Coupling between surface-polariton and edge-plasmon excitations in coupled finite half-plane superlattices

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A hydrodynamic model has been used to calculate the coupling between surface-polariton and edge-plasmon excitations in coupled finite half-plane superlattices. The numerical calculation finds the condition under which the surface-polariton mode can exist and be free from Landau damping. The strong coupling between two half-plane superlattices can destroy the surface-polariton mode and soften the edge-plasmon mode at the same time. The phase diagram separating the charge-density-wave ground state and Fermi-liquid ground state is displayed. The reduction in the number of electron layers in finite superlattices can suppress the softening of edge-plasmon mode.

The plasmon excitation of two-dimensional (2D) electron gases has received a lot of attention both theoretically and experimentally. The edge-magnetoplasmon excitation, which exists only close to the boundary of the 2D electron fluid, has been predicted.[1,2] The frequency of this edge mode varies inversely with the magnetic field as the field becomes strong. The research on the superlattice surface plasmon-polariton modes have been reviewed by Albuquerque and Cottam[3] (also see references therein). In their paper, surface plasmon-polariton excitations at zero and finite magnetic fields in superlattices with semi-infinite or finite numbers of layers were studied.[3–5]

Recently, edge-plasmon excitations on a lateral surface of an infinite half-plane superlattice and on two coupled half planes have been studied.[6,7] These excitations are of great interest because they are free from Landau damping and might be useful in surface-wave devices.

The model considered contains two coupled finite half-plane semiconductor superlattices, which are embedded in a background with different dielectric constant. This arrangement supports the surface-polariton excitation near surfaces between superlattices and background.[8] For simplicity, I assume that periodic arrays of 2D electron-fluid layers are placed in the z direction. The electron-fluid layers occupy the spaces $x < 0$ and $x > a$ of distance a separated by a barrier layer. The existence of two lateral surfaces (edges) of half-plane superlattices will support the coupled edge-plasmon modes in the regions around edges.[9] The coupled edge-plasmon modes are greatly softened as the edge coupling becomes strong.

My main interest includes the strong coupling between edge-plasmon and surface-polariton modes, the phase diagram separating the charge-density-wave ground state and Fermi-liquid ground state, and the finite-size effect on the stability of the Fermi-liquid ground state. Let me consider a rigid positive background with uniform number density n_0 and a compressible electron fluid with number density $n_0 + n$. I denote the small fluctuation in the number density and velocity field inside the plane of the jth layer by $n_j(\mathbf{r}_{||}, t)$ and $\mathbf{v}_j(\mathbf{r}_{||}, t)$. The electrostatic potential is represented by $\phi(\mathbf{r}_{||}, z, t)$, where $\mathbf{r}_{||}$ is a 2D planar vector. Since the system is translationally invariant in the y direction, a plane-wave form $\exp(ik_y y - i\omega t)$ can be assigned to n_j, $\mathbf{v}_j$, and ϕ with their amplitudes depending on x and z. Consequently, the continuity, Euler, and Poisson equations can be written as

$$-i\omega n_j + n_0 \left(\frac{\partial v_{jx}}{\partial x} + ik_y v_{jy} \right) = 0 \ , \tag{1}$$

$$-i \left(\omega + \frac{i}{\tau} \right) v_{jx} + \left(\frac{s^2}{n_0} \right) \frac{\partial n_j}{\partial x} - \left(\frac{e}{m^*} \right) \frac{\partial \phi}{\partial x} = 0 \ , \tag{2}$$

$$-i \left(\omega + \frac{i}{\tau} \right) v_{jy} + ik_y s^2 \left(\frac{n_j}{n_0} \right) - ik_y \left(\frac{e}{m^*} \right) \phi = 0 \ , \tag{3}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} - k_y^2 \right) \phi(x, z, k_y, \omega) = \frac{4\pi e}{\epsilon_s} \sum_{j=0}^{N} n_j(x, k_y, \omega) \delta(z - z_j) \times [\theta(-x) + \theta(x - a)] \ , \tag{4}$$

where $\theta(x)$ is the step function, $N + 1$ is the number of electron-fluid layers situated at $z = 0, d, \ldots, Nd$ in finite superlattices, d is the superlattice period, $\epsilon_s = 4\pi\epsilon_0\epsilon_b$ with ϵ_b denoting the dielectric constant of superlattices, s is an effective compressional wave speed, and i/τ is the phenomenological damping term.

By doing a Fourier transform in x of Poisson's equation in Eq. (4) and making use of the ansatz

$$n_j(k_x, k_y, \omega) = 2A(k_x, k_y, \omega) \cos(q_z j d) \ , \tag{5}$$

where $q_z d = 2\pi p/N$ with p being an integer, I am left with

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$$\phi(k_x,k_y,z_j,\omega) = -\frac{4\pi e}{\epsilon_s} K(k_x,k_y,q_z)A(k_x,k_y,\omega) \ , \quad (6)$$

which is independent of z_j. Here, the spectral kernel $K(k_x,k_y,q_z)$ is found to be

$$K(k_x,k_y,q_z) = \frac{S(k_x,k_y,q_z)}{2k_{||}} \ , \quad (7)$$

with $k_{||} = \sqrt{k_x^2+k_y^2}$, and

$$S(k_x,k_y,q_z) = \frac{\sinh(k_{||}d)}{\cosh(k_{||}d)-\cos(q_zd)} + \frac{a_1(k_{||},q_z)+a_2(k_{||},q_z)\{[Q+Q^2\exp(-k_{||}Nd)]/[1-Q^2\exp(-2k_{||}Nd)]\}}{2\left[\cosh(k_{||}d)-\cos(q_zd)\right]} \ , \quad (8)$$

where $Q=(\epsilon_b-\epsilon_1)/(\epsilon_b+\epsilon_1)$ with ϵ_1 being the dielectric constant of background, and

$$a_1(k_{||},q_z) = \left[1+\exp(-k_{||}Nd)\right]\left[\exp(-k_{||}d)-\cos(q_zd)\right] \ , \quad (9)$$

$$a_2(k_{||},q_z) = a_1(k_{||},q_z)\exp(-k_{||}Nd) + \left[1+\exp(-k_{||}Nd)\right]\left[\exp(k_{||}d)-\cos(q_zd)\right] \ . \quad (10)$$

By approximating the spectral kernel in Eq. (7) to be a localized one,[2] which has the same first two terms in a power series about $k_x^2=0$ and is found to work well in similar cases,[6,7,9] I get

$$K(k_x,k_y,q_z) \approx K_0(k_x,k_y,q_z) = \frac{k_y f(k_y,q_z)}{2k_y^2+k_x^2 g(k_y,q_z)} \ , \quad (11)$$

where $f(k_y,q_z)=S(k_x=0,k_y,q_z)$, and

$$g(k_y,q_z) = 1-\left[\frac{k_y}{f(k_y,q_z)}\right]\left[\frac{\partial f(k_y,q_z)}{\partial k_y}\right] \ . \quad (12)$$

In Eq. (11), $g(k_y,q_z)>0$ characterizes the screening correction to edge plasmons. The inverse Fourier transform in k_x of the equation containing $K_0(k_x,k_y,q_z)$ in Eq. (6) gives the approximate kernel in x, and then the Poisson equation in Eq. (4) is reduced to a localized one:

$$\left[\frac{\partial^2}{\partial x^2}-\frac{2k_y^2}{g(k_y,q_z)}\right]\phi(x,z_i,k_y,\omega) = \frac{4\pi e k_y}{\epsilon_s}\left[\frac{f(k_y,q_z)}{g(k_y,q_z)}\right]\sum_{j=0}^{N} n_j(x,k_y,\omega)\left[\theta(-x)+\theta(x-a)\right] \ . \quad (13)$$

In the following, I set $\tau\to\infty$ for simplicity. I also take the long-wavelength limit and let $s=0$. The remaining steps[7] include using the boundary conditions that ϕ and $\partial\phi/\partial x$ are continuous and that v_x vanishes there, together with the proper boundary behavior as $|x|\to\infty$. This yields

$$[F(k_y,q_z,\omega^2)]^2 - 2T(k_y,q_z)\coth\left\{\sqrt{[2/g(k_y,q_z)]}k_ya\right\}F(k_y,q_z,\omega^2)+[T(k_y,q_z)]^2 = 0 \ , \quad (14)$$

where

$$F(k_y,q_z,\omega^2) = [D(k_y,q_z,\omega^2)]^2\left[C(k_y,q_z,\omega^2)+\sqrt{2/g(k_y,q_z)}\right] \ , \quad (15)$$

$$T(k_y,q_z) = 2\omega_{k_y}^2\left[\frac{f(k_y,q_z)}{g(k_y,q_z)}\right]\sqrt{2/g(k_y,q_z)} \ , \quad (16)$$

$$[D(k_y,q_z,\omega^2)]^2 = 2\omega_{k_y}^2\left[\frac{f(k_y,q_z)}{g(k_y,q_z)}\right]-\omega^2 \ , \quad (17)$$

$$[C(k_y,q_z,\omega^2)]^2 = \frac{2\{\omega_{k_y}^2[f(k_y,q_z)/g(k_y,q_z)]-[\omega^2/g(k_y,q_z)]\}}{[D(k_y,q_z,\omega^2)]^2} \ , \quad (18)$$

where $\omega_{k_y}^2=(2\pi e^2 n_0/\epsilon_s m^*)k_y$ is the 2D plasma frequency. $[C(k_y,q_z,\omega^2)]^2>0$ and $[D(k_y,q_z,\omega^2)]^2>0$ from the requirement for the existence of edge-plasmon mode.[7]

The analytical solutions to Eq. (14) is obtained as

$$\omega_+^2(k_y,q_z) = 2\omega_{k_y}^2\left[\frac{f(k_y,q_z)}{g(k_y,q_z)}\right] + \frac{4\omega_{k_y}^2[f(k_y,q_z)/g(k_y,q_z)]\coth^2[k_ya/\sqrt{2g(k_y,q_z)}]}{2-g(k_y,q_z)-4\coth[k_ya/\sqrt{2g(k_y,q_z)}\,]}\ , \tag{19}$$

$$\omega_-^2(k_y,q_z) = 2\omega_{k_y}^2\left[\frac{f(k_y,q_z)}{g(k_y,q_z)}\right] + \frac{4\omega_{k_y}^2[f(k_y,q_z)/g(k_y,q_z)]\tanh^2[k_ya/\sqrt{2g(k_y,q_z)}]}{2-g(k_y,q_z)-4\tanh[k_ya/\sqrt{2g(k_y,q_z)}]}\ , \tag{20}$$

and $\omega_\pm^2(k_y,q_z) = 0$ defines the critical aspect ratio $(a/d)_c$.[9] The mode splitting is due to the coupling between two edges when a is finite. From solutions in Eqs. (19) and (20), it is easy to find the results in several limiting cases.

(1) When $a\to\infty$, I obtain the result for an uncoupled finite half-plane superlattice

$$\omega^2 = \left[\frac{2}{2+g(k_y,q_z)}\right]\omega_{k_y}^2 f(k_y,q_z)\ .$$

(2) For $a\to 0$, I have the result for a finite complete superlattice by setting $g(k_y,q_z)=2$,

$$\omega^2 = \omega_{k_y}^2 f(k_y,q_z)\ ,$$

from which the critical wave vector k_y^* for the existence of Landau-damping-free surface-plasmon mode in a semi-infinite complete superlattice ($N\to\infty$) is estimated as $k_y^*d = -\ln|Q|$.

(3) As $d\to\infty$, I find $f(k_y,q_z)\to 1$ and $g(k_y,q_z)\to 1$. This reduces to the problem of electron gases in two coupled half planes.

(4) If I first let $N\to\infty$ and then take $d\to 0$, I get $\omega_{k_y}^2 f(k_y,q_z)\to(1+Q)\Omega_p^2/2$ and $g(k_y,q_z)\to 2$ for two coupled semi-infinite half-bulks, where $\Omega_p^2 = (4\pi e^2 n_0/\epsilon_s m^* d)$ is the 3D plasma frequency.

Figure 1 shows the dispersion of strongly coupled surface-polariton modes (solid line) as a function of k_yd when $a/d = 0.6$. Two shaded regions are for coupled edge-plasmon excitations. The surface-polariton modes are free from Landau damping if their energies lie outside of these shaded regions. There is only a small range of k_yd inside the gap of edge-plasmon excitation energies, where one of surface-polariton modes at $q_zd=\pi$ can exist. The strong coupling between two edges of finite half-plane superlattices greatly suppresses the surface-polariton excitation in this case.

I display in Fig. 2 the k_yd dispersion of surface-polariton modes (solid lines) in the medium coupling regime ($a/d = 1.0$). The decrease of the edge coupling between two finite half-plane superlattices slightly favors the existence of another surface-polariton mode at $q_zd=\pi$ inside the gap of edge-plasmon excitation energies. The edge-plasmon gap becomes narrower due to smaller edge coupling. The edge-plasmon excitations are seen more localized as $k_yd \gg 1$.

When the edge coupling between two finite half-plane superlattices is further reduced ($a/d = 2.0$), I find in Fig. 3 that the coupled surface-polariton modes at $q_zd=0$ are developed and free from Landau damping in a large range of k_yd. The favored surface-polariton modes change from ones at $q_zd=\pi$ to those at $q_zd=0$. Two edge-plasmon modes at $q_zd=\pi$ only show invisible difference in their

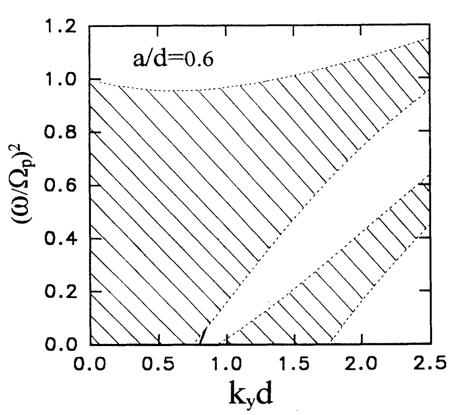

FIG. 1. The excitation energies $(\omega/\Omega_p)^2$ as a function of wave vector k_yd for surface-polariton and edge-plasmon modes. The parameters used in the calculation are $\epsilon_b = 13.0$, $\epsilon_1 = 3.25$, $N = 15$, and $a/d = 0.6$. The solid line is for the energy of the surface-polariton mode, while the shaded regions are for the excitation of edge-plasmon modes.

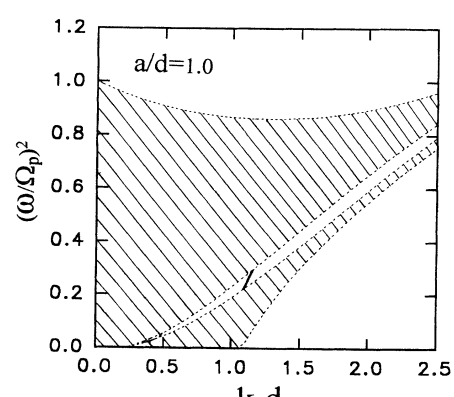

FIG. 2. The excitation energies $(\omega/\Omega_p)^2$ as a function of wave vector k_yd for surface-polariton and edge-plasmon modes. The parameters used in the calculation are the same as those in Fig. 1 except that $a/d = 1.0$. The solid lines are for the energies of surface-polariton modes, while the shaded regions are for the excitation of edge-plasmon modes.

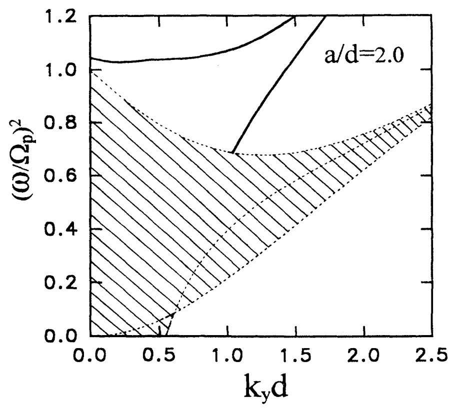

FIG. 3. The excitation energies $(\omega/\Omega_p)^2$ as a function of wave vector k_yd for surface-polariton and edge-plasmon modes. The parameters used in the calculation are the same as those in Fig. 1 except that $a/d = 2.0$. The solid lines are for the energies of surface-polariton modes, while the shaded regions are for the excitation of edge-plasmon modes.

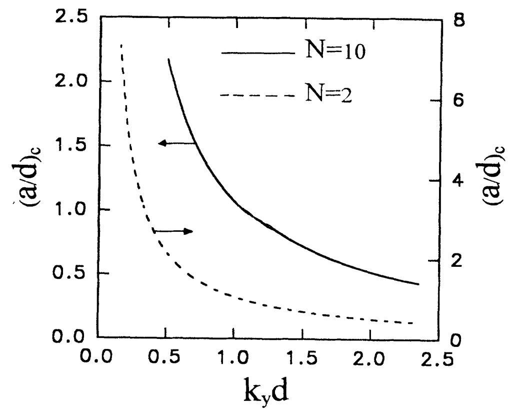

FIG. 5. The critical aspect ratio $(a/d)_c$ as a function of wave vector k_yd for two values of N. The parameters used in the calculation are $\epsilon_b = 13.0$ and $\epsilon_1 = 3.25$. Above the curve is the phase of the Fermi-liquid ground state, and below the curve is the phase of the charge-density-wave ground state. The solid line (left scale) is for $N = 10$, and the dashed line (right scale) is for $N = 2$.

energies. Both the edge-plasmon and surface-polariton modes become quite localized when $k_yd \gg 1$. Comparing Figs. 1–3, I can see the instability of Fermi-liquid ground state when the energy of the edge-plasmon mode at $q_zd = 0$ first touches zero from large values to small values of k_yd. This defines the critical aspect ratio $(a/d)_c$, which separates the charge-density-wave ground state and the Fermi-liquid ground state.

To show the finite-size effect on the softening of coupled edge-plasmon modes, I present in Fig. 4 the critical aspect ratio $(a/d)_c$ as a function of the number of electron-fluid layers N in two coupled finite half-plane superlattices. When $k_yd = 2.0$ (right scale, dashed line), $(a/d)_c$ decreases rapidly with N and becomes independent of N when $N \geq 6$. This implies that the edge-plasmon excitation becomes very localized for large values of k_yd. The whole $(a/d)_c$ curve is pushed up when $k_yd = 0.5$ (left scale, solid line). In this case, $(a/d)_c$ decreases smoothly with N in the whole range displayed because the edge-plasmon excitation is quite extended for small values of k_yd.

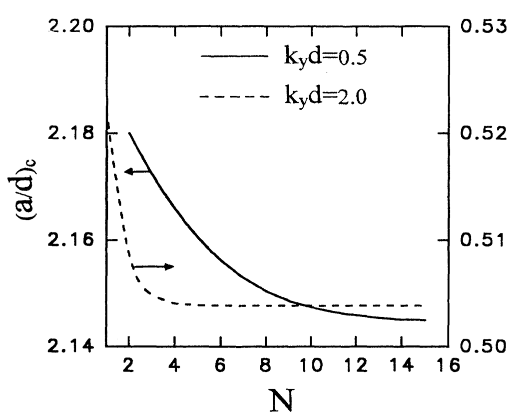

FIG. 4. The critical aspect ratio $(a/d)_c$ as a function of the number of electron-fluid layers N for two values of k_yd. The parameters used in the calculation are $\epsilon_b = 13.0$ and $\epsilon_1 = 3.25$. The solid line (left scale) is for $k_yd = 0.5$, and the dashed line (right scale) is for $k_yd = 2.0$.

Finally, I show the phase diagrams of $(a/d)_c$ as a function of k_yd for two values of N in Fig. 5. The $(a/d)_c$ curve separates two phases of the charge-density-wave ground state (below the curve) and Fermi-liquid ground state (above the curve). When $N = 10$ (left scale, solid line), the value of $(a/d)_c$ is very small, ranging from 0.5 to 2.5. This means that a slightly strong edge coupling between two finite half-plane superlattices can destroy the stability of Fermi-liquid ground state and switch to the charge-density-wave ground state due to gapless excitations. When $N = 2$ (right scale, dashed line), the $(a/d)_c$ value is greatly increased ranging from 2.0 to 8.0. From this I know that the increase in the number of electron-fluid layers in two coupled finite half-plane superlattices can enhance the coupling between the edges, and then soften the edge-plasmon modes more easily.

In conclusion, by using a hydrodynamic model I study the coupling between the surface-polariton and edge-plasmon excitations in two coupled finite half-plane superlattices. From it I find the condition for the existence of Landau-damping-free surface-polariton modes. I also show the phase diagram between the charge-density-wave ground state and Fermi-liquid ground state. The decrease in the number of electron-fluid layers in finite superlattices is found to suppress the softening of edge-plasmon modes.

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