Dynamic interaction of bulk acoustic waves with a two-dimensional electron gas at an $Al_xGa_{1-x}As/GaAs$ heterojunction in strong magnetic fields

A. G. Kozorezov* and V. W. Rampton
Department of Physics, University of Nottingham, University Park, Nottingham NG7 2RD, United Kingdom
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The results of a theoretical and experimental study of the interaction of longitudinal coherent bulk acoustic waves at 9.3 GHz with two-dimensional electron gas (2DEG) at a $GaAs/Al_xGa_{1-x}As$ heterojunction in a strong magnetic field are given. The theory of the linear 2DEG response to an incident bulk acoustic wave is developed, based on a system of quantum kinetic equations. The importance of disorder in the 2DEG and the coupling of the acoustic waves to the localized states is demonstrated. A comparison of the theoretical results with experiment is given.

I. INTRODUCTION

The use of monochromatic (coherent or incoherent) phonons can provide direct spectroscopy of the linear response of a two-dimensional electron gas (2DEG) over a substantial range of wave-vector and frequency $(\mathbf{k},\omega)$ values. Such spectroscopic measurements give information which is often otherwise inaccessible. Recently it has been shown that the absorption of high-frequency surface-acoustic waves (SAW's) reveals a noticeable contribution from the localized 2D electrons to the 2DEG linear response in strong magnetic fields.[1] At microwave frequencies (>1 GHz), this manifests itself in an additional mechanism of SAW dissipation differing markedly from what is commonly accepted as a result of the acoustoelectric interaction.[2]

If the 2DEG is expected to exhibit special behavior at frequencies lower than about 1 GHz, for instance, which is presumably the case for Wigner crystallization, then the use of SAW's gives very impressive results, showing explicitly the sharp transition to the low-temperature range $T<T_c$, where the diagonal component of the 2DEG conductivity tensor $\sigma_{xx}(\mathbf{k},\omega)$ becomes frequency dependent.[3,4] However, if one expects to observe deviations in the linear electronic response from the quasistatic values which are either due to nonlocal effects, singularities in 2DEG screening or the role played by the electronic transitions across the low energy gaps under fractional quantum Hall effect conditions, then it is more straightforward to use either incoherent phonons, (i.e., heat pulses) or coherent bulk acoustic waves (BAW's) as a probe to perturb the 2DEG system. Coherent bulk acoustic waves seem to be an extremely convenient experimental tool to do this, inasmuch as the frequency range around 10 GHz is easily accessible; the phonon energy is about 0.5 K, which is reasonably close to the characteristic energies of the 2DEG system. It is much more difficult to excite SAW's in such a high-frequency range. In spite of the fact that BAW-2DEG interaction takes place within a very thin 2DEG layer while the wave passes through it and, *a priori,* one might expect to observe very small effects, experimental results on BAW-2DEG interaction at an $Al_xGa_{1-x}As/GaAs$ heterojunction in high magnetic fields[5–8] have in fact shown an unexpectedly strong interaction, which turned out to be very sensitive to the 2DEG state.

The dynamic character of BAW-2DEG interaction was stressed in Refs. 9 and 10, where it was shown explicitly how both real and imaginary parts of the 2DEG response may affect the detected signal. It is, therefore, essential to treat BAW-2DEG interaction under the general framework of acoustic wave propagation in multilayered dissipative structures. We use the approach, based on the transfer-matrix formalism, which we generalize to account for the piezoelectric properties of the layers.[11,12] Exact expressions for the detected power at the bolometer were derived and analyzed to show the possibility of a remarkable enhancement of the bolometer sensitivity.

In this paper we start from expressions relating the acoustic fluxes to the properties of the 2DEG, which we describe via the introduction of a complex generalized stress in the plane of the 2DEG. The latter arises from the 2DEG linear response to the perturbing potential of the acoustic wave. Throughout the work, we are concerned with the 2DEG response in a high magnetic field. The simplest case is the piezoelectric coupling to the 2D electrons in extended states in the quantum Hall regime, which, for low BAW frequencies, may be found by a phenomenological treatment exactly as for the SAW-2DEG interaction. This is done in Sec. I. A comparison with well-known results for SAW-2DEG interaction makes the differences clear. These differences arise from the dynamical character of BAW-2DEG interaction.

However, our main purpose is to derive and discuss in detail another contribution to the 2DEG linear response. This arises from the electron redistribution caused by the wave, which breaks the thermal equilibrium between states belonging to different Landau levels, or between localized states in a disordered system. To our knowledge this problem has not been discussed so far. Clearly this part of the 2DEG response causes relaxation-type absorption of both BAW's and SAW's by the 2DEG.[1,8] The nontrivial problems here are (1) why is the coupling constant for relaxation absorption of the SAW's by the 2DEG in localized states on an atomic scale; (2) can piezoelectric coupling become effective in relaxation absorption; and (3) why can the dependence of the absorption coefficient on the magnetic field be accounted for by the magnetic-field dependence of the thermodynamic density of 2DEG states with the assumption that both the coupling constant and relaxation times are magnetic field independent? It appears that the answer to these questions cannot be given under the framework of the phenomenological approach used in Ref. 1.

Therefore, we derive the relevant 2DEG linear-response functions by solving the quantum kinetic equation. This is done in Sec. II for the case of deformation-potential coupling and piezoelectric coupling of the acoustic wave to the electronic states for both idealized and disordered structures. In Sec. III we discuss the experimental results and give numerical estimates, while final remarks are made in Sec. IV.

The experiments were made in transmission; the experimental arrangement is shown in Fig. 1. It shows that we must consider the phonon absorption and scattering in a multilayered dissipative structure. A simplified scheme of interaction relevant to the transmission geometry will be used for the calculations, and is shown in Fig. 2.

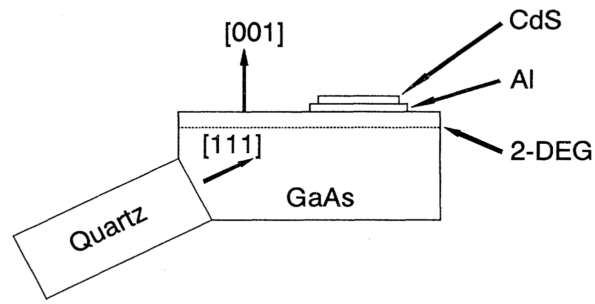

FIG. 1. The arrangement of sample and quartz ultrasonic transducer for BAW-2DEG interaction experiments.

A general expression for the relative change in the bolometer signal $\Delta S/S$ due to the 2DEG dynamic response was derived in Ref. 12. It has the form

$$\frac{\Delta S}{S}=\frac{2}{\rho\omega a_+^L}\mathrm{Im}\left(T_{xz}^{\mathrm{2DEG}}+\tfrac{1}{2}T_{zz}^{\mathrm{2DEG}}\right)+\frac{1}{\rho\omega a_+^L}\left\{\frac{A_1\mathrm{Im}\left[\left[T_{xz}^{\mathrm{2DEG}}-\frac{\kappa}{\kappa_z^S}T_{zz}^{\mathrm{2DEG}}\right]r_{12}r_{22}^*\right]+A_2\mathrm{Im}\left[\left[T_{xz}^{\mathrm{2DEG}}-\frac{\kappa}{\kappa_z^S}T_{zz}^{\mathrm{2DEG}}\right]r_{11}r_{21}^*\right]}{A_1|r_{22}|^2+A_2|r_{21}|^2-A_3\mathrm{Im}(r_{22}r_{21}^*)}-\frac{\tfrac{1}{2}A_3\mathrm{Re}\left[\left[T_{xz}^{\mathrm{2DEG}}-\frac{\kappa}{\kappa_z^S}T_{zz}^{\mathrm{2DEG}}\right](r_{11}r_{22}^*-r_{12}r_{21}^*)\right]}{A_1|r_{22}|^2+A_2|r_{21}|^2-A_3\mathrm{Im}(r_{22}r_{21}^*)}\right\}. \quad (1)$$

Here ρ is the density of the crystal, $\omega/2\pi$ is the BAW frequency, and a_+^L is the amplitude of the incident wave. T_{ij}^{2DEG} are the components of the stress tensor due to the 2DEG perturbed by the incident wave. We have taken the z axis to be normal to the plane of the 2DEG, while the x axis is in a principal direction in the 2DEG plane. $A_{1,2,3}$ are coefficients expressed in terms of the material parameters for different layers of the structure and $r_{mn}(m,n=1,2)$ are related to the elements of the transfer matrices. Exact formulas for A_l and r_{mn} are given in Ref. 12. Note that, in Eq. (1), the geometry of the multilayered structure and, therefore, all information about the reflection, transmission, change of polarization, wave interference, and dissipation which occurs, other than that due to the interaction with the 2DEG, is hidden in A_l and r_{mn}. The effect of the 2DEG has thus been separated and expressed in terms of both the real and imaginary parts of the generalized 2DEG stress components, which reflect both absorption in the 2DEG layer and the changed reflectivity of the perturbed 2DEG.

The piezoelectric interaction of the BAW with 2D electrons in extended states

We start by writing down the set of equations appropriate to the piezoelectric media:

$$\rho\frac{\partial v_i}{\partial t}=\frac{\partial T_{ik}}{\partial x_k}+\beta_{m,ik}\frac{\partial E_m}{\partial x_k}\ ,$$
$$\mathrm{div}\ \mathbf{D}=0\ , \quad (2)$$
$$D_i=\varepsilon E_i-\frac{4\pi i}{\omega}\beta_{i,ml}\frac{\partial v_m}{\partial x_l}\ ,$$

where $v_i(\mathbf{x},t)$ is the particle velocity; E_i and D_i are the electric field and electric displacement vectors, respectively; $\beta_{m,il}$ is the piezoelectric modulus tensor; and ε is the dielectric constant. The piezoelectricity results in slight changes of the eigenmodes in the stack of

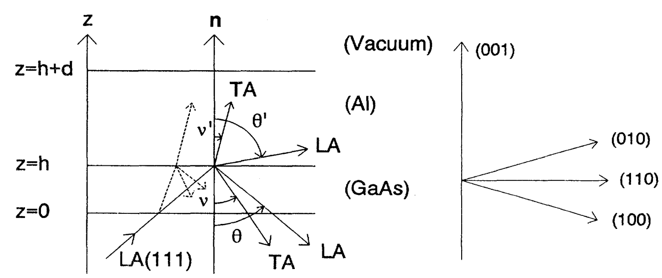

FIG. 2. The model structure used for the calculations.

$Al_xGa_{1-x}As/GaAs$ layers due to the electric fields accompanying the traveling acoustic waves, followed by corresponding changes in both reflection and transmission coefficients due to the boundary conditions for the components of the electric field and electric displacement which must be satisfied at the interfaces. This latter effect is small. Its order of magnitude is given by the value of the electromechanical coupling constant $\eta=(4\pi\beta_{14}^2/\varepsilon\mu)\ll 1$ (where μ is a Lamé coefficient), and in what follows we completely neglect such a contribution. However we keep a few terms linear in $\eta \ll 1$ which arise as a result of the 2DEG density modulation by an incident wave due to the piezoelectric coupling. The discrimination between different contributions linear in η is therefore whether they are related to the piezoelectric properties of $Al_xGa_{1-x}As/GaAs$ layers or to the 2DEG response.

To calculate the additional stress at the $z=0$ interface, induced by the perturbed 2DEG, we use an iterative procedure

$$T_{ik}^{\text{2DEG,piezo}}=\beta_{m,ik}E_m=-\beta_{m,ik}\frac{\partial\varphi}{\partial x_m}\ , \tag{3}$$

where $\mathbf{E}=-\nabla\varphi$, and φ is the electric potential. It follows immediately from (3) that only the yz and xz components of an additional 2DEG-induced stress are nonzero.

To find the electric potential φ to be used in Eq. (3) we rewrite Poisson's equation with the use of the expression for the electric displacement vector D_i [e.g., (2)]:

$$\Delta\varphi=-\frac{8\pi i}{\varepsilon\omega}\beta_{14}\left[2i\kappa\frac{\partial v_x}{\partial x}-\kappa^2 v_z\right]\ ,$$

where $\sqrt{2}\kappa$ is the BAW in-plane momentum with respect to the 2DEG plane. Solutions of this equation must satisfy the following boundary conditions:

$$E_x(-0)=E_x(+0)\ ,$$

$$D_z(+0)-D_z(-0)=4\pi e n_s\ ,$$

$$\varphi(h)=0\ ,$$

where n_s is the modulated part of the 2DEG density; the last equation is the boundary condition for the electric potential φ at a metal surface. For low BAW frequencies, the 2DEG charge density en_s can be related to the potential $\varphi(0)$ by the continuity equation $e\partial n_s/\partial t+\operatorname{div}\mathbf{j}=0$ using the 2DEG current due to mobile carriers in the form

$$j_x=\sigma_{xx}E_x+\sigma_{xy}E_y-e\bar{D}_{xx}\frac{\partial n_s}{\partial x}-e\bar{D}_{xy}\frac{\partial n_s}{\partial y}\ ,$$

$$j_y=-\sigma_{xy}E_x+\sigma_{xx}E_y-e\bar{D}_{yy}\frac{\partial n_s}{\partial y}+e\bar{D}_{xy}\frac{\partial n_s}{\partial x}\ ,$$

where σ_{ij} and $\bar{D}_{ij}$ are the 2DEG conductivity and diffusion constant tensors, respectively. To calculate $T_{ik}^{\text{2DEG,piezo}}$ to a linear approximation in $\eta \ll 1$, we can take the distribution of the elastic fields in a zeroth-order approximation.[12] Thus we find finally that

$$\varphi(0)=\frac{8\pi\beta_{14}\kappa}{\varepsilon\omega}\,\frac{a_+^L-a_-^L+2\left[\dfrac{\kappa}{\kappa^S}\right]^2\left[1-\left[\dfrac{\kappa_z^S}{\kappa}\right]^2\right]a_-^S}{1+i\dfrac{\sigma_{xx}}{\sigma_0}\dfrac{1}{1+i\omega/\omega_D}e^{-\sqrt{2}\kappa h}\sinh(\sqrt{2}\kappa h)}$$
$$\times\frac{-e^{-\sqrt{2}\kappa h}\left\{a_+^L e^{i\kappa h}-a_-^L e^{-i\kappa h}+2\left[\dfrac{\kappa}{\kappa^S}\right]^2\left[1-\left[\dfrac{\kappa_z^S}{\kappa}\right]^2\right]a_-^S e^{-i\kappa_z^S h}\right\}}{1+i\dfrac{\sigma_{xx}}{\sigma_0}\dfrac{1}{1+i\omega/\omega_D}e^{-\sqrt{2}\kappa h}\sinh(\sqrt{2}\kappa h)}\ . \tag{4}$$

In this equation, a_-^L and a_-^S are the amplitudes of the longitudinal and shear waves propagating in the backwards direction, taken in the zeroth-order approximation $\eta\to 0$, and $\sigma_0=(\sqrt{6}\varepsilon V_l/8\pi)$ and $\omega_D\equiv\frac{3}{2}(V_l^2/D_{xx})$, where ω_D is the diffusion frequency and V_l is the phase velocity of the incident wave, and κ^S and κ_z^S are the modulus of the wave vector for a shear acoustic wave at frequency ω and its projection onto the z axis. Combining (4) with (3) yields $T_{xz}^{\text{2DEG,piezo}}=-i\kappa\beta_{14}\varphi(0)$ with $\varphi(0)$ from (4), and the known amplitudes of the elastic waves in the $z\le 0$ half-space.

It is also worth showing the effect of acoustic power dissipation in the 2DEG layer due to joule heating. The joule heat power released (per unit area) is $w=\overline{\mathbf{j}\cdot\mathbf{E}}$ (the bar as before means the average over the acoustic wave period). We arrive at

$$w_{\text{joule}}=\sigma_{xx}\kappa^2|\varphi(0)|^2$$
$$\propto\eta\frac{\sigma_{xx}/\sigma_0}{1+(\sigma_{xx}/\sigma_0)^2[1/1+(\omega/\omega_D)^2]e^{-2\sqrt{2}\kappa h}\sinh^2(\sqrt{2}\kappa h)}\ . \tag{5}$$

Using $\varphi(0)$ from (4), we conclude that the order of magnitude of the relative change in the intensity of an incident wave due to power dissipation within the 2DEG layer is given by the electromechanical coupling constant η, which can be shown to be close to its value in $Al_xGa_{1-x}As$: $\eta \approx 5.1\times10^{-3}$.[13]

II. 2DEG LINEAR RESPONSE IN A STRONG MAGNETIC FIELD

A. Deformation-potential interaction between the BAW and the 2DEG

We start from a consideration of BAW-2DEG interaction via the deformation potential, but the generalization to include the piezoelectric interaction is given below. The deformation-potential coupling between the BAW and 2D electrons arises from the modulation of the 2D electron energies by the incident acoustic wave: $E_\lambda(\mathbf{x},t)=E_\lambda+\langle\lambda|\Lambda_{\alpha\beta}e^{i\kappa x}|\lambda\rangle u_{\alpha\beta}(\mathbf{x},t)$, where $\langle\lambda|$ is one of the 2DEG states, $\Lambda_{\alpha\beta}$ is the deformation-potential tensor, and $u_{\alpha\beta}(\mathbf{x},t)$ is the strain tensor. The modulation in the 2DEG occupation numbers δp_λ produces an additional stress $T_{ij}^{2\mathrm{DEG}}$ in the 2DEG plane, $z=0$. The problem thus reduces to finding the nonequilibrium contribution to the 2DEG occupation numbers, δp_λ.

Consider an idealized structure without random potentials in which the 2D states form an ideal Landau ladder (LL). Let the Fermi level lie between the Nth and $(N+1)$th LL. We discuss the specific deformation potential interaction of the BAW with electrons in the two LL closest to the Fermi level. The number of states at each LL is $1/\pi l_B^2$, where l_B is the magnetic length. The number of absorbing electrons is therefore $n_s-N(l/\pi 1_B^2)\le(n_s/N)$, where n_s is the equilibrium 2DEG density. If there is no electron redistribution due to intra-LL scattering then the only source of dissipation is the modulation of the occupation numbers in the Nth and $(N+1)$th LL induced by the incident acoustic wave followed by the relaxation of the perturbed 2DEG distribution. From a formal point of view this type of interaction looks like an interaction between the acoustic wave and two-level centers, provided the interaction of the acoustic wave with the rest of Landau ladder is small. In principle, we still can treat this case with the use of a phenomenological approach as in Ref. 1, where the necessary rearrangement in notation which refers to the relevant 2DEG states should be done. We will, however, give a full derivation of the result and show an exactly solvable extreme case, without carrying out such a rearrangement. This also throws additional light on the problem BAW-2DEG deformation-potential interaction and enables us to clarify the concept of BAW interaction with the 2DEG in localized states.

Let $|N,p\rangle$ and $|N+1,p'\rangle$ be the electronic states at Nth and $(N+1)$th LL, respectively, and p and p' the corresponding quasimomentum components in the y direction; the x coordinate defines the electron orbit center. We start our discussion with a consideration of the effect the BAW produces on the energy separation between LL's. First we consider the two 2D electron states $|N,p_y\rangle$ and $|N+1,p_y\rangle$ with the same orbit center $(p_y=p)$.

Taking $\Lambda_{\alpha\beta}$ as a constant and calculating the matrix elements $\langle N,p_y|\Lambda_{\alpha\beta}u^0_{\alpha\beta}e^{i\kappa x}|N,p_y\rangle$ and $\langle N+1,p_y|\Lambda_{\alpha\beta}u^0_{\alpha\beta}e^{i\kappa x}|N+1,p_y\rangle$ using wave functions for the ideal 2DEG as in Ref. 14 we arrive at

$$\delta E_{N,P}=I_{N,N}(\kappa_x,\kappa_y)e^{i\kappa_x X}\Lambda_{\alpha\beta}u^0_{\alpha\beta}$$

and

$$\delta E_{N+1,p}=I_{N+1,N+1}(\kappa_x,\kappa_y)e^{i\kappa_x X}\Lambda_{\alpha\beta}u^0_{\alpha\beta} \; .$$

In these expressions, $\delta E_{N,p}=E_{N,p}-\hbar\omega_c(N+\frac{1}{2})$, where ω_c is the cyclotron frequency, $u^0_{\alpha\beta}$ is the amplitude of $u_{\alpha\beta}$, and

$$I_{N,N}(\kappa_x,\kappa_y)=e^{-(1/4)(\xi_0^2+\xi_1^2)-(1/2)i\xi_0\xi_1\sum_{r=0}^{N}[-(1/2)(\xi_0^2+\xi_1^2)]^r(1/r!)^N C_{N-r}} \; ,$$

$$\xi_0=\kappa_y l_B, \quad \xi_1=\kappa_x l_B \quad \text{and} \quad X=-l_B^2 p_y$$

is the coordinate of the electron orbit center, while ${}^N C_{N-r}$ are binomial coefficients. The acoustic wave modulates the energy separation between the LL, the separation for the two states at $|N,p\rangle$, $|N+1,p\rangle$ being given by

$$\begin{aligned} E_{N+1}(x,t)-E_N(x,t) &= \hbar\omega_c+\Lambda_{\alpha\beta}u^0_{\alpha\beta}e^{i\kappa_x X} \\ &\quad\times[I_{N+1,N+1}(\kappa_x,\kappa_y)-I_{N,N}(\kappa_x,\kappa_y)] \; . \end{aligned} \tag{6}$$

It is now evident that for strong magnetic fields $(\xi \ll 1)$ the effect of the modulation of the energy separation becomes very small: $(|E_{N+1,\bar{p}}E_{N,p}-\hbar\omega_c|)/(|\Lambda_{\alpha\beta}u^0_{\alpha\beta}|)\approx\kappa^2 l_B^2 \ll 1$ because the energies of the electrons belonging to the different LL's are shifted by almost the same amount. At low magnetic fields $(\kappa l_B \gg 1)$ the scale of the LL energy modulation turns out to be exponentially small. We conclude that the effect of the modulation of the energy separation between the two 2DEG states $|N,p\rangle$ and $|N+1,p\rangle$ is appreciable at $\kappa l_B \simeq 1$ even with the same bare deformation potential. For other magnetic-field strength values it becomes insignificant.

So far we have discussed the 2DEG states with electron orbit centers at the same point. In strong magnetic fields, however, these states may not contribute to BAW-

2DEG interaction simply because the phase volume for such an interaction is small. The acoustic wave breaks the thermodynamic equilibrium in the 2DEG system by modulating the 2DEG energies in space and time. If we neglect electron-electron interactions then the acoustic wave breaks the balance between the states with orbit centers at different points in space. In thermodynamic equilibrium detailed balance can be applied; for an arbitrary interacting pair of states $|\lambda\rangle=|N,p\rangle$ and $|\lambda'\rangle=|N+1,p'\rangle$ the rate of transition from $|\lambda\rangle$ to $|\lambda'\rangle$ equals that of from $|\lambda'\rangle$ to $|\lambda\rangle$, for simplicity we consider the dominant interaction mechanism to be due to one-phonon absorption or emission processes. If $M^{\lambda,\lambda'}(\mathbf{q},s)$ is the matrix element for such an interaction, we must have

$$M^{\lambda,\lambda'}(\mathbf{q},s)[\rho^0_\lambda(1-\rho^0_{\lambda'})n_{\mathbf{q}s}-\rho^0_{\lambda'}(1-p^0_\lambda)(n_{\mathbf{q}s}+1)]\delta(\hbar\omega_{\mathbf{q}s}-\hbar\omega_c)=0 \; .$$

In this expression ($\mathbf{q}s$) are the phonon wave vector and branch index, respectively; $n_{\mathbf{q}s}$ is the Planck distribution function

$$\rho^0_\lambda=\frac{1}{e^{[(E_\lambda-E_F)/k_BT]}}+1 \; ,$$

and E_F is the chemical potential. Due to the absorption or emission of a cyclotron phonon the electron orbit center moves from $X=l_B^2p$ to $X'=l_B^2p'$, so that $\Delta X=X'-X=q_yl_B^2$. For cyclotron phonons we have $|q_y|\le(\omega_c/v_s)$, and thus a one-phonon interaction couples those 2D electrons with ΔX within the range $|\Delta X|\le x_0=(\hbar/m^*v_s)$. However, the dominant contribution (and hence the most effective coupling) comes from the exchange of phonons within a narrow conical shell with respect to the normal. That obviously restricts the value $|q_y|$ for the effective phonons and reduces the separation ΔX for the effective 2D pairs to $\approx 2\sqrt{2N+1}l_B$. For low enough magnetic fields, $x_1=2\sqrt{2N+1}l_B>(\hbar/m^*v_s)$. The last inequality holds true below about 0.6 T. The characteristic scale in space for the acoustic wave is given by the wavelength $\lambda_s\sim 1/\kappa$. Therefore, if $\kappa x_0\simeq 1$, the modulation of the 2DEG energies by the acoustic wave drives the 2DEG system far from equilibrium, even if the modulation is due to the same bare deformation potential. For instance, if at low magnetic fields when $x_0<x_1$, and $\kappa x_0\simeq\pi(x_0\simeq\lambda/2)$, the acoustic wave provides a positive shift in the energy of one of the states and a negative shift to the other. Note that for our case of BAW at 10 GHz propagating in the (111) direction $\kappa\simeq 0.63\times 10^5$ cm^{-1} and $\kappa x_0\simeq 2.5$; thus, for the model under consideration, we are very close to the optimum for BAW-2DEG interaction.

Now we turn to a quantitative treatment of the BAW-2DEG interaction. Let $\hat{\mathcal{H}}=\hat{\mathcal{H}}_0+\hat{\mathcal{H}}_{e\varphi}+\hat{\mathcal{H}}_{e\text{-ph}}$ be the full Hamiltonian of the system.

$$\hat{\mathcal{H}}_0=\sum_\lambda E_\lambda c^\dagger_\lambda c_\lambda+\sum_{\mathbf{q}s}\hbar\omega_{\mathbf{q}s}(a^\dagger_{\mathbf{q}s}a_{\mathbf{q}s}+\tfrac{1}{2}) \tag{7}$$

is the Hamiltonian for the noninteracting 2D electrons and phonons, and $c^\dagger_\lambda$, c_λ and $a^\dagger_{\mathbf{q}s}$, $q_{\mathbf{q}s}$ are the 2D electron and phonon creation and annihilation operators, respectively.

$$\hat{\mathcal{H}}_{e\varphi}=\sum_{\lambda\lambda'}c^\dagger_\lambda c_{\lambda'}\langle\lambda|\Lambda_{\alpha\beta}u_{\alpha\beta}(\mathbf{r}t)|\lambda'\rangle \tag{8}$$

is the Hamiltonian describing the 2DEG interaction with the coherent part of the phonon field (i.e., 2DEG-BAW interaction) taken as a classical field, slowly varying in space and time.

$$\hat{\mathcal{H}}_{e\text{-ph}}=\sum_{\lambda\lambda'}M^{\lambda\lambda'}(\mathbf{q}s)c^\dagger_\lambda c_{\lambda'}(a_{\mathbf{q}s}+a^\dagger_{-\mathbf{q}s}) \tag{9}$$

is the Hamiltonian describing the 2D electron-phonon interaction.

By definition, the stress $t_{ij}(x,t)$ produced by the 2DEG perturbed by acoustic wave takes the following form:

$$\begin{aligned} t_{ij}&=\sum_{\lambda\lambda'}\mathrm{Tr}\hat\rho c^\dagger_\lambda c_{\lambda'}\delta/\delta u_{ij}(x,t)\langle\lambda|\Lambda_{\alpha\beta}u_{\alpha\beta}(r,t)|\lambda'\rangle\\ &=\Lambda_{ij}\mathrm{Tr}(\hat\rho c^\dagger_\lambda c_{\lambda'})\psi^*_\lambda(x)\psi_{\lambda'}(x)\\ &\cong\Lambda_{ij}\sum_{np_yp_y'}\langle c^\dagger_{np_y}c_{np_y'}\rangle\phi^*_n(x-\tilde X)\phi_n(x-\tilde X')\\ &\qquad\times e^{-i(p_y-p_y')y}|\psi_0(z)|^2 \; , \end{aligned} \tag{10}$$

where $\hat\rho$ is the density matrix, $\langle c^\dagger_\lambda c_{\lambda'}\rangle\equiv\mathrm{Tr}(\hat\rho c^\dagger_\lambda c_{\lambda'})$, $\delta/\delta u_{ij}(x,t)$ denotes the functional derivative, $\psi_\lambda(\mathbf{x})$ is the wave function for the state $|\lambda\rangle$, and $\phi_n(x-\tilde X)$ is the harmonic-oscillator wave function. In writing the last equality in (10) we have assumed that $|\lambda'\rangle=|n,p_y'\rangle$; i.e., since $\hbar\omega_s\ll\hbar\omega_c$, there is no resonant coupling between the acoustic wave and LL.

Since we are looking at the 2DEG response in the form $t_{ij}(x,t)\propto e^{i(\kappa_xx+\kappa_yy)}$ we put $p_y'=p_y-\kappa_y$ and calculate $\langle c^\dagger_{n,p_y}c_{n,p_y-\kappa_y}\rangle$. We introduce an operator $\hat P_{N,p_y,p_y'}=c^\dagger_{n,p_y}c_{n,p_y'}$, then the statistically averaged quantity $\langle\hat P_{N,p_y,p_y'}\rangle=\langle e^\dagger_{n,p_y}c_{n,p_y'}\rangle$ obeys the quantum kinetic equation[15]

$$\frac{\partial\langle\hat P_{N,p_yp_y'}\rangle}{\partial t}=-\frac{1}{\hbar^2}\int_{-\infty}^0 dt\, e^{-\epsilon t}\times\langle[\hat{\mathcal{H}}_{ep}(t),[\hat{\mathcal{H}}_{ep},\hat P_{N,p_yp_y'}]]\rangle_Q \; . \tag{11}$$

This is the quantum kinetic equation with the collision operator taken up to the second order in the interaction, $\epsilon\to+0$. The field term in the kinetic equation is absent in as much as the commutator $[\hat{\mathcal{H}}_0,\hat P_{N,p_y,p_y'}]=(E_{N,p_y'}-E_{N,p_y})\hat P_{N,p_y,p_y'}=0$ because of LL degeneracy with respect to p_y. The operators in (11) are taken in the interaction representation, and the subscript Q in (11) means that the statistical average in the integrand is to be taken with the quasiequilibrium density matrix $\hat{\rho}_Q$.

Performing the commutator $[\hat{\mathcal{H}}_{ep},\hat{P}_{N,p_y p_y'}]$ and using a simple decoupling scheme for the statistically averaged operators, we arrive, after time integration, at the following coupled system of kinetic equations for the two functions $\delta f_{N,p_y,p_y-\kappa}$ and $\delta f_{N+1,p_y,p_y-\kappa}$ describing the nonequilibrium parts of the 2DEG distribution:

$$(-i\omega+\nu_N)\delta g_N-\nu_{N+1}(\kappa)\delta g_{N+1}=\nu_N g_N^l-\nu_{N+1}(\kappa)g_{N+1}^l\ , \quad (12)$$

$$(-i\omega+\nu_{N+1})\delta g_{N+1}-\nu_N(\kappa)\delta g_N=\nu_{N+1}g_{N+1}^l-\nu_N(\kappa)g_N^l\ .$$

The functions δg_N and δg_{N+1} are defined by $\delta f_{N,p_y,p_y-\kappa_y}=\delta g_N e^{i\kappa p_y l_B^2}$ and $\delta f_{N+1,p_y,p_y-\kappa_y}=\delta g_{N+1}e^{i\kappa p_y l_B^2}$, and are, since we are looking for the 2DEG response, proportional to $e^{i\kappa(x+y)}$.

The relaxation rates $\nu_N(\kappa)$, $\nu_{N+1}(\kappa)$, ν_N, and ν_{N+1} are given by the following expressions:

$$\nu_N(\kappa)=\sum_{\mathbf{q}s}\frac{\pi\Lambda^2(e_{\mathbf{q}s}q)^2}{\rho V\omega_{\mathbf{q}s}}|I_{N+1,N}(q_x,q_y)|^2\times\delta(\hbar\omega_{\mathbf{q}s}-\hbar\omega_c)(n_{\mathbf{q}s}+f_{N+1}^0)e^{-i(\kappa_\parallel q)l_B^2}\ ,$$

$$\nu_{N+1}(\kappa)=\sum_{\mathbf{q}s}\frac{\pi\Lambda^2(e_{\mathbf{q}s}q)^2}{\rho V\omega_{\mathbf{q}s}}|I_{N+1,N}(q_x,q_y)|^2\times\delta(\hbar\omega_{\mathbf{q}s}-\hbar\omega_c)(n_{\mathbf{q}s}+1-f_N^0)e^{-i(\kappa_\parallel q)l_B^2}\ , \quad (13)$$

$$\nu_N=\nu_N(\kappa=0)\ ,$$

$$\nu_{N+1}=\nu_{N+1}(\kappa=0)\ ,$$

where $e_{\mathbf{q}s}$ are the polarization vectors for interacting phonons, and $\kappa_\parallel$ is the projection of the BAW wave vector onto the 2DEG plane. $g_{N(N+1)}^l=f_0'(E_{N(N+1)})\Lambda_{\alpha\beta}u_{\alpha\beta}^0 I_{N,N(N+1,N+1)}(\kappa,\kappa)$ are the functions describing the deviation of local equilibrium in the strained system from thermal equilibrium.

Substituting the solution of the kinetic equations (12) into equations (10) yields

$$t_{ij}=\Lambda_{ij}\Lambda_{\alpha\beta}u_{\alpha\beta}\frac{|\psi_0(z)|^2}{\pi l_B^2}\frac{1}{-\omega^2-i\omega(\nu_N+\nu_{N+1})+[\nu_N\nu_{N+1}-\nu_N(\kappa)\nu_{N+1}(\kappa)]}$$
$$\times\{-i\omega[I_{N,N}f_0'(E_N)[\nu_N I_{N,N}-\nu_N(\kappa)I_{N+1,N+1}]+I_{N+1,N+1}f_0'(E_{N+1})[\nu_{N+1}I_{N+1,N+1}-\nu_{N+1}(\kappa)I_{N,N}]]$$
$$+[I_{N,N}^2 f_0'(E_N)+I_{N+1,N+1}^2 f_0'(E_{N+1})][\nu_N\nu_{N+1}-\nu_N(\kappa)\nu_{N+1}(\kappa)]\}\ . \quad (14)$$

$I_{N,N}$ and $I_{N+1,N+1}$ in (14) stand for $I_{N,N}(\kappa,\kappa)$ and $I_{N+1,N+1}(\kappa,\kappa)$. Both effects of the spatial modulation of the 2DEG density $[\nu(\kappa)\neq\nu]$ and the local variation of the gap separation (6) are equally important in producing the nonzero stress t_{ij}. If we take $\nu_{N(N+1)}(\kappa)\to\nu_{N(N+1)}$ and $I_{N,N}\to I_{N+1,N+1}$, then $t_{ij}\to 0$, as it should.

In the preceding analysis we have noted that the relaxation interaction of the BAW and 2DEG occurs even with the same bare potential perturbing the 2DEG states. The piezoelectricity which provides the interaction energy $e\varphi$ for all the states can be taken into account using the same approach. The interaction Hamiltonian changes to become

$$\hat{\mathcal{H}}_{e\varphi}=\sum_{\lambda\lambda'}c_\lambda^\dagger c_{\lambda'}\langle\lambda|(\Lambda_{\alpha\beta}u_{\alpha\beta}^0+e\varphi^0)e^{ikx}|\lambda'\rangle\ ,$$

where φ^0 is the amplitude of the potential. Taking a linear relationship between $e\varphi(x,t)$ and the strain $u_{\alpha\beta}(x,t)$ in the form $e\varphi(\mathbf{x},t)=\chi_{\alpha\beta}(\mathbf{k})u_{\alpha\beta}(\mathbf{x},t)$, where $\chi_{\alpha\beta}(\mathbf{k})$ is the complex linear-response function, we come to the conclusion that, to account for the piezoelectricity, we must substitute Λ_{ij} in (14) by $\Lambda_{ij}+\chi_{ij}$ and $\Lambda_{\alpha\beta}$ by $\Lambda_{\alpha\beta}+\chi_{\alpha\beta}$.

We disregard the angular dependence in (13) resulting from $(\mathbf{e}_{qs}\mathbf{q})^2$ and introduce the average value of the deformation potential $\overline{\Lambda}$ by $\overline{\Lambda}^2=\Lambda^2(\mathbf{e}_{qs}\mathbf{q})^2 1/q$. Then

$$\nu_N(\kappa)=\nu_N\nu_0(\xi_0)\quad\text{and}\quad\nu_{N+1}(\kappa)=\nu_{N+1}\nu_0(\xi_0)\ ,$$

where

$$\nu_0(\xi_0)=\frac{\int_0^{\alpha_0}dx\,|I_{N+1,N}(\sqrt{x},0)|^2 J_0(\sqrt{2}\xi_0\sqrt{x})/\sqrt{\alpha_0-x}}{\int_0^{\alpha_0}dx\,|I_{N+1,N}(\sqrt{x},0)|^2/\sqrt{\alpha_0-x}}\ .$$

$J_0(z)$ is the Bessel function and $\alpha_0=(\omega_c l_B/v_s)^2\gg 1$.

At low magnetic fields the overlap integral $I_{N+1,N}$ goes rapidly to zero if $\sqrt{x}$ exceeds $2\sqrt{2N+1}$. If $\alpha_0<4(2N+1)$ (this is equivalent to $x_0\le x_1$), then the argument of the Bessel function, $\sqrt{2}\xi_0\sqrt{\alpha_0}$ is much bigger than 1 and, under these conditions, $\nu_0(\xi_0)$ differs significantly from 1, going rapidly to zero at low magnetic fields. If $\alpha_0\gg 4(2N+1)$, then we may set the upper limit in the integrals to ∞ and in the limit of strong fields take the series expansion for the Bessel function. Thus

$$\nu_0(\xi_0)\to 1-\xi_0^2\frac{\int_0^\infty dx\, x\,|I_{N+1,N}(\sqrt{x},0)|^2}{\int_0^\infty dx\,|I_{N+1,N}(\sqrt{x},0)|^2}=1-\alpha\xi_0^2 .$$

Returning to (14), we can rewrite it in the form

$$t_{ij}=\Lambda_{ij}\Lambda_{\alpha\beta}u_{\alpha\beta}\frac{|\psi_0(z)|^2}{\pi l_B^2}[f_0'(E_N)+f_0'(E_{N+1})] \times\frac{-i\omega\nu_1(N,N+1)-\nu_2^2(N,N+1)}{-\omega^2-i\omega(\nu_N+\nu_{N+1})+\nu_N\nu_{N+1}(1-\nu_0^2)} , \quad (15)$$

where

$$\nu_1(N,N+1)=(I_{N,N}-I_{N+1,N+1})\frac{\nu_N I_{N,N}f_0'(E_N)-\nu_{N+1}I_{N+1,N+1}f_0'(E_{N+1})}{f_0'(E_N)+f_0'(E_{N+1})} +(1-\nu_0)I_{N,N}I_{N+1,N+1}\frac{\nu_N f_0'(E_N)+\nu_{N+1}f_0'(E_{N+1})}{f_0'(E_N)+f_0'(E_{N+1})}$$

and

$$\nu_2^2(N,N+1)=\nu_N\nu_{N+1}(1-\nu_0^2) \times\frac{I_{N,N}^2 f_0'(E_N)+I_{N+1,N+1}^2 f_0'(E_{N+1})}{f_0'(E_N)+f_0'(E_{N+1})} .$$

The general structure of Eqs. (14) and (15) is the same, but the frequency dependence in (14) and (15) is more complicated than that found in the phenomenological approach used in Ref. 1.

B. BAW—disordered 2DEG interactions

Now we generalize the results already obtained for the idealized model of the 2DEG in order to include the effects that disorder produces on the BAW-2DEG interaction. We consider a model in which the 2DEG states are influenced by a smooth random potential $V(r)$ with a characteristic scale $R_0 \gg l_B$, and rms amplitude $V_c=(\overline{V^2(r)})^{1/2}$, where the bar denotes the statistical average. Thus the random potential is treated as a classical field which influences the 2D electron motion. The collisionless 2D electron drift is therefore determined by the crossed random electrical and external magnetic fields. The disorder also causes significant changes in the transition probabilities. We can derive the quantum kinetic equation to describe the linear response of the disordered 2DEG to the perturbation induced by the BAW. Keeping in mind the previous results for the ideal system, we consider the piezoelectric mechanism as the major source of the interaction.

We take the Hamiltonian of the system to be $\hat{\mathcal{H}}=\hat{\mathcal{H}}_0+\hat{\mathcal{H}}_v+\hat{\mathcal{H}}_{e\varphi}+\hat{\mathcal{H}}_{e\text{-ph}}$, adding to (7)–(9) the Hamiltonian $\hat{\mathcal{H}}_v=V_c\sum_{\lambda\lambda'}c_\lambda^\dagger c_{\lambda'}\langle\lambda|v|\lambda'\rangle$ to describe the 2DEG interaction with the classical field; $v(r)=V(r)/V_c$ is the dimensionless random potential.

The quantum kinetic equation to the second order in interaction with phonons reads

$$\frac{\partial\hat{P}_{N,p_y p_y'}}{\partial t}+\frac{1}{i\hbar}\langle[\hat{\mathcal{H}}_0+\hat{\mathcal{H}}_v+\hat{\mathcal{H}}_{e-\varphi},\hat{P}_{N,p_y p_y'}]\rangle =\hat{I}\{\hat{P}_{N,p_y p_y'}\} . \quad (16)$$

The left-hand side of (16) describes the collisionless evolution of the 2DEG distribution, while $\hat{I}\{\hat{P}_{Np_y p_y'}\}$ stands for the collision integral. The disorder determines the dynamical motion of the 2D electrons and also affects the collisions by modifying their rates and making possible transitions which are absent in an ideal system.

Performing the commutator on the left-hand side of (16) we arrive at the following equation:

$$\frac{\partial\hat{P}^1_{N,p_y p_y'}}{\partial t}+\frac{l_B^2 V_c}{\hbar}\sum_{p_y''}\langle Np_y'|(\boldsymbol{\kappa}\cdot\nabla v\times\mathbf{n})|Np_y''\rangle\hat{P}^1_{N,p_y p_y''}-\frac{l_B^2}{\hbar}\langle Np_y'|(\nabla\rho^0\cdot e\mathbf{E}_{\rm alt}\times\mathbf{n})|Np_y\rangle=\hat{I}\{\hat{P}_{N,p_y p_y''}\} . \quad (17)$$

Here we have linearized the kinetic equation to keep the terms linear in the BAW amplitude. $\hat{P}^1_{N,p_y p_y'}$ is the unknown function describing the 2DEG linear response, $\mathbf{E}_{\rm alt}$ the alternating electric field induced by the BAW, and

$$\rho^0=\frac{1}{e[E_N+V(x)-E_F]/k_B T+1} .$$

As well as a basic assumption about the smoothness of the random potential, another has been made, that there is no substantial intermixing of the states defined by the different Landau levels in the vicinity of the Fermi level. If the latter is not true, then it would be necessary to consider the coupled system of kinetic equations, the coupling being due to the 2DEG phonon interactions. It is easy to show that the quantum kinetic equation (17), that we have derived, is equivalent to the result obtained by Iordansky[16] (see also Ref. 17).

Since we are looking for solutions of (17) varying in space and time as $e^{i(\kappa x-\omega t)}$, we take $\hat{P}^1_{N,p_y p_y'}$

$=g_{N,p_y p_y'} e^{i\kappa p_y l_B^2}$ and obtain

$$\left\{-i\omega+\hat{I}+\frac{l_B^2}{\hbar}V_c(\boldsymbol{\kappa}\cdot\nabla v\times\mathbf{n})\right\}g_{N,p_y p_y'} =\frac{e l_B^2}{\hbar}\langle Np_y'|(\nabla\rho^0\cdot\mathbf{E}_{\rm alt}\times\mathbf{n})|Np_y\rangle \times e^{-i\kappa p_y l_B^2}+\frac{1}{\tau}g^l_{N,p_y p_y'} , \tag{18}$$

where $g^l_{N,p_y p_y'}=(\rho^0)'(e\varphi^0-\tilde{E}_F^0)I_{N,N}(\xi_0,\xi_1)$ describes the deviation of the local equilibrium in the strained system from thermal equilibrium, τ is some characteristic 2D electron-phonon relaxation time, and $\tilde{E}_F^0$ is the amplitude of the corrections to the Fermi energy linear in the BAW amplitude. We keep the operator symbol $\hat{I}$ in the left-hand side of (18) to emphasize that, as in Eq. (12), the collision operator couples the two 2DEG states separated in space.

The formal solution of (18) has the following form:

$$g_{N,p_y p_y'}=\frac{eD\langle Np_y'|(\nabla v\cdot\mathbf{E}_{\rm alt}\times\mathbf{n})|Np_y\rangle e^{-i\kappa p_y l_B^2}(\rho^0)'_{E_F}+\frac{1}{\tau}g^l_{N,p_y p_y'}}{-i\omega+iD(\boldsymbol{\kappa}\cdot\nabla v\times\mathbf{n})+\hat{\mathbf{I}}} . \tag{19}$$

Here we have introduced $D\equiv(l_B^2/\hbar)V_c$, the 2DEG diffusion coefficient. The solution of (18) enables us to obtain the generalized stress in the 2DEG plane. In what follows we present some qualitative considerations of the BAW–disordered 2DEG interaction based on the formal solution (19).

There are two contributions to the 2DEG linear response, described by the two terms in the numerator. The first term, which is proportional to the electric field in the BAW, clearly describes that part of the 2D electron response that produces joule heating. Under conditions of low BAW frequency, $\max\{\omega,\kappa D/R_0\}\ll 1/\tau$ (where R_0 is the scale of the random potential), it gives precisely the same results as the phenomenological approach.[2] The second term gives precisely the same results as the phenomenological approach.[2] The second term has essentially the same structure as that in the right-hand side (12). It contains the contribution to the BAW-2DEG interaction coming from the modulation by the BAW of the 2DEG occupation numbers for the 2D electrons in localized states. Now consider the denominator in (19). The second-to-first-term ratio is of the order of magnitude of $(D/v_s R_0)\simeq(l_B^2 V_c/\hbar v_s R_0)$. There are two different limits of strong disorder $(l_B^2 V_c/\hbar v_s R_0)\gg 1$, and weak disorder $(l_B^2 V_c/\hbar v_s R_0)\ll 1$. Since the latter is easily reduced to the model we have already considered, of a perfect 2DEG with corrections which are to be found by expanding the denominator in a Taylor series and subsequently averaging the result over the statistical distribution of the random potential, here we treat only the case of strong disorder.

The inequality $(l_B^2 V_c/\hbar v_s R_0)\gg 1$ has a very simple physical meaning: the drift velocity of the 2D electron in a crossed random electric field and an external magnetic field is much larger than the velocity of sound. If the 2D electron is in one of the localized states performing the motion in a confined area with linear scale $R_c<L_\varphi$ (L_φ is the localization length), then it is affected by the averaged quasistatic strain in this area, produced by the acoustic wave. If $R_c>\lambda_s$, the 2D electron path goes through regions of both posiive and negative strain with the net result that the electron energy modulation is very small. If, however, $R_c\simeq\lambda_s$, then electrons confined to move in neighboring areas are influenced by essentially different strains due to the acoustic wave.

The first two terms in the denominator of Eq. (19) take into account the effects of temporal and spatial dispersions in the linear electronic response. At low acoustic wave frequencies they become unimportant. As ω increases, the spatial dispersion comes into play first, exactly as for the case of the acoustic wave interaction with free electrons. However, this is true only for the case of strong disorder. The condition, defining the frequency range at which the nonlocal 2DEG response starts, is thus $(\kappa D/R_0)\tau_{e\text{-ph}}>1$ where $\tau_{e\text{-ph}}$ is the characteristic electron-phonon relaxation time. If this criterion is fulfilled, then the dominant contribution to the interaction comes from 2D electrons, within the energy range of the thermal broadening of the Fermi distribution, which drift along trajectories defined by the balance between the two big terms ω and $D\mathbf{k}\cdot\nabla v\times n$:$\omega=D\mathbf{k}\cdot\nabla v\times n$, averaged over the random potential field; in these expressions v is the dimensionless random potential defined above. This condition replaces the well-known result $\omega=\mathbf{k}\cdot\mathbf{V}$ for the case of free carriers, where $\mathbf{V}$ is the electron velocity. The latter is the matching condition to synchronize the electron motion with the perturbing potential of the traveling wave. In the disordered system such a synchronization cannot take place along each section of the electron trajectory, so the dominant trajectories are to be found by satisfying the matching conditions on average so that the electron stays on a given trajectory (open or confined) between scattering events. Figure 3 illustrates this statement.

Open trajectories 1 and 2 and a confined trajectory 3 are shown. An electron drifts rapidly along trajectory 1 and makes the interaction with the acoustic wave ineffective, while trajectory 2 contributes greatly to the 2DEG linear response. The smaller section of curve 3 between the two crosses also contributes to the interaction.

Now we come to a qualitative consideration of the relaxation absorption arising from the modulation of the population numbers for electrons performing confined motion. At $T=0$, the 2D electrons drift along trajectories given by that section of the random potential defined by the plane $E_F=$const. At nonzero tempera-

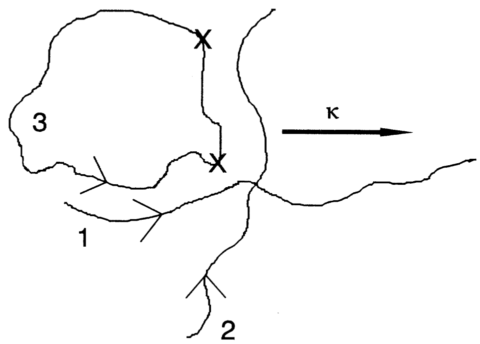

FIG. 3. A schematic example indicating the effectiveness of BAW-2DEG interaction. κ shows the projection of the BAW wave vector onto the plane of the 2DEG. Open trajectories 1 and 2 and a confined trajectory 3 are shown.

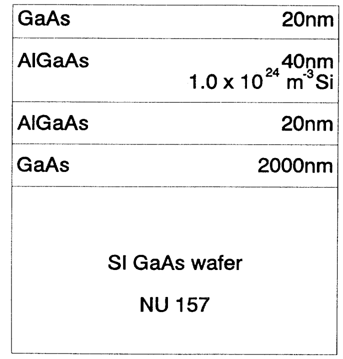

FIG. 4. Layer structure used on the GaAs sample.

tures the electrons can be activated to neighboring trajectories to be confined in an area on the scale $R_c < L_\varphi(T)$. The acoustic wave breaks the thermal equilibrium between electrons undergoing confined motion in neighboring areas, and induces a strongly nonequilibrium electronic distribution for states with $R_c \sim \lambda_s$, provided $\lambda_s < L_\varphi$. This gives rise to reasonably strong-coupling constants for such an interaction mechanism which is given in terms of the bare piezoelectric potential due to the phase difference between the strains affecting corresponding electronic states. For the dominant electronic states with $R_c \sim \lambda_s$, this coupling constant is thus relatively insensitive to the magnetic field. The number of those states which are effective in the interaction mechanism under consideration is, of course, proportional to the 2DEG thermodynamic density of states with the proportionality factor for the uncorrelated random potential field also being magnetic field independent. Since the establishment of thermal equilibrium in the 2DEG system is due to the emission and absorption of thermal phonons with relatively small displacements of the electronic orbit centers needed to couple the neighboring 2D electron trajectories, the relaxation times entering in the result of the phenomenological treatment[1] are also magnetic field independent.

III. EXPERIMENTS

A GaAs/Al_xGa_{1-x}As heterojunction, number NU157, was prepared using molecular-beam-epitaxy (MBE) on a 5-mm-thick wafer of GaAs with its top surface parallel to the (001) planes of the crystal. The sequence of layers is shown in Fig. 4. A sample was cut from the wafer and a [111]-oriented flat was cut and polished on the side of it. An x-cut cylindrical quartz rod 10 mm long and 3 mm in diameter was bonded to the [111] flat using epoxy resin adhesive. When the quartz rod was inserted in a microwave resonant cavity it was possible to generate pulses of longitudinal ultrasound at about 9.3 GHz which propagated along the [111] direction of the GaAs. The ultrasonic waves were detected after passing through the GaAs by a CdS bolometer on an evaporated aluminum film on the top surface of the wafer.[18] The arrangement was shown in Fig. 1. Ultrasonic pulses were generated using a pulse length of 500 ns and microwave peak power incident on the resonant cavity of up to 100-W, which probably gave a peak power up to 10 mW in the GaAs, corresponding to an ultrasonic intensity of 1.4 kW m^{-2}. The sample was immersed in liquid helium and could be kept at temperatures between 2 and 4.2 K. A magnetic field up to 2 T could be applied. The detected pulses from the CdS bolometer were averaged by a boxcar circuit after amplification, then digitized and stored by a microcomputer.

Electrical measurements were made, at 4.2 K, on another sample cut from the same wafer. Shubnikov–de Haas oscillations were observed and gave an electron sheet density of 4.8×10^{15} cm^{-2}, while Hall measurements gave a Hall mobility of 20 $m^2 V^{-1} s^{-1}$.

The detected signal as a function of magnetic field applied normal to the 2DEG is shown in Fig. 5. The sample was at 2.2 K. The zero of the detected signal axis is offset from the graph such that full scale on that axis represents 0.35 of the detected signal at zero magnetic field. Figure 6 shows a similar result taken at 4.2 K.

Three features are apparent: a set of quantum oscillations at fields above about 1 T, a fairly sharp minimum in

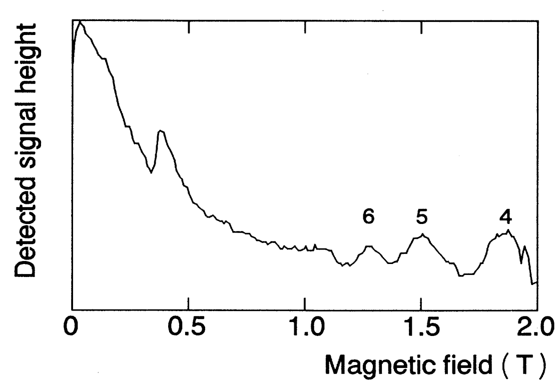

FIG. 5. Experimental results at 2 K using an ultrasonic frequency of 9.3 GHz, and showing quantum oscillations. The filling factors are given above the peaks.

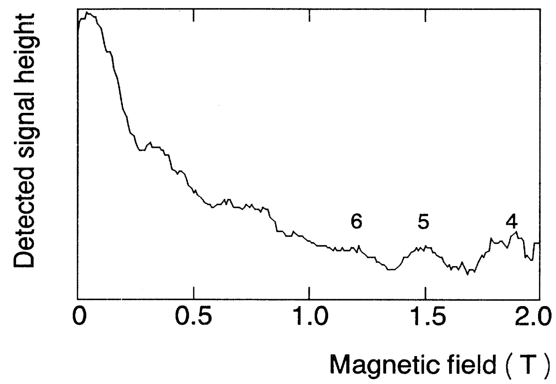

FIG. 6. Experimental results under conditions similar to Fig. 5 but at 4.2 K.

signal at 0.31 T, and an overall decrease in signal as the magnetic field increases. The quantum oscillations, which have a peak-to-peak amplitude of about 5% of the signal height in zero field, are due to piezoelectric coupling to the 2DEG and reflect Shubnikov–de Haas oscillations in the conductivity of the 2DEG. Thus we find a value for the electron sheet density of 3.8×10^{15} m^{-2} which is slightly smaller than we obtained by electrical measurements at 4.2 K on another sample from the same wafer. The minimum at 0.31 T has an amplitude of about 6% and occurs when the product of the magnetic length 1_B and the component of the ultrasonic wave vector in the plane of the 2DEG, $q_{\parallel}$, is 0.4. The overall decrease in signal by 2 T, is about 28% of the signal height in zero magnetic field.

We interpret the whole set of experimental results by assuming piezoelectric coupling to be the dominant interaction mechanism. We pay no attention to the small increase in the detected signal at low magnetic fields $B<0.2$ T. This feature is definitely not due to BAW-2DEG interaction, and appears as a result either of the experimental uncertainty in fixing the zero of magnetic field or of acoustic wave dissipation in the bulk due to the interaction with a small density of 3D carriers.

(1) We first discuss the peak in BAW attenuation at 0.31 T. Taking the cyclotron radius R_c rather than the magnetic length, we obtain the following result: $\sqrt{2}\kappa R_c=2.8$ at 0.31 T. In this expression, $\sqrt{2}\kappa$ is the in-plane component of the BAW wave vector. The cyclotron energy at 0.31 T is $\hbar\omega_c\approx5.1$ K and hence in the temperature range of our experiment electronic transitions between different Landau levels are effective since cyclotron phonons correspond to the maximum of the Planck distribution. Clearly the disorder cannot play a crucial role in BAW-2DEG interaction under such conditions, and we may use the results of Sec. II B 2 to compare the experimental data and theoretical predictions. Estimating the characteristic lengths x_1 and x_0, we have $x_1>x_0$ if $B<0.49$ T. This gives the upper limit for the location of the maximum in the attenuation of the detected signal since at higher magnetic fields the modulation-type interaction between the acoustic wave and the 2DEG occupying states in the Landau levels neighboring the Fermi level becomes very small, due both to the considerable reduction in the coupling strength according to $\kappa^2R_c^2$, and also to the growth in the characteristic relaxation time and the exponential decrease in the specific heat of the absorbing states when the energy of cyclotron phonons exceeds typical thermal energies. The lower limit may be set at about $B\simeq0.2$ T. This follows from the exponential factor e^{ξ^2} in $I_{N,N}$ and hence in expression (15) for the generalized stress. The physical meaning of that is obvious. At lower magnetic fields the linear size of the cyclotron orbit grows larger than the acoustic wavelength, and hence an electron moving in an orbit is influenced by the alternating sign of the strain, so the integrated effect goes rapidly to zero. It is worthwhile comparing the picture of the interaction under consideration with that for the acoustic geometric resonance. It is important to do this because the maximum in the attenuation of the detected signal appears very close to what one might have expected from one of the necessary conditions for the acoustic geometric resonance $R_c=\lambda_s/2$ (note that, at $B=0.31$ T, R_c is only approximately equal to $\lambda_s/2$ and differs very slightly from $\lambda_s/2$). However, what we observe on Fig. 6 is not the result of the geometric resonance. In order for the acoustic geometric resonance to take place, it is sufficient that the diameter in real space of the cyclotron orbit should be equal to multiples of the distance in real space between the wavefronts of the incident wave in the 2DEG plane. It is also necessary that the acoustic-wave-induced electric field should accelerate the electron near the turning points of its orbit. This is obviously not the case here, as the in-plane component of the alternating electric field is directed along the [110] axis perpendicular to the wavefronts. It is interesting to note that geometric arguments also play quite an important role in the interaction mechanism under consideration. This is illustrated in Fig. 7, where a schematic representation of the strain in space in the 2DEG plane is shown. + and − in the figure correspond to planes with positive and negative strain in the acoustic wage. The distance between the wavefronts in the 2DEG plane is $2\pi/\sqrt{2}\kappa$, where $\sqrt{2}\kappa$ is the in-plane component of the BAW wave vector.

If $\sqrt{2}\kappa R_c\approx\pi$, then the 2D electron effectively stays at the wavefronts longer, where it is influenced by the same sign strain to produce a net shift in the electron energy

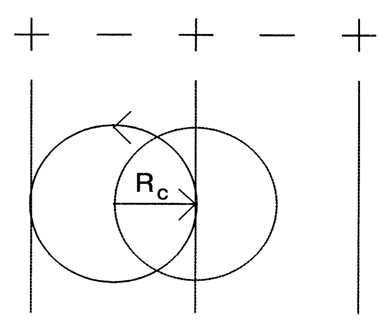

FIG. 7. An illustration of the matching conditions.

(positive for the orbit depicted by a solid line and negative for the interacting orbit shown by a dashed line). However, the orbit diameters for different Landau levels differ and hence the optimum condition for the most effective coupling does not necessarily coincide with $\sqrt{2}\kappa R_c=\pi$. The interactions with the rest of Landau level ladder, for which we have not so far accounted, can produce the peak structure. In the real structure, the disorder removes all specific details by broadening the Landau levels, leaving a single structureless peak.

(2) Here we show that the overall decrease in detected signal as the magnetic field increases (after subtracting the superimposed quantum oscillations) can be explained in essentially the same manner as was done for SAW-2DEG interaction in our previous study.[1] We may note that the smooth part of the observed curve looks very similar to that obtained for SAW-2DEG interaction. Again, at rather low magnetic fields, $B<1$ T, we observe a more rapid decrease in the signal height than we do for $B>1$ T. Now we use the results of the phenomenological description,[1] of the interaction of the BAW with the 2DEG in localized states. We take the results of the more careful consideration, given above, as a proof that there is a significant enhancement in the interaction strength that is due to piezoelectric coupling rather than deformation-potential coupling. Both the coupling constants and the relaxation times in Ref. 1 are independent of the magnetic field; the magnetic-field dependence of the smooth part of the observed curve is controlled by the magnetic-field dependence of the number of the absorbing states and hence by the thermodynamic density of states D_T. The range $B<1$ T corresponds to the low-field limit $\Gamma_N(B)<k_BT$, where $\Gamma_N(B)$ is the half-width of the Landau level. The crossover from low to high magnetic fields is shown by the change in the slope of the smooth part of the detected signal at B^*, which can be obtained from the condition $\Gamma_N(B^*)=k_BT$. Following the same arguments as in Ref. 1 we conclude that at $B<B^*$ the smooth part of the generalized stress in the 2D plane should vary linearly with B, and then for $B>B^*$ should follow a square-root dependence $\sqrt{B}$. This agrees fairly well with the experimental results of Fig. 6, with the crossover corresponding to a lower magnetic field than for the case of SAW. This is, of course, quite consistent with the model. The results on Fig. 6 were obtained at $T=2.2$ K, while the SAW results in Ref. 1 quoted above were obtained at $T=4.2$ K. Taking $\Gamma_N(B)\sim\sqrt{B}$, we arrive at $B^*=A(k_BT)^2$, with the proportionality factor A independent of both B and T, but different for different wafers. Comparing the experimental results at 2.2 and 4.2 K (Figs. 5 and 6), we conclude that the crossover at 4.2 K should be shifted by a factor of about 4 toward higher magnetic fields as compared with its position at 2.2 K. This is beyond the maximum magnetic field available in our experiment, so it is now clear why the overall decrease in the detected signal does not flatten off in our field range, in contrast to the result at the lower temperature.

(3) The quantum oscillations seen in Fig. 6 have maxima corresponding to the plateaux in the Hall data. They are quite noticeable at filling factors of 6, 5, and 4. A double-peak structure is not resolved at 2 K. This is not surprising, and we expect it to be much less pronounced than for the case of SAW-2DEG interaction even though the experiments were made at lower temperatures. In SAW experiments the transmitted intensity is governed by the SAW dissipation. The latter shows the specific behavior because the absorption coefficient has double-peak maxima close to integer fillings. In our BAW experiments both real and imaginary parts of the generalized stress in the 2DEG plane contribute to the detected signal. Moreover, the main contribution to the detected signal, as was shown in Sec. I, may come from interference between different components of the classical fields. If this is the case, then the change in the detected signal is mainly controlled by the effect the 2DEG produces on the reflection and transmission of the incident wave. The real part of the linear 2DEG response which causes the change in reflection and transmission of the incident wave for a BAW interacting with 2D electrons in extended states shows a sequence of single maxima located at integer filling factors. Thus a double-peak shape in the quantum oscillations in BAW experiments, if it could be resolved at low temperatures, would give a straightforward evaluation of the relative importance of the changes in reflectivity and dissipation.

Numerical estimates

In order to estimate the order of magnitude of the changes in the detected signal versus magnetic field, we first note that, according to our results, the coupling constant for relaxation absorption by the localized states of the 2DEG states is on an atomic scale. Moreover, piezoelectric coupling may be as efficient in this case as it is in the conventional acoustoelectric interaction with the extended states of the 2DEG. Therefore we only estimate the scale of the quantum oscillations. The order of magnitude of the smooth variation in attenuation due to the relaxation absorption is at least as large.

To account for piezoelectricity, we take $\Lambda_{ij}+\chi_{ij}$ instead of Λ_{ij} in (14). We next introduce the tensor $A^{\nu}_{\alpha\beta}$ by the expression $\varphi(r,t)=\sum_\nu A^{\nu}_{\alpha\beta}u^{\nu}_{\alpha\beta}(r,t)$, where the superscript ν denotes L_+, L_-, and S_- waves respectively. Then with the use of (4) we arrive at

$$A^{L\pm}_{\alpha\beta}=\frac{iR}{6\kappa}(1-e^{-\sqrt{2}\kappa h+i\kappa h})\begin{bmatrix}0 & \pm1 & 1\\ \pm1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}$$

and

$$A^{S_-}_{\alpha\beta}=\frac{iR}{2\kappa}\left[\frac{\kappa}{\kappa^S}\right]^2(1-e^{-\sqrt{2}\kappa h-i\kappa^S_z h})\times\begin{bmatrix}0 & -\frac{\kappa^S_z}{\kappa} & 1\\ -\frac{\kappa^S_z}{\kappa} & 0 & 1\\ 1 & 1 & 0\end{bmatrix},$$

where

$$R=\frac{8\pi\beta_{14}}{\varepsilon}\frac{1}{1+i\left[\frac{\sigma_{xx}}{\sigma_0}\right]\left[\frac{1}{1+i\left[\frac{\omega}{\omega_D}\right]}\right]e^{-\sqrt{2}\kappa h}\sinh(\sqrt{2}\kappa h)}\,.$$

Taking either $D_{xx}\sim(l_B^2/\hbar)V_c$ (Sec. II B), or alternatively $D_{xx}\sim(R_c^2/\tau)$ (Ref. 19) with $(1/\tau)\approx(\Delta E/\hbar)$, where ΔE is the Landau-level half-width, we arrive at $\omega_D<10^{10}\ \mathrm{s}^{-1}$ at $B<2$ T. In the frequency range under consideration, when $f_s\simeq 10$ GHz, it is important to notice that the effect of screening is weakened, because $\omega\gg\omega_D$, for the whole range of magnetic fields and provides strong 2DEG-BAW interaction. At this stage, however, we cannot incorporate that effect into the theory more accurately and simply stress that the effects of the BAW-2DEG interaction are quite sensitive to the particular screening mechanism.[20,21]

Note that the estimate given above, for the diffusion coefficient and diffusion frequency, is crude. We can give another, using the formula for the acoustoelectric interaction,

$$\Gamma=\frac{\eta\kappa}{2}\frac{\left[\frac{\sigma_{xx}}{\sigma_0}\right]}{1+\left[\frac{\sigma_{xx}}{\sigma_0}+\frac{\omega}{\omega_D}\right]^2}$$

and the experimental results in Ref. 2. If it is accepted that ω/ω_D is a slowly varying function of the magnetic field, then the maximum in the attenuation is given by

$$\Gamma_m=\frac{\eta\kappa}{2}\frac{1}{1+[1+(\omega/\omega_D)^2]^{1/2}}\,.$$

Taking for ω_D the frequency at which Γ_m starts to deviate from a linear law, $\Gamma_m=\eta\kappa/4$, we arrive at $\omega_D\simeq 2\times10^9\ \mathrm{s}^{-1}$. When $\omega\gg\omega_D$, $\Gamma_m\to(\eta\kappa\omega_D/2\omega)=(\eta\omega_D/2v_S)\approx 9\ \mathrm{dB\,cm}^{-1}$ over the whole range of applicability of the phenomenological approach. This value is in reasonable agreement with the results of Ref. 2. Note that the slight inequivalence of the maxima in doubled peaks may result from the weak dependence of the ω_D on the magnetic field, which is slightly different on both sides of integer filling factors.

To give a rough estimate of the effect the 2DEG produces on the detected signal, we take the first term in the general expression (1), thus omitting the possible magnification factor, due to interference effects given by the second term. We estimate the quantity $(\Delta S/S)\simeq(2/\rho\omega a_+)|T^{2\mathrm{DEG}}|$, with $|T^{2\mathrm{DEG}}|$ instead of $\mathrm{Im}T^{2\mathrm{DEG}}$, since if we omit the magnification factor the reflectivity given by the real part of the generalized stress $T^{2\mathrm{DEG}}$ also contributes an amount $(2/\rho\omega a_+)\mathrm{Re}T^{2\mathrm{DEG}}$ to the detected signal. In doing so, and estimating $A^{\nu}_{\alpha\beta}$, we arrive at

$$\left[\frac{\Delta S}{S}\right]^{\mathrm{piezo}}>\frac{16\pi}{\varepsilon}\eta\frac{e^2D_T}{\kappa}\times\frac{1}{1+\left[\frac{\sigma_{xx}}{\sigma_0}\frac{\omega_D}{\omega}e^{-\sqrt{2}\kappa h}\sinh(\sqrt{2}\kappa h)\right]^2}$$

for $\omega>\omega_D$. The second term in the denominator of the last formula is about 10^2, and $D_T\sim 5\times10^{25}\ \mathrm{erg}^{-1}\,\mathrm{cm}^{-2}$, giving $(\Delta S/S)^{\mathrm{piezo}}\approx 3.5\%$, with about the same value for the amplitude of the quantum oscillations. Such a rough estimate, which gives the correct order of magnitude of the observed effects, seems to be quite satisfactory.

IV. CONCLUSIONS

(1) The effect of dynamical 2DEG-BAW interaction has been studied theoretically within the framework of the exactly solvable model of BAW-2DEG relaxation interaction in an idealized structure without disorder. It was shown that in a proper range of sufficiently low magnetic fields, with a cyclotron energy comparable to the thermal energy, the dominant contribution comes from the interaction between the BAW and electrons occupying the highest filled and the lowest empty Landau levels. The interaction of the acoustic wave and the 2DEG breaks the thermal equilibrium between electronic states separated in space by the cyclotron radius in the 2DEG plane. Due to the phase difference, the acoustic wave strain produces a strong modulation of the cyclotron gap for the interacting states even for the same bare perturbing potential. This gives rise to a piezoelectric coupling which is much stronger, at low BAW frequencies, than that of the deformation potential, and also causes a maximum interaction which manifests itself in a single structureless peak in the detected signal, located at a low enough magnetic field to fulfill the condition for maximum coupling strength $\kappa R_c\simeq 1$.

(2) The quantum kinetic equation has been derived and solved for the case of a perfect structure, and a quantitative treatment of the relaxation-type BAW-2DEG interaction was given. The quantum kinetic equation has also been derived for the case of a smooth random potential with scale $R_0\gg l_B$. A qualitative analysis of the solutions of that equation, for the case of strong disorder and comparatively high frequencies of the acoustic wave, also reveals two main contributions to the linear 2DEG response. The first comes from the acoustoelectric interaction between the acoustic wave and the 2D electrons moving along sections of their trajectories for which the average projection of the 2D electron drift velocity onto

the acoustic propagation direction is equal to the sound velocity. This is the analog of matching conditions for the case of acoustic wave interaction with free carriers. The characteristic parameter which defines the range of low and high acoustic wave frequencies was shown to be $(\kappa D/R_0)\tau_{e\text{-ph}}$. At $(\kappa D/R_0)\tau_{e\text{-ph}} \ll 1$ the solution of the quantum kinetic equation reproduces the results of the phenomenological theory of acoustoelectric interactions. At $(\kappa D/R_0)\tau_{e\text{-ph}} \gg 1$ the effects of spatial dispersion start to play an important role, giving rise to a nonlocal linear 2DEG response.

The second contribution is a relaxation-type absorption, with the same origin as that which was discussed for the idealized case, i.e., modulation of the population numbers of the 2D electrons performing a fast drift motion along confined trajectories (localized states) and being affected by the average quasistatic strain of the acoustic wave in the area of confinement. By absorbing or emitting thermal phonons the 2D electrons may become excited into neighboring localized states, which are affected by a different average quasistatic strain, due to the spatial phase difference. This gives rise to, first, a strong piezoelectric coupling, relatively independent of the magnetic field, for the dominant interacting states provided $L_\varphi > \lambda_S$. Second, the characteristic relaxation times are independent of the magnetic field, because only thermal phonons are involved in the process of restoring the thermal equilibrium. Third, the magnetic-field dependence of the dissipation is determined by the number of absorbing states, which is proportional to the thermodynamic 2DEG density of states, with a proportionality factor for the uncorrelated random potential which is almost magnetic field independent. Fourth, the quantum oscillations, seen in the experimental results in the magnetic field range from 1 to 2 T, superimposed on the result of the relaxation absorption, have the same origin as the classical acoustoelectric interaction. However, the contribution to the detected signal comes from both the changed reflectivity and the dissipation produced by the perturbed 2DEG, and is essentially influenced by the interference of waves in the bolometer. The main contribution is that determined by dynamical BAW-2DEG interaction, the effect the 2DEG produces by changing the stiffness being at least as important as the dissipation. It may become dominant if interference enhances the bolometer sensitivity. Since the change in elastic stiffness produced by the 2DEG is determined by a single peak, located at integer filling factors, the double-peak structure (if resolved) should be much less pronounced than that of SAW-2DEG interactions.

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*Permanent address: Institute of Radioengineering and Electronics, Russian Academy of Sciences, Mokhovaya str II, Moscow, GSP-3, 103907 Russia.

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