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Coherence lengths for three-dimensional superconductors in the BCS-Bose picture

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Following an approach similar to that of Miyake or Randeria, Duan, and Shieh in two dimensions, we study a three-dimensional many-fermion gas at zero temperature interacting via some short-ranged two-body potential. To accommodate a possible singularity (e.g., the Coulomb repulsion) in the interaction, the potential is eliminated in favor of the two-body scattering t-matrix, the low-energy form of which is expressible in terms of the s-wave scattering length a_s. The BCS gap equation for s-wave pairing is then solved simultaneously with the number equation in order to self-consistently obtain the zero-temperature BCS gap Δ as well as the chemical potential μ as functions of the dimensionless coupling variable $\lambda \equiv k_F a_s$, where k_F is the Fermi momentum. Results are valid for arbitrary coupling strength, and in the weak coupling limit reproduce the standard BCS results. Finally, root-mean-square pair sizes are obtained as a function of λ and compared with experimental values.

I. INTRODUCTION: BRIEF REVIEW OF BOSE-EINSTEIN CONDENSATION IN SUPERCONDUCTIVITY

The notion of superconductivity as a Bose-Einstein (BE)-like transition of bosonic objects is not new, going back at least to Schafroth[1] in the mid-1950's. More recently, with the discovery of the short-coherence-length cuprate superconductors, the idea has experienced a rebirth. Anderson[2] envisages crystalline electrons (or holes) as pair clusters of excitations that are fermionic, chargeless "spinons" and bosonic, charged "holons"—the latter susceptible[3] to a kind of BE condensation. Schrieffer and co-workers[4] deal with a bosonic "spin bag" which is shared by two holes. Lee and co-workers[5] achieve fits to the cuprate data of Uemura *et al.*[6] with a BE-like condensation in $2+\epsilon$ dimensions[7] by assuming an effective carrier mass in the direction perpendicular to the copper-oxide planes approaching the 10^5 anisotropy reported experimentally,[8] e.g., in TlBaCaCuO. Indeed, the value of ϵ itself can be determined precisely in idealized situations,[3] and estimated[9] to be about 0.03 in cuprate superconductors, its nonzero value being a manifestation of the coupling between copper-oxide planes. Alexandrov and Mott[10] focus on a bipolaronic picture and note an amazing similarity between at least two cuprate superconductors and liquid ^{4}He as regards their empirical specific-heat singularities across T_c. Introducing a bold, fascinating idea, Fujita and co-workers[11] generalize the BCS formalism to include hole-hole (as well as particle-particle) Cooper pairs, stressing in addition that either type of pairs propagate like *massless* (but number-conserving) bosons which may BE-condense in two dimensions (2D) as well as in 3D according to very specific, unique T_c formulas which differ markedly from the familiar BE T_c formula, $T_{\rm BE}$, for *massive* bosons. Indeed, a BE *paradigm* in superconductivity would seem to be supported by the recent T_c vs $T_{\rm BE}$ data extended by Uemura *et al.*[12] to virtually *all* exotic superconductors, whether 2D-like or 3D-like, whose T_c values span almost three orders of magnitude and which reveal an intriguing universal behavior roughly parallel to but shifted *down* from $T_{\rm BE}$ in the "Uemura plot" of T_c vs T_F or $T_{\rm BE}$, thus suggesting a sort of BE mechanism somehow implicit in an appropriately generalized BCS formalism.

Based on earlier work by Eagles[13] and by Leggett,[14] Miyake[15] and later Randeria, Duan, and Shieh[16] formulated the 2D many-fermion problem at zero absolute temperature ($T=0$) within a BCS formalism whereby *both* the gap equation and the number equation are solved self-consistently[17] but conveniently without explicit reference to a (possibly singular) two-fermion interaction potential which is replaced by a t matrix. The latter in turn is then related, at low-

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scattering energies, to the s-wave "scattering length." The many-fermion formalism is thus essentially exact provided the interaction range is short compared to the average fermionic spacing. This formulation in 2D has been extended to finite T by van der Marel,[18] as well as by Drechsler and Zwerger[19] using an elegant functional integral approach which in lowest order gives a Ginzburg-Landau theory. Following Ref. 16 a generalized coherence length within the BCS-Bose picture was formulated in 1D, 2D, and 3D by Casas *et al.*,[20] and the 2D case compared with cuprate superconductor data. Their results suggested that these materials might be moderately well described as *weakly coupled* within the BCS-Bose formalism.

The 3D BCS-Bose problem was extensively analyzed by Nozières and Schmitt-Rink,[21] in fact just before the 1986 discovery[22] of high-T_c cuprate superconductivity. Its definitive formulation in two transparent papers[23,24] by Haussmann stressed the vital importance of *triple* self-consistency (viz., in the gap, number, *and* single-particle-energy equations), and leads via the Thouless criterion[25] to a (mean-field) superfluid transition temperature T_c that increases monotonically and smoothly from the weak-coupling (BCS) to the strong-coupling (Bose) extreme, the two limits exactly reproducing, respectively, the BCS T_c formula (given in terms of the s-wave scattering length) and the familiar Bose-Einstein condensation temperature formula.

In Sec. II we introduce notation by reducing the BCS number equation to dimensionless form; in Sec. III the BCS gap equation is similarly treated, after eliminating the (possibly singular) "bare" two-fermion interaction in favor of the two-body scattering t matrix which is expressible in terms of the s-wave scattering length; this is done by generalizing the 2D formulation of Ref. 16 to 3D. Section IV illustrates the 3D s-wave scattering length for a specific two-body potential, viz., the hard-core-square-well potential shape also discussed in Ref. 16 but in 2D; Sec. V displays our numerical results vs coupling strength for the gap energy, the chemical potential; and, finally, we calculate the root-mean-square pair radius which is compared to some empirical values of coherence lengths of 3D-like superconductors. Lastly, Sec. VI states our conclusions.

II. THE ZERO-TEMPERATURE NUMBER EQUATION

Consider the zero-temperature number equation from BCS theory,

$$N=2\sum_k v_k^2. \tag{1}$$

The occupation probability v_k^2 that minimizes the BCS model variational ground-state energy turns out to be

$$v_k^2=\frac{1}{2}\left(1-\frac{\xi_k}{E_k}\right), \tag{2}$$

with $\xi_k=\epsilon_k-\mu$, $\epsilon_k=\hbar^2k^2/2m$, and $E_k=\sqrt{\xi_k^2+\Delta_k^2}$ where Δ_k is the gap energy [see Eq. (8) below]. Recall that for s-wave pairing, $\Delta_k=\Delta$ for all k. Thus

$$N=\sum_k\left\{1-\frac{\xi_k}{E_k}\right\}=\frac{V}{2\pi^2}\int_0^\infty dkk^2\left\{1-\frac{\xi_k}{E_k}\right\}, \tag{3}$$

where V is the system volume. If $\epsilon=\epsilon_k=\hbar^2k^2/2m$ one can rewrite this as

$$N=\frac{V}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\int_0^\infty d\epsilon\sqrt{\epsilon}\left\{1-\frac{\xi}{E}\right\}, \tag{4}$$

with $E=\sqrt{(\epsilon-\mu)^2+\Delta^2}$. However, since for the ideal (interactionless) Fermi gas

$$N=2\frac{V}{(2\pi)^3}\int_0^{k_F}4\pi k^2dk=\frac{V}{3\pi^2}k_F^3=\frac{V}{3\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}E_F^{3/2}, \tag{5}$$

where $E_F\equiv\hbar^2k_F^2/2m$, then eliminating N from (4) and (5) gives

$$\frac{4}{3}E_F^{3/2}=\int_0^\infty d\epsilon\sqrt{\epsilon}\left\{1-\frac{\xi}{E}\right\}. \tag{6}$$

Setting $\tilde{\epsilon}=\epsilon/E_F$, $\tilde{E}\equiv E/E_F$, $\tilde{\xi}\equiv\xi/E_F$, $\tilde{\mu}\equiv\mu/E_F$, and $\tilde{\Delta}\equiv\Delta/E_F$, one arrives at *the reduced number equation*

$$\frac{4}{3}=\int_0^\infty d\tilde{\epsilon}\sqrt{\tilde{\epsilon}}\left\{1-\frac{\tilde{\epsilon}-\tilde{\mu}}{\sqrt{(\tilde{\epsilon}-\tilde{\mu})^2+\tilde{\Delta}^2}}\right\}. \tag{7}$$

This is the zero-temperature number equation in 3D but contains *two* unknowns $\tilde{\mu}$ and $\tilde{\Delta}$ so that another equation is required.

III. THE ZERO-TEMPERATURE BCS GAP EQUATION

Recall the zero-temperature BCS gap equation,

$$\Delta_k=-\sum_{k'}V_{k,k'}\frac{\Delta_{k'}}{2E_{k'}}. \tag{8}$$

To cope with a possibly ill-behaved potential $V_{k,k'}$, one can renormalize the gap equation by introducing a momentum cutoff Λ.[16] The final results turn out to be independent of Λ, so that one may take $\Lambda\to\infty$ at the very end. For $k>\Lambda$, assume $E_k\approx\epsilon_k$. Then

$$\Delta_k=-\sum_{k'}^{<}\Gamma_{k,k'}\frac{\Delta_{k'}}{2E_{k'}}, \tag{9}$$

where $<$ ($>$) indicates summation over $k'<\Lambda$ ($k'>\Lambda$), and where the pseudopotential Γ is defined by the (integral) equations

$$\Gamma_{k,k'}=V_{k,k'}-\sum_{k''}^{>}V_{k,k''}\frac{1}{2\epsilon_{k''}}\Gamma_{k'',k'}. \tag{10}$$

Now eliminate the potential V in favor of the two-body scattering t matrix via its definition in terms of Γ. In the low-energy limit one has for the lth partial wave

$$\Gamma_{k,k'}^{(l)}=t_{k,k'}^{(l)}(2E)-\int_{q<\Lambda}\frac{d^3q}{(2\pi)^3}t_{k,q}^{(l)}(2E)[\mathscr{G}_0(2E)]_{qq}\Gamma_{q,k'}^{(l)}(2E), \tag{11}$$

where the notation in Ref. 16 is followed, and where the free two-body Green function $\mathscr{G}_0$ is just

$$[\mathscr{G}_0(2E)]_{k,k'}=\frac{\delta_{k,k'}}{2(E-\epsilon_k+i\eta)}, \tag{12}$$

with the limit $\eta\rightarrow 0$ implied. The factor two has been introduced for later convenience, since the energy variable E is then given by $E=\hbar^2q^2/2m$, with q the relative momentum. For s-wave pairing ($l=0$), the gap function has no angular dependence, and furthermore $\Delta_k=\Delta$ for all k. Introduce the low-energy form for the s-wave t matrix, $t^{(0)}_{k,k'}(2E)\approx\tau_0(2E)$ to obtain

$$\Gamma^{(0)}_{k,k'}=\tau_0(2E)\left[1+\tau_0(2E)\int_{q<\Lambda}\frac{d^3q}{(2\pi)^3}\frac{1}{2(E-\epsilon_q+i\eta)}\right]^{-1}. \tag{13}$$

Substitute this into Eq. (9) and take the limit $\Lambda\rightarrow\infty$ to get

$$\frac{1}{\tau_0(2E)}=\frac{1}{4\pi^2}\int_0^\infty dq q^2\left\{\frac{1}{\epsilon_q-E-i\eta}-\frac{1}{E_q}\right\}. \tag{14}$$

Let $\epsilon=\epsilon_q=\hbar^2q^2/2m$ as before, so that this integral becomes

$$\frac{8\pi^2}{\tau_0(2E)}\left(\frac{\hbar^2}{2m}\right)^{3/2}=\int_0^\infty d\epsilon\sqrt{\epsilon}\left\{\frac{1}{\epsilon-E-i\eta}-\frac{1}{E_k}\right\}. \tag{15}$$

Recall that $\tilde{\epsilon}=\epsilon/E_F$ and note that $\tilde{E}\equiv E/E_F=(q^2/k_F^2)$; then with the same definitions of the other quantities with tildes as before, one finds

$$\frac{4\pi^2\hbar^2}{mk_F\tau_0(2E)}=\int_0^\infty d\tilde{\epsilon}\sqrt{\tilde{\epsilon}}\left\{\frac{1}{\tilde{\epsilon}-(q^2/k_F^2)-i\eta}-\frac{1}{\tilde{E}_k}\right\}. \tag{16}$$

In the low-energy limit $\tau_0(2E)$ may thus be expressed in terms of the s-wave scattering length a_s as

$$\frac{1}{\tau_0(2E)}=\frac{m}{4\pi\hbar^2}\left(\frac{1}{a_s}+qi\right), \tag{17}$$

where, as before, q refers to the transferred (or relative) momentum variable. Substituting this form into Eq. (16) gives

$$\frac{\pi}{k_F}\left(\frac{1}{a_s}+qi\right)=\int_0^\infty d\tilde{\epsilon}\sqrt{\tilde{\epsilon}}\left\{\frac{1}{\tilde{\epsilon}-(q^2/k_F^2)-i\eta}-\frac{1}{\tilde{E}_k}\right\}. \tag{18}$$

This integral is evaluated in Ref. 26, and finally gives the *reduced gap equation*

$$\frac{\pi}{\lambda}=\int_0^\infty d\tilde{\epsilon}\left\{\frac{1}{\sqrt{\tilde{\epsilon}}}-\frac{\sqrt{\tilde{\epsilon}}}{\sqrt{(\tilde{\epsilon}-\tilde{\mu})^2+\tilde{\Delta}^2}}\right\}, \tag{19}$$

with $\lambda\equiv k_Fa_s$. Note that there is no dependence on the relative momentum q, which is as it should be, since an arbitrary value may be assigned to the variable q. Equations (7) and (19) agree with the results cited in Ref. 27 without proof.

IV. SPECIFIC EXAMPLE OF AN S-WAVE SCATTERING LENGTH

We recall how the s-wave scattering length a_s appears in the basic equation of "effective range scattering theory,"[28] namely

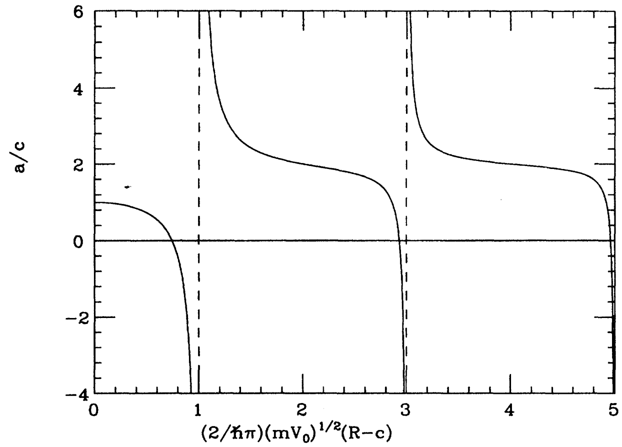

FIG. 1. Equation (21) graphed for $R=2c$, for the "hard-core-plus-square-well" potential described in the text.

$$k\cot\delta_0(k)\rightarrow-\frac{1}{a_s}+\frac{1}{2}r_0k^2+O(k^4),\quad \text{if } k\rightarrow 0 \tag{20}$$

where k is the relative wave number, $\delta_0(k)$ the s-wave scattering phase shift and r_0 the so-called "effective range" of an unspecified (or unknown) potential. Thus, low-energy scattering can empirically reveal not a *specific* potential shape but merely two characteristic lengths, a_s and r_0. For concreteness, consider the 3D "hard-core-plus-square-well" two-body potential discussed in 2D in Ref. 16 as a specific example. If r is the relative separation between the two particles, then $V(r)=\infty$ for $r<c$, $V(r)=-V_0$ for $c<r<R$, and $V(r)=0$ for $r>R$. Solutions of the two-body Schrödinger equation for positive energies $E\equiv\hbar k^2/m>0$ then yields the s-wave scattering phase shift $\delta_0(k)$ such that (20) gives[29] the explicit, closed form expression

$$\frac{a_s}{c}=\left[1+\frac{R-c}{c}\left\{1-\frac{\tan[\sqrt{mV_0}(R-c)/\hbar]}{\sqrt{mV_0}(R-c)/\hbar}\right\}\right], \tag{21}$$

which evidently reduces to unity for $V_0\rightarrow 0$ (pure hard-sphere scattering). Equation (21) is plotted in Fig. 1 for the concrete case $R=2c$; the poles denote critical attractive-well strength/range values at which bound s-wave states occur, namely, when the argument of the tangent in (21) equals $\pi/2$, $3\pi/2$, ...

Effective-range theory also relates[28] a_s and r_0 to the bound states energies, if any, of the potential, through the relation

$$\sqrt{\frac{mE_0(2)}{\hbar^2}}\rightarrow\frac{1}{a_s}+\frac{1}{2}\frac{mE_0(2)}{\hbar^2}r_0+\cdots \tag{22}$$

valid for small $E_0(2)$, which is the (positive) s-wave binding energy of the (two-body) potential. This result will be employed below.

V. RESULTS

The reduced number and gap equations (7) and (19) must now be solved simultaneously for $-\infty<1/\lambda<+\infty$, and this

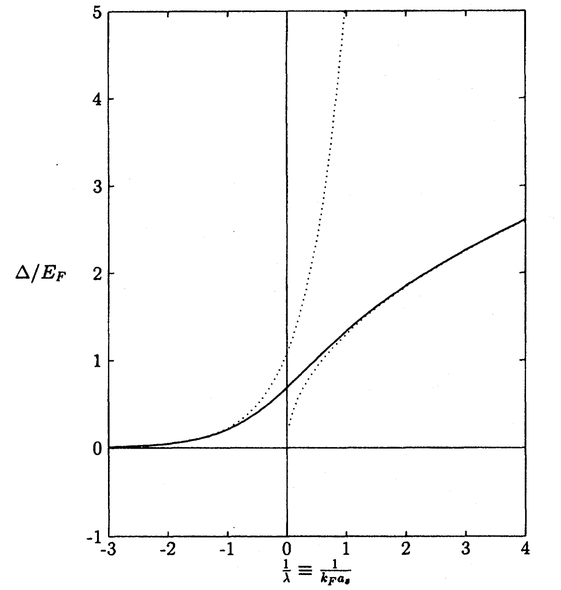

FIG. 2. Behavior of gap parameter Δ (in units of E_F) vs $1/\lambda \equiv 1/k_F a_s$, with a_s the s-wave scattering length of the two-fermion interaction, obtained by solving (7) and (19) self-consistently. Left dotted curve is the weak-coupling BCS result (25); right dotted-curve is strong-coupling limit $\Delta/E_F \to \lambda^{-1/2}$ discussed in text.

was done numerically. The results for the zero-temperature gap Δ and chemical potential μ, in units of E_F, as functions of the variable λ, are displayed in Figs. 2 and 3.

We have verified that in the weak-coupling limit $|\lambda| \ll 1$,

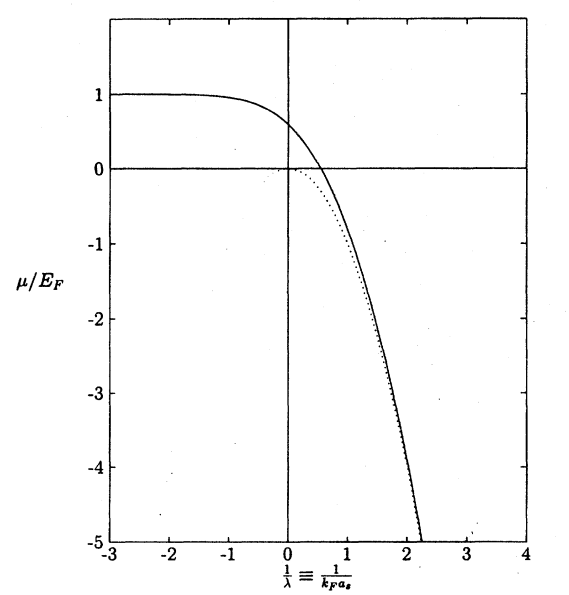

FIG. 3. Drop of μ/E_F from weak-coupling (BCS extreme) $(1/\lambda \to -\infty)$ value of unity, as coupling increases to (Bose extreme) $1/\lambda \to +\infty$. Dotted curve is strong-coupling limit of $-1/\lambda^2$ as exhibited in (28).

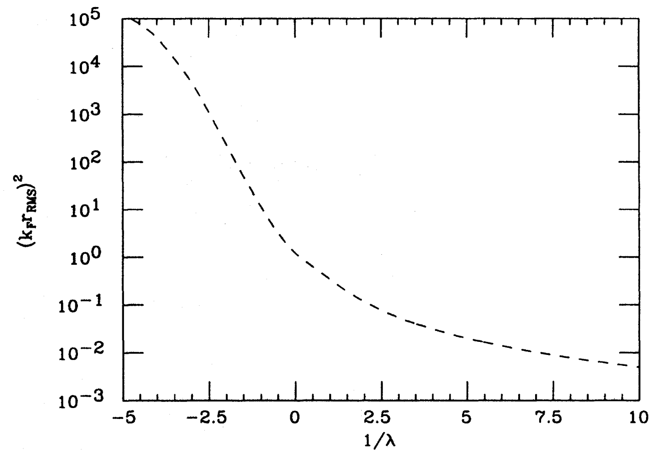

FIG. 4. Decrease of root-mean-square pair radius $r_{\rm RMS}$ as coupling is increased from the BCS extreme of large, severely overlapping, weakly bound Cooper pairs, to the Bose extreme of small, nonoverlapping, tightly bound pairs.

but for $\lambda < 0$, the results agree with the well-known analytical form, (25) below, for the BCS gap energy. For $1/\lambda = 0$, $\tilde{\Delta}$ and $\tilde{\mu}$ approach the limiting values 0.686 and 0.591, respectively, as is easily seen analytically. For weak coupling $\lambda \equiv k_F a_s \to 0^-$, assume that $\tilde{\Delta} = \Delta/E_F \ll 1$ so that the number equation (7) reduces to

$$\mu = E_F, \text{ or } \tilde{\mu} = 1. \tag{23}$$

Equation (19) then simplifies to

$$\frac{\pi}{\lambda} = \int_0^A d\tilde{\epsilon} \left\{ \frac{1}{\sqrt{\tilde{\epsilon}}} - \frac{\sqrt{\tilde{\epsilon}}}{\sqrt{(\tilde{\epsilon} - 1)^2 + \tilde{\Delta}^2}} \right\}, \tag{24}$$

where the limit $A \gg 1$ is implied. This integral too is done in Ref. 26 and finally leaves

$$\Delta = (8/e^2) E_F \exp\left\{ -\frac{\pi}{2|\lambda|} \right\}, \tag{25}$$

in agreement with the well-known result cited as Eq. (4.22) in Ref. 23. Note that in 3D a necessary condition on the two-body interaction for a many-body BCS instability is merely that it be sufficiently attractive to give rise to a *negative* (s-wave) *scattering length*, whereas in 2D the two-body interaction must be[15,16] attractive enough to actually *bind*. Indeed, as compared with 3D fluctuation effects will be more significant in 2D and in 1D. In 2D the anomalous nature of the scattering length is fortuitously circumvented automatically since, rather than $\lambda \equiv k_F a_s$, the relevant interaction strength parameter within the BCS-Bose picture turns out to be $E_0(2)/E_F \equiv \eta \geq 0$ instead. This manifests itself in the remarkably simple exactly analytical relations[15,16]

$$\tilde{\Delta} = \sqrt{2\eta} \quad \text{and} \quad \tilde{\mu} = 1 - \frac{\eta}{2} \ \ (2D). \tag{26}$$

The weak-binding result (25) for the 3D gap energy may be compared to the Cooper pair binding energy in 3D given by

TABLE I. Experimental data for seven typical 3D-like superconductors, as discussed in the text.

Superconductor	T_c (K)	k_F (Å^{-1})	ξ_0 (Å)	$(k_F\xi_0)^2$
$Ba_{1-x}K_xBiO_3$	36±4 Refs. 30,31	0.2562±0.0287 Ref. 30	53±1 Ref. 31	$(1.9\pm0.5)\times10^2$
Rb_3C_{60}	30.0±1.6 Refs. 30,32	0.5399±0.0818 Refs. 33,34	26±4 Ref. 35	$(1.8\pm0.8)\times10^2$
Nb_3Sn	17.9 Refs. 30,36	1.1160±0.0920 Ref. 30	40 Ref. 36	$(2.0\pm0.3)\times10^3$
UPt_3	0.53 Refs. 30,37	0.6146±0.0512 Ref. 30	106±14 Refs. 37,38	$(4.5\pm1.8)\times10^3$
Pb	7.22 Ref. 39	1.62 Ref. 40	830 Ref. 39	1.8×10^6
Sn	3.75 Ref. 39	1.57 Ref. 40	2300 Ref. 39	1.3×10^7
Al	1.2 Ref. 39	1.75 Ref. 40	16000 Ref. 39	7.8×10^8

$$\Delta_C=(8/e^2)E_F \exp\left\{-\frac{\pi}{|\lambda|}\right\}, \qquad (27)$$

where we note that for weak coupling $\Delta \gg \Delta_C$, as is well known. For strong coupling, $1/\lambda \to +\infty$, we verified that $\Delta/E_F \to 1/\sqrt{\lambda}$, also in accordance with the asymptotic result cited just below Eq. (4.25) of Ref. 23, and which implies that Δ itself vanishes in this limit too.

The resulting 3D behavior of μ/E_F vs coupling as depicted in Fig. 3 is qualitatively similar to the 2D *analytical* result of Ref. 16 given, according to (26), very simply by $\mu/E_F = 1 - E_0(2)/2E_F$. In this latter case weak (strong) coupling means negligence (predominance) of the term $E_0(2)/2E_F$ compared to unity. In Fig. 3 the chemical potential for *weak* coupling is just (23). On the other hand, for *strong* coupling (or alternatively small k_F) one can use (22) for small $E_0(2) \simeq \hbar^2/ma_s$ to verify that μ/E_F asymptotically becomes just the negative of one-half the (positive) binding energy of a single pair (expressed in units of E_F), which implies

$$\frac{-\frac{1}{2}E_0(2)}{E_F} = -\frac{\hbar^2/2ma_s^2}{\hbar^2k_F^2/2m} = -\frac{1}{(k_Fa_s)^2} \equiv -\frac{1}{\lambda^2}, \qquad (28)$$

the curve of which is plotted in Fig. 3 as a dotted line.

Finally, Fig. 4 depicts the behavior of $(k_F r_{\rm RMS})^2$ vs $1/\lambda$, where $r_{\rm RMS}$ is the root-mean-square pair radius obtained analytically for 1D, 2D, and 3D in Ref. 20, and which in 3D is given explicitly by

$$(k_F r_{\rm RMS})^2 = \frac{\left[\left(\tilde{\mu}^2 + \frac{5}{4}\tilde{\Delta}^2\right) + \frac{1}{2}\tilde{\Delta}\tilde{\mu}\tan\frac{\phi}{2}\right]}{2\tilde{\Delta}^2(\tilde{\mu}^2+\tilde{\Delta}^2)^{1/2}}, \qquad (29)$$

where

$$\phi = \pi + \tan^{-1}(\tilde{\Delta}/\tilde{\mu}) \quad (\text{if } \mu<0)$$

$$\phi = \tan^{-1}(\tilde{\Delta}/\tilde{\mu}) \quad (\text{if } \mu>0). \qquad (30)$$

Here the root-mean-square radius $r_{\rm RMS}$ is defined as in Ref. 16, namely

$$r_{\rm RMS}^2 \equiv \left[\int d^3k\, \psi_k^* \psi_k\right]^{-1} \int d^3k\, \psi_k^* r^2 \psi_k \qquad (31)$$

where $r^2 \to -\nabla_k^2$ and $\psi_k \equiv \Delta/2E_k$.[14,21] Formulas (6) and (7) are valid for *any* interaction leading to a k-independent gap energy Δ and *any* coupling strength. In Ref. 20 $r_{\rm RMS}^2$ from Eq. (29) was shown to reduce, in weak coupling to $(\hbar v_F/\sqrt{8}\Delta)^2$, and in strong coupling to $\hbar^2/2mE_0(2)$ where $E_0(2)$ is again the bare two-body interaction (positive) binding energy. The length $\sqrt{\hbar^2/2mE_0(2)}$ is just the size of the two-body wave function of an isolated pair bound with energy $-E_0(2)$. Note that in weak coupling $r_{\rm RMS}$ differs from the familiar Pippard coherence length $\xi_0 = \hbar v_F/\pi\Delta$ only by the factor $\sqrt{8} \approx 2.83$ vs π in the denominator, this difference being slight. As expected, a smooth shrinking of pair size is clearly observed in Fig. 4 to develop as one goes from the weak-coupling (BCS) regime to the strong-coupling (Bose) extreme.

Lastly, Table I lists some empirical data for several 3D-like superconductors with 0.5 K$\lesssim T_c \lesssim$ 40 K. The last column gives $(k_F\xi_0)^2$ with ξ_0 the reported coherence length, based largely on upper critical field $H_{c2}(0) = (\hbar/2e)/2\pi\xi_0^2$ data. All such $(k_F\xi_0)^2$ values (with the exception of the rather uncertain value for the fulleride superconductor Rb_3 C_{60}) would fall on the $(k_F r_{\rm RMS})^2$ vs $1/\lambda$ curve of Fig. 4 well *to the left* of the point $1/\lambda = 0$. Note that the elemental superconductors Pb, Sn, and Al give empirical $(k_F\xi_0)^2$ values above the scale of the figure where, according to Fig. 3 $\mu \simeq E_F$, and according to Fig. 2, Δ is small and well represented by the BCS limiting value (25) shown in Fig. 2 as the dotted curve on the left.

VI. CONCLUSIONS

As with the three (2D-like) cuprate superconductors (YBaCuO, BiSrCaCuO, and TlBaCaCuO) studied in Ref. 20, we find in the present paper that the 3D-like superconductors considered here can also be reasonably well described as moderate-to-weakly coupled materials within the BCS-Bose picture.

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