Density profile in a weakly modulated two-dimensional system in a magnetic field

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The particle density fluctuations generated by a weak unidirectional periodic potential in a two-dimensional interacting electron system, in a strong perpendicular magnetic field, are calculated using a local-density description of the exchange-correlation energy for the lowest Landau level. The correlation effects are discussed with respect to previous results obtained in the Hartree and Hartree-Fock approximations. In the spatial regions where the local filling factor is in the range 0.3–0.7, the correlation corrections cancel the exchange contributions, such that the Hartree picture is shown to be qualitatively correct. In these regions, the external potential is strongly screened by the electronic system. In the other spatial regions the density has higher order oscillations with a specific wavelength, which disappear with increasing temperature or the external potential amplitude. Possible experimental evidence of these oscillations in modulated structures is discussed.

I. INTRODUCTION

Several spectacular effects are produced by the electron-electron interactions in two-dimensional (2D) systems in a perpendicular magnetic field, the most exciting being the fractional quantum Hall effect. While this seems impossible to describe in a standard many-body theory, there are other important interaction effects which can be reasonably explained: the nonlinear dielectric response, the negative compressibility, or the exchange enhancement of the spin splitting. These properties can be understood, at least qualitatively, in one of the various forms of the Hartree or Hartree-Fock approximations.

In an external potential, assumed sufficiently smooth on the magnetic length scale, the electrons are localized on the equipotential lines of the total electrostatic potential.[1,2] In this situation, it makes sense to speak about a local filling factor (LFF) of the Landau levels, whose degeneracy is lifted. At temperatures much lower than the cyclotron energy in the spatial regions where the LFF has integer values, the electron system cannot screen the external potential. On the contrary, in the complementary regions, where the charge redistribution is allowed, the screening is strong, eventually perfect. Consequently, the resulting Landau bands have a small dispersion in the vicinity of the Fermi level, where they are almost insensitive to the variations of the external potential. This pinning effect has been obtained in the Hartree approximation for a one-dimensional periodic external potential (lateral modulation),[3] and more recently, in a more pronounced form, in the presence of an electrostatic lateral barrier.[4–6]

A more general approach, based on descriptions of the total energy in terms of the local particle density, has been used by Efros for disordered, weak external potentials,[7–10] with essentially the same results for the screening. The exchange-correlation energy has also been taken into account,[9,10] in the form proposed by Fano and Ortolani,[11] in good agreement with magnetocapacitance measurements,[12] and with the related negative compressibility for filling factors close to integers.[13–15]

In a recent paper,[16] the Landau bands for a lateral modulation are calculated in a thermodynamic Hartree-Fock approximation (HFA) and several exchange effects have been obtained. They originate in the negative exchange energy acquired by the occupied single-particle states.

First, the spin splitting of the Landau bands may be strongly enhanced at the Fermi level, in GaAs up to values comparable with the cyclotron energy.[12,17] For modulation wavelengths much longer than the magnetic length, the enhancement is strong even for a modulation amplitude much larger than the cyclotron energy, while for shorter wavelengths, the enhancement is easily suppressed by the modulation.[16]

A second exchange effect is the possible negative sign of the thermodynamic density of states D_T (which for homogeneous systems is proportional with the compressibility) even for strong modulations. For zero modulation the HFA gives also $D_T < 0$, but $1/D_T$ is always a convex function of the filling factor, in disagreement with the experimental data[13–15] and with the mentioned calculations.[9]

Another effect is the exchange broadening of the Landau bands intersecting the Fermi level. The bands width self-consistently determines the amplitude of the total (external + induced) electrostatic potential. Therefore, on one hand, the screening may be diminished to some extent, and on the other hand, its nonlinear character is enhanced.

A peculiar nonlinear effect is the appearance of additional oscillations in the particle density inside a period of a long wavelength modulation. These oscillations have their own period, about 3–5 magnetic lengths, and are very strong in the presence of a weak external potential, but they can survive even for an amplitude exceeding

several times the cyclotron energy.[16] They can also be removed by increasing the temperature. Recent calculations in models for quantum dots show similar density fluctuations, not only in the HFA,[18] but also in a more sophisticated approach derived from the current-density functional theory.[19]

For a vanishing external modulation, the Hartree-Fock equations give a constant particle density. However, this may not be a stable thermodynamic state; if a spontaneous charge-density wave (CDW) is assumed, the corresponding free energy may be lower than for the homogeneous state.[20,21] Actually, the HFA predicted an inhomogeneous ground state for any filling factor ν of the lowest Landau level, while further experimental and theoretical investigations devoted to the fractional quantum Hall effect have evidenced homogeneous states,[22] except for small filling factors, $\nu < 0.2 - 0.3$, when the Wigner crystallization is currently believed to occur.[22–25]

In the presence of a weak external modulation, those CDW's may be present as higher harmonics in the calculated density response, leading to the mentioned additional oscillations (not necessarily for the lowest Landau level). However, the HFA is expected to overestimate these oscillations, as it happens in the case without a superimposed external potential.[20,21]

The present paper is an attempt to study the density response to a weak periodic potential beyond the HFA. All the electrons are assumed in the lowest, spin polarized, Landau level. The Fano-Ortolani formula[11] is used to express the exchange-correlation energy in a local-density approximation (LDA). The particle density is calculated following the general prescriptions of the density-functional formalism,[26] i.e., by solving, approximately, the Kohn-Sham equations. In this approach, the single-particle picture is preserved and a useful comparison with the HFA can be made. Nevertheless the LDA is, in principle, limited to take into account only short-range correlations and it is not expectable to describe strongly correlated states.

The self-consistent equations for the particle density and energy spectrum are derived in Sec. II. For zero external modulation, the present LDA predicts stable homogeneous states for filling factors $0.3 < \nu < 0.7$. For the other values, roughly corresponding to the Wigner solid, the CDW instability is present up to a critical temperature. The phase diagram is calculated in Sec. III, by studying the sign of the dielectric susceptibility of the homogeneous system, which changes from negative to positive in the vicinity of a CDW state.[27–30]

In Sec. IV, the numerical results for the particle density and for the Landau band, in the presence of an external modulation with a period several times larger than the wavelength of the spontaneous CDW's, are discussed. The external potential is sinusoidal, and additional oscillations with the CDW period are possible both in the density profile and in the energy spectrum, for temperatures and filling factors compatible with the phase diagram. The conclusions are presented in Sec. V, together with a discussion of the possibility of observing these oscillations in transport or magnetocapacitance measurements on modulated structures.

II. LOCAL-DENSITY APPROXIMATION

Correlations, which — generically — mean corrections to the HFA, can be included in the framework of the density-functional theory.[26] One has to find the eigenstates of the Kohn-Sham Hamiltonian $H^{(\mathrm{KS})}$,

$$H^{(\mathrm{KS})}\psi_\alpha(\mathbf{r}) = E_\alpha\psi_\alpha(\mathbf{r}), \tag{1}$$

which incorporates an effective potential energy of the form

$$v(\mathbf{r}) = V_{\mathrm{ext}}(\mathbf{r}) + \frac{e^2}{\kappa}\int d\mathbf{r}'\frac{n(\mathbf{r}')}{|\mathbf{r}'-\mathbf{r}|} + v^{(\mathrm{xc})}(\mathbf{r}), \tag{2}$$

where V_{ext} denotes an external potential and κ the dielectric constant of the background material.

The exchange-correlation component $v^{(\mathrm{xc})}$ is given by the functional derivative of the exchange-correlation part of the total energy $\mathcal{E}^{(\mathrm{xc})}$, with respect to the particle density $n(\mathbf{r})$. In the LDA, one assumes

$$\mathcal{E}^{(\mathrm{xc})}[n(\mathbf{r})] = \int d\mathbf{r}\, n(\mathbf{r})\,\epsilon^{(\mathrm{xc})}(n(\mathbf{r}))\,, \tag{3}$$

$\epsilon^{(\mathrm{xc})}$ being an ordinary function of n, so that

$$v^{(\mathrm{xc})}(\mathbf{r}) = \frac{d}{d(n(\mathbf{r}))}[n(\mathbf{r})\epsilon^{(\mathrm{xc})}(n(\mathbf{r}))]\,. \tag{4}$$

This theory can be formally extended to finite temperatures, in this case the particle density being

$$n(\mathbf{r}) = \sum_\alpha \mathcal{F}\left(\frac{E_\alpha-\mu}{T}\right)|\psi_\alpha(\mathbf{r})|^2\,, \tag{5}$$

where μ is the chemical potential, T the temperature in energy units, and $\mathcal{F}(x) = 1/[\exp(x)+1]$ the Fermi function.

For a 2D system in a magnetic field B, Eqs. (1) and (2) can be improved by including the coupling with the current and spin densities.[31,19] However, these contributions will be here neglected. B will be assumed strong, such that the electrons populate only the lowest Landau level. The kinetic energy is thus an insignificant constant which will be further ignored.

For the homogeneous system, i.e., $V_{\mathrm{ext}} \equiv 0$, an empirical formula describing the energy per particle in terms of the filling factor $\nu = 2\pi l_B^2 n$, $0 < \nu < 1$, $l_B = (\hbar/eB)^{1/2}$ being the magnetic length, has been derived by Fano and Ortolani[11] by interpolating some trustworthy results obtained for particular filling factors, and using the electron-hole symmetry:

$$\epsilon^{(\mathrm{xc})}(\nu) = \frac{e^2}{\kappa l_B}\frac{1}{\nu}\left(-\frac{1}{2}\sqrt{\frac{\pi}{2}}\nu^2 - 0.7821\tau^{3/2} + 0.55\tau^2 - 0.463\tau^{5/2}\right), \tag{6}$$

where $\tau = \nu(1-\nu)$. Another set of numerical coefficients for Eq. (6) is also proposed, which insignificantly

changes the results below. Of course, this formula is too crude to reproduce the discontinuities responsible for the fractional quantum Hall effect. The first term is the exchange energy and the other terms describe correlation corrections. The second term is chosen to reproduce the Wigner crystal energy for small ν. The effective one-body potential $v^{(\mathrm{xc})} = d(\nu\epsilon^{(\mathrm{xc})}(\nu))/d\nu$ is plotted in Fig. 1(a) versus ν and one can see it is nearly constant around $\nu = 1/2$.

Equation (6) has reasonably reproduced experimental results on homogeneous systems, related to the thermodynamic density of states,[9,10] as mentioned in Sec. I. Here, assuming sufficiently smooth density fluctuations, the LFF, $\nu(\mathbf{r}) = 2\pi n(\mathbf{r})$ will be introduced in Eq. (6). In the rest of this paper, the energy and length units will be $e^2/\kappa l_B$ and l_B.

An external potential of the form

$$V_{\mathrm{ext}}(\mathbf{r}) = V\cos(qx) \tag{7}$$

will be considered translational invariant on the y axis. Therefore, the LFF will have the Fourier expansion

$$\nu(x) = \sum_{m\geq 0} \nu_m \cos(mqx)\,. \tag{8}$$

The magnetic field is conveniently described in the Landau gauge, by the vector potential $\mathbf{A} = (0, Bx)$. For

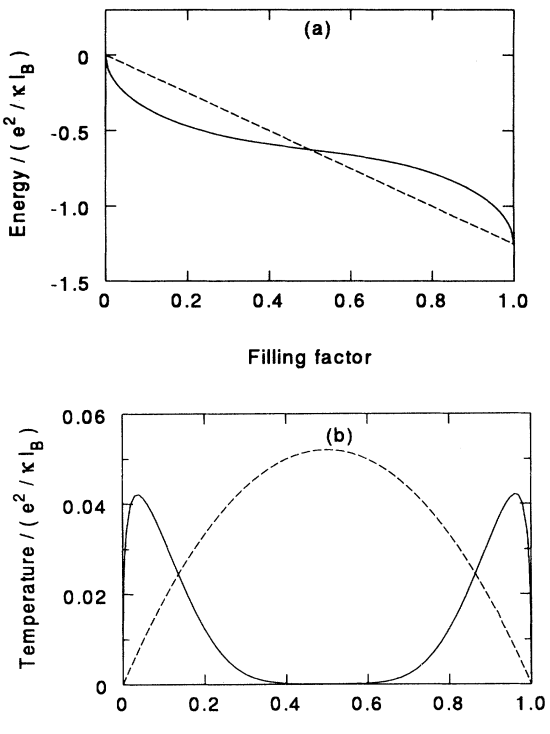

FIG. 1. (a) The effective interaction potential $v^{(\mathrm{xc})}$ vs filling factor, continuous line, and its exchange component $-\sqrt{\pi/2}\,\nu$, dashed line. (b) The homogeneous-inhomogeneous phase diagram in the LDA, continuous line, and in the local HFA, dashed line.

$V = 0$, the wave functions for the ground Landau level given by $H^{(\mathrm{KS})}$ have in this gauge the well known simple form

$$\psi_{X_0}(\mathbf{r}) = \frac{e^{iyX_0}}{L_y^{1/2}} \frac{e^{-(x-X_0)^2/2}}{\pi^{1/4}}\,, \tag{9}$$

where $X_0 = (2\pi/L_y) \times$ integer is the center coordinate, $-L_x/2 < X_0 < L_x/2$, $L_{x,y}$ being the linear dimensions of the system. The plane wave character on the y axis is exactly preserved for $V \neq 0$, but also the strong localization on the x axis is not considerably affected if $V_{\mathrm{ext}}(\mathbf{r})$ varies slowly on distances of the order l_B. In such cases the Landau wave functions, i.e., those for the homogeneous system, proved to be a good approximation of the Hartree-Fock wave functions for wavelengths $2\pi/q \approx 30 l_B$ or longer, and they could be satisfactorily used even for V several times larger than the cyclotron energy.[16] Here, the external potential (7) will have such a long wavelength, but much smaller amplitudes, and Eq. (9) should provide a reasonable approximation for the Kohn-Sham wave functions.

The single-particle energies will thus be given by

$$\begin{aligned} E_{X_0}^{(\mathrm{KS})} &= \langle\psi_{X_0}|H^{(\mathrm{KS})}|\psi_{X_0}\rangle \\ &= Ve^{-q^2/4}\cos(qX_0) + E_{X_0}^{(H)} + E_{X_0}^{(x)} + E_{X_0}^{(c)}\,. \end{aligned} \tag{10}$$

Using Eq. (8), the Hartree term becomes

$$E_{X_0}^{(H)} = \frac{1}{q}\sum_{m\geq 1} \frac{\nu_m}{m} e^{-m^2q^2/4}\cos(mqX_0)\,, \tag{11}$$

the term with $m = 0$ being excluded by a positively charged, neutralizing background. The exchange term, from the first term of Eq. (6), is then

$$E_{X_0}^{(x)} = -\sqrt{\frac{\pi}{2}}\sum_{m\geq 0} \nu_m e^{-m^2q^2/4}\cos(mqX_0)\,. \tag{12}$$

One can compare the local-density formulation of the exchange energy with the Fock form expressed with the same unperturbed wave functions.[16] The latter gives similar Fourier coefficients as in Eq. (12), but with the exponential factors replaced by the Bessel function $I_0(m^2q^2/4)$. Therefore, neglecting here the correlations, no important deviations from the HFA are expected, as long as weak, long wavelength modulations are considered. This approximation will be called local HFA.

The Fourier coefficients of the correlation energies $E_{X_0}^{(c)}$ cannot be expressed analytically, and will be calculated by numerical integration of Eqs. (4), (6), and (8). The complementary Eqs. (10) and (5), with the chemical potential fixed by the average filling factor ν_0, give self-consistently the LFF and the Landau band.

III. HOMOGENEOUS-INHOMOGENEOUS TRANSITION

In the absence of the external potential, $V = 0$, both the Hartree-Fock and the Kohn-Sham equations give

a constant particle density, but the stable states may have a spontaneous internal modulation. In the HFA, very close first order and second order phase transitions have been found, by an explicit calculation of the free energy.[20,21] The second order transitions have also been evidenced, also in the HFA, as poles in the static dielectric susceptibility,[28–30] and this simpler method will be used here to obtain the analog phase diagram in the LDA.

An external potential with only one Fourier component can always be reduced to Eq. (7). In the linear order in V, the particle density is given by

$$n(x) = V\chi(q)\cos(qx)\,, \tag{13}$$

and the susceptibility χ can be found by comparing with Eq. (5). In the latter, the linear Hartree and exchange-correlation contributions to Eq. (10) can also be expressed with the help of Eq. (13). The resulting equation for χ gives[28]

$$\chi(q) = \frac{\frac{-\nu(1-\nu)}{2\pi T}e^{-q^2/2}}{1+\frac{\nu(1-\nu)}{T}e^{-q^2/2}\{\frac{1}{q}+\frac{d^2}{d\nu^2}[\nu\epsilon^{(\mathrm{xc})}(\nu)]\}}. \tag{14}$$

The susceptibility changes sign in poles, from a homogeneous ($\chi < 0$) to a CDW state ($\chi > 0$), with a wave vector $q = q_0$, which minimizes the denominator in Eq. (14).

Neglecting the correlations, the second term in the curly brackets is simply $-\sqrt{\pi/2}$ and $q_0 \approx 1.3$, while in the HFA it is $-\sqrt{\pi/2}I_0(q^2/4)\exp(q^2/4)$ (Ref. 28) and $q_0 \approx 1.6$. The transition temperature is, therefore, reduced about two times in the local HFA, but qualitatively the resulting phase diagram, Fig. 1(b), is not changed.[20,21] The instability of the homogeneous states is a cooperative effect of the exchange with the screening. The tendency of the former is to enhance the population of the single-particle states in the vicinity of the Fermi level, by the addition of a negative amount of energy *proportionally with the filling factor*. In the meantime, the latter suppresses the long range fluctuations of the particle density; if the screening would not exist, i.e., in the absence of the term $1/q$ in Eq. (14), then $q_0 = 0$. Consequently, under both exchange and screening action, the equilibrium may be achieved by an oscillating particle density.

If the correlations are included, then the effective potential $v^{(\mathrm{xc})}$, Fig. 1(a), varies slowly in a large interval of filling factors around $\nu = 1/2$, where the correlation part increases with ν and reduces the negative slope of the exchange part. The corresponding stable states are therefore homogeneous, for any temperature, so that the phase diagram completely changes, Fig. 1(b). This is not the case for $\nu < 0.3$ and $\nu > 0.7$, where the situation is qualitatively like in the HFA. The CDW wave vector near the transition depends now on ν, $0.8 < q_0 < 2$.

The observed Wigner crystallization, characterized by small ν and very low temperatures are included in the region of inhomogeneous states at the left bottom of Fig. 1(b). Since the Wigner solid is a special CDW, endowed with a certain rigidity, associated with a shearing modulus[23] or with a high electrical resistance,[24] it is not clear whether these properties can persist with increasing temperature. Luminescence measurements[25] have evidenced the Wigner crystal characteristic spectra up to $T \approx 1.3$ K for $B = 26$ T, i.e., $T \approx 0.005$ in Fig. 1(b) (see also below, Sec. IV). Therefore, this phase diagram seems, at least qualitatively, a correct basis for the results of the present paper [compare with Fig. 4(c) of Ref. 25] on which one cannot analyze in detail the nature of the predicted CDW states.

IV. MODULATED SYSTEM

In the presence of the external potential (7), for $q \ll q_0$, the discussed CDW's may appear as strong higher harmonics in the density response. In the examples below, $q = 0.2/l_B$ will be fixed, which for magnetic fields in the range $4-10$ T corresponds to periods $400-250$ nm. The energy unit $e^2/\kappa l_B$ is then $9-14$ meV, for $\kappa = 12$ like in GaAs. The temperature will also be fixed in relative units to $T = 0.01e^2/\kappa l_B$, i.e., 1–2 K. The enhanced spin splitting is comparable with $e^2/\kappa l_B$ and the modulation amplitude will be sufficiently small, such that the lowest Landau band does not overlap with the upper one, with the reversed spin.

The numerical results are obtained after an iterative

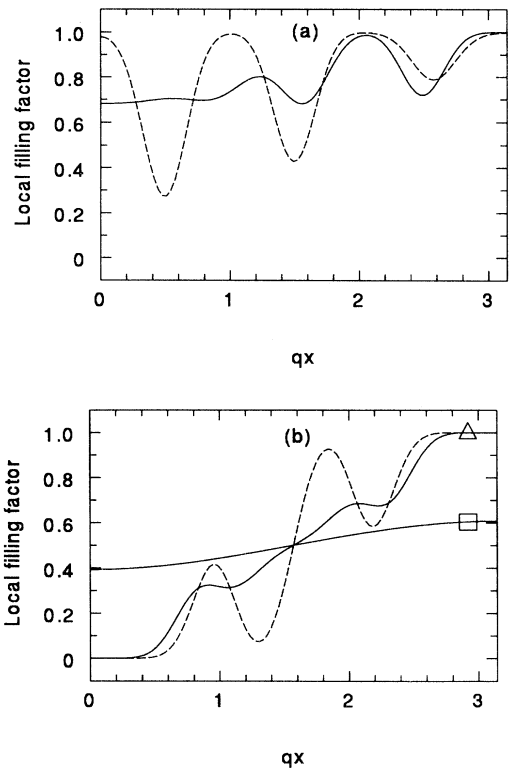

FIG. 2. The oscillations of the local filling factor in half of the period of an external potential with $q = 0.2/l_B$. The temperature is $0.01\,e^2/\kappa l_B$. (a) The average filling factor $\nu_0 = 0.8$ and the amplitude of the external potential $V = 0.5\,e^2/\kappa l_B$. The continuous line is for the LDA and the dashed line for the local HFA. (b) $\nu_0 = 0.5$ and $V = 0.5\,e^2/\kappa l_B$, marked with square (LDA only); $V = 2\,e^2/\kappa l_B$, marked with triangle (both LDA and HFA).

calculation of the coupled Eqs. (5), (8), (10), starting from the noninteracting solution. The Fourier series are truncated to at most 50 terms, the Fourier coefficients being obtained using several methods, including the high performance code described in Ref. 32.

The LFF's are shown in Fig. 2 and the related Landau bands in Fig. 3, for x and X_0 varying only between 0 and π/q, i.e., in half of the unit cell , due to the reflexion symmetry of $V_{\rm ext}$. In Fig. 2(a) the external modulation is weak, $V = 0.5$, and the average filling factor is $\nu_0 = 0.8$, i.e., close to the LDA transition value. While in the local HFA the LFF has violent oscillations, in the LDA, strips with oscillating alternate with strips with nearly constant LFF. In the latter strips, the exchange and correlations have opposite effects, in the already discussed sense [Fig. 1(a)], such that the single-particle energies are mainly given by the effective electrostatic potential, i.e., the external one strongly screened by the Hartree term, Fig. 3(a). In the other strips, also due to the screening, the LFF is bounded by the transition values, and the local maxima of the Landau band are fixed close to the Fermi level when the modulation strength is varied, unlike the behavior in the HFA.

The same external potential is, however, everywhere strongly, almost perfectly screened by the electron system for $\nu_0 = 0.5$, Figs. 2(b) and 3(b). This is similar with the results obtained in the Hartree approximation.[3] Several higher harmonics are present in the density response, but they are monotonously decreasing with their order.

The CDW oscillations can also be destroyed by increasing the external potential. In Figs. 2(b) and 3(b),

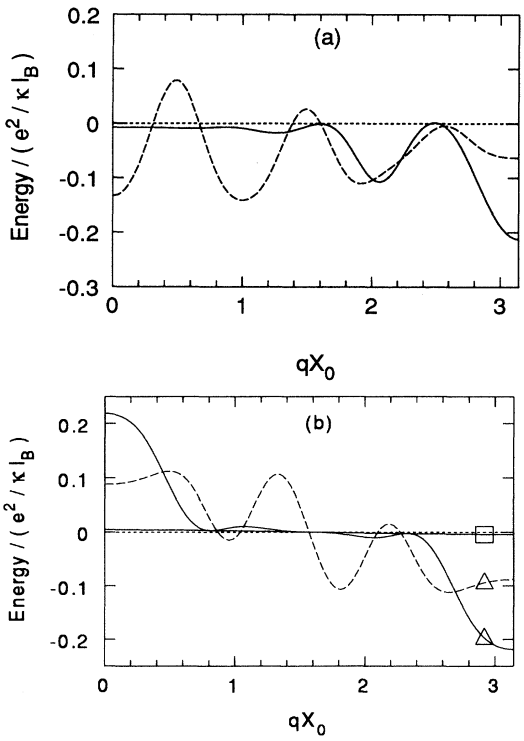

FIG. 3. The Landau bands corresponding to Fig. 2. The energies are shifted such that the chemical potential is zero, the dotted horizontal line.

$V = 2$, which is still too small in the HFA, but in the LDA it is an intermediate value. In the LDA case, this modulation strongly redistributes the electrons and the LFF may penetrate in the inhomogeneous regions of the phase diagram. Small CDW harmonics can still be observed in the transition regions where the screening can still strongly reduce the energy dispersion. The resulting weak oscillations of the Landau band are pinned around the Fermi level. For LFF's close to integers, the energy dispersion is enhanced by the cumulated effects of the strong modulation with the exchange-correlation contributions and the CDW harmonics are dominated. The Landau band looks thus again qualitatively similar with what one would expect from the Hartree approximation, i.e., alternate compressible-incompressible strips.

There is a strong discrepancy in the width of the edge compressible strips between the results in the Hartree[3–6] and in the Hartree-Fock[16,33] approximations. In the latter, it is drastically reduced by the exchange. However, the experiments seem to confirm wide edge channels.[34] Of course, the steepness of the confining potential plays an essential role in determining that width. But also, as shown here, the correlation corrections may play an important role. If the confinement is relatively smooth, they may supress the strong exchange, and change the HFA picture into the Hartree one, Fig. 3(b).

V. DISCUSSION AND CONCLUSIONS

The density response of a two-dimensional electron system in a strong magnetic field, to a weak external periodic potential with a long wavelength has been calculated including the electron-electron interaction in a local-density approximation, going beyond Hartree Fock. The results provide us with a qualitative understanding of the correlation corrections to the HFA, even if in the particular case when only the lowest Landau level is involved.

The most important consequence is the reduction of the exchange effects. In the absence of the external potential, for filling factors close to integers and low temperatures, the stable states are inhomogeneous, in qualitative agreement with the observed and expected Wigner crystallization. This instability is seen as a cooperative effect of the screening and exchange. But the latter is annihilated by the correlations for a large interval of filling factors around 1/2. The same mechanism influence the width of the compressible edge channels in the presence of a smooth lateral barrier. The present results suggest that the real width should be closer to that given by the Hartree approximation rather than by the HFA.

There is no available formula for the correlation energy corresponding to upper Landau levels. However, there is a chance to obtain similar results in a screened HFA. The exchange contribution should be strongly diminished (screened), except for filling factors close to integers (see Sec. I)

In a weakly modulated system, depending on the local filling factor, and consistently with the homogeneous-inhomogeneous phase diagram, in the present LDA the

external potential may be strongly screened in the regions where the LFF corresponds to homogeneous states, and may induce additional density oscillations, with a period of a few magnetic lengths, in the regions where the LFF corresponds to the CDW states. These oscillations have the same origin as the Wigner crystallization. The superimposed modulation only stimulates certain intrinsic harmonics of the density response.

Rigorously speaking, the single-particle energy spectrum obtained in the density-functional formalism cannot be interpreted as real excitation energies.[26] However, in the present situation, the close relationship between the oscillations of the Landau bands and those of the charge density suggest the possible observation of the higher order oscillations in the density of states (DOS), Fig. 4. The latter can be estimated using a simple self-consistent Born approximation,

$$\Sigma(E) = \Gamma^2 \frac{q}{\pi} \int_0^{\pi/q} \frac{dX_0}{E - E_{X_0}^{(\mathrm{KS})} - \Sigma(E)}, \tag{15}$$

where the impurity scattering is described by the small parameter Γ and the corresponding self-energy Σ has been simply assumed a c number.[35] The DOS is thus given by

$$D(E) = D_0 \frac{\mathrm{Im}\Sigma(E)}{2\pi^2\Gamma^2}, \tag{16}$$

with $D_0 = \kappa/e^2 l_B$.

The extrema of the one-dimensional energy bands can yield a complicated structure of van Hove singularities in the DOS at the Fermi level, D_F, in the case of a weak modulation, Figs. 4(a) and 3(a),(b). The huge peaks correspond to very flat bands, which are formed at the crossover from (quasi-)linear density response to oscillating regime. Then, several smaller van Hove maxima can be observed. For a stronger modulation, when the higher order oscillations are suppressed, only the maxima for the center and for the edges of the first Brillouin zone are present, Figs. 4(b) and 3(b), but they are not very pronounced, because of the pinning effect, which yields the very broad central maximum. Of course, the symmetry of Fig. 4, with respect to $\nu = 1/2$ (particle-hole symmetry) is only an artifact, since the Landau level mixing has been here neglected.

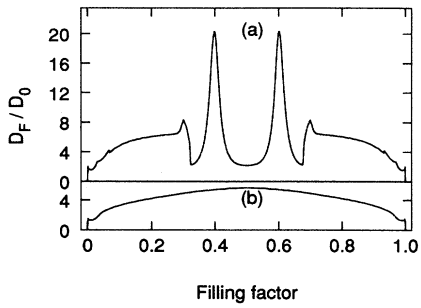

FIG. 4. The estimated DOS at the Fermi level, for $\Gamma = 0.03\, e^2/\kappa l_B$. (a) $V = 0.5\, e^2/\kappa l_B$, (b) $V = 2\, e^2/\kappa l_B$.

The Landau levels, broadened by a weak lateral external modulation, can be seen as peaks in the magnetoresistance measured perpendicular to the modulation, when the magnetic field is varied.[35,36] At high magnetic fields, these peaks have a structure determined by the spin splitting of the Landau levels and by the van Hove singularities of the D_F. The results of the present paper suggest possible observation of more than two van Hove maxima per Landau level (with definite spin), generated by the higher order oscillations discussed. The thermodynamic DOS being also sensitive to the van Hove singularities,[16] similar results are possible in the magnetocapacitance measurements. However, due to the pinning effect, the expected maxima could appear very broad, eventually overlapped.

Note added in proof. In a recent paper[37] Chklovskii obtained density profiles for edge states which look very similar to the full line marked with a triangle in Fig. 2(b). He uses a composite fermion approach, in which the density oscillations are related to the energy gaps at fractional filling factors, which are not taken into account here, but he ignores the Wigner-crystal states. Our results are therefore complementary.

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