Persistent currents in a quantum ring: Effects of impurities and interactions

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We have studied the persistent current in a quantum ring in the presence of a Gaussian impurity and/or with Coulomb interactions included. The impurity potential mixes the states, lifts the degeneracies in the energy spectrum, and reduces the persistent current from its impurity-free value. The role of interactions in the absence or presence of an impurity was found to be insignificant. We also present the results for the charge density of the system in the presence of the impurity with or without the Coulomb interactions.

Recent studies of quantum confined systems (e.g., quantum dots and quantum rings) have made it increasingly clear that electron correlations play a major role in these mesoscopic systems.[1,2] Experimental observation of the persistent current in a single mesoscopic ring[3,4] in contrast to such studies in an ensemble of 10^7 rings,[5] has opened up possibilities for exploring unambiguously several outstanding problems in this field. One such problem is how and to what extent the interelectron interactions influence the persistent current. Similarly, an unambiguous picture of the interplay between the impurity and interelectron interactions is not yet available. While the single-electron results are fairly well established,[6] no reliable quantitative theory exists, as yet, in the case of many-electron systems. A very interesting intuitive argument on the effect of interacting electrons on the persistent current was presented some time ago by Leggett.[7] Based on variational arguments and two important properties of the many-particle wave function in a mesoscopic ring, viz., the antisymmetry and the single valuedness, Leggett conjectured that, for arbitary electron-electron interactions and an arbitary external potential, the maxima and minima of the energy curves for even and odd numbers of electrons would be the same as for the noninteracting systems.

Our approach to the quantum ring, where we introduced the Coulomb interactions explicitly (in the absence of any impurity), supports this conjecture.[8] In our model,[9,10] the electron is confined to a parabolic potential and subjected to a perpendicular magnetic field. The single-electron Hamiltonian is written as

$$\mathcal{H} = \frac{1}{2m^*}\left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right)^2 + \frac{1}{2}m^*\omega_0^2(r - r_0)^2,$$

where the vector potential is $\mathbf{A} = \frac{1}{2}(-By, Bx, 0)$ (symmetric gauge). As we have demonstrated earlier,[9] our model in the appropriate limit, correctly reproduces the behavior of an ideal one-dimensional ring[6] and that of a two-dimensional electron gas. The energy spectrum for the noninteracting electrons,[9] magnetization, and susceptibility[10] were also calculated in that model.

In order to introduce the interelectron interaction, we evaluated the two-body matrix element numerically, where we employed the Coulomb interaction. Those calculations indicated that, in the lowest Landau level, the Coulomb interaction simply shifts the noninteracting energy spectrum to higher energies. There is no discernible effect of interaction on the magnetization. We found that this is due to conservation of angular momentum in the system: all close-lying states in the lowest Landau level belong to different angular momentum and the Coulomb force cannot couple them.

Similar conclusions were reached independently by Weidenmüller *et al.*,[11] who found that, in the absence of impurities, rotational invariance causes the many-electron Hamiltonian $\mathcal{H}$ and the z component of the angular momentum L_z to commute. The position of the minima of the energy parabolas, as a function of the magnetic flux Φ, is however, determined solely by the eigenvalues M of L_z and is independent of interelectron interactions. As a result, persistent current is exactly the same with and without interactions added to the Hamiltonian. On the other hand, the impurity potential destroys this rotational invariance. The persistent current is expected to reduce monotonically with increasing strength of $V^{\rm imp}(r)$. Using level-density arguments and numerical simulations, these authors showed that the interelectron interaction very effectively counteracts the tendency of $V^{\rm imp}(r)$ to drastically reduce the persistent current below the "ideal" (no impurity scattering and no Coulomb interaction) case.

At this point, we would like to contrast these results with those obtained from the one-dimensional (1D) disordered discrete-lattice ring model.[12,13] In this model, exact results from numerical diagonalization of the Hamiltonian are available for small ring sizes. In the case of long-range Coulomb interactions,[12] it was found that depending on the disorder, interaction can increase or decrease (mostly decrease) the current. There are also reports on the 1D rings of spinless fermions with short-range (nearest neighbor)[13] interaction on a lattice for

various band fillings. At half filling, the interaction was shown to induce a metal-insulator transition. Away from half filling and in the presence of impurity scattering, interaction was found to decrease the current.[13] The different conclusions reached from the two approaches (Refs. 11 and 12) were explained as due to two very different models adopted, viz., lattice and continuum models.

Obviously, the influence of the impurity interactions and/or Coulomb interactions on the persistent current is quite intricate. It is also clear that, although the work of Leggett[7] and others[11–14] provides a rather qualitative picture, for a better understanding of the underlying physics, we need to study how the energy spectrum of a quantum ring will evolve when the impurities and/or interelectron interactions are introduced. Such a study is now reported in this paper, where we have analyzed the effect of a Gaussian impurity interaction with and without the electron-electron interaction on the energy spectrum and persistent current. In a quantum ring, the wave functions are of the form

$$\psi_\lambda = R_{nl}(r)\, e^{il\theta}, \quad n = 0, 1, 2, \ldots,$$

$$l = 0, \pm 1, \pm 2, \ldots,$$

and λ represents the quantum number pair $\{n, l\}$. The impurity interaction is chosen to be of the form

$$V^{\rm imp}(r) = V_0\, e^{-(\mathbf{r}-\mathbf{R})^2/d^2},$$

where V_0 is the potential strength and d is the width. The impurity matrix element can then be written as

$$T_{\lambda,\lambda'} = 2\pi V_0\, e^{im\theta_0} \int R_\lambda(r) R_{\lambda'}(r)\, e^{-(R^2+r^2)/d^2} \times I_m\left(\frac{2rR}{d^2}\right) r\, dr,$$

where $m = l' - l$, (R, θ_0) is the impurity position, and I_m is the modified Bessel function. The two-body interaction matrix element has been already described earlier.[9] Let r_0 be the radius of the ring and $A = \pi r_0^2$ its area. The length is measured in units of r_0 and the energy in units of $h^2/2m^*\pi A$.[9] In these units the confinement potential and the Coulomb interaction are

$$U(r) = \frac{1}{2} m^* \omega_0^2 (r - r_0)^2 = 4\alpha^2 (x-1)^2\ ,$$

$$\frac{e^2}{\epsilon r} = 9.45 m^* R_0 \frac{1}{\epsilon x},$$

where $\alpha = \omega_0 m^* A/h$, $x = r/r_0$, $R_0 = 10$ is the radius of the ring in nanometers, $\epsilon = 12.9$ is the background dielectric constant, and the effective mass was chosen to be $m^* = 0.067$ appropriate for GaAs. In the impurity potential given above, the strength V_0 is expressed in this energy unit. For $\alpha = 20$, the single-electron energy spectrum closely resembles that of an ideal 1D ring, while for $\alpha = 5$, it has the characteristics of a two-dimensional electron gas.[9] In what follows, we present the numerical results for these two values of α. The noninteracting basis used in the diagonalization of the Hamiltonian was formed from the single-particle states $R_{nl}(r)\, e^{il\theta}$, where the radial wave functions $R_{nl}(r)$ were obtained by numerically solving the single-particle Schrödinger equation for the parabolic confinement potential.[9] The cutoff in the single-particle energies and correspondingly the number of the single particle states to be included into the basis was determined so that the total energies of the interacting impurity system were accurate up to six decimal places.

The results for the single-electron energy spectrum is presented in Fig. 1 for $\alpha = 20, 5$ and various values of V_0 and d. The choice of the impurity-potential parameters was made such that the spectrum is modified either very weakly or very strongly. The effect of the impurity potential, in general, is to couple all single-electron states and thereby cause level repulsion. Even for a weak impurity potential ($V_0 = 0.5, d = 0.2$) for $\alpha = 5$, the degeneracies in the spectrum are lifted and the amplitude of the oscillations in magnetization is reduced [Fig. 1(c)]. In the case of the strongest impurity potential considered here ($V_0 = 4.0, d = 0.5$) for $\alpha = 20$, the structures in the energy and magnetization curves are completely washed out [Fig. 1(b)].

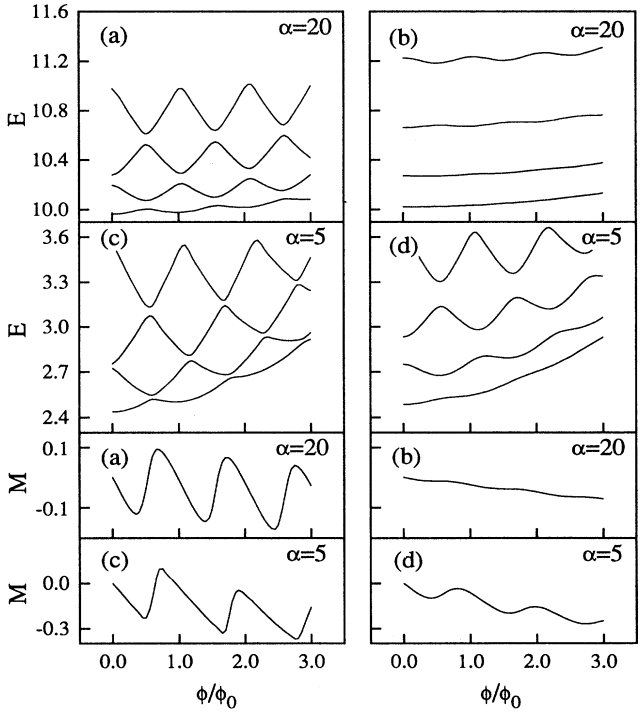

FIG. 1. Single-electron energy spectrum and magnetization (in units of energy defined in the text) vs Φ/Φ_0 for (a) $\alpha = 20$, $V_0 = 1.0$, $d = 0.2$; (b) $\alpha = 20$, $V_0 = 4.0$, $d = 0.5$; (c) $\alpha = 5$, $V_0 = 0.5$, $d = 0.2$; and (d) $\alpha = 5$, $V_0 = 1.0$, $d = 0.5$.

The effect of the impurity potential for the noninteracting and interacting systems on the energy and persistent current of a four-electron system is displayed in Fig. 2 for $\alpha = 20$ and in Fig. 3 for $\alpha = 5$. In these figures, we plot energy (left panel) and magnetization (right panel) of the (a) noninteracting and impurity-free electron systems, which are then compared with results for two different values of impurity-potential parameters V_0 and d without [(b) and (d)] and with [(c) and (e)] Coulomb interactions included. For $\alpha = 20$ and *moderate* impurity strength [inferred from the small degeneracy gap, as in Fig. 1(a)] $V_0 = 1.0, d = 0.2$, the energy spectrum shifts up slightly after impurity and interactions are added [Figs. 2(b) and 2(c)]. The amplitude of oscillations in magnetization also decreases slightly when impurity is added in the system and there is an almost insignificant further decrease when interactions are also included. For the strong impurity case, i.e., $V_0 = 4.0, d = 0.5$, the trend is the same: In a noninteracting system, but with impurity potential included, the degeneracy gap is quite large [Fig. 2(d)] and there is a rapid decrease in magnetization. Again, there is also a very minor further decrease, when the interaction is added in the presence of the impurity

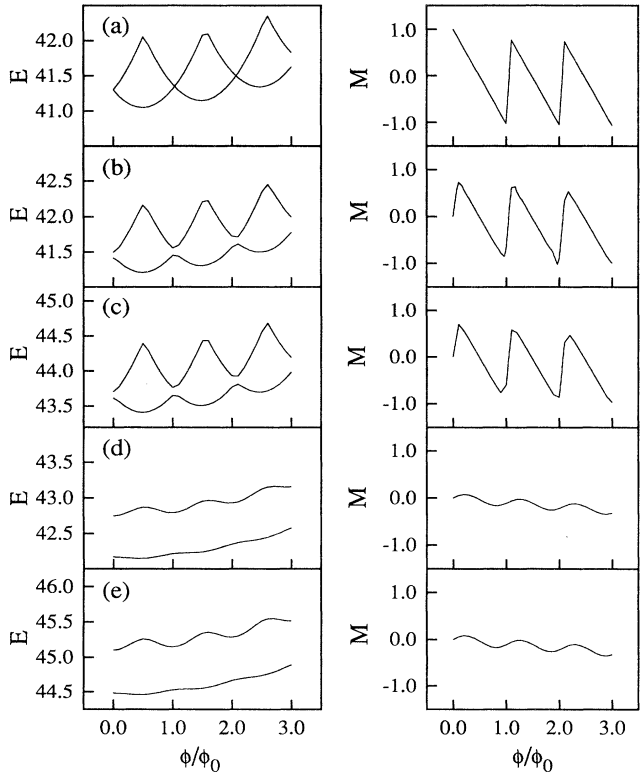

FIG. 2. The lowest two energy values and magnetization vs Φ/Φ_0 for a four-electron system at $\alpha = 20$. (a) Noninteracting and impurity-free, (b) noninteracting, but with impurity potential ($V_0 = 1.0, d = 0.2$), (c) interaction and impurity included ($V_0 = 1.0, d = 0.2$), (d) noninteracting, but with impurity included ($V_0 = 4.0, d = 0.5$), and (e) interaction and impurity included ($V_0 = 4.0, d = 0.5$).

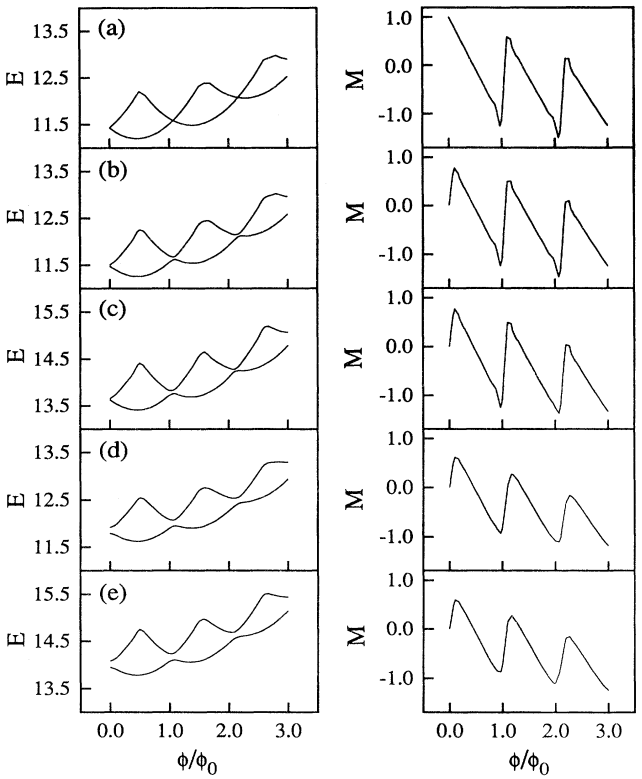

FIG. 3. Same as in Fig. 2, but for $\alpha = 5$, and $V_0 = 0.5$, $d = 0.2$ [(b) and (c)] and $V_0 = 1.0$, $d = 0.5$ [(d) and (e)].

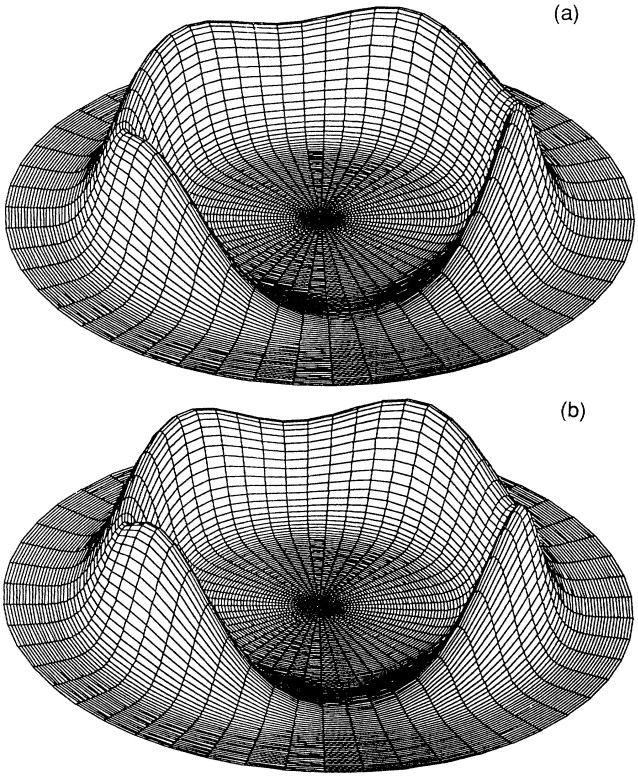

FIG. 4. The charge density for $\alpha = 20, V_0 = 4.0, d = 0.5$, with (a) noninteracting but impurity included and (b) impurity and interactions included.

[Fig. 2(e)]. The interaction simply shifts the energy spectrum to higher energies. For $\alpha = 5$, the results are qualitatively similar, except that the effects described above are somewhat weaker. We have already mentioned in the introduction that in the absence of any impurity potential, interaction has no effect on the persistent current. From these results, one can conclude that, while the persistent current is dramatically reduced in the presence of strong disorder, the role of Coulomb interactions is almost insignificant. Finally, to explore the features of the quantum ring described above somewhat further, we have presented the effect of the impurity to the electron density,

$$\rho(r,\theta) = \sum_{\lambda,\lambda'} e^{im\theta} R_\lambda(r) R_{\lambda'}(r)\, a_\lambda^\dagger a_{\lambda'} \ ,$$

in the ground state. As expected, the impurity breaks the rotational symmetry by localizing the electrons (to some extent) to positions that are mirror symmetric with respect to the axis passing through the impurity and the center of the ring [Fig. 4(a)]. This localization is slightly enhanced by the repulsive Coulomb interaction [Fig. 4(b)].

In conclusion, we calculated the effect of impurities and interactions on the persistent current in a quantum ring. We found that the persistent current is reduced when the impurity is included in the system. But even for the strongest impurity case, there was no significant effect of interaction on the persistent current. The only effect of interaction was to shift the energy spectrum to higher energies. The advantage of the present scheme is that we can investigate many other physical properties of a quantum ring, like the pair-correlation functions of the interacting systems in the presence of an impurity and the magnetoplasmon resonances, recently observed in quantum rings.[15] Such studies will be reported elsewhere.

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[8] In fact, our work also indicated that this remarkable result is valid even for a very few electron systems (Ref. 9).

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