Spin correlations in low-dimensional spin-$\frac{1}{2}$ antiferromagnets

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We study the long-range behavior of spin correlations in low-dimensional spin-$\frac{1}{2}$ antiferromagnets with both Ising anisotropy and spatial anisotropy. We represent the spin operators in terms of spin-density operators and spin-phase operators. Then for spin-$\frac{1}{2}$ antiferromagnets, within a simple harmonic approximation, we obtain expressions for the spin correlations in the ground state. For the one-dimensional linear chain, we find that the two leading terms in spin correlation are $(-1)^r c/r^\eta - c_1/r^2$ for large distance r. The power η depends on the Ising anisotropy, and $\eta=0.405$ at XY point and $\eta=1.062$ at Heisenberg point. For the two-dimensional square lattice, there is long-range staggered magnetization, and the dependence of magnetization on Ising anisotropy is given. For the crossover from one-dimensional linear chain to two-dimensional square lattice, we find that the disorder-order transition occurs as long as the interchain coupling is introduced. We believe that our method gives a unified approach to the long-range behavior of spin correlations in low-dimensional spin-$\frac{1}{2}$ antiferromagnets.

I. INTRODUCTION

In the past few years there has been increasing interest in the study of low-dimensional spin-$\frac{1}{2}$ Heisenberg antiferromagnets, because of their relevance to the undoped high-temperature superconducting cuprate. In these systems, the physical pictures obtained from the classical approach are often greatly modified or even contradicted as a result of strong quantum fluctuations and topological effects. Haldane[1] conjectured that the one-dimensional integer-spin chain with nearest-neighbor coupling has an energy gap in the spin excitation spectrum and that the spin correlation decays exponentially with distance, whereas half-odd-integer-spin chain is gapless and the spin correlation decreases algebraically with distance. In two-dimensional square lattices (SL's), it is well established that the spin excitation is gapless and there is long-range antiferromagnetic order (LRO) in the ground state, regardless of the magnitude of spins.[2] For the case of integer spins, Schwinger-boson mean-field theory[3] has proved qualitatively correct for both the one-dimensional linear chain (LC) and the SL. In the crossover from the LC to the SL, it is believed that a finite critical interchain coupling is required to undergo the transition from the disordered phase in one dimension to the ordered phase in two dimensions.[4] For the spin-$\frac{1}{2}$ case, a representative of half-odd-integer spins, there is an exact solution in one dimension, but exact calculation of the spin correlation has not been obtained. The most powerful analytical method used is the bosonization technique,[5,6] which gives the quantitatively correct behavior of the spin correlation in one dimension. However, this method is difficult to extend to higher dimensions. In the SL, the established conclusion that there is LRO in the ground state[2] validates some analytical methods based on the assumption of the existence of LRO, such as spin-wave theory. In the crossover from the LC to the SL, there are still controversies on whether there is a finite critical interchain coupling in the disorder-order transition.[7-9] Therefore a unified approach which can recover the results of both the LC and the SL is necessary to provide more reliable conclusions on this problem.

In this work, we study the spin correlations of low-dimensional spin-$\frac{1}{2}$ antiferromagnets with both Ising anisotropy Δ and spatial anisotropy α. We represent the spin operators in terms of spin-density operators and their Hermitian conjugate, spin-phase operators. For spin $\frac{1}{2}$, within a simple harmonic approximation, we obtain expressions for the spin correlations. In the ground state of the LC, the spin correlation has the form $(-1)^r c/r^\eta - c_1/r^2 + O(r^{-2-\eta})$ for large distance r, with η depending on Δ and $\eta=0.405$ at the XY point, $\eta=1.062$ at the Heisenberg point. The first term in the spin correlation was predicted by Haldane[1] with $\eta=1$ at the Heisenberg point. The dependence of η on Δ was obtained by Luther and Peschel using the Abelian bosonization technique.[5] The second term is consistent with various theoretical results.[10,11] In the ground state of the SL, it is generally believed that there is long-range staggered magnetization. In the crossover from the LC to the SL, we find that the disorder-order transition occurs as long as an interchain coupling is introduced. This conclusion is reliable, since the same approximation recovers the correct results of both the LC and the SL, the two limiting cases of the crossover from the LC to the SL. The physical reason is that the spin-$\frac{1}{2}$ LC antiferromagnet is at its critical point. Therefore it is unstable against any finite interchain coupling. We think that our treatment, which takes the ubiquitous spin fluctuations as a starting point and does not assume the existence of LRO, provides a unified approach to the long-range behavior of spin correlations in low-dimensional spin-$\frac{1}{2}$ antiferromagnets.

The rest of the paper is organized as follows. In the next section we give the formalism and the self-consistent equations within a harmonic approximation in detail. Section III gives results for the LC, the SL, and the crossover from the LC to the SL. Section IV is a summary,

II. FORMALISM AND THE HARMONIC APPROXIMATION

The low-dimensional spin-$\frac{1}{2}$ antiferromagnets we consider are defined on a two-dimensional lattice of N sites by the Hamiltonian

$$H=\sum_{i,\delta} J_\delta(S_i^xS_{i+\delta}^x+S_i^yS_{i+\delta}^y+\Delta S_i^zS_{i+\delta}^z) , \qquad (1)$$

where $\mathbf{S}_i$ are spin-S operators, $\delta=\pm x,\pm y$ denotes the four nearest-neighbor sites, and Δ represents the Ising anisotropy, $\Delta=0$ for the XY model and $\Delta=1$ for the Heisenberg model. J_δ is defined as $J_{\pm x}=1$, $J_{\pm y}=\alpha$, with α denoting the spatial anisotropy, $\alpha=0$ for the LC, $\alpha=1$ for the SL, and $0<\alpha<1$ for the crossover from the LC to the SL.

We introduce two Hermitian operators ξ_i and θ_i, with the commutation relations $[\xi_i,\xi_j]=[\theta_i,\theta_j]=0$, $[\xi_i,\theta_j]=i\delta_{ij}$; i.e., ξ_i and θ_i are conjugate with each other. Then the transformation

$$S_i^+=(-1)^{M_i}(S+\xi_i)^{1/2}\exp(-i\theta_i)(S-\xi_i)^{1/2} , \qquad (2a)$$

$$S_i^-=(-1)^{M_i}(S-\xi_i)^{1/2}\exp(i\theta_i)(S+\xi_i)^{1/2} , \qquad (2b)$$

$$S_i^z=\xi_i , \qquad (2c)$$

together with the constraint

$$\prod_{n=-S}^{S}(\xi_i-n)=0 , \qquad (2d)$$

constitutes a faithful representation of the spin operators.[12] Here $M_i=x_i+y_i$ is the Manhattan distance of the lattice. In this representation, Hamiltonian (1) becomes

$$H=\sum_{i,\delta} J_\delta\left[-\frac{1}{2}(S+\xi_i)^{1/2}e^{-i\theta_i}(S-\xi_i)^{1/2}(S-\xi_{i+\delta})^{1/2}e^{i\theta_{i+\delta}}(S+\xi_{i+\delta})^{1/2}+\text{H.c.}+\Delta\xi_i\xi_{i+\delta}+\lambda_i\prod_{n=-S}^{S}(\xi_i-n)\right] , \qquad (3)$$

where λ_i are Lagrange multipliers to ensure the constraint (2d).

Hamiltonian (3) involves the coupling of $2N$ variables. In the case of $S=\frac{1}{2}$, the constraint (2d) is quadratic, $\xi_i^2-\frac{1}{4}=0$. We make the following approximations: (a) a long-wavelength approximation of phase operators, i.e.,

$$\tfrac{1}{2}e^{i(\theta_{i+\delta}-\theta_i)}+\text{H.c.}\approx 1-\tfrac{1}{2}(\theta_{i+\delta}-\theta_i)^2 .$$

(b) The coefficient of $e^{i(\theta_{i+\delta}-\theta_i)}$ in the first term of Hamiltonian (3), which is a function of ξ_i and $\xi_{i+\delta}$, is replaced by its average. Under the constraint (2d),

$$(\tfrac{1}{2}\pm\xi_i)^{1/2}(\tfrac{1}{2}\pm\xi_{i+\delta})^{1/2}=(\tfrac{1}{2}\pm\xi_i)(\tfrac{1}{2}\pm\xi_{i+\delta})\approx\tfrac{1}{4}+\langle\xi_i\xi_{i+\delta}\rangle ,$$

and (c) the Lagrange multipliers λ_i are replaced by a static and uniform value λ. Under these approximations, apart from a constant, Hamiltonian (3) for $S=\frac{1}{2}$ is

$$H_{\text{HA}}=\sum_{i,\delta}J_\delta[(\tfrac{1}{4}+\langle\xi_i\xi_{i+\delta}\rangle)(\theta_i-\theta_{i+\delta})^2/2+\Delta\xi_i\xi_{i+\delta}+\lambda\xi_i^2] . \qquad (4)$$

This is nothing but N coupled harmonic oscillators. Accordingly, we term the above approximations as a harmonic approximation. Hamiltonian (4) can be diagonalized in momentum representation. Let

$$\xi_i=N^{-1/2}\sum_k\xi_k e^{i\mathbf{k}\cdot\mathbf{r}_i} , \quad \theta_i=N^{-1/2}\sum_k\theta_k e^{i\mathbf{k}\cdot\mathbf{r}_i} ;$$

then,

$$H_{\text{HA}}=\sum_k(A_0-A_k)\theta_k\theta_{-k}+[2\lambda(1+\alpha)+B_k]\xi_k\xi_{-k} , \qquad (5)$$

where

$$A_k=\sum_\delta J_\delta(\tfrac{1}{4}+a_\delta)e^{i\mathbf{k}\cdot\boldsymbol{\delta}} , \qquad (6a)$$

$$B_k=\sum_\delta J_\delta\Delta e^{i\mathbf{k}\cdot\boldsymbol{\delta}} , \qquad (6b)$$

and $a_\delta=\langle\xi_i\xi_{i+\delta}\rangle$. Under the transformation

$$\beta_k=(\alpha_k/2)^{1/2}(\xi_k-i\theta_{-k}/\alpha_k) ,$$
$$\beta_k^+=(\alpha_k/2)^{1/2}(\xi_{-k}+i\theta_k/\alpha_k) , \qquad (7)$$

ξ_k and θ_k can be expressed in terms of β_k and β_k^+,

$$\xi_k=(2\alpha_k)^{-1/2}(\beta_k+\beta_{-k}^+) , \qquad (8a)$$

$$\theta_k=-i(\alpha_k/2)^{1/2}(\beta_k^+-\beta_{-k}) . \qquad (8b)$$

Substituting (8a) and (8b) into (5) and choosing α_k so that the off-diagonal terms in H_{HA} vanish, then we have

$$\alpha_k=[(2\lambda(1+\alpha)+B_k)/(A_0-A_k)]^{1/2} . \tag{9}$$

And (5) becomes

$$H_{\mathrm{HA}}=\sum_k \varepsilon_k(\beta_k^+\beta_k+\tfrac{1}{2}) , \tag{10}$$

where

$$\varepsilon_k=2[(2\lambda(1+\alpha)+B_k)(A_0-A_k)]^{1/2} \tag{11}$$

is the energy spectrum of excitation.

The Z-component spin correlation in the ground state is

$$\begin{aligned}\langle S_0^z S_r^z\rangle &= \langle \xi_0\xi_r\rangle \\ &= \frac{1}{N}\sum_k \langle \xi_k\xi_{-k}\rangle e^{i\mathbf{k}\cdot\mathbf{r}} \\ &= \frac{1}{N}\sum_k \frac{1}{2\alpha_k}e^{i\mathbf{k}\cdot\mathbf{r}} .\end{aligned} \tag{12}$$

Therefore the parameters λ and a_δ for given Δ and α are determined by the self-consistent equations

$$\langle \xi_i^2\rangle=\frac{1}{N}\sum_k \frac{1}{2\alpha_k}=\frac{1}{4} , \tag{13a}$$

$$\langle \xi_i\xi_{i+\delta}\rangle=\frac{1}{N}\sum_k \frac{1}{2\alpha_k}e^{i\mathbf{k}\cdot\boldsymbol{\delta}}=a_\delta . \tag{13b}$$

The XY-component spin correlation can be obtained from

$$\begin{aligned}\langle S_0^+S_r^-\rangle &\approx (-1)^r\tfrac{1}{2}\langle e^{i(\theta_r-\theta_0)}\rangle \\ &= (-1)^r\tfrac{1}{2}e^{-\frac{1}{2}\langle(\theta_r-\theta_o)^2\rangle} ,\end{aligned} \tag{14}$$

where spin-phase correlation

$$\begin{aligned}\langle(\theta_r-\theta_0)^2\rangle &= \frac{1}{N}\sum_k \langle \theta_k\theta_{-k}\rangle[2-2\cos(\mathbf{k}\cdot\mathbf{r})] \\ &= \frac{1}{N}\sum_k \alpha_k[1-\cos(\mathbf{k}\cdot\mathbf{r})] .\end{aligned} \tag{15}$$

For given Δ and α, the parameters λ and a_δ are determined by (13a) and (13b). The spin correlations are given by (12), (14), and (15). In the next section, the solutions for the cases of the LC, the SL, and the crossover from the LC to the SL are presented in detail.

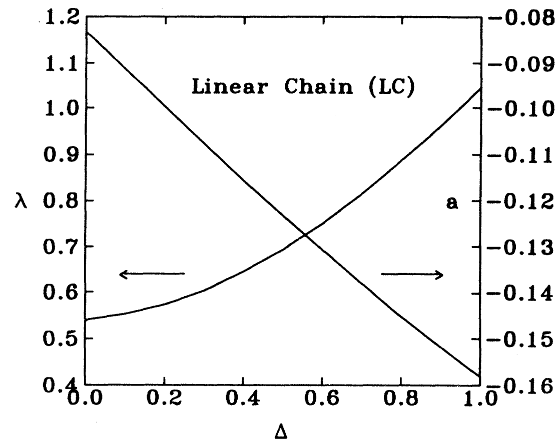

FIG. 1. Parameters λ and a vs Ising anisotropy Δ for LC's.

III. RESULTS FOR VARIOUS CASES

A. Independent linear chain

In the case of the LC, $\alpha=0$. Let $a_{\pm x}=\langle \xi_0\xi_{\pm x}\rangle=a$; then,

$$A_k=2(\tfrac{1}{4}+a)\cos k_x , \tag{16a}$$

$$B_k=2\Delta\cos k_x , \tag{16b}$$

and

$$\alpha_k=\left[\frac{\lambda+\Delta\cos k_x}{(\frac{1}{4}+a)(1-\cos k_x)}\right]^{1/2} . \tag{16c}$$

Substituting (16a)–(16c) into the self-consistent equations (13a) and (13b), we obtain λ and a for given Δ. At the XY point, $\Delta=0$, $\lambda=0.540$, $a=-0.083$, and at the Heisenberg point, $\Delta=1$, $\lambda=1.046$, $a=-0.158$. The dependence of λ and a on Δ in the interval $0\le\Delta\le1$ is shown in Fig. 1.

From (12), the Z-component spin correlation is

$$\begin{aligned}\langle S_0^zS_r^z\rangle &= \frac{1}{2N}\sum_k\left[\frac{(\frac{1}{4}+a)(1-\cos k_x)}{\lambda+\Delta\cos k_x}\right]^{1/2}e^{i\mathbf{k}\cdot\mathbf{r}} \\ &\approx -\frac{1}{4\pi r^2}\left[\frac{2(\frac{1}{4}+a)}{\lambda+\Delta}\right]^{1/2}+O(r^{-3}) , \quad r\gg1 ,\end{aligned} \tag{17}$$

and the spin-phase correlation is

$$\begin{aligned}\langle(\theta_0-\theta_r)^2\rangle &= \frac{1}{N}\sum_k\left[\frac{\lambda+\Delta\cos k_x}{(\frac{1}{4}+a)(1-\cos k_x)}\right]^{1/2}[1-\cos(\mathbf{k}\cdot\mathbf{r})] \\ &\approx \frac{2}{\pi}\left[\frac{\lambda+\Delta}{2(\frac{1}{4}+a)}\right]^{1/2}\left[C+\ln(\pi r)+\frac{(-1)^r}{(\pi r)^2}+O(r^{-4})\right] , \quad r\gg1 ,\end{aligned} \tag{18}$$

where $C \approx 0.5772$ is Euler's constant. So the XY-component spin correlation is

$$\langle S_0^+ S_r^- \rangle \approx (-1)^r \frac{c}{r^\eta} \left[1 - \eta \frac{(-1)^r}{(\pi r)^2} + O(r^{-4}) \right], \quad r \gg 1 , \tag{19}$$

where exponent η and constant c are

$$\eta = \frac{1}{\pi} \left[\frac{\lambda + \Delta}{2(\frac{1}{4}+a)} \right]^{1/2}, \tag{20a}$$

$$c = \frac{1}{2} \exp \left[-\frac{1}{\pi} \left[\frac{\lambda+\Delta}{2(\frac{1}{4}+a)} \right]^{1/2} (C + \ln \pi) \right]. \tag{20b}$$

Figure 2 shows the dependence of the exponent η on Δ for $0 \le \Delta \le 1$. At the XY point, $\Delta = 0$, $\eta = 0.405$, $c = 0.249$. At the Heisenberg point, $\Delta = 1$, $\eta = 1.062$, $c = 0.161$.

From (17) and (19), the total spin correlation, to the first two leading terms for a large distance r, is readily expressed as

$$\langle \mathbf{S}_0 \cdot \mathbf{S}_r \rangle \approx (-1)^r \frac{c}{r^\eta} - \frac{c_1}{r^2} + O(r^{-2-\eta}) , \quad r \gg 1 . \tag{21}$$

Such a long-range power law decay of the spin correlation with distance was predicted by Haldane for the half-odd-integer-spin LC at the Heisenberg point with $\eta = 1$.[1] At the XY point, McCoy first obtained the exact value $\eta = \frac{1}{2}$.[13] Using the Abelian bosonization technique, Luther and Peschel obtained $\eta = \frac{1}{2}[(\pi + 2\Delta)/(\pi - 2\Delta)]^{1/2}$ in the continuum limit and $\eta = \frac{1}{2} + (1/\pi)\arcsin\Delta$ in the discrete limit.[5] The second term in (21), which comes from the Z-component spin correlation, is in agreement with the numerical results of Liang[10] and the analytical results of Affleck.[11]

B. Square lattice

In the case of the SL, $\alpha = 1$. From the symmetry of (13b), let $a_x = \langle \xi_0 \xi_{\pm x} \rangle = a_y = \langle \xi_0 \xi_{\pm y} \rangle = a$; then,

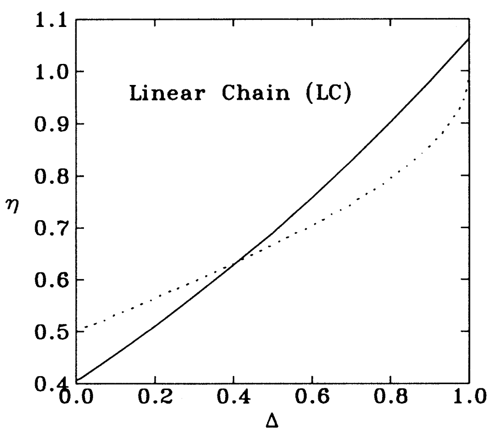

FIG. 2. Exponent η vs Δ for LC's. The solid line is our result. The dashed line is the result in Ref. 5, $\eta = \frac{1}{2} + (1/\pi)\arcsin\Delta$.

$$A_k = 4(\tfrac{1}{4} + a)\gamma_k , \tag{22a}$$

$$B_k = 4\Delta \gamma_k , \tag{22b}$$

and

$$\alpha_k = \left[\frac{\lambda + \Delta\gamma_k}{(\frac{1}{4}+a)(1-\gamma_k)} \right]^{1/2}, \tag{22c}$$

where $\gamma_k = (\cos k_x + \cos k_y)/2$. Substituting (22a)–(22c) into (13a) and (13b), we get λ and a for a given Δ. For example, at the XY point, $\Delta = 0$, $\lambda = 0.784$, $a = -0.037$, and at the Heisenberg point, $\Delta = 1$, $\lambda = 1.040$, $a = -0.082$. The dependence of λ and a on Δ is shown in Fig. 3.

From (12), we have

$$\lim_{r \to \infty} \langle S_0^z S_r^z \rangle = \lim_{r\to\infty} \frac{1}{2N} \sum_k \left[\frac{(\frac{1}{4}+a)(1-\gamma_k)}{\lambda + \Delta\gamma_k} \right]^{1/2} e^{i\mathbf{k}\cdot\mathbf{r}} = 0 . \tag{23}$$

From (15), we have the spin-phase correlation

$$\langle (\theta_\infty - \theta_0)^2 \rangle = \lim_{r\to\infty} \langle (\theta_r - \theta_0)^2 \rangle = \frac{1}{N} \sum_k \left[\frac{\lambda + \Delta\gamma_k}{(\frac{1}{4}+a)(1-\gamma_k)} \right]^{1/2}, \tag{24}$$

and the XY-component spin correlation

$$\lim_{r\to\infty} \langle S_0^+ S_r^- \rangle \approx (-1)^r \tfrac{1}{2} \exp(-\tfrac{1}{2} \langle (\theta_\infty - \theta_0)^2 \rangle) . \tag{25}$$

From (23)–(25), for $r \to \infty$, we find that the spin correlation is determined by its XY component. It does not vanish for two infinitely separated sites; i.e., there is LRO in the ground state. The corresponding staggered magnetization is

$$m \approx (\tfrac{1}{2})^{1/2} \exp(-\tfrac{1}{4} \langle (\theta_\infty - \theta_0)^2 \rangle) . \tag{26}$$

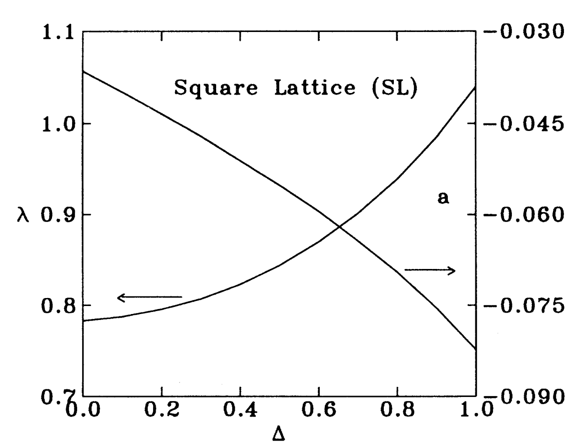

FIG. 3. Parameters λ and a vs Ising anisotropy Δ for SL's.

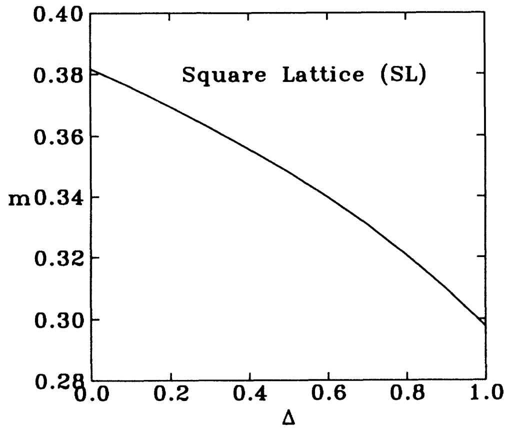

FIG. 4. Dependence of staggered magnetization m on Δ for SL's.

In the ground state of the SL, the existence of LRO was exactly proved by Kubo and Kishi for $0\leq\Delta\leq 0.13$.[2] At the XY point, $\Delta=0$, $m=0.382$, and at the Heisenberg point, $\Delta=1$, $m=0.297$, which is very close to the second-order spin wave results $m=0.3007$.[14] The dependence of staggered magnetization m on Δ is shown in Fig. 4.

C. Crossover from the LC to the SL

Since for the spin-$\frac{1}{2}$ Heisenberg model LRO does not exist in the ground state of the LC, but does exist in the ground state of the SL, it is natural to ask whether there is a finite interchain coupling α_c at which the disorder-order transition occurs. Sakai and Takahashi,[7] combining a mean-field treatment with one-dimensional exact diagonalization, found that once the interchain coupling was introduced, LRO persists; i.e., $\alpha_c=0$. Azzous,[8] using a mean-field approximation in the Wigner-Jordan fermion representation, reached the same conclusion. Very recently, Parola, Sorella, and Zhang[9] proposed that there is a disorder-order transition at a finite interchain coupling $\alpha_c\approx 0.1$.

Since our method gives the correct results of both the LC and the SL, we naturally use it to deal with the case of the crossover from the LC to the SL. Without loss of generality, we only confine our discussions to the Heisenberg model, i.e., let $\Delta=1$. To find out the behavior of spin correlations in the crossover from the LC to the SL, i.e., $0<\alpha<1$, set $a_x=\langle\xi_0\xi_{\pm x}\rangle$ and $a_y=\langle\xi_0\xi_{\pm y}\rangle$; then,

$$A_k=2(\tfrac{1}{4}+a_x)\cos k_x+2\alpha(\tfrac{1}{4}+a_y)\cos k_y\ , \tag{27a}$$

$$B_k=2(\cos k_x+\alpha\cos k_y)\ , \tag{27b}$$

and

$$\alpha_k=\left[\frac{\lambda(1+\alpha)+\cos k_x+\alpha\cos k_y}{(\frac{1}{4}+a_x)(1-\cos k_x)+\alpha(\frac{1}{4}+a_y)(1-\cos k_y)}\right]^{1/2} . \tag{27c}$$

From (27c) and the self-consistent equations (13a) and (13b), λ, a_x, and a_y can be determined once α is given. For example, $\alpha=0.1$, $\lambda=1.0457$, $a_x=-0.1302$, and $a_y=-0.0311$. For $0<\alpha<1$, the values of a_x and a_y interpolate between the values of the LC and the SL.

Similar to those of the SL, we have

$$\lim_{r\to\infty}\langle S_0^zS_r^z\rangle=0\ , \tag{28}$$

$$\langle(\theta_\infty-\theta_0)^2\rangle=\frac{1}{N}\sum_k\left[\frac{\lambda(1+\alpha)+\cos k_x+\alpha\cos k_y}{(\frac{1}{4}+a_x)(1-\cos k_x)+\alpha(\frac{1}{4}+a_y)(1-\cos k_y)}\right]^{1/2} . \tag{29}$$

The long-range behavior of the spin correlation is also dominated by the XY component. The staggered magnetization is

$$m\approx(\tfrac{1}{2})^{1/2}\exp(-\tfrac{1}{4}\langle(\theta_\infty-\theta_0)^2\rangle)\ . \tag{30}$$

Note that as long as $\alpha\neq 0$, both a_x and a_y do not vanish, and the spin correlation for two infinitely separated sites approaches a finite value. The staggered magnetization m is nonzero for arbitrarily small interchain coupling. Therefore the critical value of the interchain coupling is equal to zero. Any finite value of α will give rise to a finite staggered magnetization m. For example, $\alpha=1.0\times10^{-5}$, $\lambda=1.046$, $a_x=-0.158$, $a_y=-8.14\times10^{-6}$, and $m=0.02$. Figure 5 shows the dependence of m on α.

Our result is in agreement with those of Sakai and Takahashi[7] and Azzouz.[8] We think that our result is more reliable since the same method gives the quantitatively correct behavior of the two opposite limiting cases of the crossover from the LC to the SL. The spin-$\frac{1}{2}$ results are different from those of integer spins, in which a finite critical value of interchain coupling is required to establish the ordered phase. The physical reason is as follows. For the spin-$\frac{1}{2}$ LC, the gapless excitation and algebraical decay of the spin correlation imply that the spin correlation length is infinite. Although there is no true LRO, it is at the critical point and unstable against any finite interchain coupling. But for integer-spin linear chains, there is an excitation gap, and the spin correlation

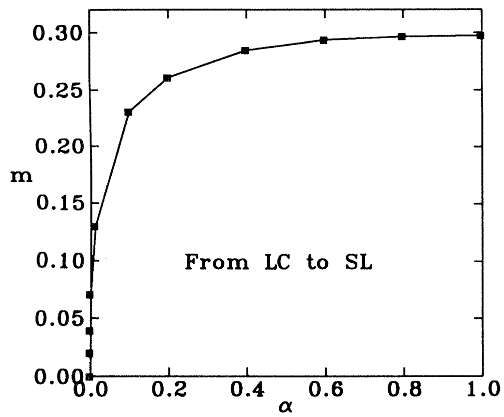

FIG. 5. Staggered magnetization m vs spatial anisotropy α for the crossover from LC's to SL's.

decreases exponentially with distance. Therefore a finite interchain coupling is required to make the gap to vanish and LRO to appear.

IV. SUMMARY

We studied the spin correlations for the LC, the SL, and the crossover from the LC to the SL. Within a simple harmonic approximation, we found that the spin correlation decreases algebraically with distance for the LC, whereas for the SL, there is LRO in the ground state. For the crossover from the LC to the SL, there is LRO for any finite interchain coupling.

In our harmonic approximation, the starting point is the existence of local spin fluctuations, which are ubiquitous in low-dimensional antiferromagnetic systems. Although we do not assume the existence of LRO, it does emerge for the SL. Our method is particularly applicable to low-dimensional antiferromagnetic systems, where the existence of LRO is not yet as clear as in three dimensions. Unlike the bosonization technique, which represents the Wigner-Jordan fermions in terms of boson operators, we directly use the boson nature of the spin operators. Therefore it is readily valid in dimensions higher than 1, as our results indicated. The remaining problem concerning the extension to spins higher than $\frac{1}{2}$ is that the constraint (2d), although different for half-odd-integer spins and integer spins [i.e., the constraint (2d) is even power of spin density for half-odd-integer spins and is odd power of spin density for integer spins], is a power of a spin density larger than 2 for $S > \frac{1}{2}$. This may invalidate the simple harmonic approximation.

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