Influence of electron temperature and carrier concentration on electron–LO-phonon intersubband scattering in wide $GaAs/Al_xGa_{1-x}As$ quantum wells

S.-C. Lee, I. Galbraith, and C. R. Pidgeon
Physics Department, Heriot-Watt University, Riccarton, Edinburgh EH14 4AS, United Kingdom
(Received 18 January 1995)

In view of very disparate relaxation times measured in experiments on wide $GaAs/Al_xGa_{1-x}As$ quantum wells, we have calculated the electron – LO-phonon intersubband scattering rate with a thermal distribution of electrons in the quantum well subbands. The intersubband transition can proceed through LO-phonon emission even at wide well widths through the high energy tail of the thermal distribution. Our results show that at low electron temperature the scattering rate has a very sensitive dependence on electron temperature, resulting in a wide range of lifetimes, from picosecond to over a nanosecond. This sensitivity of the scattering rate to electron temperature makes it possible to account for the large variation in decay times that have been measured in similar wide wells. We find that at room temperature the lifetimes are around 1 ps at all well widths. Detailed results are given of the dependence of the scattering rate on temperature and carrier concentration.

I. INTRODUCTION

There is interest in elucidating the nature of electron relaxation mechanisms in quantum wells both as fundamental physical processes, and for technological applications as an understanding of carrier relaxation is needed to model optical devices such as infrared lasers and detectors. Many experiments[1–8] have measured the scattering rates for intersubband relaxation between the second and the first subbands in $GaAs/Al_xGa_{1-x}As$ quantum wells, with measured relaxation times ranging from subpicosecond to over a nanosecond. Suggested mechanisms for the relaxation are longitudinal-optical- (LO-) phonon interactions, acoustic phonon interactions, or carrier-carrier scattering.

A distinction has been made between wide wells, where the intersubband energy separation ΔE_{21} is less than the LO-phonon energy E_{LO}, and narrow wells where $\Delta E_{21} > E_{LO}$. There remains a debate over the reason for the very different lifetimes measured for the intersubband relaxation in similar wide $GaAs/Al_xGa_{1-x}As$ quantum wells, where both long lifetimes of hundreds of picoseconds[1,2] and much shorter lifetimes, around 20–40 ps,[3,4] have been measured. It has been suggested that in these wide wells, where $\Delta E_{21} < E_{LO}$, the transition would take place through the emission of an acoustic phonon giving lifetimes of hundreds of picoseconds,[9] but the shorter lifetimes of tens of picoseconds remain unexplained by this mechanism.

In this paper, we present the results of calculations showing the strong influence of electron temperature on the electron – LO-phonon scattering rate in wide wells. The relaxation lifetime is shown to vary from 1 ps to well over 1 ns with a small change in electron temperature. This dramatic variation in lifetime could account for both the short lifetimes, 20 – 40 ps, and long lifetimes, hundreds of picoseconds, observed in wide wells, where the transition energy is less than the LO-phonon energy. We also show the influence of lattice temperature and carrier concentration on the scattering rate. The effects of state blocking can be seen at high carrier concentration.

The experiments on wide wells include a time-resolved Raman experiment[1] on 215 Å wells, with an estimated excitation density of $\simeq 4.0 \times 10^{11}$ cm^{-2}. A lifetime between 325 ps and 570 ps was reported in this experiment. Saturated absorption measurements[2] with the Santa Barbara free electron laser on a sample with a wide 400 Å well and doped barriers found a lifetime between 180 ps to 1.6 ns. Short lifetimes, 20 ps and 35 ps, were reported in an interband excite-probe experiment[3] with well widths of 225 Å and 240 Å. An intersubband excite-probe experiment[4] with an infrared free electron laser on a uniformly doped sample with wide 270 Å wells also measured a lifetime around 40 ps.

It has been suggested[3,4] that the short lifetimes in wide wells, 20–40 ps, could be explained if there is a thermal distribution of electrons in the second subband. Electrons in the high energy tail of the distribution would have sufficient energy to relax to the first subband through the emission of an optical phonon rather than an acoustic phonon so reducing the lifetime. Levenson *et al.* calculated[3] the electron – LO-phonon scattering rate, using a simple analytical model derived[10] in the momentum conservation approximation (MCA), and they took into account the higher energy electrons by averaging the scattering rate over a Maxwell-Boltzmann distribution of carriers. Their results, plotted as a function of well width, show that the abrupt cutoff in the scattering rate when the intersubband energy drops below the LO-phonon energy is smoothed out by the average over the thermal distribution. There is a finite electron – LO-phonon scattering rate even for $\Delta E_{21} < E_{LO}$, since there is a significant fraction of electrons with enough kinetic energy in the second subband to emit LO phonons.

0163-1829/95/52(3)/1874(8)/$06.00

Their results show the overall trends of the scattering mechanism, but no details of the carrier concentration or temperature dependencies of the scattering rate are given.

We give here detailed results of a more complete calculation (without the MCA) making explicit the dependence of the scattering rate on electron and lattice temperatures, carrier concentration, and well width. We find that the scattering rate is particularly sensitive to electron temperature below 100 K at wide well widths, and also in narrower well widths at high carrier densities. At room temperature it is seen that the lifetimes are around 1 ps at all well widths, making clear that LO-phonon emission can occur even when $\Delta E_{21} < E_{\rm LO}$, if there are electrons high in the second subband with enough energy to emit a LO phonon.

We mention here that in narrow wells, where $\Delta E_{21} > E_{\rm LO}$, the relaxation lifetime, due to electron – LO-phonon scattering is expected to be subpicosecond. This subpicosecond lifetime has been measured in several experiments.[7,8]

II. THEORY

In a quantum well with parabolic, conduction subbands, the scattering rate W of an electron in a subband to another state in the same subband (intrasubband scattering) or to a different subband (intersubband scattering) can be derived from Fermi's golden rule:

$$W = \frac{2\pi}{\hbar} \sum_f |\langle f|\hat{H}|i\rangle|^2 \delta(E_f - E_i), \tag{1}$$

where f (i) labels the final (initial) state. If the electron makes the transition through the absorption or emission of an LO phonon, the interaction Hamiltonian $\hat{H}$ is given by

$$\hat{H} = \sum_{\mathbf{q}} [\alpha(\mathbf{q}) e^{-i\mathbf{q}\cdot\mathbf{r}} \hat{b}^\dagger_{\mathbf{q}} + \text{H.c.}], \tag{2}$$

where $\alpha(\mathbf{q})$ is the strength of the electron – LO-phonon (Fröhlich) interaction and

$$\alpha(\mathbf{q}) = i\left[\frac{\hbar\omega_{\rm LO} e^2}{2\epsilon_p V q^2}\right]^{\frac{1}{2}}. \tag{3}$$

$\mathbf{q}$ is the phonon wave vector, $\hat{b}^\dagger_{\mathbf{q}}$ ($\hat{b}_{\mathbf{q}}$) is the phonon creation (annihilation) operator, $\hbar\omega_{\rm LO} = E_{\rm LO}$ is the LO-phonon energy, V is the crystal volume, and $\epsilon_p^{-1} = \epsilon_\infty^{-1} - \epsilon_s^{-1}$. ϵ_∞, ϵ_s are the high-frequency and static absolute permittivities. Evaluating Eq. (1), with the sum converted to an integral over final states, and assuming bulk phonons and an infinite square well gives the scattering rate[11]

$$W_{nn'} = \frac{W_0}{2}\left[n(\omega_{\rm LO}) + \frac{1}{2} \pm \frac{1}{2}\right] \times \int_{-\infty}^{\infty} \frac{|G(k_z', k_z, q_z)|^2 dq_z}{\left[\frac{(\hbar\omega^*)^2}{E_1} + 2q_z^2(2E_\parallel \mp \hbar\omega^*) + q_z^4 E_1\right]^{\frac{1}{2}}}, \tag{4}$$

where $n(\omega_{\rm LO})$ is the phonon occupation number given by the Bose-Einstein distribution and is fixed by the lattice temperature $T_{\rm lattice}$. q_z is the z component of the phonon wave vector and k_z (k_z') is the z component of the electron wave vector in its initial (final) state. The z direction is taken parallel to the growth direction. E_1 is the first subband edge energy and $E_\parallel$ is the in-plane kinetic energy of the electron in its initial state measured from the subband edge of that state. $\hbar\omega^* = E_{\rm LO} \mp (E_n - E_{n'})$, where E_n, $E_{n'}$ are subband edge energies of the initial and final states of the electron. Energies are scaled to the LO-phonon energy and wave vectors are scaled to the z component of the electron wave vector in the first subband, $k_1 = \pi/L$, where L is the well width. The upper (lower) sign is for emission (absorption) of a phonon. The overlap integral

$$G(k_z', k_z, q_z) = \frac{2}{L}\int_0^L e^{iq_z z} \sin k_z' z \sin k_z z \, dz, \tag{5}$$

and the scale of the rate is set by the prefactor,

$$W_0 = \frac{e^2}{4\pi\epsilon_p \hbar}\left(\frac{2m^*\omega_{\rm LO}}{\hbar}\right)^{\frac{1}{2}}, \tag{6}$$

where m^* is the effective mass of the electron. For GaAs, $W_0 = 8.5 \times 10^{12}$ s^{-1}, with $m^* = 0.067 m_e$, $\hbar\omega_{\rm LO} = 36.7$ meV, and $\epsilon_p = 65\epsilon_0$ ($\epsilon_\infty = 10.9\epsilon_0$, and $\epsilon_s = 13.1\epsilon_0$). The scale of the lifetime is given by $\tau = \frac{1}{W_0} \sim 0.1$ ps. We are primarily interested in scattering from the second subband to the first, so setting $E_n = E_2 = 4E_1$, $E_{n'} = E_1 = (\hbar\pi)^2/(2m^* L^2 E_{\rm LO})$, $k_z = k_2 = 2$, and $k_z' = k_1 = 1$ (scaled energies and wave vectors) in Eq. (4) and integrating Eq. (4) numerically gives the scattering rate, W_{21}, from subband 2 to 1.

What we have found up to this point is the empty-subband to empty-subband scattering rate for a single electron. We also need to consider the filling of the states in the bands by other electrons. Instead of the single-electron scattering rate, we can find an average scattering rate for all the electrons in the second subband. The different single-electron scattering rates corresponding to different initial in-plane kinetic energies are weighted with a Fermi distribution and averaged to give the average scattering rate:

$$W_{21}^{\rm av} = \frac{1}{\tau_{\rm calc}} = \frac{\int \rho_E f(E, T_e)[1 - f(E \mp E_{\rm LO}, T_e)] W_{21}(E) dE}{\int \rho_E f(E, T_e) dE}, \tag{7}$$

where ρ_E is the two-dimensional (2D) density of states and the Fermi distribution:

$$f(E, T_e) = \frac{1}{1 + \exp\left(\frac{E-\mu}{k_B T_e}\right)}. \tag{8}$$

$E = E_2 + E_\parallel$ is the initial energy of the electron in the second subband and $E \mp E_{\rm LO}$ is its final energy in the first subband with the upper (lower) sign for emission

(absorption). The chemical potential μ is calculated by assuming the electrons are distributed over the three lowest subbands and are in thermal equilibrium with electron temperature T_e. The parameters needed to calculate $W_{21}^{\rm av}$ are the well width, the lattice temperature, the electron temperature, and the areal carrier concentration n_e.

We note that the electron temperature may differ from the lattice temperature. When the electrons are first excited into the quantum well subbands they have a nonequilibrium distribution, which rapidly thermalizes to a Fermi distribution with a high T_e. The thermalization occurs in tens of femtoseconds through electron-electron scattering in which no energy is transferred to the lattice so that the electron temperature remains greater than the lattice temperature. The electrons then lose energy to the lattice through intrasubband/intersubband electron – LO-phonon interactions and settle down to a Fermi distribution at a lower T_e, which may still be higher than the lattice temperature. We show later that intrasubband scattering times become very long at low T_e, thus limiting the cooling of the electron plasma to around 50 K, despite a lower lattice temperature.

III. RESULTS

In this section, the results of calculations of the average scattering rate of electrons from the second to the first subband are presented. The behavior of the scattering rates and relaxation times (inverse scattering rates) in different parameter regimes is described and discussed. Unless otherwise stated, the calculated scattering rates are for transitions from the second to the first subband through the emission of an LO phonon. At low lattice temperatures the phonon occupation number $n(\omega) \simeq 0$. The $2 \rightarrow 1$ transition occurs only through the emission of an LO phonon, since there are no phonons to be absorbed. At high lattice temperatures, $n(\omega) > 0$ and the transition can take place either through emission or absorption of phonons. In Figs. 1–3, average scattering rates are plotted as a function of well width for different carrier densities at lattice temperature 15 K with $T_e = 15$ K, 77 K, and 300 K. The fractional population of carriers in the first subband is also plotted. At this low lattice temperature the scattering is due to emission only.

In Fig. 1, $T_{\rm lattice} = T_e = 15$ K and at a low carrier concentration 10^{10} cm^{-2}, the rate increases with increasing well width to a maximum at 215 Å and then drops off sharply beyond this point. The well width 215 Å marks the crossover where $\Delta E_{21} = E_{\rm LO}$. At this low electron temperature there are few electrons high enough in the second subband with enough energy to emit a phonon. At higher concentrations, 10^{11} and 10^{12} cm^{-2}, the chemical potential moves above the first subband edge and the first subband begins to fill. The rate then cuts off at narrower well widths (< 215 Å), when the energy separation between the second subband edge and the chemical potential is less than $E_{\rm LO}$. At all concentrations, the lifetimes are subpicosecond at well widths where LO-

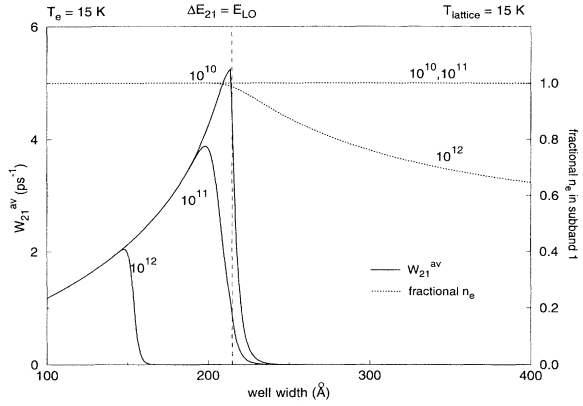

FIG. 1. Plot of $W_{21}^{\rm av}$ (solid lines) and the fractional carrier density in the first subband (dotted lines) against well width for 2D carrier densities 10^{10}, 10^{11}, and 10^{12} cm^{-2}. The intersubband transition is due to emission only. The curves are labeled by the carrier density. The vertical dashed line marks the well width, where $\Delta E_{21} = E_{\rm LO}$. $T_{\rm lattice} = 15$ K, $T_e = 15$ K.

phonon emission is permitted from the second subband edge, but rise sharply when LO-phonon emission is not allowed, either because of energy considerations or state filling. At these long lifetimes, beyond $\simeq 1$ ns, other decay mechanisms, such as acoustic phonon emission and carrier-carrier scattering, would also have to be considered.

In Fig. 2 with $T_{\rm lattice} = 15$ K and $T_e = 77$ K, electrons at all concentrations from $10^{10} - 10^{12}$ cm^{-2} begin to occupy the higher subbands. There is now a larger rate for emission even when $\Delta E_{21} < E_{\rm LO}$, because there are electrons with enough energy in the second subband to emit an LO phonon. The lifetimes in narrow wells ($\Delta E_{21} > E_{\rm LO}$) are still subpicosecond, but lifetimes in wider wells have dropped to tens of picoseconds, $\simeq 10 - 70$ ps. In Fig. 3 with $T_{\rm lattice} = 15$ K and $T_e = 300$ K, there is a substantial fraction of electrons in the higher subbands at almost all well widths. The rate curves become broader and flatter with all maxima at 215 Å. The rate at wider well widths beyond 215 Å is now compara-

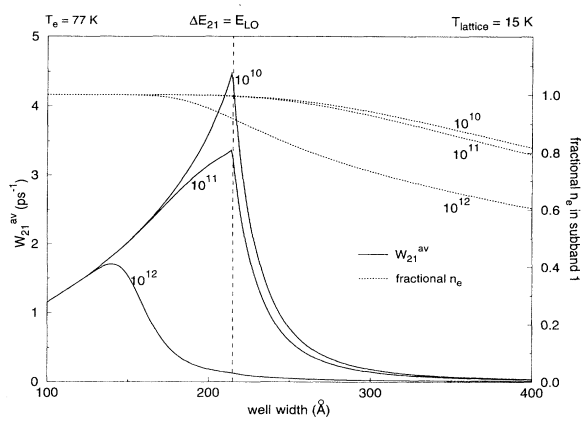

FIG. 2. As in Fig. 1, except $T_e = 77$ K.

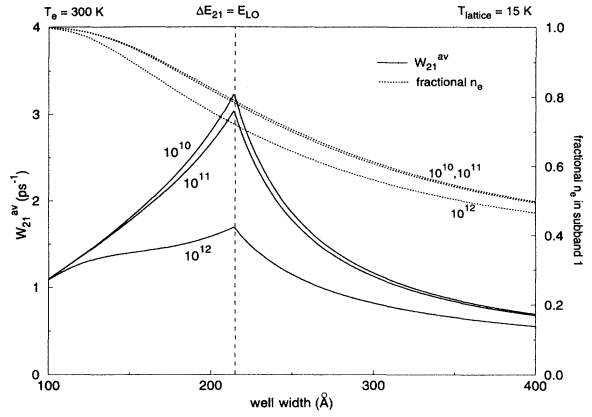

FIG. 3. As in Fig. 1, except $T_e = 300$ K.

ble to the rate at narrower well widths below 215 Å. The lifetimes at all well widths are now around 1 ps. This implies that whether or not $\Delta E_{21} >< E_{LO}$ the rate at 300 K is roughly the same over a wide range of well widths. This result is somewhat surprising, but is brought about because $k_B T \simeq E_{LO}$. At 300 K band filling is not very important, unlike at 15 K.

In Fig. 4, the scattering rate was calculated with both the lattice temperature and electron temperature at 300 K. At this higher lattice temperature, the shape of the rate curves for transitions, due to LO-phonon emission, is the same as those at low $T_{lattice}$, with the same T_e. The rates are slightly larger due to the phonon occupation factor $n(\omega)+1$, where $n(\omega) > 0$ at high $T_{lattice}$. The main difference at a higher lattice temperature is that there is a contribution to the scattering rate from the absorption of an LO phonon, which occurs at an almost constant rate at all well widths and concentrations. The scattering, due to absorption, dominates the scattering rate when the emission scattering tends to zero and limits the lifetimes to around 1 ps or less at room temperature. The scattering rate curves shown in Fig. 4 include both absorption and emission. The shape of the rate curves is similar to those in Fig. 3 at $T_{lattice} = 15$ K, with comparable scattering rates above and below 215 Å. The scattering rates are slightly faster than those in Fig. 3 with subpicosecond lifetimes at all well widths.

In Fig. 5, the lifetime is plotted as a function of electron temperature at carrier concentrations 10^{10} and 10^{12} cm^{-2} and, for each carrier concentration, at well widths 200 Å (ΔE_{21} slightly greater than E_{LO}), 250 Å (ΔE_{21} slightly less than E_{LO}), and 400 Å (a wide well). The lattice temperature is fixed at 15 K. Looking first at the dependence of lifetime on carrier concentration for a given well width, we see that lifetimes at high concentrations (e.g., 10^{12} cm^{-2}) are longer than lifetimes at low concentrations (e.g., 10^{10} cm^{-2}). The increased lifetime at high carrier concentration demonstrates the effect of state blocking when fewer electrons in the second subband are able to make the $2 \rightarrow 1$ transition, because the final states in the first subband are filled. Looking next at the dependence of lifetime on well width at a fixed carrier concentration, we find a similar trend: the lifetime is shorter for narrow wells and increases as the well width increases. The longer lifetimes in wider wells occur because, again, fewer electrons in the second subband can scatter from $2 \rightarrow 1$, but the obstacle is now due to energy considerations rather than state blocking. In the wider wells $\Delta E_{21} < E_{LO}$ and the electrons at the bottom of the second subband have insufficient energy to make the transition. In both cases, high concentrations or wide wells, electrons near and at the edge of the second subband are unable to scatter into the first subband and the $2 \rightarrow 1$ transition must proceed through the higher energy electrons in the second subband. This reduces the average scattering rate, increasing the lifetime, because fewer electrons participate in the scattering, but the average is taken over all the electrons in the second subband. In addition, the single-electron scattering rate given by Eq. (4)

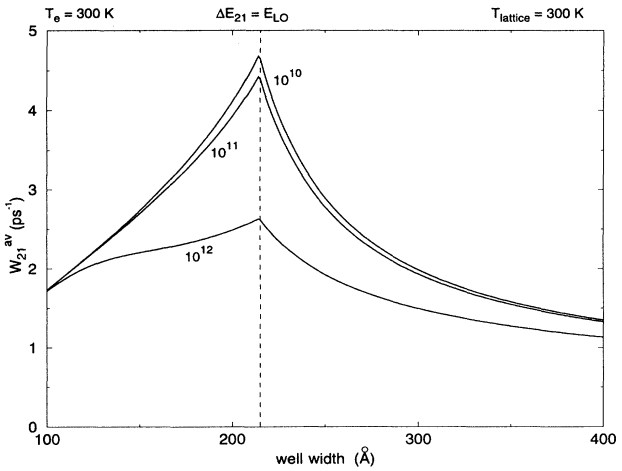

FIG. 4. As in Fig. 1, except $T_{lattice} = 300$ K, $T_e = 300$ K, and the scattering includes both absorption and emission. The fractional carrier densities are not shown as they are identical to the ones in Fig. 3.

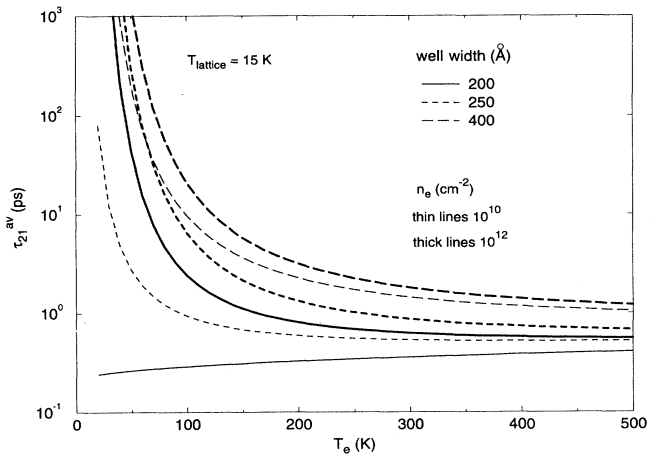

FIG. 5. Plot of τ_{21}^{av} against T_e at carrier densities 10^{10} cm^{-2} (thin lines) and 10^{12} cm^{-2} (thick lines), and for each carrier density at well widths 200 Å (continuous line), 250 Å (dashed line), and 400 Å (long dashed line). $T_{lattice} = 15$ K.

decreases when the initial in-plane kinetic energy of the electron increases so that the maximum single-electron scattering rate occurs from the edge of the second subband. If the electrons at the second subband edge are unable to participate in the transition, as is the case at high concentrations and wide well widths, the absence of their higher scattering rates would be a further cause of the slowing down of the average scattering rate in this parameter region.

Turning now to the dependence of lifetime on electron temperature, we find that the behavior of the lifetime divides into two regions. As the electron temperature increases above 200 K towards high values (e.g., 500 K), all the lifetimes, regardless of carrier concentration and well width, tend to $\sim$ 1 ps or less. Conversely, in the region below $\simeq$ 100 K, there is a more divergent behavior with the lifetime showing more sensitivity to well width and carrier concentration. The lifetime varies strongly with electron temperature below 100 K and ranges from over a nanosecond below $\simeq$ 50 K to picoseconds at higher T_e near 100 K. As T_e increases beyond 100 K, the lifetime levels off and decreases more gradually. The strong variation in lifetime seen below $T_e = 100$ K occurs mainly in wider wells ($>$ 215 Å), but at high carrier concentrations (10^{12} cm^{-2}) this strong variation, with very long lifetimes at low T_e, appears also in narrow wells (200 Å) demonstrating the effect of final state blocking. This is the same effect seen in Fig. 1 where the scattering rate cuts off at narrower well widths at high carrier concentrations. Thus, we see that the strong dependence of lifetime on electron temperature occurs at low T_e in the parameter region, narrow wells with high carrier density and wide wells, where LO-phonon emission cannot take place from the second subband edge, but can proceed through higher energy electrons in the second subband. Conversely, in narrow wells at low carrier concentrations (e.g., 200 Å, 10^{10} cm^{-2}) where LO-phonon emission can occur from the edge of the second subband the lifetime is subpicosecond at all T_e, even below 100 K, and remains almost constant, increasing very gradually with increasing electron temperature.

IV. DISCUSSION

We now discuss in detail the application of our calculation to published experimental results. The main aim is to show that in certain parameter regimes scattering rates, due to the LO-phonon interaction, have a far wider range than previously thought (i.e., relaxation times not limited to subpicosecond), and that the scattering rates have a marked sensitivity to electron temperature making it possible to account, with a single mechanism, for very disparate lifetimes measured in experiments. Including a thermal distribution of electrons has introduced a flexibility in the range of achievable lifetimes that was previously absent in the single-electron – LO-phonon scattering rate.

There is a practical difficulty in that some experimental parameters needed for the calculation are difficult to measure and are, therefore, unknown or have great uncertainty. In particular, the electron temperature, which is crucial to the calculation, is seldom estimated and its value is rarely reported in the experimental literature. The 2D carrier density in experiments with photoexcited carriers is another parameter which is difficult to estimate accurately. We, therefore, stress that the fits we give to experimental results are only to demonstrate the extensive range of lifetimes realizable through the LO-phonon interaction. They cannot be taken as definitive fits without precise estimates of the experimental parameters.

To model the experimental lifetimes, the lattice temperatures and carrier densities reported in the experiments were used in the calculations. The well widths used were so-called effective well widths, i.e., the widths of infinite wells giving the same intersubband separation ΔE_{21} as measured in the experiment. Table I shows the lifetimes measured in the wide well experiments, as well as our calculated lifetimes. The intersubband separation ΔE_{21} in these wells was less than $E_{\mathrm{LO}} = 36.7$ meV. The parameters reported in the experiments are also displayed in the table. In general, the same parameters were used in the calculations. If a different parameter was used, the number is enclosed in brackets beside the experimental parameter. The well widths in brackets are the effective well widths. If an experimental parameter is missing, it was not reported in the cited reference.

The first two experiments in the table measured long lifetimes ($>$ 100 ps). The first experiment,[1] carried out by Oberli *et al.*, had a photoexcited carrier density distributed over the first two subbands. The lifetime measured in the experiment was 325 ± 25 ps for the decay of electrons from the second subband. A range of lifetimes, 325 to 570 ps, is quoted because Oberli *et al.* interpreted

TABLE I. Table of experimental and calculated lifetimes for $2 \rightarrow 1$ intersubband transitions in wide GaAs/Al$_x$Ga$_{1-x}$As wells. Experimental parameters are also displayed. Numbers in brackets are parameters used in the calculation. If there is no bracketed number the experimental parameter was used. If an experimental parameter is missing, it was not reported in the cited reference. Well widths in brackets are effective well widths, i.e., the widths of infinite wells with the same intersubband separation as reported in the experiment.

Ref.	L (Å)	n_e (cm^{-2})	T_{lattice} (K)	T_e (K)	ΔE_{21} (meV)	τ_{exp} (ps)	τ_{calc} (ps)
1	215 (251)	4×10^{11}	5	30 – 40 (30–33)	26.8	325 – 570	629–293
2	400 (394)	8×10^{10}	10	(44)	10.9	180 ps – 1.6 ns	557
3	225 (290)	7×10^{11}	15	(78)	20	20 ± 2	20.6
	240 (290)	7×10^{11}	15	(70)	20	35 ± 2	35.4
4	270 (307)	1.5×10^{11}	(10)	(53)	18	40 ± 5	39.8

the decay as including radiative recombination in addition to intersubband scattering. They deduced that including this additional process could increase the lifetime, due to intersubband scattering, to 570 ± 70 ps. This was the only experiment where an estimate of the electron temperature (30 – 40 K) was given. We found that using a temperature range between 30 – 33 K gave lifetimes varying between $\simeq$ 630 – 290 ps. (The lifetime decreases as temperature increases.) When the temperature is increased to 40 K, the calculated lifetime falls sharply to 79 ps. Since the carrier density was photoexcited and the estimate of its value uncertain, we also explored the effect of varying the carrier concentration. With the electron temperature fixed at 31 K, the carrier density was varied between 3.8×10^{11} and 4.2×10^{11} cm^{-2}. We found the calculated lifetimes to vary between $\simeq$ 380 – 600 ps, showing that the carrier concentration also has a strong influence on the lifetime. Thus, with parameters close to those reported in the experiment, we were able to obtain lifetimes in the same range as the measured lifetimes.

The second experiment[2] in the table was on a modulation-doped sample with a very wide 400 Å well. This experiment involved saturation spectroscopy measurements, which is an indirect way of finding the scattering rate unlike the direct measurement made in excite-probe experiments. Because of a factor of 3 uncertainty in the intensity the lifetime was estimated to be 600 ps within the same factor leading to the large lifetime range seen in the table. As in all the remaining entries in the table, no estimate was given of the electron temperature. Using a value of 44 K, for the electron temperature in the calculation, gives a lifetime of 557 ps close to the estimated lifetime in the experiment. We return to this choice of electron temperature later. In both experiments discussed thus far, the long lifetimes measured were attributed to acoustic phonon emission, because the intersubband separation ΔE_{21} was less than the LO-phonon energy $E_{\rm LO}$. The results we present here show that LO-phonon emission can occur even when $\Delta E_{21} < E_{\rm LO}$, if there is a thermal distribution of electrons in the second subband, and this process gives lifetimes of similar magnitudes to acoustic phonon emission and thus also needs to be considered when calculating relaxation times in wide wells.

The remaining experiments in the table measured short lifetimes in the tens of picoseconds. The experiment[3] by Levenson *et al.* carried out on two samples with different well widths were quoted to have the same intersubband separation $\simeq$ 20 meV, so the same effective well width was used in the calculation for both samples. Using electron temperatures between 70 and 80 K gave calculated lifetimes similar to the measured lifetimes. We stress again that since the experimental electron temperature is unknown, these numbers are given only to show that lifetimes close to the experimental result can be obtained using a plausible electron temperature and are not meant to be true fits. As in the experiment by Oberli *et al.*, the carriers in the wells were photoexcited, leading to an uncertainty in the carrier density. If a lower carrier density is used in the calculation, similar lifetimes can be obtained with lower electron temperatures. The last experiment[4] in the table was carried out on a uniformly doped sample. Using an electron temperature of 53 K gives a calculated lifetime of 39.8 ps close to the experimental result. Decreasing the temperature to 45 K gives a lifetime of 94 ps, again showing the sensitivity of lifetime to electron temperature. Since the wells in this sample were doped, there could also be impurity scattering, which would also cause a reduced lifetime.

We have shown in this discussion on wide wells that the sensitive dependence of lifetime on electron temperature at low electron temperature and the extensive range of possible lifetimes obtained in the calculation (Fig. 5) makes it possible to attribute *both* the short ($\simeq$ 20 – 40 ps) and long (hundreds of picoseconds) measured lifetimes in wide wells to electron–LO-phonon scattering.

We now look briefly at experiments performed on narrow wells. In these wells, where $\Delta E_{21} > E_{\rm LO}$, the suggested mechanism for the intersubband transition is the electron–LO-phonon interaction. In an experiment with a 146 Å well (effective well width 182 Å) Tatham *et al.* observed[7] a decay time of $\sim$ 1 ps. Our calculation of the lifetime gave a value of 0.32 ps, due only to the $2 \rightarrow 1$ transition through the emission of an LO phonon. But uniquely in this experiment the first four subbands are excited and the lifetime in the second subband could be increased by the relaxation of electrons from the higher subbands into the second subband. When the scattering of electrons from the third and fourth subbands is included in our calculation, the calculated lifetime increases to 0.9 ps close to the value observed in the experiment.

Longer lifetimes around 10 ps have also been observed in narrow wells. This is more than an order of magnitude greater than lifetimes expected from LO-phonon emission. An experiment[3] by Levenson *et al.* on a narrow 120 Å well (effective well width 161 Å) found a lifetime of 8 ± 2 ps. Using the reported carrier density of 7×10^{11} cm^{-2} and an electron temperature of 30 K gives a calculated lifetime of 0.5 ps. However, the carrier density was photoexcited making its value indefinite. If the carrier density is increased to 1.1×10^{12} cm^{-2}, the calculated lifetime increases to $\sim$ 7 ps. This variability in calculated lifetimes again demonstrates the need for more accurate and reliable estimates of the electron temperature and carrier concentration in experiments. Further experiments with narrow wells and doped barriers[5,6] found lifetimes between 10 – 15 ps. Using the reported experimental parameters in our calculation, we always find subpicosecond lifetimes. Since the experiment by Seilmeier *et al.* was carried out at 300 K, we have explored the effect of absorption of phonons. In calculating the lifetime the transitions from $2 \rightarrow 1$ and $1 \rightarrow 2$, due to absorption of an LO-phonon, and the transition from $1 \rightarrow 2$, due to emission of an LO phonon, were also included:

$$\frac{1}{\tau_{21}} = -W_e^{\rm av}(2 \rightarrow 1) - W_a^{\rm av}(2 \rightarrow 1) + W_e^{\rm av}(1 \rightarrow 2) + W_a^{\rm av}(1 \rightarrow 2),$$

where τ_{21} is the $2 \rightarrow 1$ intersubband relaxation time and the subscript e (a) stands for emission (absorption). The

last three terms on the right-hand side were found to be negligible compared to the first term so that even at high temperature the population change in the second subband due to electron – LO-phonon interactions is dominated by the scattering from $2 \rightarrow 1$, due to emission of an LO phonon. As discussed in Refs. 12 and 13, the increased lifetimes in these narrow wells could be explained by another process: the transfer of electrons into the barrier because of the potential minimum created in the barrier by the ionized donors and the delayed decay back into the first subband in the well. This additional complication is beyond the scope of our calculation.

Finally, we discuss the value of the electron temperature. Oberli *et al.* have mentioned[1] that a hot electron plasma in thermal equilibrium will rapidly cool down via the emission of LO phonons in intrasubband transitions, resulting in a final electron temperature between 30 and 60 K that is still somewhat above the lattice temperature. To calculate the single-electron intrasubband scattering rate W_{11} (W_{22}) in the first (second) subband, we set $E_n = E_{n'} = E_1$ (E_2) and $k_z = k'_z = k_1$ (k_2) $= 1$ (2) in Eq. (4). Using these rates in Eq. (7) (with the energy E adjusted accordingly), we find the average intrasubband scattering rates, and hence the intrasubband relaxation times $\tau_{11}^{\rm av}$ and $\tau_{22}^{\rm av}$. In Fig. 6, we plot $\tau_{11}^{\rm av}$ and $\tau_{22}^{\rm av}$ against T_e for a 200 Å well width and carrier density 10^{11} cm^{-2}. The intersubband relaxation time $\tau_{21}^{\rm av}$ at the same well width and carrier density is also shown for comparison.

Comparing $\tau_{11}^{\rm av}$ and $\tau_{22}^{\rm av}$ in Fig. 6, we see that the curves are almost identical with the intrasubband emission in the first subband being slightly faster than that in the second subband. We also find (not shown here) that the intrasubband emission in both subbands is almost insensitive to carrier concentration and well width, the rate decreasing slightly only at high carrier concentrations (e.g., 10^{12} cm^{-2}) and at wider well widths.

Comparing intrasubband and intersubband relaxation times, we find that the intrasubband curves at all concentrations and well widths have a similar shape to the $\tau_{21}^{\rm av}$ curve at high concentrations or wide well widths (seen in Fig. 5): short subpicosecond lifetimes at high T_e and a dramatic variation of lifetime at low T_e, tending to very long lifetimes (> 1 ns) below 50 K. But for the same moderate well width and carrier density (Fig. 6), there is a marked difference between the intersubband and intrasubband scattering rates at low T_e. $\tau_{21}^{\rm av}$ at this lower well width and carrier density is subpicosecond and almost constant at all T_e. In contrast, the intrasubband relaxation times are only subpicosecond above $\sim$ 250 K. Below 250 K, these lifetimes increase greatly becoming much greater than $\tau_{21}^{\rm av}$. At this well width and concentration, the intrasubband emission times are longer than $\tau_{21}^{\rm av}$. This is in marked contrast to the single-electron scattering rates where $\tau_{21} > \tau_{11}$. The average intrasubband scattering rate is reduced compared to the single-electron intrasubband scattering rate, because only electrons higher up the subband, with $E_{\|} > E_{\rm LO}$, take part in the scattering. Since there are fewer electrons with large $E_{\|}$, and these electrons have a smaller single-electron scattering rate than those with smaller $E_{\|}$, but the average is taken over all the electrons in the subband, this reduces the overall average intrasubband scattering rate.

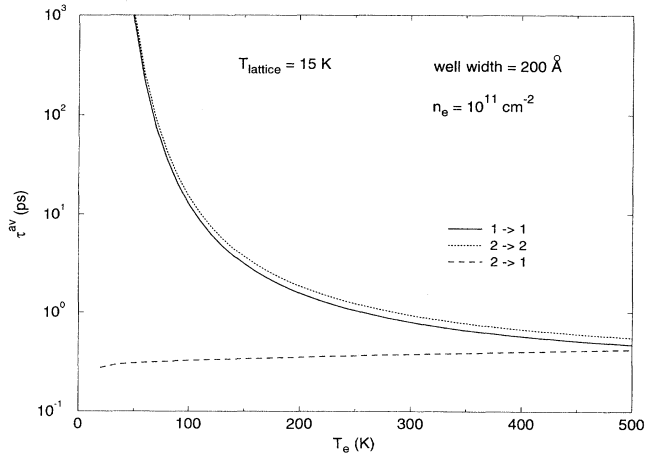

FIG. 6. Plot of $\tau_{11}^{\rm av}$ (solid line), $\tau_{22}^{\rm av}$ (dotted line), and $\tau_{21}^{\rm av}$ (dashed line) against T_e for a well width of 200 Å and $n_e = 10^{11}$ cm^{-2}.

In summary, we see that above $T_e \simeq 150$ K, the electrons cool through both intrasubband and intersubband scattering, which occur on similar time scales. The electrons can lose energy rapidly to the lattice, because of the rapid scattering rates at high T_e (with subpicosecond decay times). But below $\simeq$ 100 K, the intrasubband scattering rates decrease enormously with decay times in the tens to hundreds of picoseconds. The intersubband process in wide wells and at high concentrations undergoes a similar slowing down. Only the intersubband process for moderate well widths and densities remains fast. The carriers in the first and second subbands are in thermal equilibrium and since more electrons are in the first subband, we expect most of the cooling to occur through intrasubband emission from these carriers. Since the intrasubband process slows down so greatly below $\simeq$ 100 K this explains the limiting behavior of T_e, where T_e drops rapidly down to around 50 K, but does not decrease further, down to the lattice temperature.

V. CONCLUSION

In conclusion, the very strong influence of electron temperature and carrier concentration on the $2 \rightarrow 1$ intersubband electron–LO-phonon scattering rate has made it possible to account for a wide range of observed lifetimes with a single mechanism: the emission of LO phonons from a thermal distribution of electrons in the second subband. Calculations with this mechanism result in a wide range of decay times for the $2 \rightarrow 1$ intersubband transition, with a very sensitive dependence on the electron temperature and carrier concentration. This wide variation from subpicosecond to over a nanosecond occurs mainly at wide well widths (> 215 Å), where $\Delta E_{21} < E_{\rm LO}$ and at low electron temperatures below 100 K, but at high carrier concentrations increased lifetimes are also seen at narrower well widths as states in the

first subband begin to fill. In view of this very sensitive dependence of lifetime on electron temperature, the disparate lifetimes observed in similar wide wells in different experiments is not surprising. We have been able to account for both short and long lifetimes measured in wide wells with this mechanism. We also find that at high temperatures (e.g., 300 K) the lifetimes are around 1 ps at all well widths. This is remarkable when we remember that the general consensus has been that intersubband transitions in wide wells cannot occur through LO-phonon emission. Our results show that LO-phonon emission is a process that should also be considered in wide wells. Although other processes may also occur (e.g., acoustic phonon emission or the suggested band bending in the experiment by Seilmeier *et al.*) the far-ranging effect of the electron – LO-phonon interaction in a wider parameter region than previously thought should also be given due consideration. To further explore the dependence of lifetime on electron temperature and carrier concentration, more experimental measurements of the lifetime as a function of electron temperature, carrier concentration, and well width are essential. To date most experimental reports have not given an estimate of the electron temperature. Although difficult to measure, more precise estimates of this parameter are needed in order to correctly interpret the results.

ACKNOWLEDGMENT

We wish to acknowledge financial support from The Royal Society and the EPSRC.

[1] D. Y. Oberli *et al.*, Phys. Rev. Lett. **59**, 696 (1987).
[2] K. Craig *et al.*, Semicond. Sci. Technol. **9**, 627 (1994).
[3] J. A. Levenson, G. Dolique, J. L. Oudar, and I. Abram, Phys. Rev. B **41**, 3688 (1990).
[4] B. N. Murdin *et al.*, Semicond. Sci. Technol. **9**, 1554 (1994).
[5] A. Seilmeier *et al.*, Phys. Rev. Lett. **59**, 1345 (1987).
[6] F. H. Julien *et al.*, Appl. Phys. Lett. **53**, 116 (1988).
[7] M. C. Tatham, J. F. Ryan, and C. T. Foxon, Phys. Rev. Lett. **63**, 1637 (1989).
[8] D. Cui *et al.*, Phys. Rev. B **47**, 6755 (1993).
[9] R. Ferreira and G. Bastard, Phys. Rev. B **40**, 1074 (1989).
[10] B. K. Ridley, J. Phys. C **15**, 5899 (1982).
[11] B. K. Ridley, in *Hot Carriers in Semiconductor Nanostructures: Physics and Applications*, edited by J. Shah (Academic Press, San Diego, 1992).
[12] A. Seilmeier, U. Plödereder, and G. Weimann, Semicond. Sci. Technol. **9**, 736 (1994).
[13] S. M. Goodnick and P. Lugli, in *Hot Carriers in Semiconductor Nanostructures: Physics and Applications*, edited by J. Shah (Academic Press, San Diego, 1992).