Electron wave transport in coherently absorptive random media

Abhijit Kar Gupta*
Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Calcutta 700064, India

A. M. Jayannavar†
Institute of Physics, Sachivalaya Marg, Bhubaneswar 751005, India
(Received 13 January 1995; revised manuscript received 22 March 1995)

We study the statistics of reflection and transmission coefficients across a one-dimensional disordered electronic system in the presence of absorption. The absorption is introduced via a uniform imaginary part in the site energies in the disordered segment. Our results for the stationary distribution of the backscattered reflection coefficient differ qualitatively from recently obtained analytical results. We point out the source of this discrepancy by studying the phase evolution of the complex reflection amplitude. The limited range of validity of earlier known analytical results in the parameter space of the strength of disorder and absorption are also discussed.

I. INTRODUCTION

Wave propagation in a lossless random medium has been studied for over several decades.[1] Some of these studies include electron transport in the presence of random impurity potential, light or electromagnetic wave propagation in randomly distributed polarizable particles, acoustic (or sound) propagation in an inhomogeneous elastic medium, seismic or water wave propagation, etc. Results on these different types of wave propagation complement each other since all these different types of wave propagation in an appropriate limit follow a similar mathematical equation, namely, the Helmholtz type equation. It is the wave character giving interference and diffraction which is the common operative feature. In the context of electron wave transport in one-dimensional random media (in the absence of absorption), it is now well established that coherent interference effects, due to elastic scattering by the serial static disorder, lead to a strong localization of all eigenstates for arbitrary weak disorder.[2] From the point of view of scattering these positive-energy localized states in a disordered system play a qualitatively distinctive role. This localized nature of eigenstates manifests as an exponential increase of ensemble averaged resistance with the sample length. Moreover, the transmission across a sample is very sensitive to the spatial realization of impurity configuration. In fact this leads to a non-self-averaging behavior in the resistance (or transmission) in that the resistance fluctuations over the ensemble of macroscopically identical samples dominate the ensemble average. The presence of inelastic scatterings (phonons) leads to the loss of phase memory of the wave function and suppresses these fluctuations, and much attention has been paid to these studies.[2]

As compared to the studies on wave propagation in a static nonabsorptive medium, the problem of wave propagation in an absorptive medium has received less attention. In recent years several results have been obtained in this field.[3–6] This problem is also of physical interest as there are several physical situations where one encounters the absorption of elementary particles or excitations due to impurities in a medium. One recent example is the light (photon) propagation in a lossy dielectric medium.[1,7–9] The absorption corresponds to the actual removal of particles by recombination processes (or energy in the case of electromagnetic wave propagation). To allow the possibility of inelastic decay on otherwise coherent tunneling through potential barriers and to study the crossover from coherent to sequential tunneling several studies invoke the absorption[10,11] mechanism.

In earlier studies it was widely thought that the effect of absorption on classical wave propagation is analogous to that of inelastic scattering of electrons.[7,10] However, recent studies have indicated that absorption does not provide a cutoff length scale (similar to an inelastic scattering length) for the renormalization of wave propagation in random media, i.e., the absorption does not reestablish the diffusive behavior of the wave propagation by destroying the localization of eigenfunctions.[3–5] The transport remains nondiffusive even in the presence of absorption. It has been predicted that in higher dimensions ($d = 3$) a genuine mobility edge can exist in a sufficiently strongly disordered medium even in the presence of a significant degree of absorption and several scaling properties of the transmitted waves at the mobility edge are predicted.[4] Absorption induced coherence has also been discussed in a separate study.[5] In a related development Rubio and Kumar[12] have emphasized the dual role played by the absorption. For double barrier structures they have shown that the mismatch caused by absorptive potential (or imaginary potential) leads to a nonmonotonic dependence of the total absorption on the strength of the potential. The absorptive potential being imaginary causes enhanced reflection due to the potential dispersion, and absorption without reflection is not possible. In a Schrödinger equation to describe the absorption one introduces imaginary potentials. In that case the Hamiltonian becomes non-Hermitian and leads

0163-1829/95/52(6)/4156(6)/$06.00

to the absorption of the probability current. This approach is quite well known in nuclear physics wherein imaginary potentials are referred to as optical potentials. For the simple case of a purely absorptive δ-function potential $V(x) = -i\eta\delta(x)$, the corresponding reflection r, transmission t, and absorption σ coefficients are given by[12]

$$r = \frac{\eta^2}{(\hbar^2 k/m + \eta)^2}, \tag{1a}$$

$$t = \frac{(\hbar^2 k/m)^2}{(\hbar^2 k/m + \eta)^2}, \tag{1b}$$

$$\sigma = \frac{2\hbar^2 k\eta/m}{(\hbar^2 k/m + \eta)^2}, \tag{1c}$$

where k is the wave vector, m the mass, and η the strength of the absorptive potential. One can readily notice from Eq. (1c) that σ is a nonmonotonic function of η for fixed k or incident energy. It initially rises and then, after exhibiting a maximum, decreases towards zero. It should also be noted that as $\eta \to \infty$ the reflection coefficient tends to unity (total backscattering). In the vicinity of the absorber the particle experiences mismatch in the potential, and tries to avoid this region by enhanced back reflection. These discussions clearly indicate that imaginary potential plays a dual role as absorber and reflector. We would also like to emphasize here that unlike the absorption and reflection, transmission decreases monotonically as we increase η.

In a recent study on localization of light in coherently amplifying random media Pradhan and Kumar[13] have obtained an analytical expression for the stationary distribution of a coherently backscattered reflection coefficient (r) in the presence of both absorption and amplification. Their results for the stationary probability distribution $P_s(r)$ for r in the absorptive case is given by

$$P_s(r) = \frac{|\,D\,|\exp(|\,D\,|)\exp\left(-\frac{|D|}{1-r}\right)}{(1-r)^2}, \tag{2}$$

where D is proportional to η/d (η and d being the strength of absorptive potential and disorder, respectively). It can readily be noticed from Eq. (2) that in the limit of η large $P_s(r)$ does not tend to a delta function distribution at $r = 1$ (or total reflection). Instead their results indicate that the value of $P_s(r)$ at $r = 0$ gets more pronounced as we increase η. As already pointed out in the limit of large η the absorber acts as a reflector and hence $P_s(r)$ should tend to $\delta(r-1)$. This clearly indicates that the distribution obtained earlier is in general not correct. They have obtained the distribution on the assumption of random-phase approximation. We show explicitly below by numerical simulations that the random-phase approximation breaks down in the limit of large disorder and absorption coefficient. The expression (2) is valid only in the limited parameter domain of weak disorder and small η.

In our present work we have studied numerically, following a transfer-matrix approach, the statistics of reflection and transmission coefficient in one-dimensional electronically random absorptive system. The electronic motion is governed by a tight-binding one-band Hamiltonian. We show that the transmission across a random absorptive system introduces a length scale ξ in the problem which is smaller than the localization length with no absorption (l) and the absorption length in an ordered system (l_a). We have also studied the evolution of probability distribution for transmission (t) and reflection (r) coefficients obtained by averaging over a large number of realizations of random potentials in each case. We show that in the asymptotic regime (large length of disordered segment $L >> \xi$) the probability distribution of the reflection coefficient $P(r, L)$ tends to a stationary distribution $P_s(r)$. We have studied the properties of this stationary distribution by varying parameters associated with the disorder and absorption. The obtained stationary distribution differs qualitatively from that given by Eq. (2) and also exhibits several different features. Finally, we have studied the stationary phase distribution of the complex reflection coefficient and shown that it is nonuniform. The stationary distribution of the phase is uniform only in the limit of small disorder and weak absorption. In the next section we define our model Hamiltonian and numerical procedure. The sections after that are devoted to our results and conclusions.

II. HAMILTONIAN AND NUMERICAL PROCEDURE

We consider a tight-binding one-band Anderson model with diagonal disorder with a nearest-neighbor transfer-matrix element, namely

$$H = \sum \epsilon_n \mid n\rangle\langle n \mid + V \sum (\mid n\rangle\langle n+1 \mid + \mid n\rangle\langle n-1 \mid), \tag{3}$$

where the state $\mid n\rangle$ is an atomiclike orbital (Wannier orbital) with one degree of freedom per atom and the sites n form a one-dimensional lattice with a lattice spacing a, which we set to be unity. The site energies ϵ_n can be written in an equivalent form of $\epsilon_n - i\eta$, with real part of site energies ϵ_n assumed to be independent random variables distributed uniformly over a range $-W/2$ to $+W/2$ (width W) for $1 < n < N$ and zero otherwise, so we are considering a disordered chain of N atoms embedded in an infinite ordered chain. The imaginary part in the site energy ($-i\eta$) makes the Hamiltonian non-Hermitian and consequently leads to absorption of quasiparticles. We have taken the absorption parameter η to be a constant across a disordered segment $1 < n < N$ and zero otherwise. V is the nearest-neighbor hopping matrix element. Since all the relevent energies can be scaled by V, we can set V to unity without loss of generality.

We calculate t, r, and σ for the sample chain through a well-known transfer-matrix formalism.[14] Below we briefly describe the method. In the transfer-matrix geometry, the sample is connected with two semi-infinite perfect leads which are connected to two heat baths (reservoirs).

All nonequilibrium processes and inelastic scatterings, etc., are supposed to occur in the baths only. A plane wave (e^{ikn}, n is the site index) sent through a perfect lead undergoes only elastic scattering inside the sample. The eigenvalue equation of (3) reads as

$$(E - \epsilon_n)c_n = V(c_{n+1} + c_{n-1}),$$

where E is the Fermi energy and c_n's are the coefficients of expansions of the wave functions in a basis of atomic orbitals ($\mid n\rangle$). Now one can easily obtain a set of these coefficients for any length of the chain from the initial ones through a sequential product of transfer matrices as in the following:

$$\begin{pmatrix} c_N \\ c_{N-1} \end{pmatrix} = \prod_{i=1}^{N} T_i \begin{pmatrix} c_1 \\ c_0 \end{pmatrix},$$

where

$$T_i = \begin{pmatrix} (E - \epsilon_i)/V & -1 \\ 1 & 0 \end{pmatrix}.$$

The solutions on the two sides of the sample are related by a product matrix

$$M = \omega S^{-1} \prod_{i=1}^{N} T_i S,$$

where

$$\omega = \begin{pmatrix} e^{ik(N+1)} & 0 \\ 0 & e^{-ik(N+1)} \end{pmatrix}, \quad S = \begin{pmatrix} e^{-ik} & e^{ik} \\ 1 & 1 \end{pmatrix}.$$

The transmission (t) and reflection (r) coefficients are immediately obtainable from the matrix elements as

$$t = \frac{1}{\mid M_{11} \mid^2}, \quad r = \frac{\mid M_{12} \mid^2}{\mid M_{11} \mid^2}.$$

Note that usually where there is no absorber in the system, we have $\det M = 1$ and the complex product matrix (M) is Cayley type so that $\mid M_{11} \mid^2 - \mid M_{12} \mid^2 = 1$, which leads to $r + t = 1$. Here in our problem M does not remain the Cayley type matrix, so we have $r + t \neq 1$ and hence absorption $\sigma = 1 - (r + t)$.

III. RESULTS AND DISCUSSIONS

All results are shown for $E = 0$, i.e., we choose the Fermi energy of the incident beam to be at a midband energy. Physics of the problem does not change if one chooses some other energy. The figures are mostly self-explanatory. Below in all cases we take 5000 configurations to obtain an ensemble average of a quantity or its distribution.

In Fig. 1 we have shown the behavior of t for ordered absorptive medium ($W = 0, \eta = 0.1$), ensemble averaged disordered nonabsorptive medium ($W = 5.0, \eta = 0$), and disordered absorptive medium ($W = 5.0, \eta = 0.1$) as a function of length (L). In all these cases transmission decays exponentially. Figure 1 represents a case where $l < l_a$.

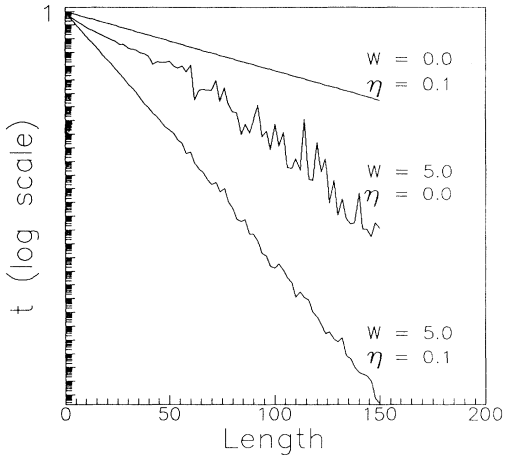

FIG. 1. Transmission coefficient (t) against system length (L), shown in a semilog plot to compare different length scales. Values of W and η are written in the figure for all three cases. Here $l < l_a$, where l is the localization length without absorbtion and l_a is that for absorption only.

In Fig. 2 we have given a transmission plot for a case where $l > l_a$. The parameter values are given in the figures themselves. From these two figures it is clear that irrespective of $l > l_a$ or $l < l_a$, the absorption induced length scale ξ in random medium associated with the decay of the transmission coefficient is always less than both l_a and l. We would like to point out that at the center of the band (i.e., at $E = 0$) l scales as $96V^2/W^2$,[15] and l_a scales as V/η. In the limit of weak disorder and absorption ($W/V < 1$ and $\eta/V < 1$), ξ scales[16] as $ll_a/(l + l_a)$. All the lengths are in units of the lattice spacing. The results obtained in Ref. 16 are based on random-phase approximation. Our numerical results are in conformity with earlier known analytical results[15,16] in the appro-

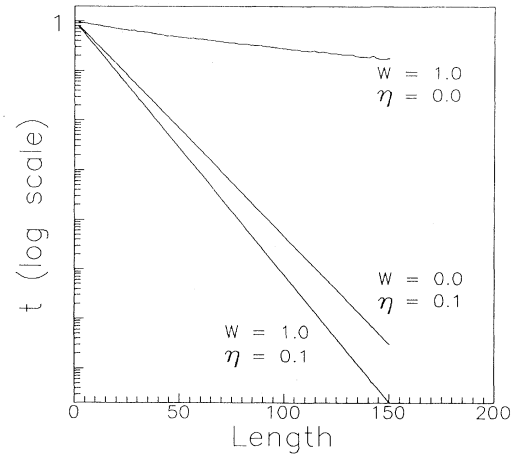

FIG. 2. Transmission coefficient (t) against system length (L), shown in a semilog plot to compare different length scales. Values of W and η are written in the figure for all three cases. In this case $l > l_a$.

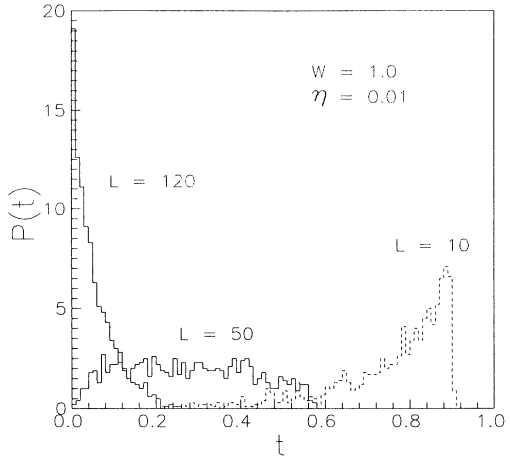

FIG. 3. Distribution of transmission coefficient $[P(t)]$ for three different lengths (L) for a fixed W and η.

priate domain of physical parameters.

In Fig. 3 we show the distribution $P(t)$ of transmission (t) for various lengths (L) of the sample for a very small value of $\eta = 0.01$ and a high degree of disorder $W = 1.0$. For small L, due to dominance of resonance transmission, along with small η $(= 0.01)$, as expected we observe $P(t)$ to peak at a higher value of t. In fact $P(t) = \delta(t-1)$ in the limit $L \to 0$. As we increase the length of the sample due to the interference between the multiply scattered electrons (which eventually leads to localization in the large length limit) the traversal time for an electron to diffuse across the sample increases thus increasing the absorption. This enhancement of absorption along with concomitent reflection due to disorder, $P(t)$ broadens and its magnitude at a large value of t gets suppressed. Also, as expected, for $L \to \infty$ (i.e., for $L > \xi$), $P(t) \to \delta(t)$.

In Fig. 4 we have plotted the stationary distributions

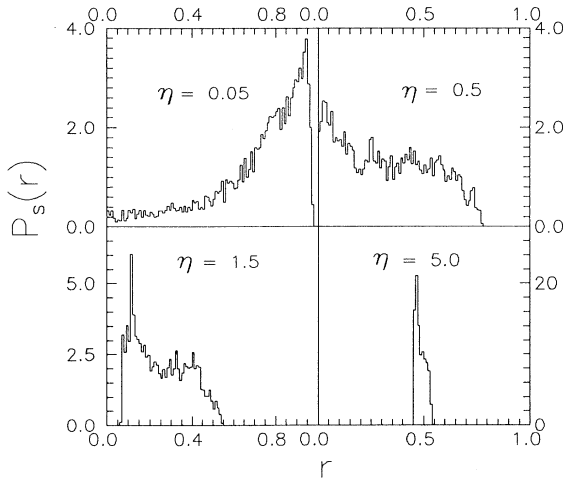

FIG. 4. Stationary distribution of reflection coefficient $[P_s(r)]$ for various values of absorption coefficient η for a fixed value of $W = 5.0$. 5000 configurations were taken for each time.

of $P_s(r)$ of r for various values of η. The length scale which dominates the evolution of $P(r)$ towards the stationary distribution is again given by ξ;[13] numerically we obtain stationary distribution for $L > \xi$. In the small parameter range of η, i.e., for $\eta = 0.05$, we see that the distribution has a single peak at larger value of $r = r_{\max}$ (note that r is bounded between 0 and 1) and $r_{\max}$ shifts to a lower side as we increase η. These graphs agree qualitatively with the behavior obtained from Eq. (2). As we increase η further $P_s(r)$ shows a double peak structure for a range of values of η and with further increase in η the peak at a lower value of r becomes narrow and the peak at the higher value of r disappears. The single narrow peak distribution now shifts to a higher value of r as η is increased again and finally in the limit of $\eta \to \infty$ this becomes a delta function $\delta(r-1)$. In the limit of large η the absorber acts as a reflector and disorder plays a subdominant role.

In Fig. 5 we have plotted the averaged value of absorption $\langle\sigma\rangle$ and reflection $\langle r\rangle$ obtained from the stationary distribution as a function of η, for a disorder $W = 5.0$. As we increase the strength of η the averaged absorption $\langle\sigma\rangle$ first increases and after exhibiting a maximum value at $\eta_{\max}$, $\langle\sigma\rangle$ decreases monotonically. For the value of $\eta > \eta_{\max}$ the absorber plays a dominant role as a reflector. This is also self-evident from looking at the behavior of $\langle r\rangle$ in Fig. 6. The dual role played by the absorber is already mentioned in the Introduction. It is interesting to note that the double peak structure in $P_s(r)$ appears for values of η close to $\eta_{\max}$. For given amount of disorder the total absorption increases with η for $\eta < \eta_{\max}$. This is because, as explained earlier in the presence of disorder due to multiple reflections the particle spends a large amount of time in the sample before getting reflected. This enhances the total absorption and the peak of $P_s(r)$ shifts to a lower value of r. However, as we approach $\eta_{\max}$ (beyond which the absorber plays a dominant role as a reflector), η plays a dual role of absorber as well as reflector in the crossover regime, and an additional peak in the $P_s(r)$ arises at lower r. As we increase η further the absorber plays a dominant role as a reflec-

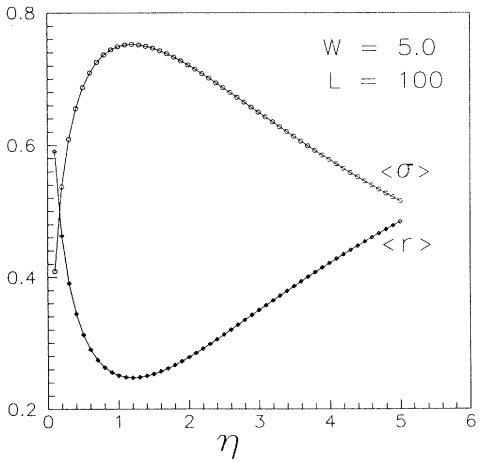

FIG. 5. This figure shows the variation of the absorption (σ) and reflection (r) coefficients (averaged over 5000 configurations) against η for a fixed value of W.

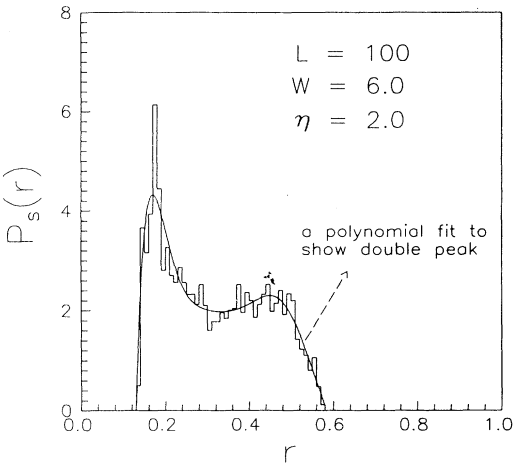

FIG. 6. This figure shows clearly the occurrence of a double peak in the stationary distribution of the reflection coefficient.

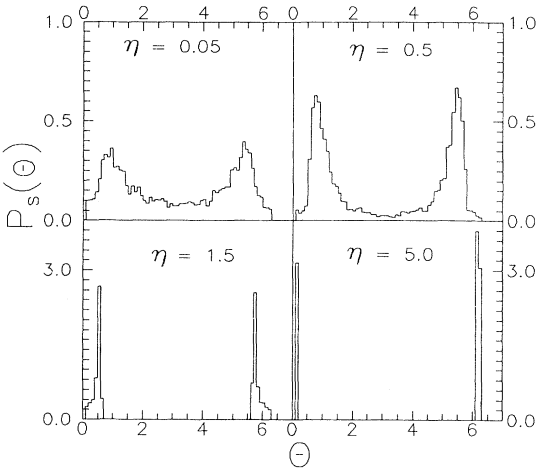

FIG. 7. Stationary distribution of a phase angle for the complex reflection coefficient for the corresponding cases for Fig. 6. $W = 5.0$.

tor thus suppressing the peak in $P_s(r)$ associated with disorder in conjunction with coherent absorption. The suppression in the peak in $P_s(r)$ associated with disorder can also be explained by noting that as η increases ξ decreases and average delay time in reflection decreases. The average delay time of the reflected electron is approximately given by the time taken by the electron to traverse (or diffuse) a distance of the order of ξ. Earlier studies[13] cannot describe our numerical observations: the existence of a double peak structure in $P_s(r)$ around $\eta \approx \eta_{\max}$ and a shift of peak in $P_s(r)$ to a large value of r for $\eta > \eta_{\max}$. For example, Eq. (2) can show at most one value of peak and for higher values of η it leads to an exponential behavior with a maximum value at $r = 0$, contrary to the physical expectations. Figure 6 shows a double peak structure in $P_s(r)$ for $\eta = 2.0$ and $W = 6.0$. A smooth line is drawn (a polynomial fit) as a guide to the eye to show the occurrence of two peaks.

In Fig. 7 we have plotted the stationary distribution of the phase $P_s(\theta)$ of the complex reflection amplitude for the parameters same as in Fig. 6. In general the phase distribution[17] should be uniform over 2π for small values of disorder ($W \ll 1$) and for $E \neq 0$ if one takes the system size to be greater than a typical length called the phase randomization length. But here one can readily notice that the phase distribution is highly nonuniform from the very beginning for even small values of η (where $W = 5.0$). As we increase η the phase distribution changes to have two prominent peaks and eventually they get separated. Ultimately as $\eta \to \infty$ the phase distribution tends to $\delta(\theta)$ and $\delta(\theta + 2\pi)$. One can readily calculate the phase of the complex reflection amplitude in the Schrödinger equation by putting $V = 0$ for $-\infty < x < 0$ and $V = -i\eta$ for $0 < x < \infty$ and verify that the phase tends to 0 or 2π in the limit $\eta \to \infty$. Figure 7 clearly suggests that the nonuniform phase distribution is a generic property of random systems in the presence of absorption. It is also well known in the earlier literature that the nonuniform phase distribution is a general property in a random system in the high disorder limit in the absence of absorption.[17,18] This fact has played an important role in determining scaling properties of resistance.[19] Now, the qualitative difference between our results and that given by (2) (results based on random-phase approximation) can be understood in terms of the breakdown of random-phase approximation. The results based on random-phase approximation are valid only in the limit $W < 1$ along with $\eta < 1$ such that $\eta/W < 1$.

IV. CONCLUSIONS

By studying the statistics of reflection and transmission coefficient in absorptive media we have shown that the stationary distribution for $P_s(r)$ for r differs qualitatively from the earlier known analytical results.[13] The source of this discrepancy is related to the random-phase approximations made in earlier studies. By studying the stationary distribution of the phase of the complex reflection coefficient we have shown that this distribution is in general not uniform. We recover uniform phase distribution only in the limit of weak disorder $W/V < 1$ and $\eta/V < 1$. In the presence of both disorder and absorption, the length ξ associated with the behavior of the average transmission coefficient is always less than l and l_a. We would also like to point out that the breakdown of random-phase approximation is valid even for the case of an amplifying medium which we have checked separately. In a recent work[6] it has been pointed out, by studying the higher moments of the transmission coefficient $\langle t^m \rangle$ ($m \gg 1$), that a scale (ξ_m) appears, such that $l < \xi_m < l_a$. This result is interesting and at the same time surprising. Again this study is based on the random-phase approximation. To establish the validity of this result more work beyond random-phase approximation has to be carried out. Work along this direction is in progress.

ACKNOWLEDGMENTS

One of us (A.K.G.) thanks Institute of Physics, Bhubaneswar, for hospitality, where a major part of this work was carried out.

* Electronic address: abhi@hp1.saha.ernet.in
† Electronic address: jayan@iopb.ernet.in

[1] See *Scattering and Localization of Waves in Random Media*, edited by Ping Sheng (World Scientific, Singapore, 1990).
[2] P.A. Lee and T.V. Ramakrishnan, Rev. Mod. Phys, **57**, 287 (1985).
[3] R.L. Weaver, Phys. Rev. B **47**, 1077 (1993).
[4] M. Yosefin, Europhys. Lett. **25**, 675 (1994).
[5] A.M. Jayannavar, Phys. Rev. B **49**, 14 718 (1994).
[6] V. Freilikher, M. Pustilnik, and I. Yurkevich, Phys. Rev. Lett. **73**, 810 (1994).
[7] S. John, Phys. Rev. Lett. **53**, 2169 (1984).
[8] A.Z. Genack, Phys. Rev. Lett. **58**, 2043 (1986).
[9] A.Z. Genack and Garcia, Phys. Rev. Lett. **66**, 2064 (1991).
[10] A.D. Stone and P.A. Lee, Phys. Rev. Lett. **54**, 1196 (1985).
[11] Y. Zohta and H. Ezawa, J. Appl. Phys. **72**, 3584 (1992).
[12] A. Rubio and N. Kumar, Phys. Rev. B **47**, 2420 (1993).
[13] P. Pradhan and N. Kumar, Phys. Rev. B **50**, 9644 (1994).
[14] See, e.g., Y. Liu and K.A. Chao, Phys. Rev. B **34**, 5247 (1986); P.K. Thakur, C. Basu, A. Mookerjee, and A.K. Sen, J. Phys. Condens. Matter **4**, 6095 (1992).
[15] E.N. Economou, *Green's Functions in Quantum Physics*, 2nd ed. (Springer-Verlag, Berlin, 1983), p. 174.
[16] R. Rammal and B. Doucot, J. Phys. (Paris) **48**, 509 (1987).
[17] A.D. Stone, D.C. Allan, and J.D. Joannopoulos, Phys. Rev. B **27**, 863 (1983).
[18] A.M. Jayannavar, Solid State Commun. **73**, 247 (1990).
[19] A.M. Jayannavar, Pramana J. Phys. **36**, 611 (1991).