Oscillations in the diffusion thermopower of a two-dimensional electron gas

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The phonon drag thermopower of a two-dimensional electron gas at a $GaAs/Ga_{1-x}Al_xAs$ heterojunction has been strongly attenuated by growing the sample on a heavily doped substrate and by working in the liquid ^{3}He temperature range. As a consequence the diffusion thermopower dominates at $T < 0.5$ K, at least at zero magnetic field. The low-field oscillations in the resistivity and thermopower have been investigated. The former behave essentially as predicted. The amplitude of the oscillations in the Nernst-Ettingshausen coefficient shows the field and temperature dependence expected for diffusion, although the absolute magnitude is about a factor of 2 too large. The amplitude of the longitudinal thermopower oscillations does not behave as expected for diffusion, although the oscillations exhibit the predicted change in phase as a function of magnetic field.

I. INTRODUCTION

Oscillations in the thermopower of two-dimensional electron gases (2DEG's) as a function of magnetic field have been studied extensively both experimentally and theoretically; a recent review[1] gives many references. Most of the work has been done at high magnetic fields, the quantum Hall regime, where the Landau levels are well separated, i.e., $2\Gamma < \hbar\omega_c$ where ω_c is the cyclotron frequency and Γ is the half-width of the Landau levels. If the temperature is not too high, i.e., if $\hbar\omega_c > k_BT$, then typical data taken under these conditions shows a series of peaks separated by zeros. The aim of this work is to investigate the opposite limit of $2\Gamma > \hbar\omega_c$.

There are two components of the thermopower, diffusion and phonon drag, but most of the previously observed oscillations have been dominated by phonon drag, especially in the case of heterojunctions. Indeed, there has been no clear evidence that diffusion oscillations have been observed at all in this case.

The present experiments were designed to suppress phonon drag so that the diffusion component can be seen. The simplest way of doing this is to move to lower temperatures where phonon drag rapidly decreases as the number of phonons decreases. With this in mind, the present experiments have been carried out at liquid ^{3}He temperatures. However, in typical heterojunction samples at zero field, it is found that phonon drag remains large down to rather low temperatures and is usually similar in magnitude to the diffusion component at 0.5 K. One would need $T < 0.3$ K to adequately suppress phonon drag, and under these conditions the experimental signals are very small.

We have also used another technique to reduce phonon drag, that of growing the 2DEG on a doped substrate. The effect has been seen experimentally[2,3] and is implicit in the theoretical expressions for phonon drag thermopower.[4] We have used much heavier doping than in these previous experiments and at 4 K phonon drag has been reduced by more than an order of magnitude, although by 0.3 K the reduction is perhaps only a factor of 3; these numbers refer to the zero-field values. Nevertheless, the combination of reduced temperature and substrate doping is very effective and by 0.5 K the magnitude of the phonon drag component is only ~10% of the total. The ratio of the oscillation amplitudes of phonon drag and diffusion does not have the same temperature dependence as the zero-field ratio but qualitatively we expect the ideas to remain valid.

This paper concentrates on the low-field oscillations where we believe the separation of diffusion and phonon drag should be the clearest. In the next section it will be shown that the low-field diffusion oscillations are predicted to be much larger in the Nernst-Ettingshausen coefficient, i.e., the transverse thermopower, than in the longitudinal thermopower. This is the opposite of what is expected with the phonon drag part. As a result, and with the help of the experimental techniques we have used to reduce phonon drag, we believe that the diffusion component of the oscillatory thermopower in the Nernst-Ettingshausen coefficient has been clearly observed.

II. THEORY

When subjected to an electric field $\mathbf{E}$ and temperature gradient ∇T, the current density $\mathbf{J}$ in the 2DEG has two components:

$$\mathbf{J} = \sigma\,\mathbf{E} - \epsilon\,\nabla T, \tag{1}$$

where σ and ϵ are the conductivity and thermopower tensors, respectively. The thermopower tensor S is defined by $\mathbf{E} = S\,\nabla T$ measured under the condition $\mathbf{J} = \mathbf{0}$ and

is thus given by $S = \sigma^{-1}\epsilon = \rho\epsilon$, where ρ is the resistivity.

The theory of ϵ and S to be presented primarily concerns the diffusion component, although reference to phonon drag will sometimes be made. There have been many calculations of the thermoelectric coefficients in the quantum Hall regime. In this limit, the diffusion oscillations in $-S_{xx}$ are in phase with, and resemble, those in ρ_{xx} with zeros when the Landau levels are full, and peak values at half filling, assuming no spin splitting, given by[1]

$$S_{xx} = -\frac{k_B \ln 2}{e(n+\frac{1}{2})}. \tag{2}$$

In this equation, k_B is Boltzmann's constant, e is the magnitude of the electronic charge, and n is the number of completely full Landau levels.

In the case of S_{yx} we will not reproduce the details[1] since they are not too useful in the present experiment, which focuses on the low-field behavior.

The low-field oscillations in ρ have been extensively studied (see Coleridge *et al.*[5] and reference therein) in the case of 2DEG's and are predicted to be given by

$$\tilde{\rho}_{xx} = 4\,\bar{\rho}_{xx}\,D(X)\,\exp\left(-\frac{2\pi^2 k_B T_D}{\hbar\omega_c}\right)\,\cos\left(\frac{2\pi f}{B}-\pi\right), \tag{3a}$$

$$\tilde{\rho}_{yx} = -\frac{2}{\omega_c^2\tau_t^2}\,\bar{\rho}_{yx}\,D(X)\,\exp\left(-\frac{2\pi^2 k_B T_D}{\hbar\omega_c}\right) \times\cos\left(\frac{2\pi f}{B}-\pi\right). \tag{3b}$$

We have used $\bar{\rho}$ and $\tilde{\rho}$ to represent the monotonic and oscillatory parts of the resistivity, respectively: τ_t is the transport relaxation time, T_D is the Dingle temperature, f is the frequency of the oscillations ($f/B = \varepsilon_F/\hbar\omega_c$), and B is the magnetic field. The quantity $D(X)$ is the thermal damping factor defined by $D(X) = X/\sinh X$ where $X = 2\pi^2 k_B T/\hbar\omega_c$ with T the temperature. The Dingle temperature is related to the Landau level broadening factor Γ and to the quantum lifetime τ_q by $\Gamma = \pi k_B T_D = \hbar/2\tau_q$. The equations are expected to be valid only for $\omega_c\tau_q < 1$, or equivalently $2\Gamma > \hbar\omega_c$, although Coleridge *et al.*[5] gave a simple argument why $\tilde{\rho}_{xx}$ seems to follow Eq. (3a) to higher fields than expected. Experimentally the same authors show that Eq. (3a) is accurately obeyed, but Eq. (3b) is valid only up to $\omega_c\tau_q \sim 0.5$; for $\omega_c\tau_q > 1$, it is empirically observed that the phase of $\tilde{\rho}_{yx}$ is shifted by $\pi/4$, the effective Dingle temperature increases by a factor of 2, and the amplitude is greatly increased.

The above equations were originally derived for the case of short-range scattering, which does not distinguish τ_t and τ_q, and were later extended using nonrigorous methods by Coleridge *et al.*[5] to the case with different τ_t and τ_q. More recently, Laikhtman and Altshuler[6,7] have given a self-consistent theory for $\tilde{\rho}_{ij}$, which does distinguish τ_t and τ_q, and they confirm the form of Eq. (3a). Bockelmann *et al.*[8] have suggested different equations but these are very inconvenient for our purposes. The experimental results[5] appear to be in favor of Eq. (3), so we will use these in the following.

It is important to realize that at low temperatures, diffusion thermopower and resistivity are both controlled by impurity scattering, unlike phonon drag which is driven by the electron-phonon interaction. Thus, it is not surprising that one finds[9–11] the low-field oscillations in the diffusion part of ϵ are simply related to those in σ; the relation is

$$\tilde{\epsilon} = -i\left(\frac{\pi k_B}{e}\right)\frac{D'(X)}{D(X)}\,\tilde{\sigma} = \beta\tilde{\sigma}, \tag{4}$$

where $D'(X)$ is the derivative of $D(X)$ with respect to X and represents the thermal damping factor for thermoelectric oscillations. We have used i to represent a phase factor that shifts the phase of the oscillations by $\frac{\pi}{2}$. Havlová and Smrčka[11] use an expansion to find $\tilde{\epsilon}$, the first term of which is the one given above. They also retain a further term, which has a thermal damping factor involving $D''(X)$ and a phase shift of zero or π relative to the resistivity oscillations. Experimentally, the coefficient of this term is not known. However, it appears that it should have a magnitude of order $k_B T/\varepsilon_F$ smaller than the leading term given above and so it is not included in the following.

With these results, the expected low-field behavior of the diffusion thermopower oscillations $\tilde{S}$ can be evaluated. Using

$$S = (\bar{\rho}+\tilde{\rho})(\bar{\epsilon}+\tilde{\epsilon}) \simeq \bar{\rho}\,\bar{\epsilon} + \tilde{\rho}\,\bar{\epsilon} + \bar{\rho}\,\tilde{\epsilon}, \tag{5}$$

where we have dropped the term $\tilde{\rho}\tilde{\epsilon}$, which is assumed small and in any event does not contribute to the fundamental oscillation frequency, and using Eq. (4) together with the approximate relation $\bar{\rho}\,\tilde{\sigma} \simeq -\tilde{\rho}\,\bar{\sigma}$ in Eq. (5), gives

$$\tilde{S} = \tilde{\rho}(\bar{\epsilon} - \beta\bar{\sigma}). \tag{6}$$

The first term $\tilde{\rho}\,\bar{\epsilon}$ is negligible compared to the second, so only the latter need be considered, i.e., $\tilde{S} = -\beta\tilde{\rho}\,\bar{\sigma}$. We can now evaluate the two independent components of $\tilde{S}$, $\tilde{S}_{xx}$, and $\tilde{S}_{yx}$, which are given by

$$\tilde{S}_{xx} = -\beta(\tilde{\rho}_{xx}\,\bar{\sigma}_{xx} + \tilde{\rho}_{xy}\,\bar{\sigma}_{yx}), \tag{7a}$$

$$\tilde{S}_{yx} = -\beta(\tilde{\rho}_{xx}\,\bar{\sigma}_{yx} + \tilde{\rho}_{yx}\,\bar{\sigma}_{yy}). \tag{7b}$$

Using Eqs. (3) and free electron values for $\bar{\sigma}$ finally results in

$$\tilde{S}_{xx} = \frac{2}{1+\omega_c^2\tau_t^2}\left(\frac{\pi k_B}{e}\right)D'(X)\exp\left(-\frac{2\pi^2 k_B T_D}{\hbar\omega_c}\right) \times\sin\left(\frac{2\pi f}{B}-\pi\right), \tag{8a}$$

$$\tilde{S}_{yx} = \frac{4\omega_c\tau_t}{1+\omega_c^2\tau_t^2}\left(\frac{\pi k_B}{e}\right)D'(X)\exp\left(-\frac{2\pi^2 k_B T_D}{\hbar\omega_c}\right) \times \sin\left(\frac{2\pi f}{B}-\pi\right). \quad (8b)$$

In the second equation we have dropped the term arising from $\tilde{\rho}_{yx}$, which is smaller by a factor $1/2(\omega_c\tau_t)^2$; in other words, only the term with $\tilde{\rho}_{xx}$ in Eq. (7b) contributes to $\tilde{S}_{yx}$. In contrast, the two terms in Eq. (7a) are similar in magnitude.

An unambiguous signature of low-field diffusion thermopower oscillations would seem to be the observation of the phase shift by $\frac{\pi}{2}$, which is not seen with phonon drag oscillations, but there are other interesting features about this result. Unlike the resistivity components in Eq. (3), the diffusion oscillation amplitude does not depend on the monotonic part of the thermopower, nor on the electron density. In addition, one typically finds $\tau_t \sim 10\tau_q$ in heterojunction samples (in the present case the ratio is ~ 9) and, since the oscillations usually become visible only when $\omega_c\tau_q \sim 1$, then the factor $1+\omega_c^2\tau_t^2$ will cause a large reduction in amplitude. Finally, graphs of $D'(X)$ have been given by Young,[9] and Havlová and Smrčka[11] where it is seen that the function has a maximum at $X = 1.62$ and goes linearly to zero at small X, thus giving a distinctly different behavior to $\tilde{S}_{ij}$ as compared to $\tilde{\rho}_{xx}$.

There is no general theory of the low-field oscillations in the phonon drag for comparison purposes, but experiments[12,13] on $\tilde{S}_{xx}$ indicate that the amplitude monotonically increases with B/T in a manner similar to the case of $\tilde{\rho}_{xx}$ but with an effective Dingle temperature, which is almost a factor of 2 higher than the actual value, presumably reflecting the fact that an energy $\sim k_BT$ is involved at each phonon-electron scattering event. This should help to accentuate and distinguish the diffusion oscillations at low fields. It is also found that the oscillations in $-S_{xx}$ are accurately in phase with those in ρ_{xx} at all fields. The monotonic part of the phonon drag decreases roughly as the inverse of the electron density,[4] and it seems reasonable to assume that the oscillation amplitude increases with the magnitude of the monotonic component, particularly at low fields. This implies that high-density samples would probably be the most useful for examining diffusion oscillations. However, the present sample has a rather low density.

III. EXPERIMENTS

The experiments were carried out in the temperature range 0.3–6 K using a charcoal pumped ^{3}He cryostat, although most of the oscillatory data were taken only below 1 K. The techniques were similar to those described previously.[14] The single GaAs/$Ga_{1-x}Al_xAs$ heterojunction with the 2DEG was grown on a conducting GaAs substrate doped with approximately 4×10^{19} cm^{-3} of Zn. The growth sequence is similar to that used by Das *et al.*[15] The substrate can be used as a gate but we have kept it shorted to one point of the sample at all times. The final substrate dimensions were $3 \times 10 \times 0.4$ mm^3. The 2DEG sample was about 2.8×0.2 mm in size, with limbs that extended the overall width to 1.0 mm. The 2DEG density was $\sim 1.9 \times 10^{15}$ m^{-2} and the transport mobility $\mu_t = 40$ m^2/Vs, with variations of a few percent between different cooldowns.

Field sweep data were obtained with both dc and ac techniques, in the latter case using a detection frequency of 4 Hz. Even at this low frequency, the low thermal conductivity led to a loss of signal of about 10–20% in the range 0.3–0.8 K. The system was calibrated using the zero-field thermopower measured at dc.

It was found that there exists a field-dependent dc output signal from any pair of voltage contacts even in the absence of a heat current. This signal oscillates at the de Haas–Shubnikov frequency and so becomes entangled with the desired oscillations. With dc, the thermoelectric signal is larger than the spurious signal at $T > 0.7$ K but the opposite is the case by 0.3 K (although only at the relatively low fields of interest here). The spurious signal is noticeably temperature dependent becoming larger at lower temperatures; this implies that ac techniques cannot fully eliminate it since the sample temperature varies during the ac cycle, though ac data are influenced far less than dc data. Of course, traces taken with ac detection but no heat input, e.g., for ρ measurement, do not show this signal.

Such signals have been noticed in our previous experiments, as well as by others,[16,17] and their origin is not fully understood. In the present case they are reproducible over time intervals of a few hours, but appear to change in amplitude over longer time intervals, e.g., a day, though retaining the same basic field dependence. Within experimental error they are even under reversal of the magnetic field for both transverse and longitudinal pairs of leads, which shows that they are not due to an extraneous heat current through the sample. The magnitude of the spurious signals on the various pairs are similar and are much smaller than those described by others;[16,17] in the present case the maximum amplitude is about 200 nV at $B = 1$ T and $T = 0.35$ K. The fact that they are not constant over long time periods might suggest that they are associated with noise input from outside the cryostat, perhaps radio frequency interference, although all the leads are well shielded and filtered over a very wide range of frequencies.

Because these signals are even in field, they can be eliminated in the case of S_{yx} by subtracting data taken at $\pm B$; this is so for both ac or dc data. However, for S_{xx} we can only rely on the ac technique to reduce the spurious signal. Regardless of this problem, field sweeps at $\pm B$ were always taken for both S_{ij} coefficients and the results suitably averaged to eliminate any admixture of S_{yx} and S_{xx}. This was not required for ρ_{ij}. Resistivities were measured with a current of about 60 nA, which was found to give negligible self-heating.

Most of the data were recorded using sweeps for which $1/B$ was approximately linear with time: Points were taken at equal increments of $1/B$, typically with ~ 10 points per oscillation. This enabled Fourier transform filtering to be used to separate the oscillations from the

monotonic background. The amplitude of the oscillations was determined by a program that fits all the points in a complete oscillation.[18] The technique is insensitive to any second harmonic components.

The phases were also determined automatically[13] using the field differences in the zero crossing points between the oscillations in $\tilde{\rho}_{xx}$ and $\tilde{S}_{ij}$. Resistivity and thermoelectric oscillatory data were taken under the same sweep conditions, including lock-in time constants, so that phase comparisons remain valid as far as possible.

Unfortunately, when measuring $\tilde{S}_{yx}$ it was found that different pairs of transverse contacts gave somewhat different phases for the signal even though the amplitudes were very similar; the differences are at the level of about $\frac{\pi}{2}$, which is the same range as that expected for the phase differences between $\tilde{\rho}_{xx}$ and $\tilde{S}_{yx}$. Given that S_{yx} is measured over a transverse section of the sample whereas ρ_{xx} is for a longitudinal section, the phase differences probably result from inhomogeneities in the 2DEG at the level of 1%. The problem was likely aggravated by the relatively large sample size. Such differences should be less significant for $\tilde{S}_{xx}$, because it monitors essentially the same region as $\tilde{\rho}_{xx}$.

IV. RESULTS AND DISCUSSION

Heavily doping the substrate led to a large reduction in the thermal conductivity as shown in Fig. 1, where the thermal conductivity of an undoped substrate of similar dimensions is also given for comparison. The doping reduces the thermal conductivity by almost 2 orders of magnitude at 4 K, although by 0.5 K the factor is only about 6. It is possible that other dopants might be more effective at low temperatures.

In a typical sample, phonon drag completely dominates the thermopower in the ^{4}He temperature range but, in the present case, doping the substrate results in a large decrease in phonon drag thermopower so that diffusion is always significant. The diffusion thermopower should be linear in T and given by $S = -(\pi^2 k_B^2 T/e\varepsilon_F)(1 + p)$ where $p = \partial \ln \tau_t / \partial \ln \varepsilon_F$. Figure 2 shows the zero-field thermopower of the present sample; the data are divided by T so that the ratio of the diffusion and phonon drag components is easily seen. The horizontal line in Fig. 2 corresponds to the best estimate of the diffusion term giving $p = 1.20 \pm 0.15$. Both theory and previous experiments[1,3,19] give similar values of p.

According to Eq. (3a), the amplitude of the resistivity oscillations relative to the zero-field resistivity should be given by $\tilde{\rho}_{xx}/[D(X)\ \bar{\rho}_{xx}] = 4\exp(-2\pi^2 k_B T_D/\hbar\omega_c)$. Figure 3 shows $\tilde{\rho}_{xx}/D(X)\bar{\rho}_{xx}$ near 0.3 K plotted as a function of $1/B$. The line has an intercept of 3.3 on the vertical axis, which is acceptably close to the expected value of 4. The slope yields $T_D = 0.70 \pm 0.05$ K giving a quantum mobility $\mu_q = \pi/T_D = 4.5$ m^2/Vs. Since the transport mobility is about 40 m^2/Vs, their ratio is 9, which seems typical of reasonably high-mobility samples. We would expect the theory to be valid for $\mu_q B < 1$, or $\omega_c \tau_q < 1$, which implies $B < 0.22$ T, but the oscillation amplitude follows the theory reasonably well for fields of at least twice this value.

Figure 4 shows the behavior of a selection of the data on S_{yx} at various temperatures for fields up to about 1.5 T. These data have not been averaged for $\pm B$ but the

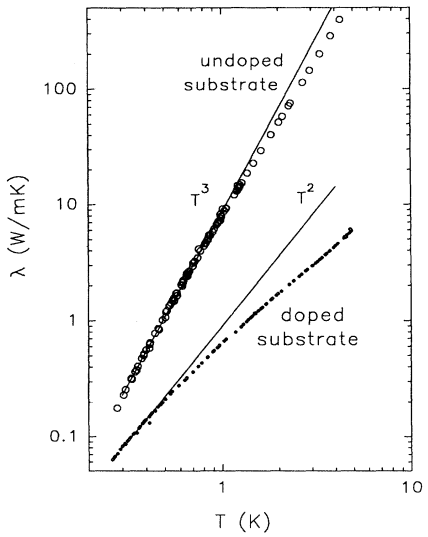

FIG. 1. The measured thermal conductivity λ of the doped substrate as a function of temperature T. The thermal conductivity of a typical undoped substrate of similar dimensions is given for comparison. The lines give the closest integer power laws to the low-temperature limiting behavior of the data.

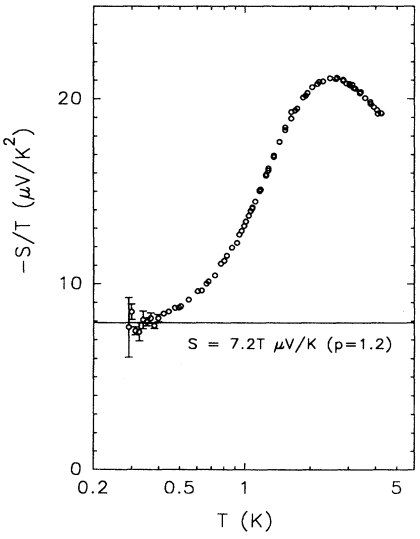

FIG. 2. The zero-field thermopower of the sample divided by T as a function of T. The horizontal line is the best experimental estimate of the diffusion component, the remainder being phonon drag. At 4 K, the thermopower is about 80 μV/K, of which about 40% is diffusion; for comparison, the thermopower of a similar 2DEG sample grown on an undoped substrate is roughly 1.0 mV/K and diffusion would be negligible.

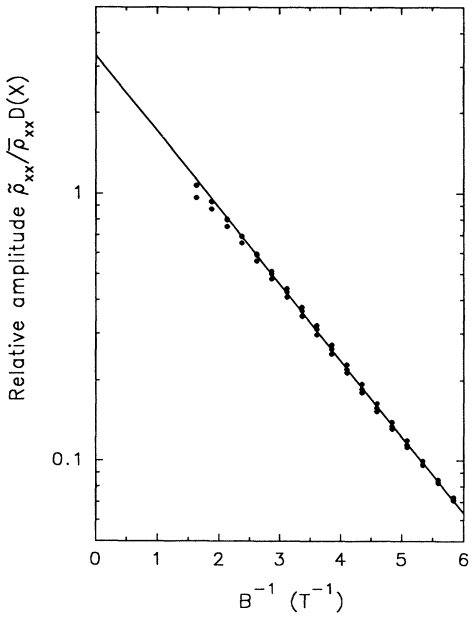

FIG. 3. The relative amplitude of the oscillatory resistivity $\tilde{\rho}_{xx}$, divided by the zero-field resistivity $\bar{\rho}_{xx}$ and the thermal damping factor $D(X)$, as a function of $1/B$ at 0.29 K. The slope of the line corresponds to a Dingle temperature T_D of 0.7 K.

spurious signal is not too large in this particular case. There is little information available in the literature on the low-field behavior of the phonon drag component of $\tilde{S}_{yx}$, although it seems to be significantly smaller than that in $\tilde{S}_{xx}$. Recalling that the diffusion component of $\tilde{S}_{yx}$ should be larger than that of $\tilde{S}_{xx}$, at least at low fields, we expect to see this latter component more easily in $\tilde{S}_{yx}$. The burst of oscillations below about 0.6 T (primarily seen at temperatures of < 0.55 K in this figure) is believed to be due to diffusion. Notice that the oscillations at higher fields become smaller, as is expected for diffusion but not for phonon drag.

Using Fourier transform techniques, $\tilde{S}_{yx}$ was separated from the monotonic background. Noting that both Eqs. (3) and (8) contain the same term describing the effects of Landau level broadening, i.e., $\exp(-2\pi^2 k_B T_D/\hbar\omega_c)$, the amplitude data for $\tilde{S}_{yx}$ were scaled to give $\tilde{S}_{yx}(1+\omega_c^2\tau_t^2)/\omega_c\tau_t D'(X)$ as a function of $1/B$. Some of the scaled and unscaled data from one set of contacts are shown in Fig. 5. Other sets of data taken on different days or with different transverse pairs of contacts are all consistent with these results. The scaled data seem to conform to a straight line and the average slope, as shown on the graph, gives $T_D = 0.7 \pm 0.1$ K, in agreement with that from $\tilde{\rho}_{xx}$. If phonon drag was important we would expect an effective T_D much higher than this, assuming $\tilde{S}_{yx}$ behaves in a similar manner to $\tilde{S}_{xx}$.[12,13] The intercept on the vertical axis is 2.3±1.0 mV/K, which is about a factor of 2 larger than the expected value of $4\pi k_B/e = 1.08$ mV/K.

The intercept scales almost linearly with the value of μ_t used in the analysis. A value of $\mu_t \sim 20$ m^2/Vs, rather than the measured value of 40 m^2/Vs, would give good agreement with theory. As with $\tilde{\rho}_{xx}$, the data seem to fit the straight line to higher fields than we would expect. As mentioned in Sec. III, the experimental phase of $\tilde{S}_{yx}$ proved to be different for different pairs of con-

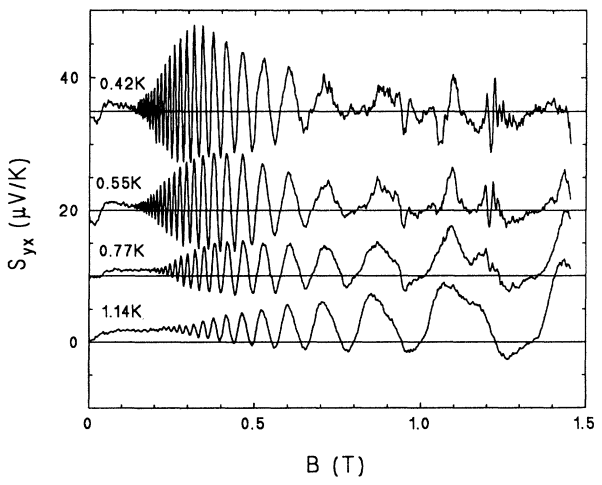

FIG. 4. The Nernst-Ettingshausen coefficient S_{yx} as a function of magnetic field B at various temperatures. The curves have been offset for clarity, and the true zeros are given by the horizontal lines through the data.

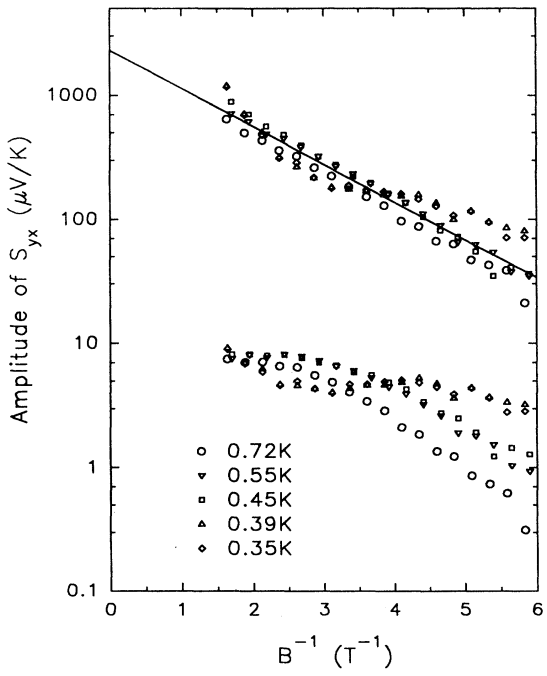

FIG. 5. The amplitude of $\tilde{S}_{yx}$ as a function of $1/B$. The upper set of data has been normalized by multiplying by $(1+\omega_c^2\tau_t^2)/[\omega_c\tau_t D'(X)]$. The line through the data has a slope corresponding to a Dingle temperature of 0.7 K, which is the same as that in Fig. 3.

tacts. Because of this the only result that can be quoted with confidence is that for any particular pair of contacts: The phase is the same for all data in the range 0.35 K$< T <$ 0.75 K with $1/B < 6$ T^{-1}.

Data for $\tilde{S}_{xx}$ as a function of $1/B$ are shown in Fig. 6. At the lowest temperatures, the oscillations show a maximum amplitude near 0.3 T, which is similar to that seen with $\tilde{S}_{yx}$. However, this feature has disappeared by 0.6 K and, unlike $\tilde{S}_{yx}$, there are strong oscillations at higher fields at all temperatures. The oscillations are not so clearly identified with diffusion as was the case with $\tilde{S}_{yx}$.

The behavior of the phase of the low-temperature oscillations is shown in Fig. 7. At high fields $-\tilde{S}_{xx}$ and $\tilde{\rho}_{xx}$ are in phase as would be expected for both phonon drag and diffusion. Near 0.25 T, where $\mu_q B \sim 1$, there is a change of phase. Phonon drag shows no phase changes whereas the phase difference between the diffusion component of $-\tilde{S}_{xx}$ and $\tilde{\rho}_{xx}$ should be $-\frac{\pi}{2}$ at low fields. Thus, the results suggest that the oscillations are due to diffusion, at least for $T < 0.6$ K and at the lowest fields. By 0.8 K, the amplitude is too small to follow the phase at fields below 0.25 T, but at high fields there are no phase differences, which is consistent with either mechanism.

The data have been analyzed in a similar manner to that for $\tilde{S}_{yx}$. Following Eq. (8a), we plot $\tilde{S}_{xx}(1 + \omega_c^2\tau_t^2)/D'(X)$ as a function of $1/B$ in Fig. 8. In contrast to Fig. 5, the data are not grouped around a single line. All the points seem to follow a common curve at higher fields and temperatures, but show strong systematic departures to a set of curves of lower slope at a field, which depends on the temperature. In the light of the results on $\tilde{S}_{yx}$, a line with the slope appropriate to $T_D = 0.7$ K

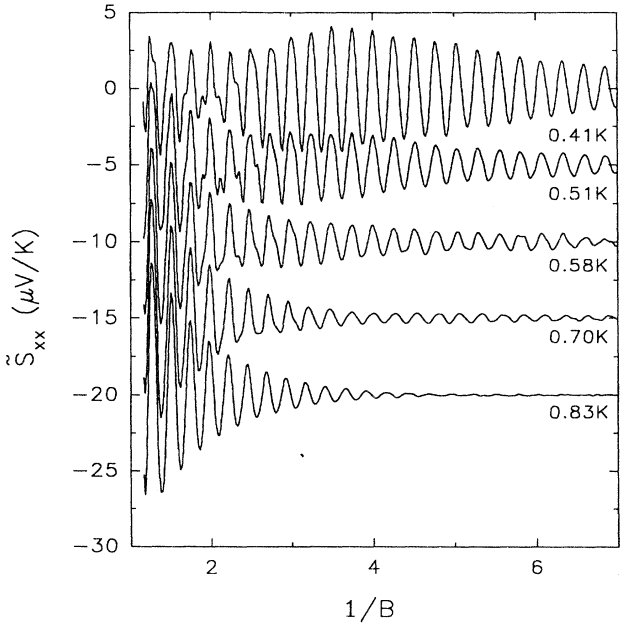

FIG. 6. The longitudinal oscillatory thermopower $\tilde{S}_{xx}$ as a function of inverse magnetic field $1/B$ at various temperatures. The monotonic backgrounds have been removed and the curves have been offset for clarity.

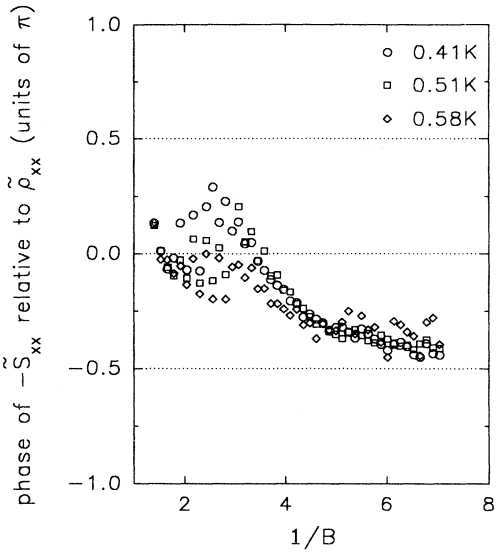

FIG. 7. The phase of the low-temperature data on $-\tilde{S}_{xx}$ relative to that of ρ_{xx} as a function of $1/B$. The phase is expected to be zero at high field and, if diffusion dominates, shift to $-\frac{\pi}{4}$ at low fields.

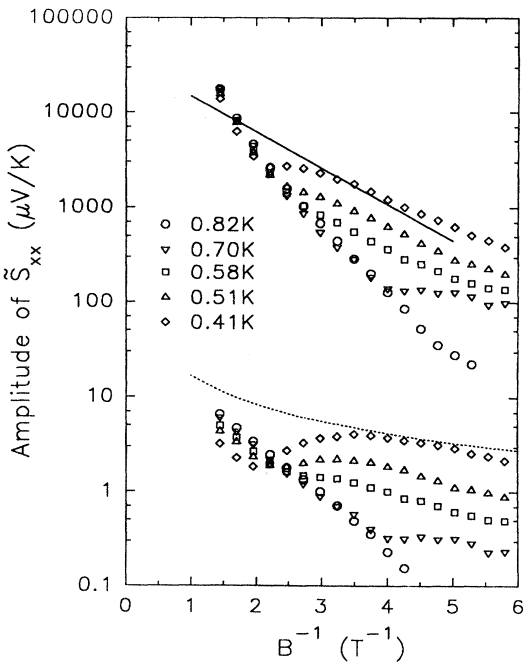

FIG. 8. The amplitude of $\tilde{S}_{xx}$ as a function of $1/B$. The upper set of curves has been multiplied by $(1+\omega_c^2\tau_t^2)/D'(X)$ and the data are then expected to lie on a single curve with the same slope as Figs. 3 and 5. This slope, equivalent to a Dingle temperature of 0.7 K, is shown as the full line. The high-field, high-temperature limit of the diffusion component, according to Eq. (2) and which is $\sim B$, is shown as a dotted line.

has also been shown, but this is not consistent with any of the data. The expected intercept on the vertical axis, assuming Eq. (8a) to be valid, is $2\pi k_B/e = 0.54$ mV/K, which is also inconsistent with all the data.

Clearly the experimental amplitude is far larger than predicted at all temperatures. Because of the factor $(1+\omega_c^2\tau_t^2)$ in the denominator of Eq. (8a), a very small amplitude is predicted for $\tilde{S}_{xx}$ at fields for which $\mu_t B = \omega_c\tau_t > 1$, which applies at all fields here. According to Eq. (8), we should find that $\tilde{S}_{yx} \sim 2\omega_c\tau_t\tilde{S}_{xx}$ at all fields, and given that $2\omega_c\tau_t = 20$ at $1/B = 4$ T^{-1}, then the diffusion component should probably be too small to see at all in this sample. We conclude that the observed amplitude cannot be described by Eq. (8a) in spite of the the phase change expected for the diffusion component being observed.

Some of the discrepancy may be connected with the behavior of $\tilde{\rho}_{yx}$ when $\omega_c\tau_q > 1$ as mentioned in Sec. II. As we noted there, the observed amplitude of $\tilde{\rho}_{yx}$ increases very strongly in this region with an effective Dingle temperature of twice the low-field value. If Eq. (7b) remains approximately valid in this region, then $\tilde{S}_{xx}$ would also exhibit a very large increase with a similar Dingle temperature, essentially following $\tilde{\rho}_{yx}$. All of these expectations are qualitatively in agreement with the observations. There would also be a phase change in keeping with that actually observed, i.e., $-\tilde{S}_{xx}$ would move into phase with $\tilde{\rho}_{xx}$, but this is also the case with other explanations. According to Eq. (7b), $\tilde{S}_{yx}$ would be affected to a much lesser extent because the second term is weighted by a factor of order $1/2(\omega_c\tau_t)^2$ compared with the first.

On the unscaled data in the lower half of Fig. 8, we have also shown a dotted line that corresponds to the expected amplitude of $\tilde{S}_{xx}$ given by Eq. (2). All the data appear to be approaching this curve at high fields, which is reasonable since this is where the equation should become valid. However, the fact that the dotted line lies close to the lowest temperature data at low fields is presumably coincidental since these data do not satisfy the necessary conditions for Eq. (2) to be valid, nor would we expect a minimum in amplitude as shown by the data.

V. CONCLUSIONS

A survey of the thermoelectric properties of a 2DEG sample grown on a strongly doped substrate has been made at zero and low magnetic fields. The doping led to a large decrease in the thermal conductivity of the substrate and also the phonon drag thermopower, although the resistive properties of the sample were not affected in any noticeable way.

The oscillations in the resistivity behave essentially as was found in previous work, which indicates that there are no fundamental problems with the sample. At low temperatures, the low-field oscillations in the Nernst-Ettingshausen coefficient S_{yx} show the expected field and temperature dependence for diffusion, although the amplitude is about a factor of 2 larger than predicted. However, even the amplitude of the simpler resistivity oscillations do not agree perfectly with theory, so we take a factor of 2 as being reasonable agreement. The fact that thermopower oscillations yield the same Dingle temperature as the resistivity oscillations makes the identification with diffusion especially convincing because phonon drag oscillations have a much higher effective Dingle temperature.

In contrast, the observed oscillations in the longitudinal thermopower $\tilde{S}_{xx}$ are far larger than expected. As mentioned in Sec. III, there was a spurious signal present in all the low-field data, which may be responsible for some of the discrepancy. At higher fields, it seems likely that $\tilde{S}_{xx}$ follows the observed behavior of $\tilde{\rho}_{yx}$ and has a much larger amplitude than the present theory predicts.

The use of strongly doped substrates to reduce phonon drag seems to be a technique that should be useful over a wide temperature and field range. We hope to examine the high-temperature properties of this sample in the near future.

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