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Two-dimensional magnetic polarons: Anisotropic spin structure of the ground state and magneto-optical properties

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Magnetic polarons formed by holes in semimagnetic quantum wells are studied theoretically. The theory is based on the formalism of the pseudospin and the anisotropic g-factor tensor of the hole. It yields the direction of the total moment and energy of the polaron as functions of external magnetic fields. Polarization characteristics of light- and heavy-exciton luminescence are shown to be strongly influenced by the magnetic polaron formation.

I. INTRODUCTION

The exchange interaction of charge carriers with spins of magnetic ions results in a variety of physical phenomena in diluted-magnetic (semimagnetic) semiconductors (DMS's).[1] Those most widely recognized are associated with pronounced spin-dependent shifts of band edges with a magnetic field applied. This so-called giant Zeeman splitting stems from the strong exchange field, acting on carriers, which is produced by polarized local spins. This feature of DMS's became particularly valuable after DMS heterostructures had been grown, since it made possible quantum tailoring of confined electron and hole states just by tuning the magnetic field or temperature. Methods of optical spectroscopy, widely practiced in studies of conventional semiconductors, have been successfully applied to the resulting structures. By this means a substantial dependence has been revealed of the Zeeman pattern of free excitons in DMS quantum wells on the magnetic-field direction.[2,3] The anisotropy results from strong spin-orbit interaction in the valence band. At the conditions of size quantization this interaction splits hole states into two subbands with spin projections on the growth axis equal to $\pm\frac{1}{2}$ (light holes) and $\pm\frac{3}{2}$ (heavy holes). Both subbands demonstrate an anisotropic response to the exchange field, due to unequal spin components in the quantum-well plane and normal to it. The theoretical model, accounting for subband mixing and exciton spin rearrangement in the exchange field,[4,5] has provided a good fit to the experimental data.[2,3]

Another class of exchange phenomena widely studied in bulk and low dimensional DMS's is due to a counterinfluence of localized carriers and excitons on spins of magnetic ions in their vicinity. The correlation of ion and carrier spins leads to the appearance of clouds of polarized ion spins usually referred to as magnetic polarons.[6] Magnetic polarons (MP's) in bulk semimagnetic crystals have been studied experimentally by Raman-scattering spectra,[7] and by photoluminescence due to recombination of impurity-bound carriers[8,9] and excitons trapped at alloy fluctuations.[10-13] Theoretical models of three-dimensional MP's in magnetic and semimagnetic semiconductors with various types of interactions involved[14-19] were built during the last three decades after the pioneering work by De Gennes.[14]

Exciton two-dimensional magnetic polarons in DMS-based quantum wells have been detected by the photoluminescence Stokes shift.[20] The structure symmetry has manifested itself in a significant difference of characteristic magnetic fields required to suppress the polaron shift in Faraday and Voigt geometries.[21] This result has been explained in the framework of the simplest model, assuming the components of the heavy-hole spin in the quantum-well plane to be equal to zero.[22] Although the model gives a good approximation of the MP energy in moderate magnetic fields, it is evidently of limited utility. For instance, it cannot describe the MP behavior in strong magnetic fields, when mixing of the states in the heavy-hole Kramers doublet becomes essential. Also, it does not allow an analysis of selection rules for optical transitions, which are expected to change as a result of this mixing. In addition, manufacturing of strained structures, with the light subband being the deepest,[23] has made topical a theoretical treatment of the light magnetic polaron. The present work is aimed at studying these problems.

In Sec. II the simplest model of the ground state of the two-dimensional (2D) magnetic polaron formed by magnetic ion spins and a hole with anisotropic g factor is developed. Section III is devoted to an analysis of polarization, and the probability of optical transitions in an external magnetic field. In Sec. III some ways to compare the results with experimental data are proposed.

II. A MODEL OF THE GROUND STATE OF THE LOCALIZED MAGNETIC POLARON FORMED BY A 2D HOLE

To find out the Stokes shift and polarization of luminescence from DMS quantum wells as functions of magnetic field, it is necessary to know the structure of magnetopolaron states. In this section a theory of these states is developed. In Sec. II A the general form of the Hamiltonian is obtained, which describes the exchange interaction of two-dimensional electrons and holes with magnetic ions. It is shown that although in bulk cubic

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DMS's a hole has a spin $J=\frac{3}{2}$, it is natural to ascribe a pseudospin $j=\frac{1}{2}$ to the hole states in a quantum well (QW). This pseudospin is subjected to an exchange field $\mathcal{B}$ which results from the polarization of magnetic ions, but does not retrace the direction of their average spin. In Sec. II B the simplest model of a magnetic polaron is considered, where the hole is assumed to interact with the total spin $\mathbf{I}$ of all the ions inside its orbit. Within this model the general expression for the MP energy is found, as well as the relation between $\mathbf{I}$ and an external magnetic field $\mathbf{B}$. These expressions are legitimate for all types of $\mathcal{B}(\mathbf{I})$ dependence. In Sec. II C a linear approximation is introduced for the function $\mathcal{B}(\mathbf{I})$ by use of the anisotropic hole g factor. The model allows one to identify all characteristic features of the dependences of the polaron energy and spin on magnetic field. The symmetry of the MP spin state appears to change at some value of the external magnetic field. Finally, in Sec. II D this approach is refined for the heavy-hole subband, where some components of the g-factor tensor are equal to zero. It is shown that in this case the basic results of Sec. II C remain valid, if small nonlinear components of $\mathcal{B}(\mathbf{I})$ are taken into account. The threshold value of the external magnetic field in which the symmetry of the MP spin states changes is estimated.

A. Exchange Hamiltonian for 2D carriers

In bulk diamondlike DMS's the exchange interaction of charge carriers with magnetic ions is usually described by the Hamiltonian

$$\hat{H}_{\rm exc}=a\sum_n(\mathbf{J}\cdot\mathbf{I}_n)\delta(\mathbf{r}-\mathbf{r}_n)\ , \tag{1}$$

where a is an exchange integral, $\mathbf{I}_n$ is the spin operator of a magnetic ion with the position vector $\mathbf{r}_n$, and $\mathbf{J}$ and $\mathbf{r}$ are the spin operator and position vector of the carrier. The value of J equals $J_e=\frac{1}{2}$ for electrons and $J_h=\frac{3}{2}$ for holes, and zone-center states are twofold and fourfold degenerate, respectively. For definiteness sake, we consider most widely used compounds which incorporate ions of manganese (Mn^{2+}).

In quantum wells the states of electrons and holes throughout the 2D bands are twofold degenerate. Hence both electrons and holes can be considered as quasiparticles with spin $\frac{1}{2}$ (for holes the term pseudospin $j=\frac{1}{2}$ will be used below). The exchange Hamiltonian of a 2D electron keeps the form given by Eq. (1), whereas one can expect that the strong spin-orbit interaction in the valence band results in a more complex Hamiltonian for a hole. This Hamiltonian, being a scalar function of the Pauli matrices, has the following general form:

$$\hat{H}^h_{\rm exc}=H_{VV}(\mathbf{I}_1\ldots\mathbf{I}_n)-(\mathcal{B}(\mathbf{I}_1\ldots\mathbf{I}_n)\cdot\mathbf{j})\ . \tag{2}$$

Here H_{VV} is the part of the Hamiltonian which depends on the magnetic-ion spin state only, and leads to an exchange-induced energy shift of both degenerate hole levels, bearing some resemblance to the Van Vleck paramagnetism.[24] The estimated contribution of this shift to the magnetic polaron energy is usually small; hereafter we will omit this term. The exchange field $\mathcal{B}$, being responsible for the splitting of the two hole states and for the rearrangement of the two-component hole wave function, plays the main role in magnetic polaron formation. It will be shown below that the anisotropy of $\mathcal{B}$ results in a behavior of the 2D hole magnetic polaron ground state which is dramatically different from anything known for isotropic polarons in bulk cubic DMS's. It should be noted that Hamiltonian (2) is also valid for wurzitelike bulk DMS's, where the valence-band structure is similar. Therefore all results obtained below can be also applied to these crystals.

B. General consideration

Following the procedure suggested in Ref. 18, we consider a hole interacting with the total spin $\mathbf{I}=\sum_n\mathbf{I}_n$ of all ions inside the hole orbit. This corresponds to the formal assumption that the hole wave function Ψ is constant within the localization region ($\Psi=1/\sqrt{V}$, where V is the localization volume), and equal to zero outside it. Then, assuming $I\gg1$, we neglect the quantum uncertainty of its components. The spin Hamiltonian of the magnetic polaron in the external magnetic field $\mathbf{B}$ can thus be written as follows:

$$\hat{H}=g_{\rm Mn}\mu_B(\mathbf{B}\cdot\mathbf{I})-(\mathcal{B}(\mathbf{I})\cdot\mathbf{j}) \tag{3}$$

where $g_{\rm Mn}$ is the g factor of Mn^{2+} and μ_B is the Bohr magneton. In Eq. (3) the magnetic energy of the hole ($\mu_Bg_h\mathbf{B}\cdot\mathbf{J}$) is omitted, as it is small as $J/I\ll1$.

States of the polaron given by Eq. (3) are characterized by the value and direction of $\mathbf{I}$, and by the projection of the pseudospin (m) on the exchange field $\mathcal{B}(\mathbf{I})$. The energy of the ground state should be minimal, and therefore in the ground state $m=+\frac{1}{2}$. Thus the ground-state energy is

$$\varepsilon=g_{\rm Mn}\mu_B(\mathbf{B}\cdot\mathbf{I})-\tfrac{1}{2}|\mathcal{B}(\mathbf{I})|\ . \tag{4}$$

The field $\mathcal{B}(\mathbf{I})$ usually increases with I, so that the minimum of ε is achieved at the maximal value of the total ion spin $I=I_0$. Therefore, the polaron ground state can be found from the system of equations

$$\begin{aligned}0&=\frac{\partial\varepsilon}{\partial I_\alpha}=g_{\rm Mn}\mu_BB_\alpha-\frac{1}{2}\frac{\partial\mathcal{B}_\beta}{\partial I_\alpha}\frac{\mathcal{B}_\beta}{|\mathcal{B}|}+\lambda\frac{I_\alpha}{I}\ ,\\ I&=I_0\end{aligned} \tag{5}$$

where λ is a Lagrange multiplier. As can be readily seen, λ gives the strength of an effective field which is a sum of the external field B_α and the exchange field $-(1/\mu_Bg_{\rm Mn})(\partial\mathcal{B}_\beta/\partial I_\alpha)\langle j_\beta\rangle$ produced by the hole. The physical meaning of Eqs. (5) is thus quite clear: it follows from this relation that the total ion magnetic moment $\mathbf{M}=-\mu g\mathbf{I}$ in the ground state is directed along this total field. The system (5) permits one to find the connection between the ion spin $\mathbf{I}$ and the external magnetic field easily:

$$\mathbf{B}=\mathbf{B}_t(\mathbf{I})+\mathbf{B}_l\ , \tag{6}$$

where

$$\mathbf{B}_t(\mathbf{I}) = \frac{1}{2\mu_B g_{\mathrm{Mn}}} \frac{[[\nabla_I |\mathcal{B}(\mathbf{I})| \times \mathbf{I}] \times \mathbf{I}]}{I^2} \tag{7}$$

is perpendicular to **I**; the component of **B** along **I**, B_l, may be of an arbitrary value. Therewith the polaron energy takes the form

$$\varepsilon = g_{\mathrm{Mn}} \mu_B B_l I - \tfrac{1}{2} |\mathcal{B}(\mathbf{I})| \ . \tag{8}$$

From symmetry considerations one can conclude that B_t vanishes if **I** is normal to the quantum-well plane or lies in this plane (the latter is strictly true if account is not taken of the crystal anisotropy). Solid lines in Fig. 1 give a schematic sketch of the B_t dependence on the direction of **I**. It is worthwhile to note that B equals $[B_t^2 + B_l^2]^{1/2}$, and at given **I** is not less than $B_t(\mathbf{I})$. Regions corresponding to forbidden combinations of **I** and **B** are shown by dashed lines in Fig. 1.

C. Polaron states in a magnetic field

To find out the polaron ground state along Eqs. (7) and (8), it is necessary to know the explicit expression for the field $\mathcal{B}(\mathbf{I})$. In the simplest and most demonstrative case this field is related to **I** by a linear dependence

$$\mathcal{B}(\mathbf{I}) = \frac{a}{V} \hat{g} \mathbf{I} \ , \tag{9}$$

where $\hat{g}$ is the tensor of hole g factor, which connects, neglecting Van Vleck paramagnetism, the hole spin **J** with its pseudospin **j**:

$$\hat{J}_\alpha = g_{\alpha\beta} j_\beta \ . \tag{10}$$

If hole localization does not lead to an additional symmetry reduction, the tensor $\hat{g}$ is diagonal, with $g_{xx} = g_{yy} = g_\perp \neq g_{zz}$ (the axis Z is chosen normal to the quantum-well plane). In this case the vectors **B** and **I** and

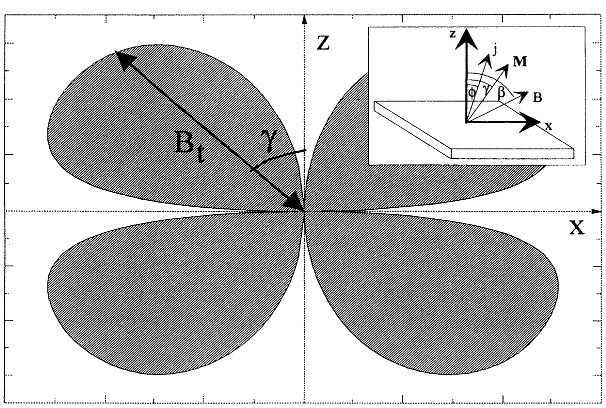

FIG. 1. Polar plot of the forbidden values of the external magnetic field vs the direction of **I** (dashed). Solid lines denote the value of transversal component of the magnetic field, B_t, as a function of the angle γ between $\mathbf{M} = -\mu g \mathbf{I}$ and Z. Calculations are made for the case of the heavy hole. The inset shows a sketch of vectors $\langle \mathbf{j} \rangle$, **M**, and **B** in respect to the structure axis Z.

the Z axis evidently lie in one plane. Taking into account this feature, one can find the relation between angles of **B** and the ion magnetic moment $\mathbf{M} = -\mu g \mathbf{I}$ to Z (β and γ respectively):

$$\sin(\beta - \gamma) = \frac{1}{2\mu_B g_{\mathrm{Mn}} B} \frac{a}{V} \frac{(g_{zz}^2 - g_\perp^2) \sin\gamma \cos\gamma}{\sqrt{g_{zz}^2 \cos^2\gamma + g_\perp^2 \sin^2\gamma}} \tag{11}$$

and the expression for the polaron energy in terms of these angles:

$$\varepsilon = -\mu_B g_{\mathrm{Mn}} B I \cos(\beta - \gamma) - \frac{1}{2} \frac{a}{V} \sqrt{g_{zz}^2 \cos^2\gamma + g_\perp^2 \sin^2\gamma} \ . \tag{12}$$

It is seen from Eqs. (11) and (12) that the case of $g_{zz} < g_\perp$ can be brought to its opposite ($g_{zz} > g_\perp$) by adding $\pi/2$ to both angles and simultaneously interchanging g_{zz} and $g_\perp$. Taking that into account we restrict consideration to the case of $g_{zz} > g_\perp$, which is realized for heavy holes in quantum wells. An analysis of Eq. (11) yields four ranges of the external field B, where the dependences $\gamma(\beta)$ are qualitatively distinct (see Fig. 2). These ranges are separated by critical fields B_1, B_2, and B_3:

$$B_1 = \frac{a}{2\mu_B g_{\mathrm{Mn}} V} (g_{zz} - g_\perp) \ , \tag{13}$$

$$B_2 = \frac{a}{2\mu_B g_{\mathrm{Mn}} V} \frac{g_{zz}^2 - g_\perp^2}{g_{zz}} \ , \tag{14}$$

$$B_3 = \frac{a}{2\mu_B g_{\mathrm{Mn}} V} \frac{g_{zz}^2 - g_\perp^2}{g_\perp} \ . \tag{15}$$

At low fields [$B < B_1$, Fig. 2(a)] the right-hand side of Eq. (11) appears to be greater than 1 at some values of γ. Therefore, corresponding directions of **I** cannot be realized. These are precisely the combinations of B and **I** which fall into the dashed regions in Fig. 1. For each direction of the magnetic field there are four solutions of Eq. (11) which are stationary states with different energies. In the deepest state the deflection of **I** from the z axis is minimal, as well as the difference between the angles β and γ.

At $B > B_1$ [Fig. 2(b)] intervals of β with only two polaron states appear. As the magnetic field overcomes B_2 [Fig. 2(c)], the dependence $\gamma(\beta)$ suffers additional qualitative changes. However, $\gamma(\beta)$ for the polaron ground state still has a discontinuity at $\beta = \pi/2$. It is not until B exceeds B_3 [Fig. 2(d)] that this dependence becomes a smooth function like one for a polaron formed by a carrier with isotropic g factor. Nevertheless, the difference between γ and β at any $\beta \neq n\pi/2$ vanishes only with $B \to \infty$.

The most spectacular feature of the anisotropic MP, given by the above analysis, is the coexistence of several degenerate states in the case when the magnetic field $B < B_3$ is applied along the direction corresponding to the minimal g-factor component [$\beta = \pi/2$ in Figs. 2(a)–2(c)]. It is worth noting that in this geometry a qualitative difference appears between the cases of $g_{zz} > g_\perp$ and $g_{zz} < g_\perp$. In the first case the ground state is

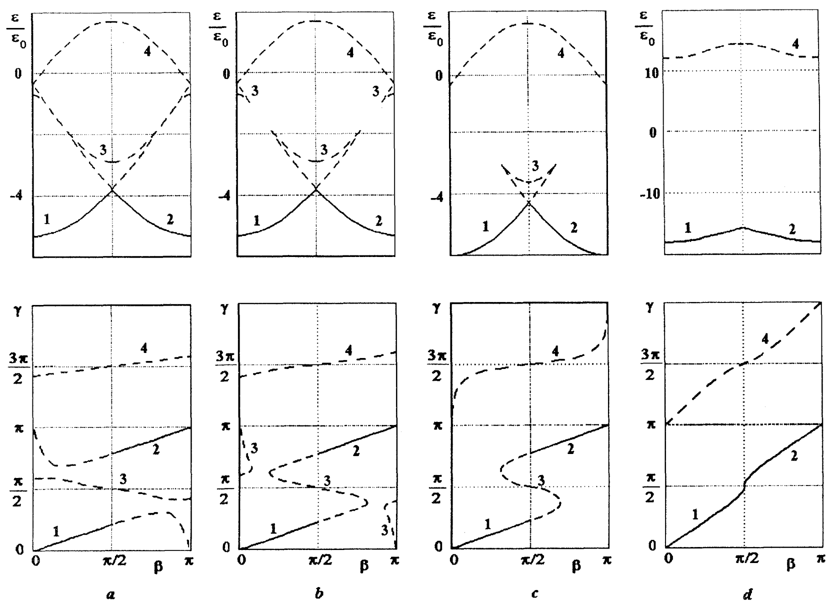

FIG. 2. The solutions of Eqs. (11) and (12): the direction of the ion total spin (angle γ, lower plots) and energy (upper plots) vs the direction of external magnetic field (angle β); $a - B < B_1$, $b - B_1 < B < B_2$, $c - B_2 < B < B_3$, and $d - B > B_3$. Different branches of solutions are numerated. The states with the minimal energy (ground states) are shown by the solid line. The curves are calculated for $g_\perp / g_{zz} = 0.2$, and magnetic fields were taken equal to (a) $2.3B_0$, (b) $2.45B_0$, (c) $3B_0$, and (d) $15B_0$, where $B_0 = (a/2\mu_B g_{\rm Mn} V)$. The characteristic exchange energy $\varepsilon_0 = aI/V$.

twofold degenerate (the magnetic-ion total spin lies in the plane specified by the structure axis and the external field). If $g_{zz} < g_\perp$ and B is applied along Z, the vector $\mathbf{I}$ can lie in any plane which contains this axis, the ground state thus being infinitely degenerate. When the magnetic-field strength reaches B_3, the degenerate states merge to one state with the polaron magnetic moment directed along the external field. The appearance of components normal to $\mathbf{B}$ of this moment at low fields ($B < B_3$) can be considered a spontaneous symmetry break resulting from the nonlinearity of the tight-bound spin system of the polaron.

In quantum wells the case of $g_{zz} < g_\perp$ is realized for hole states near the bottom of the light subband, where the z projection of the real spin J is equal to $\pm\frac{1}{2}$. It is easy to find that in these states $g_\perp = 2g_{zz} = 2$. All characteristic fields B_1, B_2, and B_3 therefore appear to be of the same order of magnitude, $B_n \approx (1/2\mu_B g_{\rm Mn})(a/V)$.

D. Heavy-hole polarons

In the states at the bottom of the heavy subband ($J_z = \pm\frac{3}{2}$) at a zero approximation $g_\perp = 0$, $g_{zz} = 3$. This gives an infinitely high value for the field B_3 at which the twofold degeneration of the ground state vanishes. To refine this value we complement the expression for the field $\mathcal{B}$ by terms cubic in components of $\mathbf{I}$, which are small comparative to the first linear term:

$$\mathcal{B} = g_{zz} I_z \mathbf{e}_z + G_\perp I_\perp^3 \mathbf{e}_x \ , \tag{16}$$

where $G_\perp$ is determined by the ratio of the characteristic exchange energy a/V and the splitting of light- and heavy-hole subbands. For its relation to experimentally available quantities, see Sec. IV below. In Eq. (16), for definiteness' sake, we consider the external field and the spin $\mathbf{I}$ to lie in the plane (ZX); $\mathbf{e}_z$ and $\mathbf{e}_x$ are orts along the respective axes.

By substitution of this expression into Eq. (7), we find the modified relation between β and γ:

$$\sin(\beta-\gamma) = \frac{1}{2\mu_B g_{\rm Mn} B} \frac{a}{V} \times \frac{(g_{zz}^2 - 3G_\perp^2 I^4 \sin^4\gamma)\sin\gamma\cos\gamma}{\sqrt{g_{zz}^2\cos^2\gamma + G_\perp^2 I^4 \sin^6\gamma}} \ . \tag{17}$$

Since $G_\perp I^2 \ll g_{zz}$, the term containing $G_\perp$ in the numerator can be omitted. The corresponding term in the denominator appears to be significant only in the narrow range of angles $|\pi/2-\gamma| \ll 1$, where $\cos^2\gamma \ll 1$ and $\sin^6\gamma \approx 1$. Here Eq. (17) coincides with Eq. (11) if $g_\perp$ is replaced by $G_\perp I^2$. An estimation for the parameter B_3 can thus be found as:

$$B_3 \approx \frac{a}{2\mu_B g_{\rm Mn} V} \frac{g_{zz}^2}{G_\perp I^2} \ . \tag{18}$$

Therefore, the theory developed gives the desired dependences of the direction of the total ion moment $\mathbf{I}$ and of the MP ground-state energy on the strength and direction of an external magnetic field, for heavy as well as light holes. Recall that the direction of the hole pseudospin, which is another important characteristic of the polaron state, is connected with $\mathbf{I}$ by the simple formula

$$\langle \mathbf{j} \rangle = \frac{1}{2} \frac{\mathcal{B}(\mathbf{I})}{|\mathcal{B}(\mathbf{I})|} \ . \tag{19}$$

Since the vector $\mathbf{M} = -\mu g \mathbf{I}$ lies between $\mathcal{B}$ and $\mathbf{B}$, in the case of strong anisotropy the hole pseudospin is deflected from the direction of the external field more strongly than the total magnetic moment of the magnetic ions. Relation (19) will be used below in a description of magnetopolaron luminescence.

III. INTENSITY AND POLARIZATION OF LUMINESCENCE DUE TO RECOMBINATION OF ELECTRON-HOLE PAIRS IN THE GROUND MAGNETOPOLARON STATE

Photoluminescence studies have made feasible extensive information on two-dimensional magnetic polarons formed from localized excitons.[21] The anisotropy of the 2D hole spin state viewed above is expected to reveal itself in the polarization characteristics of the exciton magnetic polaron luminescence, as well as in its intensity dependence on a magnetic field. In this section, we derive expressions for these dependences. In Sec. III A the probabilities of optical transitions are obtained as functions of the direction of the ion total spin **I** and the external magnetic field **B**. In Sec. III B the peculiarities in luminescence intensity and polarization in an external magnetic field, resulting from the anisotropic structure of the magnetic polaron, are discussed.

A. Probabilities of optical transitions

To treat the optical manifestations of the MP anisotropy, proper allowance must be made for the exchange interaction of the electron with magnetic ions which generally can complicate the entire MP structure. Here we consider two limiting cases defined by the value of the electron localization radius R_e.

If R_e in the localized exciton state is less than or approximately equal to the radius of the hole wave function R_h, then in the context of our model the electron spin **S** can be regarded as interacting with the total ion moment **I** in the hole polaron dealt with in Sec. III. Since the electron g factor is isotropic, the ground state of the exciton magnetic polaron corresponds to $\mathbf{S}\|\mathbf{I}$. On that account, the exchange interaction with the electron does not change the direction of **I**, and, consequently, the expressions for the angle γ remain valid. The electron exchange energy $-\frac{1}{2}(a_e/V)I$ (a_e is the conduction-band exchange constant) must therewith be added to the polaron energy.

In the opposite limiting case of $R_e \gg R_h$, the electron spin is oriented along the average magnetization of the magnetic ions outside the hole orbit, which follows the direction of the external field given by the angle β. The electron in this case does not contribute to the polaron energy, because the electron density at each ion is negligible.

To obtain the matrix elements of optical transitions, one needs expressions for spin parts of wave functions of both electron (ψ_e) and hole (ψ_h). Since the magnetic field **B** affects the carriers much more weakly than their exchange interaction with the Mn^{2+} spins, these functions are determined by the directions of average magnetization of ions in the polaron and outside it, which are given by the angles γ and β from Eqs. (11) and (17). For this reason we begin by finding the dependences of polarization and intensity of optical transitions on γ and β, to apply them later on to the MP ground state.

The electron wave function can be found by the use of finite rotation matrices for the spin $\frac{1}{2}$:

$$\psi_e = |+1/2\rangle \cos\frac{\varphi_e}{2} + |-1/2\rangle \sin\frac{\varphi_e}{2} , \tag{20}$$

where $|+1/2\rangle$ and $|-1/2\rangle$ are spin functions related to projections of S on z equal to $+\frac{1}{2}$ or $-\frac{1}{2}$, respectively, and φ_e is equal to $\gamma+\pi$ if $R_e \lesssim R_h$, and to $\beta+\pi$ if $R_e \gg R_h$. Accepting here the representation of a 2D hole as a particle with the pseudospin $j=\frac{1}{2}$ allows one to write the spin function of the hole in the same manner as for the electron:

$$\psi_h = |+1/2,h\rangle \cos\frac{\varphi_h}{2} + |-1/2,h\rangle \sin\frac{\varphi_h}{2} , \tag{21}$$

where $|\pm 1/2,h\rangle$ corresponds to $j_z=\pm\frac{1}{2}$, φ_h being the angle of $\langle \mathbf{j}\rangle$ to Z which generally does not coincide with $\gamma+\pi$. It is easy to deduce from Eq. (19) that $\tan\varphi_h = \mathcal{B}_\perp/\mathcal{B}_z$, which for light holes gives

$$\tan\varphi_{\mathrm{lh}} = \frac{g_\perp}{g_{zz}} \tan\gamma , \tag{22}$$

and, for heavy holes,

$$\tan\varphi_{\mathrm{hh}} = \frac{G_\perp I^2}{g_{zz}} \sin^2\gamma \tan\gamma . \tag{23}$$

As it was already mentioned, $j_z=\pm\frac{1}{2}$ relates to $J_z=\pm\frac{1}{2}$ for light holes and to $J_z=\pm\frac{3}{2}$ for heavy holes. By the use of well-known selection rules for the transitions between these states and the state of the conduction band[25] combined with Eqs. (20) and (21) we obtain the matrix elements for recombination of heavy holes ($\mathbf{d}_{\mathrm{hh}}$) and light holes ($\mathbf{d}_{\mathrm{lh}}$) with electrons:

$$\mathbf{d}_{\mathrm{hh}}(\varphi_h,\varphi_e) \propto -\mathbf{e}_x \cos\left[\frac{\varphi_e+\varphi_h}{2}\right] - i\mathbf{e}_y \cos\left[\frac{\varphi_e-\varphi_h}{2}\right] , \tag{24}$$

$$\mathbf{d}_{\mathrm{lh}}(\varphi_h,\varphi_e) \propto \frac{2}{\sqrt{3}}\mathbf{e}_z \cos\left[\frac{\varphi_e-\varphi_h}{2}\right] - \frac{1}{\sqrt{3}}\mathbf{e}_x \sin\left[\frac{\varphi_e-\varphi_h}{2}\right] - \frac{1}{\sqrt{3}} i\mathbf{e}_y \sin\left[\frac{\varphi_e+\varphi_h}{2}\right] . \tag{25}$$

Here $\mathbf{e}_x$, $\mathbf{e}_y$, and $\mathbf{e}_z$ are orts along X, Y, and Z, respectively. The frame is chosen so that the external magnetic field lies in the plane ZX.

In experiments with quantum wells the radiation, which propagates along the structure axis Z, is usually studied. Equations (24) and (25) result in the following expressions for the intensity $\mathcal{J}$ and degrees of circular (ρ_c) and linear (ρ_l) polarization in this case:

$$
\begin{aligned}
&\mathcal{J}_{\rm hh}(\varphi_h,\varphi_e)\propto 1+\cos\varphi_e\cos\varphi_h\ ,\\
&\mathcal{J}_{\rm lh}(\varphi_h,\varphi_e)\propto \frac{1-\cos\varphi_e\cos\varphi_h}{3}\ ,\\
&\rho_l^{\rm hh}(\varphi_h,\varphi_e)=-\frac{\sin\varphi_e\sin\varphi_h}{1+\cos\varphi_e\cos\varphi_h}\ ,\\
&\rho_c^{\rm hh}(\varphi_h,\varphi_e)=\frac{\cos\varphi_h+\cos\varphi_e}{1+\cos\varphi_e\cos\varphi_h}\ ,\\
&\rho_l^{\rm lh}(\varphi_h,\varphi_e)=\frac{\sin\varphi_e\sin\varphi_h}{1-\cos\varphi_e\cos\varphi_h}\ ,\\
&\rho_c^{\rm lh}(\varphi_h,\varphi_e)=\frac{\cos\varphi_h-\cos\varphi_e}{1-\cos\varphi_e\cos\varphi_h}\ .
\end{aligned}
\tag{26}
$$

Here positive signs of ρ_c and ρ_l correspond to the right-hand circular polarization and the electric vector of light parallel to X, respectively.

B. Luminescence of anisotropic magnetic polarons

Relations (26) can immediately be used to describe experiments if by some reason magnetopolaron effects are absent, so that the ion magnetic moment is parallel to **B** both inside the hole orbit and outside it ($\gamma+\pi=\varphi_e=\beta+\pi$). For instance, this may be realized with free carriers, high temperature, or strong magnetic field ($B\gg B_3$). Keeping in mind the dependences $\varphi_h(\gamma)$ [Eqs. (22) and (23)], one can conclude that both heavy and light excitons demonstrate pronounced peculiarities in luminescence when **B** is close to the direction which corresponds to the minimal hole g-factor component. At $|(\pi/2)-\beta|\le(g_\perp/g_{zz})$ the linear polarization of the heavy-exciton luminescence rises sharply with a simultaneous decrease of the circular polarization [Fig. 3(a)]. For light excitons, near $\beta=0$ $\rho_c\approx-0.6$ is insensitive to small changes of the direction of **B** [see Fig. 3(b)]. In the opposite situation the plane of linear polarization is determined by small normal to Z components of **B**, $|\rho_l|$ being approximately equal to 0.8. The light-exciton luminescence intensity vanishes with $\beta\to n\pi$.

If a MP is formed, a theoretical description of experimental data on photoluminescence polarization and intensity requires an account for the fact that the direction of **I** is governed not only by the external field, but also by the hole anisotropy. The appropriate analysis for the MP ground state can be done in the framework of the model developed in the Sec. II.

In the polaron formed by a hole from the heavy subband, as was already mentioned, on the magnetic field being tilted to Z, the ion magnetic moment lags behind it. At strong fields ($B>B_3$) this just leads to a more pronounced nonlinearity of ρ_l and ρ_c as functions of β. At moderate fields ($B<B_3$) the situation changes qualitatively. The point is that under this condition even in the Voigt geometry ($\beta=\pi/2$), the polaron moment is not aligned in the quantum-well plane. Therefore, the luminescence polarization does not reach 100%. The circular polarization at $\beta=\pi/2$ is nevertheless expected to be zero due to averaging over the two degenerate ground states with opposing projections of **I** on $Z(\gamma_1=\pi-\gamma_2)$.

For a hole belonging to the light subband, optical transitions from the ground state with emitted photons along Z are forbidden at high fields ($B>B_3$). The polaron formation at $B<B_3$ allows the transition, with the luminescence circularly polarized. The polarization degree has the opposite sign as compared to heavy-exciton luminescence. Its value reaches $0.6(R_e\le R_h)$ or $1(R_e\gg R_h)$ at $B\approx B_3$, and vanishes linearly with decrease of B at $B\to 0$: $\rho_c\approx-2\mu_B g_{\rm Mn}VB/3a(R_e\le R_h)$. If $R_e\gg R_h$, ρ_c keeps close to -1, while the magnetic field is strong enough to saturate the orientation of the electron spin.

The qualitative peculiarities listed exemplify the possibility of detecting magnetic polarons in quantum wells not only by Stokes shift, but also by the dependences of photoluminescence polarization and intensity on the direction and strength of applied magnetic fields.

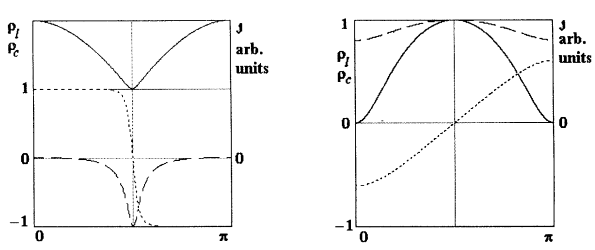

FIG. 3. Degrees of linear (dashed) and circular (dotted) polarizations and the intensity of optical transitions (solid), vs the direction of the ion total moment: (a) conduction-band–heavy-hole subband ($G_\perp I^2/g_{zz}=0.1$); (b) conduction-band–light-hole subband.

IV. DISCUSSION

It is worthwhile to compare the properties of two- and three-dimensional magnetic polarons with respect to their possible optical manifestations.

A. Qualitative considerations

In optical experiments MP's are usually detected by a photoluminescence Stokes shift which is suppressed by applied magnetic fields and decreases with temperature elevation. Suppression of a 3D MP in magnetic fields is due to saturation of the magnetic-ion polarization, which results in the ion spin density being leveled off inside and outside the carrier (exciton) orbit. The characteristic value of the field saturating the ion magnetization is determined by the lattice temperature T and the effective temperature T_0 of the ion spin system: $B_s\approx k(T+T_0)/\mu_B g_{\rm Mn}$ (T_0 allows for spin-spin interaction among ions[1]). In the demonstrative ultimate case of $T=T_0=0$ the 3D polaron is completely suppressed by a field as weak as is wished. The 2D hole polaron under these conditions behaves quite differently. The quantity which gives the Stokes shift in optics, i.e., the difference between the hole exchange energies in the MP ground state and with no polaron formed,

$$\Delta\varepsilon = \frac{1}{2}\frac{Ia}{V}\left(\sqrt{g_{zz}^2\cos^2\gamma + g_\perp^2\sin^2\gamma} - \sqrt{g_{zz}^2\cos^2\beta + g_\perp^2\sin^2\beta}\right) \tag{27}$$

does not vanish except when **B** is parallel to one of the g-tensor axes.[26] Provided this is realized, the field in addition must not be less than B_3, unless it is oriented along an axis with the maximal g-factor component. It is in the latter case only when the 2D polaron can be suppressed by an infinitely weak field B, like the 3D one. This result is not quite unexpected, since a similar situation is met with in magnetic polarons (ferrons) in antiferromagnets,[16] or in Van Vleck paramagnets,[19] where the polaron is formed due to deflection of magnetic-ion spins from the direction prescribed by some built-in effective field, instead of the external field in our case.

The strong anisotropy of field suppression of 2D magnetic polarons was well documented experimentally.[21,22] It was also described theoretically[22] assuming the profound anisotropy ($g_\perp = 0$) of the hole g factor. As is noted above, this is a good zero approximation for heavy holes considered in Ref. 22, valid at moderate fields $B \ll B_3$, where B_3 can be found with Eq. (18).

No reliable experimental evidence of the light-hole polaron has been reported so far, since the heavy-hole subband is the deepest in the overwhelming majority of available quantum-well structures. Hopes for finding it are pinned on ZnSe-based strained quantum-well structures,[23,27] where the ground state of a free hole is one with $J_Z = \pm\frac{1}{2}$. The analysis done in the present work shows how essential the g-factor anisotropy is for understanding the physics of this object: the hole moment is not quantized along the structure axis even without an external magnetic field. This allows a transition which is forbidden without the magnetopolaron effect, thus providing the possibility to observe this kind of polaron in photoluminescence spectra.

B. Relations between polarization and Stokes shift

The peculiarities related to magnetic polarons in the magnetic-field dependences of luminescence intensity and polarization, described in Sec. III, suggest some additional ways to obtain experimental insight into the structure of 2D magnetopolaron states (for reference, in the 3D case the luminescence intensity and polarization are virtually insensitive to MP formation). At the end of this section we will show that within the model developed above it is possible to construct an expression which links a set of parameters of the heavy-hole MP, available in optical experiments: exchange-induced splittings of free heavy-hole levels in the Voigt (ε_V) and Faraday (ε_F) geometries, the polaron shift of the PL line ($\Delta\varepsilon$), and the PL linear polarization in the Voigt geometry.

First we note that ε_F and ε_V are related to parameters of the model by the following transparent relations:

$$\varepsilon_F = \frac{1}{2}\frac{aI}{V}g_{zz}, \quad \varepsilon_V = \frac{1}{2}\frac{aI^3}{V}G_\perp . \tag{28}$$

Therefore, the polaron shift [Eq. (27)] in the Voigt geometry is given by the simple formula

$$\Delta\varepsilon = \tfrac{1}{2}\sqrt{\varepsilon_F^2\cos^2\gamma + \varepsilon_V^2\sin^6\gamma} - \tfrac{1}{2}\varepsilon_V , \tag{29}$$

which allows one to find the angle γ. If the condition $\varepsilon_V \ll \varepsilon_F$ is satisfied, which is true in narrow quantum wells, we get

$$\cos^2\gamma \approx \frac{4\Delta\varepsilon(\Delta\varepsilon+\varepsilon_V)}{\varepsilon_F^2} . \tag{30}$$

On the other hand, Eq. (23) for γ and φ_h, on being combined with Eq. (28), takes the form

$$\tan\varphi_h = \frac{\varepsilon_V}{\varepsilon_F}\sin^2\gamma\tan\gamma . \tag{31}$$

By this means both angles γ and φ_h, which govern the value of linear polarization, are uniquely determined by $\Delta\varepsilon$, ε_F, and ε_V.

After substitution of Eqs. (30) and (31) into the formula for the linear polarization ρ_l^{hh} [Eq. (26)], we achieve the expression for ρ_l^{hh} as a function of the polaron shift $\Delta\varepsilon$. For $R_e \le R_h$ it takes the form

$$\rho_l^{\mathrm{hh}}(\Delta\varepsilon) \approx \frac{\varepsilon_V[\varepsilon_F^2 - 4\Delta\varepsilon(\Delta\varepsilon+\varepsilon_V)]^2}{\varepsilon_F^3[4\Delta\varepsilon(\Delta\varepsilon+\varepsilon_V)+\varepsilon_F(2\Delta\varepsilon+\varepsilon_V)]} , \tag{32}$$

and, for $R_e \gg R_h$,

$$\rho_l^{\mathrm{hh}} \approx \frac{\varepsilon_V[\varepsilon_F^2 - 4\Delta\varepsilon(\Delta\varepsilon+\varepsilon_V)]^{3/2}}{\varepsilon_F^3(2\Delta\varepsilon+\varepsilon_V)} . \tag{33}$$

Equations (32) and (33) give lower and upper bounds for the linear polarization degree for a heavy MP luminescence. They contain no fitting parameters and can be used to compare the above theory with experimental data.

In summary, we have developed a theory of the ground state of a two-dimensional magnetic polaron, taking into account an anisotropic hole g factor. The model has yielded magnetic-field dependences of the polaron energy, and the polarization of the magnetopolaron luminescence, which allow direct comparison with experimental data. The problem of a light magnetic polaron has been considered theoretically. The results of the theory can also be applied to magnetic polarons in bulk wurtzitelike DMS's.

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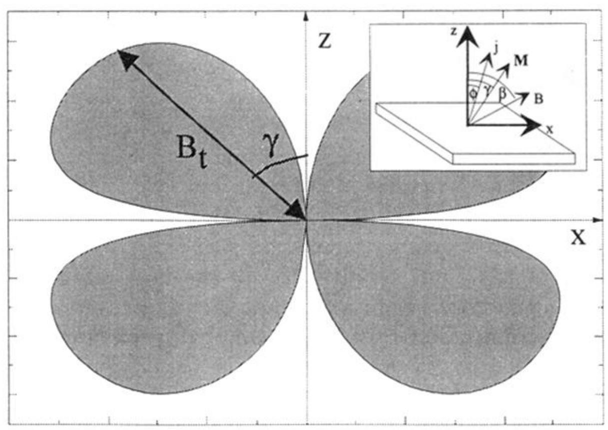

FIG. 1. Polar plot of the forbidden values of the external magnetic field vs the direction of **I** (dashed). Solid lines denote the value of transversal component of the magnetic field, B_t, as a function of the angle γ between $\mathbf{M}=-\mu g\mathbf{I}$ and Z. Calculations are made for the case of the heavy hole. The inset shows a sketch of vectors $\langle \mathbf{j} \rangle$, $\mathbf{M}$, and $\mathbf{B}$ in respect to the structure axis Z.