mu_3\,,\quad \nu=\nu_1+\nu_2+\nu_3\,,$$

and

$$f_{ijk}(\delta)=\tanh\left\{\beta J_a\left[(1-\delta)(k-2i)+(1+\delta)(N-k-2j)+\frac{h}{J_a}\right]\right\}, \tag{17}$$

$$U_{p_1,p_2,p_3,k_1,k_3}=\frac{1}{2^N C_{p_3}^{N-2}}\sum_{i_1=0}^{k_1}\sum_{i_3=0}^{k_3}C_{i_3}^{k_3}\sum_{l=0}^{1}\sum_{j_1=0}^{1-k_1}\sum_{j_3=0}^{N-1-k_3}C_{j_3}^{N-1-k_3}\sum_{\mu_1=0}^{i_1}\sum_{\mu_3=0}^{i_3}\sum_{\nu_1=0}^{j_1}\sum_{\nu_3=0}^{j_3}\sum_{\varphi=0}^{l}(-1)^{\mu+\nu+\varphi}C_{\mu_3}^{i_3}C_{\nu_3}^{j_3}C_{p_1-\mu_1-\nu_1}^{1-i_1-j_1}$$
$$\times C_{p_3-\mu_3-\nu_3}^{N-1-i_3-j_3}C_{p_2-\varphi}^{1-l}g_{ijkl}(\delta)\,, \tag{18}$$

where

$$g_{ijkl}(\delta)=\tanh\left\{\beta J_a\left[(1-\delta)(k-2i)+(1+\delta)(N-1-k-2j)+(1-2l)r_1+\frac{h}{J_a}\right]\right\}, \tag{19}$$

with

$$i=i_1+i_3\,,\quad j=j_1+j_3\,,\quad \mu=\mu_1+\mu_3\,,\quad \nu=\nu_1+\nu_3\,,\quad k=k_1+k_3\,,\quad r_1=\frac{J_{ab}}{J_a}\,,$$

$$V_{p_1,p_2,p_3}=\frac{1}{2^N C_{p_3}^{N-2}}\sum_{i=0}^{N-1}\sum_{j=0}^{1}C_i^{N-1}C_j^1\sum_{\mu=0}^{i}\sum_{\nu=0}^{j}(-1)^{\mu+\nu}C_\mu^i C_\nu^j C_{p-\mu-\nu}^{N-i-j}f_{ij}\,, \tag{20}$$

where

$$f_{ij}=\tanh\left\{\beta J_a\left[r_1(1-2j)+r_2(1-2i)+\frac{h}{J_a}\right]\right\} \tag{21}$$

and

$$r_2=\frac{J_b}{J_a}\,,$$

$$B_{p_1,p_2,p_3}=\frac{1}{2^N C_{p_3}^{N-2}}\sum_{i_1=0}^{1}\sum_{i_2=0}^{1}\sum_{i_3=0}^{N-2}C_{i_3}^{N-2}\sum_{\mu_1=0}^{i_1}\sum_{\mu_2=0}^{i_2}\sum_{\mu_3=0}^{i_3}(-1)^{\mu_1+\mu_2+\mu_3}C_{\mu_3}^{i_3}C_{p_1-\mu_1}^{1-i_1}C_{p_2-\mu_2}^{1-i_2}C_{p_3-\mu_3}^{N-2-i_3}f_i\,, \tag{22}$$

with

$$f_i=\tanh\left\{\beta J_a\left[r_2(N-2i)+\frac{h}{J_a}\right]\right\},\quad i=i_1+i_2+i_3\,, \tag{23}$$

and $C_n^m=m!/[n!(m-n)!]$ are the binomial coefficients.

Using the simplest approximation of the Zernike decoupling of the type

$$\langle\sigma_i^{z-1}\sigma_{i'}^{z-1}\cdots\sigma_j^z\sigma_{j'}^z\cdots\sigma_k^{z+1}\cdots\rangle=\langle\sigma_i^{z-1}\rangle\langle\sigma_{i'}^{z-1}\rangle\cdots\langle\sigma_j^z\rangle\langle\sigma_{j'}^z\rangle\cdots\langle\sigma_{ik}^{z+1}\rangle\cdots\quad\text{for } i\neq i',j\neq j',\ldots,$$

and seeing that the number of elements of the group $\{\sigma\}_{p_1,p_2,p_3}$ is equal to $C_{p_3}^{N-2}$, then the averaged magnetizations $m_z=\langle\langle\langle\sigma_0^z\rangle_C\rangle_D\rangle$ of the layer z are given by the following: for $z\in A\,(z=2,\ldots,n-1)$,

$$m_z=\frac{1}{2^N}\sum_{k_1=0}^{1}\sum_{k_2=0}^{1}\sum_{k_3=0}^{N-2}C_{k_3}^{N-2}\sum_{p_1=0}^{1}\sum_{p_2=0}^{1}\sum_{p_3=0}^{N-2}A_{p_1,p_2,p_3,k_1,k_2,k_3}(m_{z-1})^{p_1}(m_z)^{p_3}(m_{z+1})^{p_2}C_{p_3}^{N-2}\ , \tag{24}$$

$$m_n=\frac{1}{2^{N-1}}\sum_{k_1=0}^{1}\sum_{k_3=0}^{N-1}C_{k_3}^{N-1}\sum_{p_1=0}^{1}\sum_{p_2=0}^{1}\sum_{p_3=0}^{N-2}U_{p_1,p_2,p_3,k_1,k_3}(m_{n-1})^{p_1}(m_n)^{p_3}(m_{n+1})^{p_2}C_{p_3}^{N-2}\ , \tag{25}$$

$$m_1=m_n\ , \tag{26}$$

and, for $z\in B(z=n+2,\ldots,n+m-1)$,

$$m_{n+1}=\sum_{p_1=0}^{1}\sum_{p_2=0}^{1}\sum_{p_3=0}^{N-2}V_{p_1,p_2,p_3}(m_n)^{p_1}(m_{n+1})^{p_3}(m_{n+2}^{p_2})C_{p_3}^{N-2}\ , \tag{27}$$

$$m_z=\sum_{p_1=0}^{1}\sum_{p_2=0}^{1}\sum_{p_3=0}^{N-2}B_{p_1,p_2,p_3}(m_{z-1})^{p_1}(m_z)^{p_3}(m_{z+1})^{p_2}C_{p_3}^{N-2}\ , \tag{28}$$

$$m_{n+m}=m_{n+1}\ . \tag{29}$$

Then the total magnetization of the model (1) is given by

$$M=\frac{1}{n+m}\left|\sum_z m_z\right|\ . \tag{30}$$

III. RESULTS AND DISCUSSION

Equations (24)–(29) are solved by iteration. For $\delta<\delta_1$ and for a fixed value of the magnetic field h, different initial guesses can lead to two solutions, namely, ($m_z>0$ for $z\in B$; $m_z<0$ for $z\in A$) or ($m_z>0$ for $z\in B$; $m_z>0$ for $z\in A$), for $h>0$, and ($m_z<0$ for $z\in B$; $m_z>0$ for $z\in A$) or ($m_z<0$ for $z\in B$; $m_z<0$ for $z\in A$), for $h<0$.

But from a ground-state study ($T=0$ and $\delta=0$) of the model (1), it is easy to show the existence of a critical value ($h_c\approx 3|J_{ab}|/2n$) of the magnetic field h, such that, for $|h|<h_c$, the solution ($m_z>0$ for $z\in B$; $m_z<0$ for $z\in A$) for $h>0$ [($m_z<0$ for $z\in B$; $m_z>0$ for $z\in A$) for $h<0$] is selected as a stable configuration, while, for $|h|>h_c$, the solution ($m_z>0$ for $z\in B$; $m_z>0$ for $z\in A$) for $h>0$ [($m_z<0$ for $z\in B$; $m_z<0$ for $z\in A$) for $h<0$] is the stable configuration.

For $\delta>\delta_1$, only one solution persists ($m_z>0$ for $z\in B$; $m_z>0$ for $z\in A$) for $h>0$ [($m_z<0$ for $z\in B$; $m_z<0$ for $z\in A$) for $h<0$]. From the above it is clear that, for $|h|<h_c$, δ_1 is the transition point between the configuration with $m_z>0$ and the configuration with $m_z<0$, for $z\in A$. The dependence of the total magnetization $|M|$ of the model (1) on the amorphization δ [Fig. 2(a)] shows that the magnetization undergoes a finite gap at a critical value $\delta_1\approx 0.33$, which is due to the transition of the magnetization of the amorphous film from a positive value to a negative one. This transition is due to the competition between the magnetic field h, the antiferromagnetic coupling interaction J_{ab}, and the amorphization δ. Indeed, for a sufficiently weak amorphization ($\delta<\delta_1$), the effect of the perpendicular antiferromagnetic exchange J_{ab} is more important that the effect of the magnetic field; then, the states where the magnetizations of layers of type A and the magnetizations of layers of type B are of opposite sign are considered. While for sufficiently large values of the amorphization $\delta>\delta_1$ the effect of the antiferromagnetic exchange becomes negligible in front of the external magnetic field h, the configuration where the magnetizations of layers of type A and layers of type B are of the same sign is the dominant state. We remark that the value of δ_1 depends on the value of the external magnetic field and is independent on the thickness d_A of films of type A (amorphous film), because only the two adjacent layers (one of type A and one of type B) are coupled antiferromagnetically. But the strength of the gap at $\delta=\delta_1$ decreases when increasing d_A. For $\delta<\delta_1$ the absolute value of the total magnetization decreases when increasing δ, while for $\delta_1<\delta<\delta_2$ the magnetization increases with δ, passes through a maximum at $\delta=\delta_2$, and then decreases when increasing the amorphization δ. We note that for $|h|>h_c$ the gap disappears, because the sign of the magnetization of type A does not change when δ increases [Fig. 2(b)]. Figure 2 shows another critical value $\delta_I\approx 2.30$ (called inversion amorphization) below which $|M|$ increases when increasing the thickness d_A of the amorphous layers for a fixed value of the temperature, because for $\delta<\delta_I$ the amorphous film is still ordered and it is clear that the total magnetization increases when increasing the number of layers in such a film, while for $\delta>\delta_I$ and for a sufficiently low temperature ($T<T_I$) the disorder due to the amorphization becomes more important than the order induced by the external magnetic field. Hence the average magnetization of the amorphous film is weak in front of the magnetization of layers B and then the average magnetization ($|M|$) of the system is approximately reduced to $|M|_B/(n+m)$ (with $|M|_B=\sum_{z\in B}m_z$). Then the total magnetization decreases when increasing the thickness ($d_A=n$) of the amorphous layers (Fig. 3) for $T<T_I$, while for $T>T_I$ such situation is reversed (i.e., $|M|$ increases when increasing d_A, since this region corresponds to a paramagnet in a magnetic field) in agreement with experiments on (Fe-Si)/Pd (Fig. 2 of Ref. 23). At $\delta=\delta_I$, the effect of the external magnetic field on the

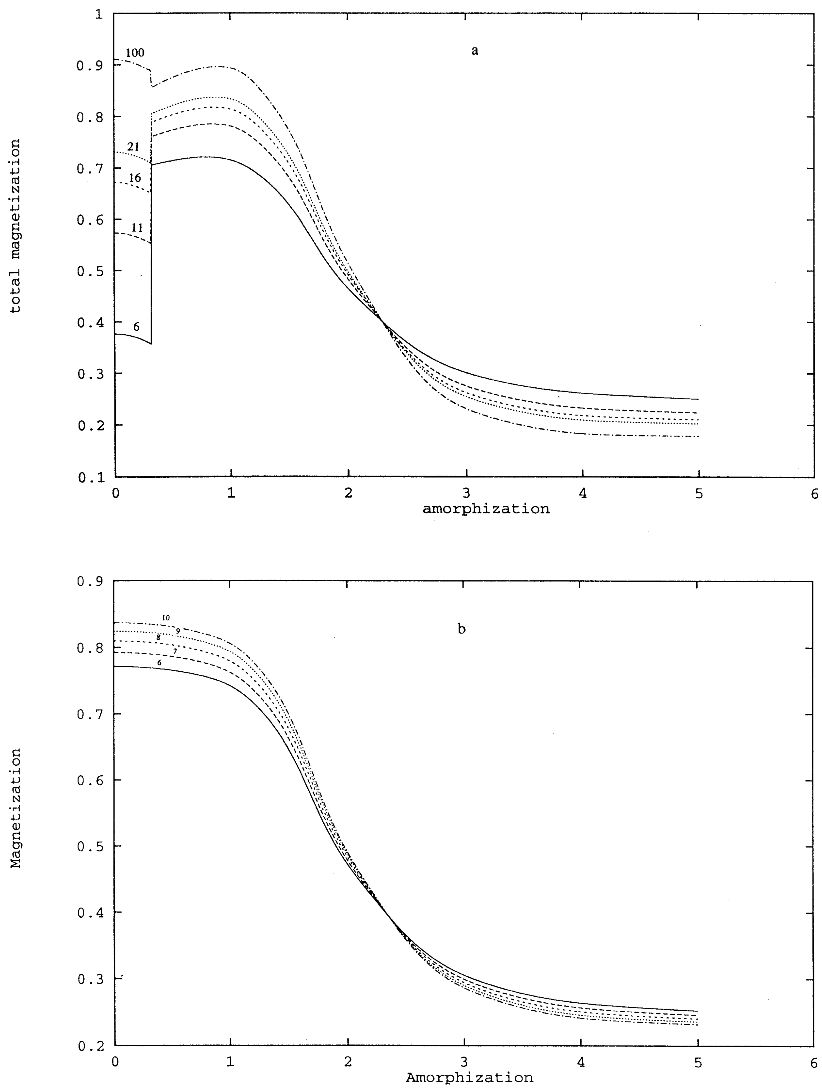

FIG. 2. Dependence of the absolute value of the total magnetization on the amorphization at $T/J_a=1$, $J_{ab}/J_a=-1$, $J_b=0$, and in (a) $h/J_a=0.1$, (b) $h/J_a=0.5$. The number accompanying each curve denotes the value d_A of the thickness of the amorphous layers.

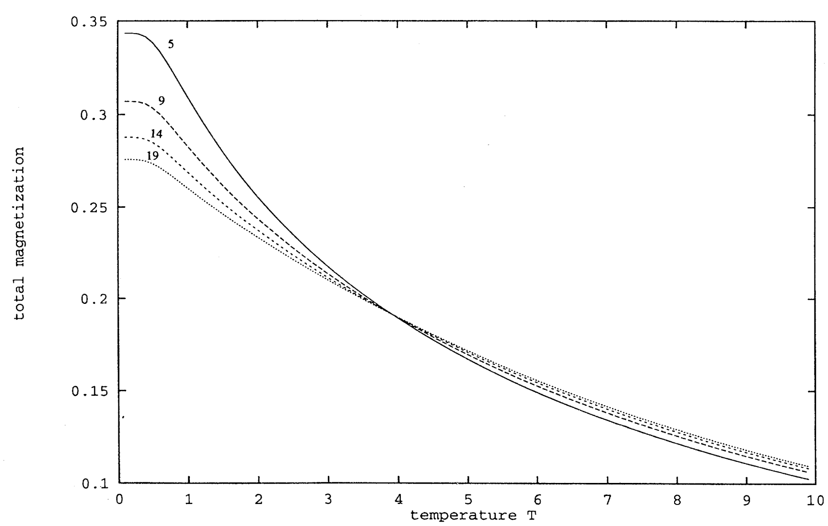

FIG. 3. Dependence of the absolute value of the total magnetization on the temperature at $d_B=4$, $h/J_a=0.1$, $\delta=3$, $J_{ab}/J_a=-1$, $J_b=0$. The number accompanying each curve denotes the value d_A of the thickness of the amorphous layers.

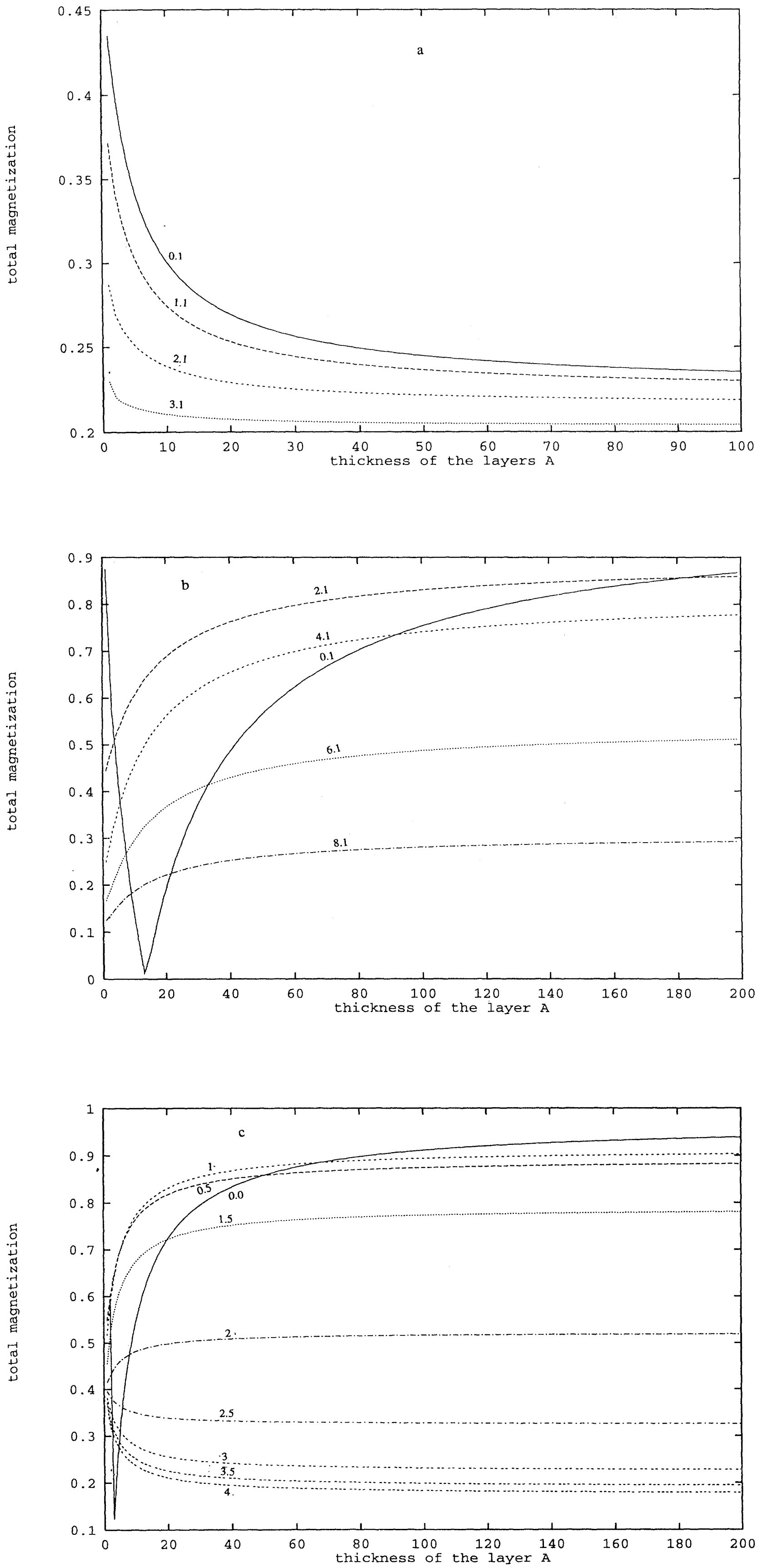

FIG. 4. Dependence of the absolute value of the total magnetization on the thickness d_A of the amorphous layers at a fixed value of $h/J_a=0.1$, $J_{ab}/J_a=-1$, and $J_b=0$. (a) $\delta=3$, $d_B=4$, (b) $d_B=15$, $\delta=0.1$, (c) $d_B=5$, $T/J_a=1$. The number accompanying each curve in (a) and (b) denotes the value of the temperature T/J_a, while in (c) this number denotes the value of the amorphization δ.

amorphous layers and the effect of the amorphization are compensated. In view of the above results, it should be of a great interest to represent the absolute value of the total magnetization with the thickness d_A of the amorphous layers. For the region in which the amorphization $\delta > \delta_I$ and $T < T_I$, the total magnetization decreases when increasing the thickness of the amorphous layers A, and for each fixed value of the thickness d_A the magnetization decreases with the temperature [Fig. 4(a)]. Such a situation is experimentally observed in (Fe-Si)/Pd multilayered films (Fig. 4 of Ref. 23). For $\delta < \delta_I$ the dependence of the magnetization is represented in Fig. 4(b) in which the total magnetization increases when increasing the thickness d_A. The dependence of the magnetization $|M|$ as a function of the thickness d_A for several values of the amorphization is represented in Fig. 4(c) in which two different kinds of behavior of the magnetization are obtained (in agreement with Fig. 2) in which the magnetization increases with d_A for $\delta < \delta_I$ and decreases when increasing d_A for $\delta > \delta_I$.

The dependence of the total magnetization $|M|$ of the model (1) on the thickness d_B of layers of type B for several values of the temperature T/J_a and, at fixed values of the magnetic field h, the amorphization δ and the thickness d_A of layers of type A is presented in Fig. 5 for $J_B = 0$ (if we assume that layers of type B are nonmagnetic). A compensation thickness d_{BC} appears only in the region $\delta < \delta_1$, where the magnetizations $m_{z \in A}$ and $m_{z \in B}$ are of the opposite sign. This is due to the effect of the antiferromagnetic coupling. Then the compensation thickness is due to the competition between the magnetizations $m_{z \in A}$ and $m_{z \in B}$. Indeed, for a fixed thickness of the amorphous film and for a sufficiently small thickness of the nonmagnetic layers, the total magnetization

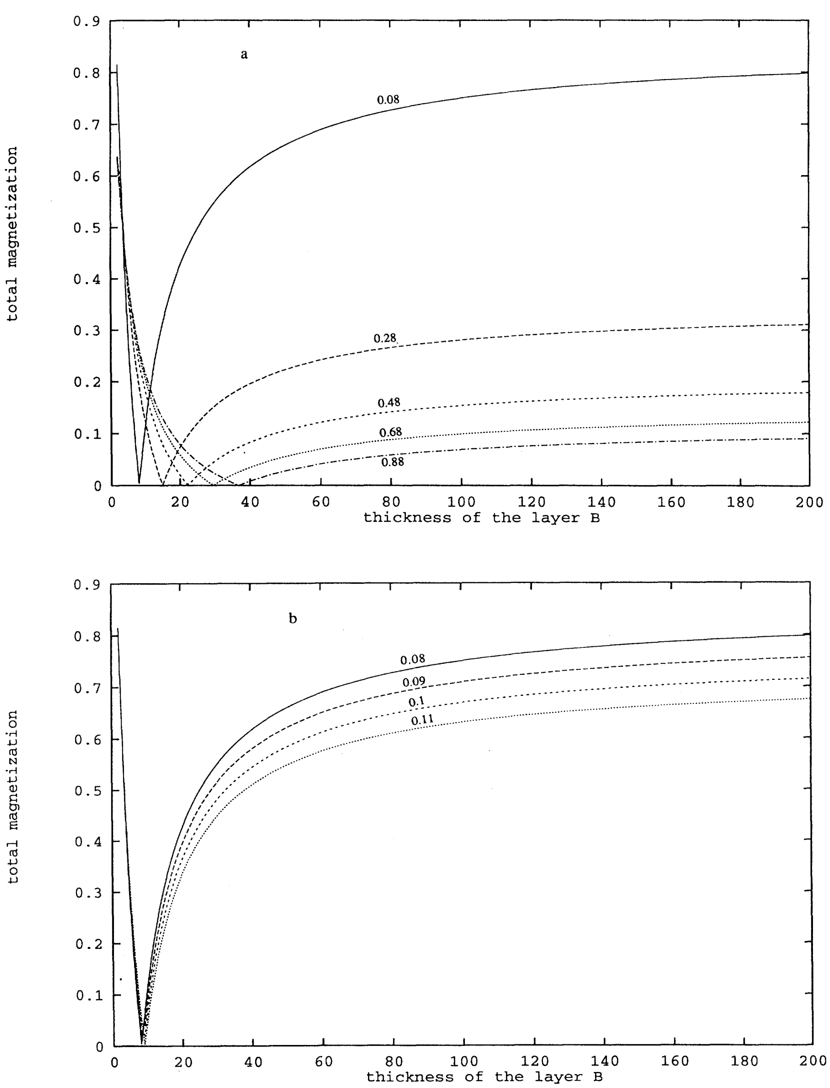

FIG. 5. Dependence of the absolute value of the total magnetization on the thickness d_B of layers of type B for several values of the temperature at $d_A = 4$, $h/J_a = 0.1$, $\delta = 0.1$, $J_{ab}/J_a = -1$, and $J_b = 0$. (a) At large temperature, (b) At very low temperature.

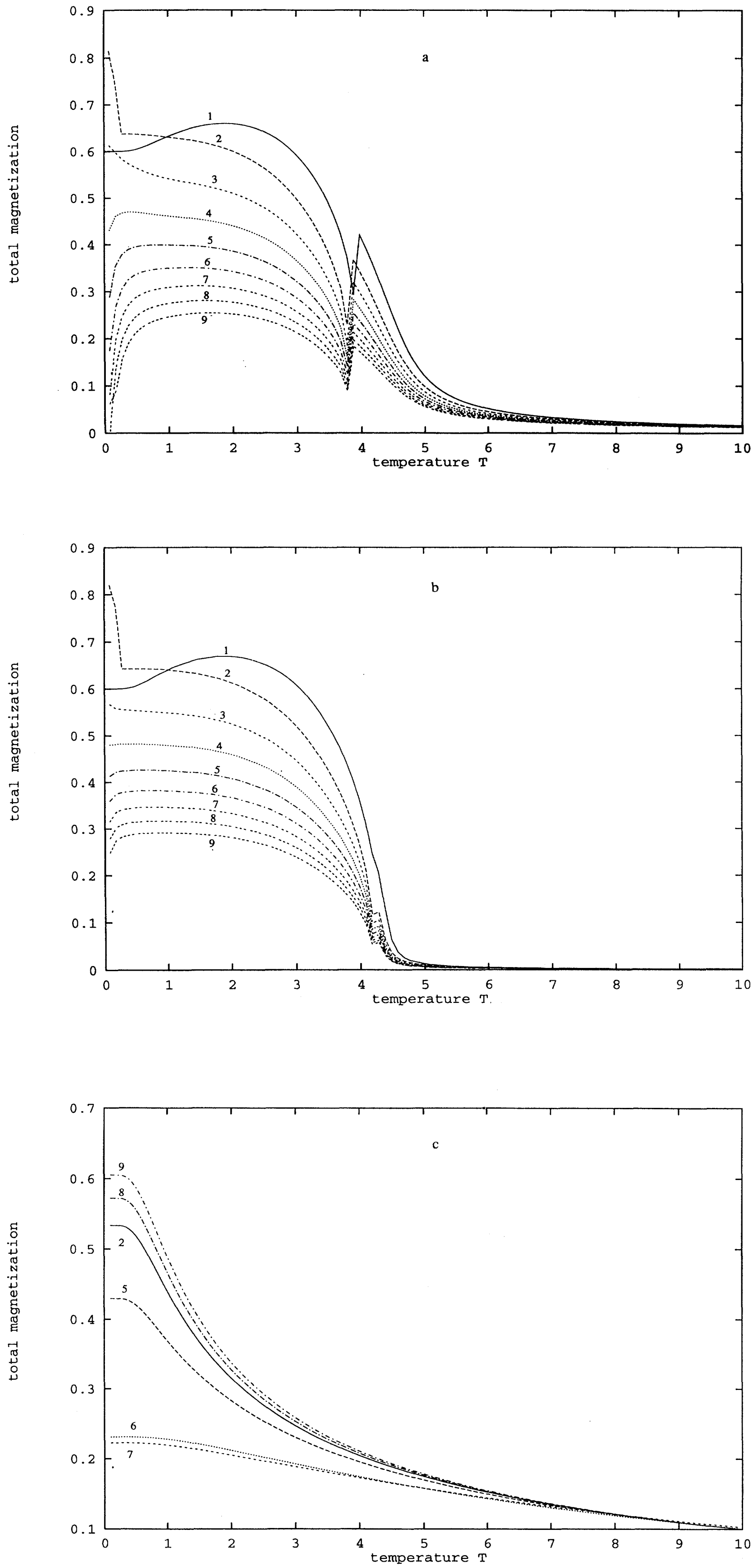

FIG. 6. Dependence of the absolute value of the total magnetization on the temperature for several values of the thickness d_B of layers of type B at $d_A=4$, $\delta=0.1$, $J_{ab}/J_a=-1$, $J_B=0$. (a) $h/J_a=0.1$, (b) $h/J_a=0.01$, (c) $\delta=2.3$, $h/J_a=0.01$.

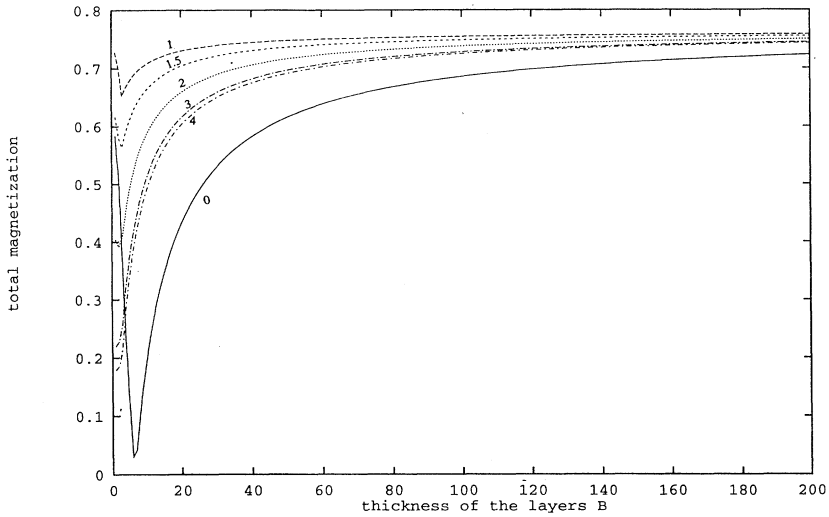

FIG. 7. Dependence of the absolute value of the magnetization on the thickness d_B of layers of type B obtained for several values of the amorphization δ and at $d_A=4$, $h/J_a=0.1$, $T/J_a=1$, $J_{ab}/J_a=-1$, and $J_B=0$.

$$|M|=\frac{1}{n+m}\left|\sum_{z\in B} m_z+\sum_{z\in A} m_z\right|$$

is dominated by A. But when increasing the thickness d_b of the B layers (i.e., adding magnetizations with opposite sign to the magnetizations $m_{z\in A}$), the total magnetizations decreases, passes through a minimum at $d=d_{BC}$, and then increases rapidly when $d>d_{BC}$, and for sufficiently large values of the thickness of the nonmagnetic layers, the total magnetizations is dominated by B. Such behavior is experimentally observed in (Fe-Si)/Pd multilayered films (Fig. 6 of Ref. 23). It is seen from Fig. 5(a) that the critical thickness d_{BC} depends strongly on the value of the temperature at sufficiently high temperatures, while this dependence is not considerable at sufficiently low temperatures [Fig. 5(b)]. Indeed, d_{BC} increases when increasing the temperature. At very large values of the thickness d_B of layers of type B, the saturation value of the total magnetization decreases when increasing the temperature. Since the behavior of the total magnetization $|M|$ depends on the values of the temperature, it should be of great interest to represent the dependence of the total magnetization on the temperature T/J_a (Fig. 6) for several values of the thickness d_B of layers of type B. Form Fig. 6(a), it is clear that the magnetization decreases when increasing the temperature and exhibits a jump at a critical temperature (where the spontaneous magnetization vanishes), and then undergoes a saturation value when the temperature becomes very large. The jump is due to the presence of the magnetic field. In fact, this jump disappears at a sufficiently low magnetic filed [Fig. 6(b)], in agreement with experiments on (Fe-Si)/Pd multilayered films (Fig. 5 of Ref. 23). In Fig. 6(c) the magnetization decreases with the thickness d_B of layers of type B, passes through a minimum, and then increases with d_B, in agreement with experiments on (Fe-Si)/Pd multilayered films (Fig. 3 of Ref. 23).

Another important feature is obtained in the dependence of the total magnetization as a function of the thickness of type B (Fig. 7) for several values of the amorphization δ. Figure 7 shows that the compensation thickness decreases when increasing the amorphization δ, and for a sufficiently large value of δ the compensation thickness disappears and the total magnetization increases with the thickness of layers of type B. This result means that the order in amorphous layers of type A favors the appearance of the compensation thickness d_{BC}.

IV. CONCLUSION

In this paper we have studied theoretically the magnetic properties of a superlattice with amorphous mulitlayered films. For reasonable values of the parameters of such system, our theoretical results are in agreement with an experiment on (Fe-Si)/Pd multilayered films.[23] New theoretical results are also obtained in further regions of the space parameters, which can be an area for future experimental work.

ACKNOWLEDGMENTS

This work has been done in the framework of the collaboration between the CNR (Morocco) and the CNRI (Italy). We would like to thank both organizations. One of the authors (A.B.) would like to thank Pr. L. Peliti of the department of theoretical physics, University "Federico II" Napoli for hospitality. The authors thank Pr. L. Peliti for illuminating discussions.

[1] R. Pandit, M. Schick, and M. Wortis, Phys. Rev. B **26**, 8115 (1982).
[2] C. Ebner, C. Rottman, and M. Wortis, Phys. Rev. B **28**, 4186 (1983).
[3] S. Dietrich and M. Schick, Phys. Rev. B **31**, 4718 (1985).
[4] M. Wortis, in *Fundamental Problems in Statistical Mechanics VI*, edited by E. G. D. Cohen (North-Holland, Amsterdam, 1985), p. 87.

[5]A. Patrykiejew, D. P. Landau, and K. Binder, Surf. Sci. **238**, 317 (1990).
[6]A. Benyoussef and T. Kaneyoshi, Phys. Lett. A **173**, 411 (1993).
[7]A. Benyoussef and H. Ez-Zahraouy, Physica A **206**, 196 (1994).
[8]A. Benyoussef and H. Ez-Zahraouy, J. Phys. (France) I **4**, 393 (1994).
[9]Proc. 13th Int. Colloq. on Magnetic Films and Surfaces, Glasgow, 1991 (unpublished).
[10]K. Schröder, J. Appl. Phys. **57**, 3666 (1985).
[11]F. J. A. den Broeder, H. C. Donkerrsloot, H. J. G. Draaisma, and W. J. M. Jonge, J. Appl. Phys. **61**, 4317 (1987).
[12]B. Heinrich, Z. Celinski, K. Myrtle, J. F. Cochran, A. S. Arrott, and J. Kirschner, J. Magn. Magn. Mater. **93**, 75 (1991).
[13]Z. Celinski and B. Heinrich, J. Magn. Magn. Mater. **99**, L25 (1991).
[14]S. S. P. Parkin, N. More, and K. P. Roche, Phys. Rev. Lett. **64**, 2304 (1990).
[15]J. Mathon, M. Villeret, and D. M. Edwards, J. Phys. Condens. Matter **4**, 9873 (1992).
[16]L. L. Hinckey and D. L. Mills, J. Appl. Phys. **59**, 3698 (1985).
[17]T. Kaneyoshi and H. Beyer, J. Phys. Soc. Jpn. **49**, 1306 (1980).
[18]E. L. Alnguergue, E. F. Sarmento, and D. R. Tilley, Solid State Commun. **58**, 41 (1986).
[19]A. F. Ferrenberg and D. D. Landau, J. Appl. Phys. **70**, 6215 (1991).
[20]T. Hai, Z. Y. Li, D. L. Lin, and T. F. George, J. Magn. Magn. Mater. **79**, 227 (1991).
[21]T. Tejada, F. Badia, B. Martinez, and J. H. Ruiz, J. Magn. Magn. Mater. **101**, 181 (1991).
[22]A. Khater, G. le Gal, and T. Kaneyoshi, Phys. Lett. A **171**, 237 (1992).
[23]Y. H. Liu, X. D. Ma, and L. M. Mei, J. Phys. Condens. Matter **4**, 9893 (1992).
[24]N. Boccara, Phys. Lett. **94A**, 185 (1983).
[25]A. Benyoussef and N. Boccara, J. Phys. C **16**, 1143 (1983).
[26]T. Kaneyoshi, *Amorphous Magnetism* (CRC Press, Boca Raton, 1984); *Introduction to Amorphous Magnets* (World Scientific, Singapore, 1992).
[27]B. B. Prasad, A. K. Bhtnager, and R. Jagannathan, Solid State Commun. **36**, 661 (1980).
[28]P. Tomczak, E. F. Sarmento, A. F. Siqueira, and A. R. Ferchmin, Phys. Status Solidi B **142**, 551 (1987).