

Quantum effects in a superconducting-glass model

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(Received 2 June 1995)

We study disordered Josephson-junction arrays with long-range interaction and charging effects. The model consists of two orthogonal sets of positionally disordered N parallel filaments (or wires) Josephson coupled at each crossing and in the presence of a homogeneous and transverse magnetic field. The large charging energy (resulting from small self-capacitance of the ultrathin wires) introduces important quantum fluctuations of the superconducting phase within each filament. Positional disorder and magnetic-field frustration induce spin-glass-like ground states, characterized by not having long-range order of the phases. The stability of this phase is destroyed for sufficiently large charging energy. We have evaluated the temperature vs charging energy phase diagram by extending the methods developed in the theory of infinite-range spin glasses, in the limit of large magnetic field. The phase diagram in the different temperature regimes is evaluated by using a variety of methods, to wit: semiclassical WKB and variational methods, Rayleigh-Schrödinger perturbation theory, and pseudospin effective Hamiltonians. Possible experimental consequences of these results are briefly discussed.

I. INTRODUCTION AND MODEL HAMILTONIAN

Josephson-junction arrays (JJA) have been the subject of considerable recent research activity.¹ Such systems consist of superconducting grains embedded in a nonsuperconducting host and coupled together by the Josephson effect. Recently there has been a surge of both theoretical and experimental interest in studying disordered Josephson-coupled systems in an applied magnetic field.²⁻⁵ Theoretically, it has been shown that in the presence of sufficiently strong magnetic fields the system may freeze into a state exhibiting spin-glass-like type order among the superconducting grains.⁶ From the experimental viewpoint the interest is motivated by the existence of irreversibility lines in the temperature-field diagram, in virtually all high- T_c superconductors, thus suggesting the existence of a glassy phase.⁷

For sufficiently small grains, the behavior of each junction in the JJA is modified by the quantum effects which arise from the small capacitances that lead to large charging energies. The competition between phase coherent ordering and charging effects in periodic JJA has been the subject of a number of studies⁸ and it is by now well established that for sufficiently large charging energy quantum phase fluctuations lead to the complete suppression of long-range superconducting order.⁹

In disordered JJA with small capacitances thermal, random, and quantum fluctuations will determine the physics of the system and an interesting question arises regarding the competition between them. The problem we would like to address in this paper is then: What is the effect of having a competition between the thermal, quantum, and random fluctuations on the long-range properties of a Josephson-junction network? This is a highly nontrivial question in general but partial answers can be obtained in certain limits. Since there are only very limited studies on this issue (see, e.g., Ref. 10), the purpose of this paper is to investigate these quantum-fluctuation effects systematically in a mean-field-like approximation, to be specified below. We shall study a quantum model of a disordered JJA system which, in principle, could

be realized experimentally and simultaneously allowing for a detailed theoretical treatment thus constituting an attractive setting for the study of the complicated interplay between quenched disorder, interactions, and quantum fluctuations.

To be specific, we will study a stack of two sets of N mutually perpendicular parallel wires (or filaments) Josephson coupled at nodes (see Fig. 1). Since each horizontal (vertical) filament of the system is directly coupled to every other vertical (horizontal) filament, the number of nearest neighbors z in this model is $z=N$. This is then a realization of a JJA with *long-range* (infinite-range, in the thermodynamic limit $N \rightarrow \infty$) interactions which differ from the conventional two-dimensional (2D) Josephson-junctions arrays. Furthermore, we assume that the distance between neighboring parallel wires varies randomly around some average value l . Finally, the system is placed in a transverse magnetic field B and we shall assume that the Josephson couplings are sufficiently small so that the induced magnetic fields are negligible in comparison with B so that the phase gradient along

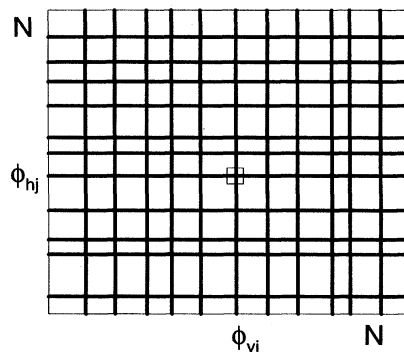


FIG. 1. Disordered Josephson-coupled array of $2N$ superconducting wires (straight horizontal and vertical lines, square box denotes the junction at a node). Each wire has small self-capacitance C and is characterized by the superconducting phase ϕ_i . The magnetic field B is applied perpendicular to the array.

any filament results only from the presence of the external magnetic field. The dynamical variables of this system are the superconducting phases associated with each wire. The properties of the (classical) model defined above have been studied some time ago.² Very recently the thermodynamical properties of this system (both periodic and positionally disordered version) have been investigated theoretically and experimentally.^{11–13}

An important ingredient in our present considerations is that we allow for *quantum* phase fluctuation within a wire assuming the node junctions have a sufficiently small self-capacitance C . In this case the charging energy becomes a dominant quantity considerably affecting the properties of the array in the low-temperature regime. More precisely, we are interested in the behavior where the quantum fluctuations compete with the formation of the superconducting glassy phase due to randomness and magnetic frustration.

In the low-temperature regime we expect to have a glassy phase with randomly frozen superconducting phases. To examine the extent to which the system is frozen, it is convenient to introduce the Edwards-Anderson order parameter¹⁴ defined by

$$q_{EA} \equiv [\langle \mathbf{S}_i \rangle_T^2]_{av}, \quad (1)$$

where

$$\mathbf{S}_i = [S_i^x, S_i^y] \equiv [\cos(\phi_i), \sin(\phi_i)], \quad (2)$$

while $\langle \dots \rangle_T$ and $[\dots]_{av}$ denote thermodynamic and configurational averaging, respectively. In a disordered state the phases will randomly sample their entire phase space and $q=0$, while in a frozen system $q \neq 0$ and $[\langle \mathbf{S}_i \rangle_T]_{av} = 0$ indicating the absence of long-range order. The spin-glass-like ground state may be destabilized by quantum fluctuations.¹⁵ As one varies the strength of the charging energy (i.e., increasing the strength of the quantum fluctuations) there can be a phase transition at zero temperature between the glassy phase and the paraconsistent disordered ground states. This *quantum phase transition* in random spin systems has received much attention recently.¹⁶ However, its corresponding nature in quantum disordered Josephson-coupled systems has not been previously studied. The goal of this paper is to examine this problem in some detail starting from a quantum gauge-glass model defined in Eq. (3) below.

As discussed in the main body of the paper, we need to use different complementary calculational techniques to attack the problem, for each one of them is by nature approximate and their regions of validity are different. It is also important that using only one of these techniques by itself may lead to spurious results that can only be validated by an independent check. A case in point will be the reentrant transition found in the variational calculation, which is valid in the semiclassical region, and which is not found in our low-temperature expansion.

We now describe the main body of the paper. In Sec. II we define the model Hamiltonian, we perform the quenched average over the disorder, followed by a functional integral formulation of the mean-field theory and the derivation of the saddle-point equations in the $N \rightarrow \infty$ limit. This constitutes an *exact* self-consistency condition for the glass order parameters. Section III presents the phase diagram for the

model preceded by an exhaustive investigation of the self-consistency equations via a variety of approaches including semiclassical WKB and variational methods, Rayleigh-Schrödinger perturbation theory, and pseudospin effective Hamiltonians in a truncated charge Hilbert space. Finally, in Sec. IV, we discuss our results and present our conclusions.

II. THE MODEL

The *quantum* Hamiltonian of the disordered system of Josephson-coupled wires is given by

$$H = H_C + H_J, \quad (3)$$

$$H_C = -\frac{K}{2} \sum_{i=1}^N \left(\frac{\partial^2}{\partial \phi_{hi}^2} + \frac{\partial^2}{\partial \phi_{vi}^2} \right),$$

$$H_J = \frac{E_J}{\sqrt{N}} \sum_{i=1}^N \sum_{j=1}^N [1 - \cos(\phi_{hi} - \phi_{vj} - f_{ij})].$$

Here, $\hat{n}_i = (2e/i)\partial/\partial\phi_i$ is the charge operator while ϕ_{hi} (ϕ_{vj}) represent the superconducting phase of the i th horizontal (j th vertical) wire, respectively; E_J is the Josephson coupling and $K = 4e^2/C$ is the charging energy, respectively. In order to have a well defined thermodynamic limit we have to scale the Josephson coupling energy by a factor \sqrt{N} . Furthermore, $f_{ij} = (2\pi/\Phi_0) \int_{\mathbf{r}_i}^{\mathbf{r}_j} \mathbf{A} \cdot d\mathbf{l}$ is the line integral of the vector potential \mathbf{A} and Φ_0 is the elementary flux quantum.

It is clear that E_J is a positive quantity (and it may depend only on the distance between wires which we are neglecting) and it does not introduce any frustration in the system. Magnetic field *and* random location of the wires are what generate variations of the phase parameters f_{ij} , thus allowing for the random frustration present in the system.¹⁷ The relevant quantity is thus the effective random coupling matrix in the Josephson part of the Hamiltonian (3)

$$\mathbf{J}_{ij} = \frac{E_J}{\sqrt{N}} \begin{pmatrix} 0 & e^{if_{ij}} \\ e^{-if_{ji}} & 0 \end{pmatrix}, \quad (4)$$

whose density of the eigenvalues is given by

$$\rho(E) = -(1/\pi) \overline{\text{Im}[E \delta_{ij} - \mathbf{J}_{ij} + i0]_{ii}^{-1}}, \quad (5)$$

where the bar denotes averaging over the positional disorder. The behavior of (5) varies with increasing the strength of the magnetic field. In the case of large magnetic field, so that the flux per average plaquette $\Phi = Bl^2$ is much larger than the elementary flux quantum $\Phi/\Phi_0 \gg 1$, the frustration parameters f_{ij} acquire random values and fill the interval $(0, 2\pi]$ uniformly. In this limit the effects of disorder become especially apparent and the behavior moves toward the asymptotic regime which is essentially field independent.² In this case correlations between the matrix elements \mathbf{J}_{ij} vanish and the density of states (5) approaches the Wigner semicircular law²

$$\rho(E) \rightarrow \rho_W(E) = (\pi E_J)^{-1} \sqrt{1 - (E/2E_J)^2}, \quad (6)$$

implying that the random matrix (4) belongs to the Gaussian unitary ensemble with only the second moment nonvanishing,

$$\langle \mathbf{J}_{ij} \mathbf{J}_{kl} \rangle_{\text{av}} = \frac{E_J}{N} \delta_{ik} \delta_{jl}. \quad (7)$$

Therefore, in the high-field regime, we can implement a mean-field theory of our quantum gauge-glass problem in a way closely resembling the infinite-range interaction Sherrington-Kirkpatrick magnetic spin-glass model.¹⁸ However, the resulting formulation is not just a quantum extension of the planar spin-glass model. Unlike the random bond XY spin glass, the improper global rotation $\phi_i \rightarrow -\phi_i$ (i.e., time reversal) is *not* a symmetry of our quantum gauge-glass model. To make this observation apparent we introduce the two component ‘‘spin’’ vector (2) so that the Josephson part of the Hamiltonian (3) reads

$$H_J = -\frac{E_J}{2\sqrt{N}} \sum_{i=1}^N \sum_{j=1}^N [e^{if_{ij}} (\mathbf{S}_{hi} \cdot \mathbf{S}_{vj} - i\hat{\mathbf{z}} \cdot \mathbf{S}_{hi} \times \mathbf{S}_{vj}) + \text{H.c.}], \quad (8)$$

where $\hat{\mathbf{z}}$ is a unit vector perpendicular to the plane containing \mathbf{S}_i . We note that in addition to the conventional XY coupling $\mathbf{S}_{hi} \cdot \mathbf{S}_{vj}$ there is a cross term $\mathbf{S}_{hi} \times \mathbf{S}_{vj}$ which is the analog of the spin-orbit [i.e., Dzialoshinsky-Moriya (DM)] interaction in magnetic systems — essentially violating the time-reversal symmetry.¹⁹ Thus, the present quantum gauge-glass problem formally resembles more closely the quantum spin-glass formulation in the presence of the DM anisotropy.²⁰

III. DISORDER AVERAGE AND MEAN-FIELD FORMULATION

In a random system we need to calculate the average of the free-energy density $\mathcal{F} = -(1/\beta) \ln Z/2N$ over the disorder (7). This is done by using the replica method permitting to average the replicated partition function Z^n instead of $\ln Z$,

$$\beta \mathcal{F} = -\lim_{N \rightarrow \infty} \lim_{n \rightarrow 0} \frac{1}{nN} (\langle Z^n \rangle_{\text{av}} - 1). \quad (9)$$

It is convenient to express the replicated partition function $Z^n = \text{Tr} \exp(-\sum_{\alpha} \beta H^{\alpha})$ in the interaction representation as

$$Z^n = \text{Tr} e^{-\beta \sum_{\alpha} H_0^{\alpha}} T_{\tau} \exp \left[-\sum_{\alpha} \int_0^{\beta} d\tau H_I^{\alpha}(\tau) \right], \quad (10)$$

with the interaction picture Hamiltonian

$$H_I^{\alpha}(\tau) = e^{\tau H_0^{\alpha}} H_I^{\alpha} e^{-\tau H_0^{\alpha}}, \quad (11)$$

and the free part

$$H_0^{\alpha} = -\frac{K}{2} \sum_i \left(\frac{\partial^2}{\partial \phi_{hi}^{\alpha 2}} + \frac{\partial^2}{\partial \phi_{vi}^{\alpha 2}} \right), \quad (12)$$

where $H^{\alpha} = H_0^{\alpha} + H_I^{\alpha}$ is the total Hamiltonian. For the interaction Hamiltonian one has explicitly

$$H_I^{\alpha}(\tau) = \frac{E_J}{2\sqrt{N}} \left\{ \sum_{i=1}^N \sum_{j=1}^N e^{if_{ij}} [\mathbf{S}_{hi}^{\alpha}(\tau) \cdot \mathbf{S}_{vj}^{\alpha}(\tau) - i\mathbf{z} \cdot \mathbf{S}_{hi}^{\alpha}(\tau) \times \mathbf{S}_{vj}^{\alpha}(\tau)] + \text{H.c.} \right\}, \quad (13)$$

so that the statistical average can be taken in the ensemble given by H_0^{α} . Here T_{τ} is the Matsubara ‘‘imaginary time’’ ordering operator allowing us to treat the time-dependent operators $\mathbf{S}_i^{\alpha}(\tau) = e^{-\tau H_0^{\alpha}} \mathbf{S}_i^{\alpha} e^{\tau H_0^{\alpha}}$ as c numbers within the time-ordered exponential (10). Consequently, the Gaussian average (7) readily gives

$$\langle Z^n \rangle_{\text{av}} = \text{Tr} e^{-\sum_{\alpha} \beta H_0^{\alpha}} T_{\tau} \exp \left[-\int_0^{\beta} d\tau \int_0^{\beta} d\tau' \Omega(\tau, \tau') \right], \quad (14)$$

where

$$\Omega(\tau, \tau') = \frac{E_J^2}{4N} \sum_{i=1}^N \sum_{j=1}^N \sum_{\alpha\beta} \{ [\mathbf{S}_{hi}^{\alpha}(\tau) \cdot \mathbf{S}_{vj}^{\alpha}(\tau)] [\mathbf{S}_{hi}^{\beta}(\tau') \cdot \mathbf{S}_{vj}^{\beta}(\tau')] + [\mathbf{S}_{hi}^{\alpha}(\tau) \times \mathbf{S}_{vj}^{\alpha}(\tau)] \cdot [\mathbf{S}_{hi}^{\beta}(\tau') \times \mathbf{S}_{vj}^{\beta}(\tau')] \}. \quad (15)$$

To make further progress we utilize the vector identity $(\mathbf{A} \times \mathbf{B}) \cdot (\mathbf{C} \times \mathbf{D}) = (\mathbf{A} \cdot \mathbf{C})(\mathbf{B} \cdot \mathbf{D}) - (\mathbf{B} \cdot \mathbf{C})(\mathbf{A} \cdot \mathbf{D})$ to reduce mixed vector products along with integrations over auxiliary variables according to the formulas

$$\exp(\pm ab) = \int \frac{dx dy}{2\pi i} \exp[\pm(xy - ax - by)], \quad \exp(a^2/2) = \int \frac{dx}{\sqrt{2\pi}} \exp(-x^2/2 - ax), \quad (16)$$

allowing to decouple various quartic interactions in Eq. (15) and reduce Eq. (14) to the effective single-filament problem. In terms of the functional integrals

$$\langle Z \rangle_{\text{av}} = \int \prod_{\alpha\beta} \prod_{\mu\nu} DQ_{\mu\nu}^{\alpha\beta} DR^{\alpha\beta} DP_{1\mu\nu}^{\alpha\beta} DP_{2\mu\nu}^{\alpha\beta} \exp(-2NL[\mathbf{P}_1, \mathbf{P}_2, \mathbf{Q}, \mathbf{R}]), \quad (17)$$

involving the nonlocal (in time) tensor fields $X_{\mu\nu}^{\alpha\beta}(\tau, \tau')$ ($\mathbf{X} \equiv \mathbf{P}_1, \mathbf{P}_2, \mathbf{Q}$) and $R^{\alpha\beta}(\tau, \tau')$ where $\mu, \nu = x, y$. The effective local Lagrangian reads

$$L[\mathbf{P}_1, \mathbf{P}_2, \mathbf{Q}, \mathbf{R}] = \text{Tr} \mathbf{P}_1 \mathbf{P}_2 + \text{Tr} \mathbf{Q}^2 + \text{Tr} \mathbf{R}^2 - \ln \Phi[\mathbf{P}_1, \mathbf{P}_2, \mathbf{Q}, \mathbf{R}], \quad (18)$$

with

$$\text{Tr}\mathbf{XY} = \int_0^\beta d\tau \int_0^\beta d\tau' \sum_{\alpha\beta} \sum_{\mu\nu} X_{\mu\nu}^{\alpha\beta}(\tau, \tau') Y_{\nu\mu}^{\beta\alpha}(\tau', \tau), \quad \text{Tr}\mathbf{R}^2 = \int_0^\beta d\tau \int_0^\beta d\tau' \sum_{\alpha\beta} R^{\alpha\beta}(\tau, \tau') R^{\beta\alpha}(\tau', \tau). \quad (19)$$

Here,

$$\Phi[\mathbf{P}_1, \mathbf{P}_2, \mathbf{Q}, \mathbf{R}] = \text{Tr} e^{-\beta H_0 T_\tau} \exp \left[- \int_0^\beta d\tau \int_0^\beta d\tau' \hat{H}_{\text{eff}}(\tau, \tau') \right] \quad (20)$$

is the effective time-dependent single-filament Hamiltonian describing interactions between replicas

$$\hat{H}_{\text{eff}}(\tau, \tau') = -E_J \sum_{\alpha\beta} \sum_{\mu\nu} \left[Q_{\mu\nu}^{\alpha\beta}(\tau, \tau') - \frac{1}{2} P_{1\mu\nu}^{\alpha\beta}(\tau, \tau') + R^{\alpha\beta}(\tau, \tau') \delta_{\mu\nu} - \frac{1}{2} P_{2\nu\mu}^{\alpha\beta}(\tau, \tau') \right] S_\mu^\alpha(\tau) S_\nu^\beta(\tau'). \quad (21)$$

In the thermodynamic limit, $N \rightarrow \infty$, the steepest descent method can be used which amounts to finding the stationary points $X_{0,\mu\nu}^{\alpha\beta}$ and $R_0^{\alpha\beta}$ determined by the extremal conditions

$$\delta L[\mathbf{P}_1, \mathbf{P}_2, \mathbf{Q}, \mathbf{R}] / \delta X_{\mu\nu}^{\alpha\beta} = 0, \quad \delta L[\mathbf{P}_1, \mathbf{P}_2, \mathbf{Q}, \mathbf{R}] / \delta R^{\alpha\beta} = 0. \quad (22)$$

Thus, one obtains

$$G_{0,\mu\nu}^{\alpha\beta}(\tau - \tau') = \frac{E_J}{2} G_{\mu\nu}^{\alpha\beta}(\tau - \tau'), \quad R_0^{\alpha\beta}(\tau - \tau') = \frac{E_J}{2} \sum_{\mu} G_{\mu\mu}^{\alpha\beta}(\tau - \tau'), \quad P_{0,1\mu\nu}^{\alpha\beta}(\tau - \tau') = P_{0,2\nu\mu}^{\alpha\beta}(\tau - \tau') = \frac{E_J}{2} G_{\mu\nu}^{\alpha\beta}(\tau - \tau'), \quad (23)$$

where the correlation function describing the dynamic self-interaction is

$$G_{\mu\nu}^{\alpha\beta}(\tau - \tau') = \frac{\text{Tr} e^{-\beta H_0 T_\tau} S_\mu^\alpha(\tau) S_\nu^\beta(\tau') \exp \left[- \int_0^\beta d\tau_1 \int_0^\beta d\tau_2 \hat{H}_{\text{eff}}(\tau_1, \tau_2) \right]}{\text{Tr} e^{-\beta H_0 T_\tau} \exp \left[- \int_0^\beta d\tau_1 \int_0^\beta d\tau_2 \hat{H}_{\text{eff}}(\tau_1, \tau_2) \right]}. \quad (24)$$

Equation (24) represents an *exact* self-consistency condition for the replica-dependent matrix Green function $G_{\mu\nu}^{\alpha\beta}(\tau - \tau')$ of the quantum disordered Josephson model (3). For classical spin glasses, this is a matrix $G^{\alpha\beta}$, where the off-diagonal components of $q_{\alpha\beta}$ can be related to the spin-glass-like Edwards-Anderson order parameter

$$q_{\text{EA}} = \max_{\alpha \neq \beta} (q^{\alpha\beta}), \quad q^{\alpha\beta} = G^{\alpha\beta}(\tau - \tau') \delta_{\mu\nu}, \quad (25)$$

which is purely static (i.e., “imaginary-time” independent) and vanishes on the superconducting-glass-paracoherent phase boundary. However, for the quantum problem the time-dependent fluctuations of the replica-diagonal components $G_{\mu\nu}^{\alpha\alpha}(\tau - \tau')$ must be considered in the “imaginary” Matsubara time τ . More precisely, the replica-diagonal part $G^{\alpha\alpha}$ is not an order parameter because its expectation value is nonzero on both sides of the transition; nonetheless it has to be determined self-consistently along with the glass order parameter $q^{\alpha\beta}$.

IV. CALCULATION OF THE PHASE DIAGRAM

A general solution of the self-consistency equation (24) poses a rather difficult problem since the quantum-mechanical nature of the problem requires that the

time dependence of replica-diagonal dynamic parameters $G_{\mu\nu}^{\alpha\alpha}(\tau - \tau')$ have to be determined self-consistently. Therefore, we employed here the static ansatz (cf. Ref. 21) which retains only the $\omega_{\neq 0}$ Fourier component $r \delta_{\mu\nu} = (1/\beta) \int_0^\beta G_{\mu\nu}^{\alpha\alpha}(\tau)$ of the dynamic self-interaction. Furthermore, since we are here interested mainly in the critical line separating the glass and paracoherent phases (where $q_{\text{EA}} = 0$) we employ the replica-symmetry assumption ($q^{\alpha\beta} \equiv q$ for $\alpha \neq \beta$). In this way we avoid the subtle intricacies of the replica symmetry breaking (which, however, will be important *inside* the glass-phase region). Therefore,

$$G_{\mu\nu}^{\alpha\beta}(\tau) = [r \delta_{\alpha\beta} + q(1 - \delta_{\alpha\beta})] \delta_{\mu\nu} \quad (26)$$

and the effective Hamiltonian becomes

$$\hat{H}_{\text{eff}}(\tau, \tau') = - \frac{E_J^2}{2} \sum_{\alpha\beta} \sum_{\mu} [2r \delta_{\alpha\beta} + 2q(1 - \delta_{\alpha\beta})] S_\mu^\alpha(\tau) S_\mu^\beta(\tau'). \quad (27)$$

Consequently, the averaged free-energy density (9), in the replica $n \rightarrow 0$ limit, becomes

$$\beta \mathcal{F}_{\text{av}} = \frac{1}{2} (\beta E_J)^2 (r^2 - q^2) - \int_0^\infty \sigma d\sigma e^{-\sigma^2/2} \ln Z(\sigma),$$

$$Z(\sigma) = \int_0^\infty \rho d\rho e^{-\rho^2/2},$$

$$\mathcal{Z}(\sigma, \rho), \mathcal{Z}(\sigma, \rho) = \text{Tr}_\phi \exp [-\beta \mathcal{H}_\phi(\sigma, \rho)],$$

$$\mathcal{H}_\phi(\sigma, \rho) = -\frac{K}{2} \frac{\partial^2}{\partial \phi^2} - E_J \sqrt{2} (\sigma \sqrt{q} + \rho \sqrt{r-q}) \cos(\phi). \quad (28)$$

Here we have employed integrations over the auxiliary variables σ, ρ and $\text{Tr}_\phi \dots = \sum \langle \Psi | \mathcal{L}(\phi) | \dots | \Psi \rangle$, with $|\Psi\rangle$ being the eigenstates of the operator $\mathcal{H}_\phi(\sigma, \rho)$ and

$$q = \int_0^\infty \sigma d\sigma e^{-\sigma^2/2} \langle \cos(\phi) \rangle_\sigma^2, \\ r = \int_0^\infty \sigma d\sigma e^{-\sigma^2/2} \langle \cos^2(\phi) \rangle_\sigma \quad (29)$$

where

$$\langle \dots \rangle_\sigma = \frac{1}{Z(\sigma)} \int_0^\infty \rho d\rho e^{-\rho^2/2} \\ \times \text{Tr}_\phi \dots \exp [-\beta \mathcal{H}_\phi(\sigma, \rho)]. \quad (30)$$

The freezing temperature, i.e., the onset of the glassy phase, is marked by a nonzero value of the spin-glass order parameter q . We can now establish the equation for the critical line $T_c(K)$ by expanding the free energy in powers of q and equating the coefficient of q^2 to zero. We find that

$$\beta_c E_J r(\beta_c, K) = 1, \quad (31)$$

and the self-consistency equation for r is [cf. Eq. (28)]

$$r = \frac{\int_0^\infty \rho d\rho e^{-\rho^2/2} \text{Tr}_\phi \cos^2(\phi) \exp [-\beta \mathcal{H}_\phi(\rho, 0)]}{\int_0^\infty \rho d\rho e^{-\rho^2/2} \text{Tr}_\phi \exp [-\beta \mathcal{H}_\phi(\rho, 0)]} \quad (32)$$

with the effective single site quantum rotor Hamiltonian

$$\mathcal{H}_\phi(\rho, 0) = -\frac{K}{2} \frac{\partial^2}{\partial \phi^2} - \mu \cos(\phi), \quad (33)$$

where $\mu = E_J \rho \sqrt{2r}$. Finally, using partial integrations it is convenient to represent the self-consistency equation for r as

$$\int_0^\infty \rho d\rho e^{-\rho^2/2} \mathcal{Z}(\rho, 0) [2(\beta E_J)^2 r^2 + 3 - \rho^2] = \mathcal{Z}(0, 0), \quad (34)$$

where $\mathcal{Z}(0, 0) = \theta_3(0, \beta K/2)$, and $\theta_3(0, \beta K/2)$ is the theta function

$$\theta_3(z, t) = 1 + 2 \sum_{k=1}^{\infty} \cos(2kz) e^{-k^2 t}. \quad (35)$$

To proceed, we have to explicitly evaluate $\mathcal{Z}(\rho, 0)$ which amounts to finding the eigenenergy of the effective quantum rotor Hamiltonian (33) as a function of (arbitrary positive) ρ . This is a standard eigenvalue problem involving a Mathieu-type differential equation. Unfortunately, it is difficult to obtain a useful analytical solution for general values of K and μ in this way. Therefore, we treat the problem in various charging energy-temperature regimes using a combination of calculational approaches to construct the entire phase diagram.

A. Semiclassical WKB limit

For $k_B T \gg E_J$, $K/E_J \ll 1$ the charging energy is expected to play a subdominant role. Our approach here is to consider the ‘‘potential energy’’ $\mu \cos(\phi)$ in the quantum rotor Hamiltonian (33) as the leading contribution and treat charging energy perturbatively. At lowest order in K one obtains²²

$$\mathcal{H}_\phi(\rho) \approx -E_J \rho \sqrt{2r} [1 - (\beta K/24)] \cos(\phi). \quad (36)$$

Furthermore, by using the identity

$$I_n(\lambda) = \int_0^{2\pi} \frac{d\phi}{2\pi} e^{\lambda \cos(\phi)} \cos(n\phi), \quad (37)$$

where $I_n(\lambda)$ is the modified Bessel function of n th order. Using Eq. (34), the critical boundary for small K is readily obtained from

$$\int_0^\infty \rho d\rho e^{-\rho^2/2} (5 - \rho^2) I_0 \left[\rho \sqrt{2\beta E_J} \left(1 - \frac{\beta K}{24} \right) \right] = 1. \quad (38)$$

For $K=0$ at the classical critical point we have $k_B T_c / E_J = \sqrt{2}/2 \approx 0.71$ (cf. Ref. 23).

B. Variational method

The quantum-mechanical partition function $\mathcal{Z}(\rho, 0)$ in Eq. (34) can be approximated by an effective classical function via the path-integral formalism using a variational method. Following Refs. 24,25 (see the Appendix for details pertaining to this problem) we obtain the critical boundary between the glassy and paracoherent phases from

$$\int_0^\infty d\rho \rho e^{-\rho^2/2 + \xi(\rho)} \frac{\beta K \Omega_c(\rho) (5 - \rho^2)/2}{\sinh [\beta K \Omega_c(\rho)/2]} I_0 \left[\rho \sqrt{2\beta E_J} \right] \\ \times \exp \left(-\frac{a_c^2(\rho)}{2} \right) = 1, \quad (39)$$

where ξ_c , a_c^2 , and Ω_c^2 are determined (for a given value of ρ) from the set of self-consistency equations:

$$\xi_c(\rho) = \frac{\beta_c K}{2} \Omega_c^2 a_c^2, \\ a_c^2 = \frac{1}{\beta K \Omega_c^2} \left[\frac{\beta K \Omega_c}{2} \coth \left(\frac{\beta K \Omega_c}{2} \right) - 1 \right], \\ \Omega_c^2 = \rho \frac{\sqrt{2\beta_c E_J}}{\beta_c K} \exp \left(-\frac{a_c^2}{2} \right). \quad (40)$$

Numerical evaluation of Eq. (39) reveals a reentrance in the low-temperature region from the glassy phase back to the paracoherent state (see Fig. 2) and the zero-temperature critical value of reduced charging energy $\alpha_c = K/E_J \approx 1.1$. In the opposite semiclassical limit, $K \rightarrow 0$ Eq. (39) reduces to Eq. (38) (see the Appendix). The important question arises as to whether the predicted reentrant feature is a genuine property of the model or is an artifact of the approximation. We note that the variational method implementation^{24,25} considers that the paths must satisfy the boundary condition $\phi(0) = \phi(\beta)$, appropriate for a quantum particle in a periodic potential rather than a quantum rotor system [Eq. (33)] with 2π periodic wave functions. Therefore, the proper boundary conditions for the latter must be $\phi(0) = \phi(\beta) + 2\pi m$ (where $m = 0, \pm 1, \pm 2, \dots$ are the winding numbers) which appear not to be accounted for consistently in this approach. This might have consequences for the low-temperature behavior of the system and we are therefore motivated to look for an alternative method to study the quantum $T \rightarrow 0$ limit.

As we shall see next, the reentrant transition found in the variational approximation is not found in the low-temperature expansion. Thus it is most likely an artifact of the variational approach, which is strictly a high-temperature approximation.

C. Perturbation expansion about the quantum limit

Assuming that $K \gg E_J$, the potential energy of the quantum rotor (33) $\mu \cos(\phi)$ may be treated perturbatively using the standard Rayleigh-Schrödinger approach. The unperturbed part is then $-(K/2)\partial^2/\partial\phi^2$ with eigenfunctions

$$|\Phi_m^0(\phi)\rangle = \frac{1}{\sqrt{2\pi}} \exp(im\phi). \quad (41)$$

To lowest nontrivial order we get

$$E_m(\rho) = \frac{K}{2} \left[m^2 - \frac{4(E_J/K)^2 r \rho^2}{1 - 4m^2} \right], \quad (42)$$

so that

$$\sum_{m=-\infty}^{m=+\infty} e^{-m^2 \beta_c K/2} \frac{16m^4 + 8m^2[3(E_J/K) - 1] - 8(E_J/K)^2 - 6(E_J/K) + 1}{[4m^2 + 4(E_J/K) - 1]^2} = 0. \quad (44)$$

At $T=0$ the $m=0$ term is the only one that contributes and thus

$$\frac{1}{1 - 4[E_J/K_c(T=0)]} \left[5 - \frac{2}{1 - 4[E_J/K_c(T=0)]} \right] = 1, \quad (45)$$

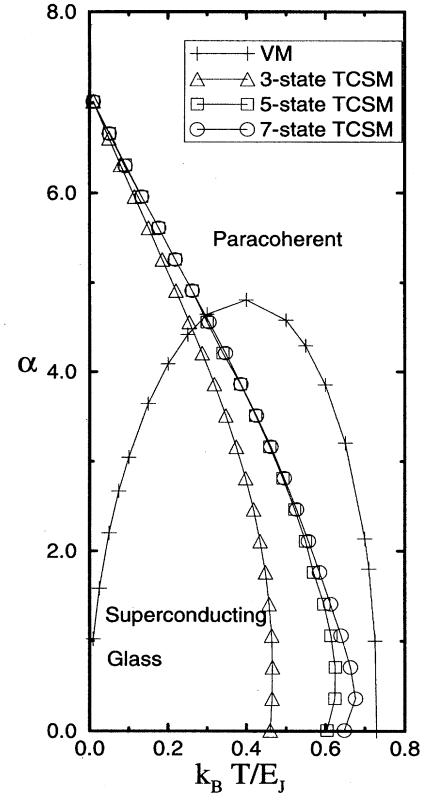


FIG. 2. Temperature-charging energy, $T-\alpha$, phase diagram of the disordered Josephson-coupled array of superconducting wires with self-charging energies. Here $\alpha = K/E_J$, with K the charging energy and E_J the Josephson coupling energy. The results were obtained from variational method (VM) and truncated charge state models (TCSM) for different numbers of charge states as indicated. We note, as discussed in the text, that the reentrant behavior obtained from the variational calculation does not emerge from the low-temperature analysis and thus it must be an artifact of the variational approximation.

$$\mathcal{Z}(\rho) = \sum_{m=-\infty}^{m=+\infty} e^{-\beta E_m(\rho)}, \quad (43)$$

and the critical boundary condition is implemented after performing the integration obtaining

that gives $\alpha_c(T=0) = (6 + 2\sqrt{17})/2 \approx 7.1231$ which differs quite significantly from the critical value of α obtained previously by the variational method.^{11,12} We attribute this difference again to the fact that in the present formulation of the variational method^{24,25} the boundary conditions for the superconducting phase variables are not compatible with the discrete nature of the charge-transfer process which becomes

especially important as $T \rightarrow 0$. As a result, a discrepancy in the low-temperature limit is expected, whereas in the classical limit ($K \rightarrow 0$) T_c is reproduced correctly.

D. Truncated charge space projection method

The expansion about the quantum limit from the previous subsection is not sufficient to treat the critical boundary away from the quantum critical point $\alpha_c(T=0)$, where the condi-

tion $\alpha \gg 1$ is no longer valid. A nonperturbative approach is sought and of particular interest are the truncated charge state models (TSCM) spanned by the charge states of the operator $-(K/2)\partial^2/\partial\phi^2$ in a restricted, finite-dimensional, Hilbert space. This can be interpreted as an approximation of a suitable quasispin model. Using the eigenstates of the charge operator (41) one can readily calculate the matrix elements of the operators involved:

$$N_{km} = \langle \Phi_k(\phi) | \hat{n}(\phi) | \Phi_m(\phi) \rangle = \int_0^{2\pi} \frac{d\phi}{2\pi} \exp(-ik\phi) \left(\frac{2e}{i} \frac{\partial}{\partial\phi} \right) \exp(im\phi) = 2em\delta_{km},$$

$$[S_x]_{km} = \langle \Phi_k(\phi) | \cos(\phi) | \Phi_l(\phi) \rangle = \int_0^{2\pi} \frac{d\phi}{2\pi} \exp[-i(k-m)\phi] \cos(\phi) = \frac{1}{2}(\delta_{k-m-1,0} + \delta_{k-m+1,0}). \quad (46)$$

In particular, for $k, m = 0, \pm 1$, one has

$$N_{km} = [\mathcal{S}_z]_{km}, \quad [S_x]_{kl} = \left[\frac{\mathcal{S}_x}{\sqrt{2}} \right]_{kl}, \quad (47)$$

where $\mathcal{S}_a (a = z, x)$ are given by

$$\mathcal{S}_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad \mathcal{S}_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \quad (48)$$

Consequently, the lowest-order quasispin model belongs to $\mathcal{S} = 1$ and its Hilbert space is spanned by the charge states $|0\rangle, |\pm 1\rangle$. It is now useful to recast the quantum rotor Hamiltonian (33) in the $\mathcal{S} = 1$ pseudospin language as

$$H = (K/2)\mathcal{S}_z^2 - (\mu/\sqrt{2})\mathcal{S}_x. \quad (49)$$

In the context of magnetic spin glasses the first charging energy term in Eq. (49) refers to a single-ion crystal anisotropy, which opposes ordering in the x - y plane.²⁶ Thus the system will exhibit a phase transition driven by quantum fluctuations of the transverse (pseudo)spin component. Correspondingly, for the statistical sum $\mathcal{Z}(\rho, 0)$ we have

$$\mathcal{Z}(\rho, 0) \equiv \mathcal{Z}_{\mathcal{S}=1}(\rho, 0) = \exp\left(-\frac{\beta_c K}{2}\right) + 2 \exp\left(-\frac{\beta_c K}{4}\right) \cosh\left(\frac{\sqrt{(\beta_c K)^2 + (\beta_c E_J)^2} \rho^2}{4}\right), \quad (50)$$

while

$$\int_0^\infty \rho d\rho e^{-\rho^2/2} \mathcal{Z}(\rho, 0) (5 - \rho^2) = \mathcal{Z}(0, 0), \quad (51)$$

and the critical line $T_c(\alpha)$ can be readily calculated numerically using Eq. (51) (see Fig. 2). We found $\alpha_c(T=0) = 7$ in good agreement with the perturbative approach. The classical value $T_c(\alpha=0)$ is underestimated as a result of restricting

the original charge state Hilbert space. We have also examined models spanned by five ($|0\rangle, |\pm 1\rangle, |\pm 2\rangle$) and seven ($|0\rangle, |\pm 1\rangle, |\pm 2\rangle, |\pm 3\rangle$) charge states which better approximate the behavior at high temperatures. For both cases it is possible to derive critical line equations analogous to Eq. (50) analytically. However, the corresponding formulas are too lengthy to reproduce here. We found that close to $T_c(\alpha=0)$ TCSM exhibit a small reentrance for a certain interval of α 's. However, as the number of charge states m_{charge} increases this interval became narrower and presumably disappears for $m_{\text{charge}} \rightarrow \infty$ indicating that this might be a spurious feature due to the restriction imposed on the original infinite-dimensional Hilbert space of charge states.

V. CONCLUSIONS

In this paper we have studied the competition between quantum and thermal fluctuations in a superconducting glass state using a positionally disordered model of ultrathin Josephson coupled wires in a transverse magnetic field. We focused our attention in the regime where the magnetic field produces large frustration in a system of randomly spaced Josephson microjunctions with a large number of nearest neighbors $z = N$. We investigated this model by spin-glass-inspired techniques in the $N \rightarrow \infty$ limit, showing that the model is a superconducting analog of a quantum spin glass with Dzialoshinsky-Moriya time-reversal breaking interaction.

We studied the phase boundary separating the paracoherent from glassy phases as a function of the charging energy associated with the small capacitance of an individual wire. We found that, for sufficiently large charging energy, the glassy ground state is destabilized due to the strong quantum fluctuations. This is reminiscent of the scenario found in ordered Josephson-junction arrays, where charging energy effects can lead to the destruction of long-range phase coherence by zero-point quantum fluctuations. However, this analogy is not complete as in the glassy phase there is no long-range order and, therefore, the destructive role of quantum fluctuations is less transparent. We note that it is not

easy to improve our static ansatz analytically. We expect that a direct quantum Monte Carlo calculation, where the imaginary time direction is discretized, will yield information inside the phase boundary. The shortcoming of this approach is that one cannot get too close to the $T=0$ region. We leave this problem for the future.

The results presented here can be tested experimentally in artificially fabricated disordered arrays of ultrathin filaments provided that the charging energy of a wire is large enough to produce substantial superconducting quantum phase fluctuations. For example, $T_c(\alpha)$ should be observable by resistivity measurements. In obtaining our results for the phase diagram we considered the solution which does not break the replica symmetry. Although a broken replica symmetry solution is not required to trace the critical phase boundary (where the order parameter vanishes) it is important when distinguishing equilibrium from nonequilibrium properties of the glassy phase. In the low-temperature phase, as usual, the strongest signatures of glassiness are in the dynamical properties. Similar to the magnetic spin glasses, nonergodic behavior is likely to manifest itself in differences between field-cooled and zero-field prepared samples via typical effects like hysteresis, remanence, aging effects, etc. In the classical limit of our model (vanishing of charging energy) all these history-dependent signatures of a glassy state have

been observed.^{12,27,5} The corresponding nonequilibrium metastable states might be probed, e.g., via ac conductivity measurements determining the barriers separating metastable states and the associated distribution of relaxation times as a function of charging energy.

Although the system of superconducting ultrathin wires constitutes an interesting experimental setting to test the predictions of the infinite-range interaction theory, the investigation of the behavior of disordered quantum truly 2D array would require knowledge relevant to short-range spin glasses and presents a difficult subject for further study.

ACKNOWLEDGMENTS

This work has been partially supported by NSF Grants Nos. DMR-95-21845 and PHYS-94-07194 (ITP at UC, Santa Barbara), and by the U.S. Information Agency.

APPENDIX: VARIATIONAL METHOD

In this appendix we give the specific details of the derivation of Eq. (39) based on the variational approach.^{24,25} The path-integral formulation of the partition function involves an infinite product of periodic paths $\phi(\tau) = \phi_0 + \sum_{\ell=1}^{\infty} (\phi_{\ell} e^{i\omega_{\ell}\tau} + \text{c.c.})$ in the form

$$\mathcal{Z}(\rho, 0) = \int \frac{d\phi_0}{\sqrt{2\pi\beta K}} \prod_{\ell=0}^{\infty} \left[\int \frac{d\phi_{\ell}^{\text{re}} d\phi_{\ell}^{\text{im}}}{\pi / (\beta K \omega_{\ell}^2)} \right] \times \exp \left\{ -\beta K \sum_{\ell=1}^{\infty} \omega_{\ell}^2 |\phi_{\ell}|^2 - \mu \int_0^{\beta} d\tau \cos \left[\phi_0 + \sum_{\ell=1}^{\infty} (\phi_{\ell} e^{-i\omega_{\ell}\tau} + \text{c.c.}) \right] \right\}. \quad (\text{A1})$$

The essence of the variational method^{24,25} is to approximate (52) by an effective *classical* partition function

$$\mathcal{Z}(\rho, 0) = \int \frac{d\phi_0}{\sqrt{2\pi\beta K}} e^{-\beta V_{\text{eff}}(\phi_0)}. \quad (\text{A2})$$

Here, the associated effective potential V_{eff} is given by

$$V_{\text{eff}} = \frac{1}{\beta} \ln \left[\frac{\sinh(\beta K \Omega / 2)}{\beta K \Omega / 2} \right] + V_{a^2} - \frac{K}{2} \Omega^2 a^2. \quad (\text{A3})$$

The unknown functions $a^2(\phi_0)$, $V_{a^2}(\phi_0)$, and $\Omega(\phi_0)$ are determined by using the extremal principle from the self-consistency conditions

$$\begin{aligned} a^2 &= \frac{1}{\beta K \Omega^2} \left[\frac{\beta K \Omega}{2} \coth \left(\frac{\beta K \Omega}{2} \right) - 1 \right], \\ V_{a^2} &= -\mu \int_{-\infty}^{+\infty} \frac{d\phi}{\sqrt{2\pi a^2}} \exp \left[-\frac{(\phi - \phi_0)^2}{2a^2(\phi_0)} \right] \cos(\phi), \\ &= -\mu \exp \left(-\frac{a^2}{2} \right) \cos(\phi_0), \end{aligned}$$

$$\Omega^2 = \frac{\mu}{K} \exp \left(-\frac{a^2}{2} \right) \cos(\phi_0). \quad (\text{A4})$$

For low temperatures we seek for the uniform solution (i.e., with $\phi_0=0$) of Eq. (A4) [cf. Eq. (40)]. In the regime of high temperature and small quantum fluctuations by using the expansion

$$\frac{1}{x} \left[\frac{x}{2} \coth \left(\frac{x}{2} \right) - 1 \right] = \frac{x}{12} - \frac{x^3}{720} + O(x^4), \quad (\text{A5})$$

we have for $V_{\text{eff}}(\phi_0)$

$$\begin{aligned} \mathcal{H}(\rho, 0) &\rightarrow -E_J \rho \sqrt{2r} \exp \left(-\frac{\beta K}{24} \right) \cos(\phi_0), \\ &\approx -E_J \rho \sqrt{2r} \left(1 - \frac{\beta K}{24} \right) \cos(\phi_0), \end{aligned} \quad (\text{A6})$$

i.e., $V_{\text{eff}}(\phi_0)$ reduces to the semiclassical WKB result (36).

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