Breakdown of correlated diffusion in quasiballistic quantum wires at high magnetic fields

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We study the universal conductance fluctuations (UCF) observed in the low-temperature magnetoconductance of high-mobility $GaAs/Al_xGa_{1-x}As$ quantum wires. As the magnetic field is increased, such that the cyclotron orbit becomes smaller than the wire width, both the amplitude δg and correlation field B_c of the fluctuations are found to increase linearly with magnetic field. The linear increase in B_c is in agreement with the results of recent nonlocal studies of the UCF, and is thought to demonstrate a transition to edge-related transport. Similarly, the increase in δg is argued to result from a magnetically induced breakdown of correlated diffusion, which occurs as the edge states begin to be resolved. Such a breakdown was not observed in previous studies of dirty wires, in which the electron motion remains diffusive up to very high fields, but is instead thought to result from the quasiballistic nature of the wires we study.

I. INTRODUCTION

Universal conductance fluctuations (UCF) result from an electron interference effect, and have been widely observed in a variety of disordered mesoscopic systems.^{1,2} Their zero-temperature amplitude is independent of both the sample size and the degree of disorder, a characteristic which in turn implies a special correlation between the different conduction channels of the device. The basic idea is that, in a quantum-mechanical picture of transport, the conductance of a system can be directly expressed, in terms of interchannel scattering probabilities:¹

$$g = [e^2/h][\Sigma T_{ii}], \qquad (1)$$

where T_{ij} is the probability for an electron incident in channel *i* to be transmitted via channel *j*, and the sum is taken over all channels. Given such an expression, it can then be shown that the assumption of independent channels leads to a nonuniversal fluctuation, the amplitude of which decreases with increasing channel number.¹ In contrast, a universal amplitude of fluctuation can only be obtained by assuming the various transmission coefficients to be correlated. While the exact origin of this correlation is not clearly understood, it is thought to be a consequence of the diffusive nature of motion in disordered systems, which effectively mixes the channels by coherently scattering electrons between them.

Another remarkable feature of the UCF in disordered metals is their persistence to extremely high magnetic fields, a characteristic which in turn results from the inability to induce distinct Landau quantization in these systems.¹ In disordered semiconductors, however, welldefined Landau levels are readily resolved, and recent studies have shown the UCF to exhibit a marked magnetic-field dependence. In particular, the correlation field of the fluctuations has been found to increase as a monotonic function of the magnetic field, while the amplitude of fluctuation remains simultaneously unaltered. $^{3-7}$ Treating transport as a bulk-related process, it is possible to account for the increased correlation field by invoking a magnetically induced reduction in the diffusion constant.⁸ Such a model is unable to explain the field-independent amplitude of fluctuation, however, and it has therefore been argued that the magnetic field induces a breakdown of the UCF.⁴ Subsequent numerical studies suggest that this in turn results from a transition to spatially inhomogeneous current flow, as skipping orbits form at high magnetic fields.⁶

The purpose of this paper is to demonstrate that, by studying much higher mobility wires than those investigated to date, a very different breakdown of the UCF can occur at high fields. In particular, we consider the UCF observed in quasiballistic wires, in which electron scattering occurs predominantly at the wire boundaries. At low magnetic fields, reproducible UCF are observed in the

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magnetoconductance of the wires, the average characteristics of which are independent of magnetic field. As the magnetic field is increased, however, the cyclotron orbit shrinks inside the wire width, and both the correlation field and amplitude of the fluctuations increase as linear functions of the magnetic field. At the highest fields, the relative increase in amplitude can correspond to more than an order of magnitude, with the resulting fluctuations significantly exceeding the expected universal value.¹ We associate this previously unreported effect with the different nature of electron scattering in disordered and quasiballistic wires. In particular, while spatially inhomogeneous, electronic motion in dirty wires is expected to remain diffusive up to very high fields,⁶ so that the interchannel correlation mentioned above should also be preserved. In contrast, in quasiballistic wires, well-defined edge states should begin to form once the cyclotron orbit fits inside the wire width.⁹ The absence of significant bulk disorder should then ensure that increasing the magnetic field leads to a gradual suppression of inter-edge-state scattering. In other words, application of a quantizing field should cause a quenching of diffusion and, on the basis of this, we propose that the observed behavior results from a breakdown of the interchannel correlation at high fields. In order to demonstrate this, we consider the statistical properties of the fluctuations, and discuss our results in relation to recent experimental and theoretical reports.

Before continuing, we point out that some of the results discussed here appeared in an earlier, preliminary study of the UCF.¹⁰ The purpose of this report is to provide a more quantitative description of the behavior observed in that study, by presenting previously unpublished results obtained in further experimental investigations of the same wires.

II. DEVICE FABRICATION AND CHARACTERIZATION

Quantum wires were fabricated in molecular beam epitaxy (MBE)-grown GaAs/Al_xGa_{1-x}As heterojunction material, using standard UV lithographic techniques, and a subsequent wet etch.¹¹ The mesa pattern enabled fourterminal resistance measurements to be performed on narrow wires of length $L = 30 \,\mu m$, while the etching process resulted in significant sidewall depletion. This reduced the effective conducting width of the wires W from their etched values W_e , and, in the experiments we discuss here illumination from a red light-emitting diode (LED) was used to vary W crudely at low temperatures. The samples were clamped to the mixing chamber of a dilution refrigerator, and audio frequency resistance measurements were made in magnetic fields of up to 1.2 T. These employed standard current biasing, with the current I adjusted to produce a voltage drop of less than $k_B T/e$ across the quantum wires (I = 0.2 - 5.0 nA).

Since a detailed discussion of the electrical characterization of the wires has appeared elsewhere,¹⁰ here we simply summarize the main results (Table I). At temperatures around a degree Kelvin, the low-field deviation of

TABLE I. Transport properties of the quantum wires ($L = 30 \mu$ m), measured at 1 K.

Width W., W	Carrier density	Mean free path	Mobility	Diffusion	1,
(µm)	(10^{15} m^{-2})	(µm)	$(m^2/V s)$	$(m^2 s^{-1})$	(µm)
1.8,0.55	3.73	0.97	9.6	0.128	4
1.8,0.74	3.71	1.31	12.8	0.177	6
1.8, 1.25	4.56	3.60	32.3	0.527	6
1.5, 1.00	2.86	3.23	36.4	0.373	10
1.0,0.44	2.13	3.06	40.0	0.312	9

the Shubnikov-de Haas (SdH) oscillations, from an inverse-field behavior, enabled W to be determined.¹² An important characteristic of the wires, was that the resulting mean free path typically exceeded W; transport was therefore quasiballistic, with electron scattering occurring predominantly at the wire walls.¹³

III. EXPERIMENTAL RESULTS

A. Basic observations

In Fig. 1 we show the evolution of the low-temperature magnetoconductance of a single quantum wire, as its effective width is progressively increased by LED illumination. For relatively weak illumination [Figs. 1(a) and 1(b)], average features in the magnetoconductance are obscured by aperiodic fluctuations, which persist across the entire range of magnetic field. As the wire width is increased, however, the average features clearly emerge, and the high-field fluctuations are seen to be superimposed upon well-defined SdH oscillations [Fig. 1(c)]. Considering the magnetic-field dependence of the fluctuations, at low fields their average amplitude and period are field independent, while at much higher fields both of these quantities are found to increase (Fig. 2). More specifically, the field dependence of the fluctuations is found to begin once the cyclotron orbit size $hk_F/\pi eB$ becomes comparable to, or smaller than W [arrows in Figs. 1(a) - 1(c)].

In order to analyze the characteristics of the fluctuations, it is first necessary to subtract a background, due to ensemble averaged magnetotransport, from the raw data. At low magnetic fields, this is a relatively simple process, since the fluctuations are superimposed upon a monotonic background, associated with electron-electron interaction effects.¹⁰ Accounting for this, the phase-breaking length in the wires 1_{ϕ} can then be determined from the amplitude of the remaining fluctuations, ^{10, 13} and the results of this analysis are summarized in Table I. In all cases 1_{ϕ} is found to be significantly shorter than the wire length, a conclusion supported by the observed symmetry of the fluctuations, on reversing magnetic field¹⁴ (Fig. 3). At higher magnetic fields, however, the background is oscillatory and, before continuing our discussion, we first describe the technique used to subtract this from the raw data.



FIG. 1. Conductance fluctuations in a wire of etched with $W_e = 1.8 \ \mu m$, in which the effective conducting width W was increased by illumination from a red LED at low temperatures. The resulting conducting widths were calculated from the observation of one-dimensional subband oscillations at higher temperatures, which give the values (a) $W=0.54 \ \mu m$, (b) $W=0.74 \ \mu m$, and (c) $W=1.22 \ \mu m$. The arrows mark the magnetic field at which the cyclotron orbit is calculated to be comparable to W.



FIG. 2. Expanding the magnetic-field scale of Fig. 1(b) $(W_e = 1.8 \ \mu m, W = 0.74 \ \mu m)$ to achieve greater sensitivity, a significant increase in both the average amplitude and period of the UCF is observed, as the magnetic field is increased.



FIG. 3. Symmetry of the magnetoconductance of a quantum wire ($W_e = 1.8 \ \mu m$, $W = 0.55 \ \mu m$) with respect to field reversal. For the sake of clarity, the *B* trace has been shifted upwards by $0.75e^2/h$. The arrow marks the magnetic field at which the cyclotron orbit is calculated to be comparable to *W*.

B. High-field background subtraction

In previous high-field studies of the UCF, various techniques have been applied to eliminate the influence of SdH oscillations. Possibly the most successful of these involves the use of nonlocal measurements, which yield no SdH contribution, and so allow an unambiguous study of the UCF.^{3,4} Alternatively, in four-terminal measurements which employ a local geometry, an ingenious technique can be applied when transport is phase coherent.⁷ This involves subtracting pairs of magnetoresistance traces, obtained on reversing magnetic field, to leave an aperiodic component with no trace of SdH oscillations.¹⁴ Unfortunately, such an approach cannot be applied in our studies, where inelastic scattering ensures the symmetry of the magnetoconductance (Fig. 3), and we therefore account for the background by using a different technique.

In particular, we exploit the fact that the fluctuations are typically washed out, at much lower temperatures than the SdH oscillations [Fig. 4(a)]. Consequently, there is usually a significant temperature range in experiments, over which the main effect of a variation in temperature, is simply to induce a change in the amplitude of the oscillations. In order to remove the oscillatory component from the low-temperature fluctuations, we therefore subtract one of these high-temperature traces from the raw data. Before doing this, however we first scale the hightemperature trace by an appropriate factor, to account for the temperature-dependent amplitude of the oscillations Δ :

$$\Delta(B,T) = \Delta(B,0)(\gamma T) / \sinh(\gamma T) , \qquad (2)$$

where $\gamma = 4\pi^3 k_B / h\omega_c$ and ω_c is the cyclotron frequency.¹⁵ Plotting the amplitude of a single oscillation, over the temperature range where it is not obscured by the fluctuations, we typically obtain excellent agreement with Eq. (2), by treating $\Delta(B,0)$ and γ as adjustable parameters [Fig. 4(b)]. Furthermore, since the relative change in Δ is only on the order of a few percent at the lowest temperatures, we use the resulting fit parameters to generate



FIG. 4. (a) The magnetoconductance of a quantum wire $(W_e = 1.8 \ \mu m, W = 1.22 \ \mu m)$, measured at two distinct temperatures. The aperiodic fluctuations are no longer observed above roughly 0.5 K, while the SdH oscillations remain clearly resolved. For the sake of clarity, the 560-mK trace has been shifted upwards by $10e^2/h$. (b) The temperature dependence of the SdH oscillations, observed in the wire of (a). The dependence is that of a single oscillation, whose minimum is located at roughly 0.97 T, while the normalization is performed with respect to the amplitude at 0.13 K. The analysis is not extended to temperatures lower than this, since the amplitude became obscured by the high-field fluctuations. The solid line is a two-parameter least-squares fit to the form of Eq. (1), which yields the parameter $\Delta(0)=0.997$ and $\gamma=1.6 \ K^{-1}$.

an extrapolation function, which in turn enables us to determine the scaling factor mentioned above. From Fig. 5(a) it can be seen that this approach is really very accurate.

While the technique described above is far from simple, we emphasize that it is derived from well-defined physical arguments. We are therefore confident that the background can be satisfactorily accounted for, and with respect to this issue we make the following points: first, the correlation field of the resulting fluctuations is found to increase linearly at high magnetic fields, in good agreement with the results of a recent, nonlocal study;⁶ second, the temperature dependence of the resulting fluctuations is found to be independent of magnetic field, and to be very different from that of the SdH oscillations (Sec. III O below); finally, and in order to confirm our conclusions, we also have used much lower-temperature traces as our reference background (≤ 0.1 K). The SdH oscillations are typically independent of temperature in this range



FIG. 5. Subtraction of the oscillatory background at high magnetic fields ($W_e = 1.8 \ \mu m$, $W = 1.22 \ \mu m$). (a) The line shape of the SdH oscillations at 560 (line with crosses) and 58 mK (plane line). To generate this figure, a fourth-order polynomial was subtracted from the raw magnetoconductance data [Fig. 4(a)], to leave traces centered around zero. The 560-mK trace was then scaled by a factor determined from Eq. (1), to give the line shape shown. (b) Conductance fluctuations obtained after subtracting the two traces in (a). The arrow marks the magnetic field at which the cyclotron orbit is calculated to be comparable to W.

[Fig. 4(b)], and the background subtraction can therefore be performed directly. The disadvantage of this approach, however, is that the reference trace then also contains a small, aperiodic component. Consequently, the amplitude of the resulting fluctuations is found to be somewhat smaller than that obtained by subtracting a background containing only smooth oscillations. Nonetheless, this latter approach is also found to give rise to fluctuations, whose amplitude increases with magnetic field [Fig. 5(b)], so that the conclusions we discuss below remain essentially unchanged.

C. Magnetic-field-dependent characteristics

Having removed the background from the raw data, we now consider the magnetic-field dependence of the resulting fluctuations. In particular, in previous studies it has been found beneficial to express the statistical properties of the fluctuations via the correlation function

$$F(\Delta B) = \langle [g(B) - \langle g(B) \rangle] [g(B + \Delta B) - \langle g(B) \rangle] \rangle ,$$
(3)



FIG. 6. The magnetic-field dependence of the correlation field B_c in a quantum wire of etched width $W_e = 1.8 \ \mu m$ ($W = 0.74 \ \mu m$, $T = 41 \ m$ K: open circles, and $W = 1.22 \ \mu m$, $T = 58 \ m$ K: filled circles). Each point was obtained by evaluating B_c over a fixed magnetic-field range $\Delta B = 0.05 \ T$. The arrows mark the magnetic field at which the cyclotron orbit is calculated to be comparable to W, while the solid line at high fields indicates the slope $B_c \propto B$.

where g(B) is the magnetoconductance in units of e^2h , and the angled brackets indicate an average over a suitably large field range.¹ The advantage of this approach becomes apparent when we consider the basic properties of the fluctuations. For example, the correlation field B_c represents the average period of fluctuation, and is simply defined via the half-width $F(B_c)=0.5F(0)$. Similarly, the root-mean-square amplitude of fluctuation δg is trivially related to the variance of the correlation function $\delta g = \sqrt{F}(0)$.

Applying the above analysis to our experimental results, we find that at magnetic fields where the cyclotron orbit size exceeds the wire width, B_c is essentially independent of magnetic field. Once the orbit shrinks



FIG. 7. Magnetic-field dependence of the amplitude of fluctuation δg , in a quantum wire of etched width $W_e = 1.8 \ \mu m$ $(W=0.74 \ \mu m, T=41 \ mK:$ open circles; and $W=1.22 \ \mu m$, $T=58 \ mK:$ filled circles). The vertical arrows mark the field at which the cyclotron orbit is calculated to be comparable to W, while the horizontal arrow marks the universal amplitude predicted for a quasi-one-dimensional wire. The solid lines at high magnetic fields indicate the slope $\delta g \alpha B$.



FIG. 8. Temperature dependence of the conduction fluctuations in a quantum wire, measured over two distinct magneticfield ranges ($W_e = 1.8 \ \mu m$, and $W = 1.22 \ \mu m$). For magnetic fields less than roughly 0.2 T the Landau levels are unresolved, while in the vicinity of 1 T distinct ScH oscillations are observed. The solid lines are straight-line fits to the data.

within the width, however, B_c is found to increase significantly, showing an approximately linear dependence at high magnetic fields (Fig. 6). Observation of the same linear dependence in all wires studied demonstrated the origin of this increase to be intrinsic, in agreement with previous observations in diffusive semiconductor wires.⁶ As for the amplitude analysis, the most interesting feature of this paper is the observation of a linear increase in δg , which also begins once the cyclotron orbit shrinks inside the wire width [Figs. 5(b) and 7]. The increase in amplitude was observed in all wires studied, although it was found to be most pronounced in the widest wires, in which the SdH oscillations were also clearly resolved. In particular, $\delta g(B)$ was found to significantly exceed the universal value¹ at high magnetic fields, where the highfield enhancement $[\delta g(B)/\delta g(0)]$ could correspond to as much as an order of magnitude. As for the temperature dependence of δg , as mentioned above, this was found to be independent of magnetic field, and to be very different from that of the SdH oscillations (Fig. 8).

IV. DISCUSSION

In previous studies of dirty wires, a monotonic increase in B_c was observed at high magnetic fields, while δg remained field independent.³⁻⁷ Modeling transport as a bulk-related process, it is only possible to account for this behavior, by invoking a breakdown of the scaling properties of the UCF.⁴ More recently, however, it has been shown that a strong magnetic field induces a transition to spatially inhomogeneous transport in dirty wires, as diffusive skipping orbits form near the walls. These short out the bulk diffusion to dominate interference, giving rise to a magnetically induced increase in the correlation field, as the orbits become progressively confined near the edges:⁶

$$B_{c}(B) = B_{c}(0)(1_{0}/1_{\phi})(eB\tau/m^{*}), \qquad (4)$$

where 1_0 is the mean free path, τ is the electron scattering time, and m^* is the effective mass. A surprising consequence of this approach is that it can actually be argued that the scaling properties of the UCF are preserved at high fields, even though the current flow is highly inhomogeneous. In particular, since the phase breaking length of electrons diffusing via the skipping orbits is expected to be independent of magnetic field,⁶ universal scaling then predicts a linear increase in B_c [Eq. (4)] and a field invariant δg : characteristics which are actually in agreement with the results of experiment.³⁻⁷

In our experiment, we also observe a linear increase in B_c , suggestive of a transition to edge-related transport. Given such a dependence, and considering the arguments above, we might then expect to observe a field-invariant amplitude δg . In contrast, however, we find that δg actually increases with magnetic field; it is therefore clear that, even taking into account a transition to edge-related motion, a breakdown of universal scaling must occur in our quasiballistic wires. In order to account for this, we emphasize an additional effect due to edge-related transport, namely a breakdown of correlated diffusion, which we expect to occur as well-defined edge states form in the quasiballistic wires. Such a breakdown was not considered in previous studies of dirty wires, 3^{-7} in which the strong bulk disorder present ensures that motion remains diffusive up to very high fields. In quasiballistic wires, the situation is very different, however, since diffusive motion is largely established by boundary scattering at zero magnetic field. As the magnetic field is increased, such that edge states begin to form, we might therefore expect a breakdown of the interchannel correlation (Sec. I) to occur, as diffusive motion gradually becomes suppressed.

A breakdown of the interchannel correlation should give rise to a change in the statistical properties of the UCF, whose universal amplitude can only be explained by assuming that correlation exists.¹⁶ In particular, if transmission via the various channels is uncorrelated, a nonuniversal amplitude of fluctuation results:¹

$$\delta g \propto g_{\rm av} / N$$
, (5)

where g_{av} is the average conductance, and N is the number of channels. At high magnetic fields there is a direct correspondence between the number of channels and the Landau-level index $N_L = n_s h/2eB$, so that for independent edge states we expect a linear field dependence to δg . This is precisely what we observe experimentally (Fig. 7), and we therefore suggest that the nonuniversal scaling characteristics we observe, indeed result from a breakdown of correlated diffusion at high fields. This conclusion is supported by the fact that the breakdown is

most clearly observed in the wider wires, in which the overlap between oppositely propagating edge states is expected to be strongly reduced [Figs. 1(a)-1(c)].

On the basis of the above discussion, we suggest that our experiments probe the intermediate magnetic-field regime, where the edge states begin to be resolved, but where there is still weak scattering between them.¹⁷⁻¹⁹ In support of this we note that, while we were ultimately able to observe well-defined SdH oscillations, we did not have sufficient magnetic-field range to access the quantum Hall limit. In this intermediate regime, we imagine that the weak scattering is insufficient to establish a strong interchannel correlation, so that nonuniversal fluctuations result. Increasing the magnetic field from zero, we therefore envisage three distinct regimes of transport. Close to zero field the motion is diffusive, resulting in a quantum-mechanical correlation between the different channels, and the observation of UCF. At intermediate magnetic fields we have the regime of nonuniversal fluctuations, as the interchannel correlation breaks down. Finally, as the magnetic field is further increased, the edge states ultimately propagate without any scattering, and in this regime the fluctuations are expected to quench completely.²⁰

V. CONCLUSIONS

We have studied the UCF observed in the low-temperature magnetoconductance of high-mobility GaAs/Al_xGa_{1-x}As quantum wires. As the magnetic field is increased, such that the cyclotron orbit becomes smaller than the wire width, both the amplitude and correlation field of the fluctuations are found to increase, as linear functions of the magnetic field. The linear increase in B_c is in agreement with the results of recent nonlocal studies of the UCF, and is thought to demonstrate a transition to edge-related transport. Similarly, the increase in δg is thought to result from a magnetically induced breakdown of correlated diffusion, which occurs as the edge states begin to be resolved. Such a breakdown was not observed in previous studies of dirty wires, in which the electron motion remains diffusive up to very high fields, but is instead thought to result from the quasiballistic nature of the wires we study. With regards to this possibility, the field dependence of δg was shown to be in quantitative agreement with theoretical predictions for fluctuations due to uncorrelated channels. In future studies, it would be of interest to confirm the phenomena presented here, using nonlocal measurements of similar quasiballistic wires, in which the absence of a SdH-related contribution would simplify the analysis.

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