Nonlinear transport of electrons in a quasi-one-dimensional channel on the liquid-helium surface

Sviatoslav S. Sokolov
Departamento de Física, Universidade Federal de São Carlos, 13565-905 São Carlos, São Paulo, Brazil and B. I. Verkin Institute for Low Temperature Physics and Engineering, Academy of Sciences of Ukraine, 310164, Kharkov, Ukraine

Guo-Qiang Hai and Nelson Studart
Departamento de Física, Universidade Federal de São Carlos, 13565-905 São Carlos, São Paulo, Brazil
(Received 12 April 1995)

The nonlinear transport of electrons bound in quasi-one-dimensional channels on the surface of liquid helium is studied theoretically in the case of large clamping fields normal to the surface. It is shown that, due to the weakness of the electron-ripplon interaction, the nonlinear effects in the electron mobility arise at driving electric fields near 10^{-2} V/cm. The mobility is an increasing function of the driving field for all clamping fields considered. The calculations are carried out in the two limiting regimes corresponding to different roles of the electron-electron correlations. The results in the two approximations differ significantly, which allows us to check out the role of the electron-electron interaction in the quasi-one-dimensional charge system through a comparison with experimental results.

I. INTRODUCTION

In the last years the studies of quasi-one-dimensional (Q1D) electron systems over the liquid-helium surface became one of most developing trends in the physics of low-dimensional charge systems. Q1D electron systems, which are realized near a superfluid helium surface with a finite curvature, have all the essential features of quasi-two-dimensional surface electrons (SE) over bulk helium such as cleanness, homogeneity and the possibility of modifying the charge density and the strength of both the electron-electron and electron-ripplon interactions by varying the clamping electric field perpendicular to the surface. Furthermore, electrons can move freely in one direction, along the channel axis, due to the lateral potential imposed by geometrical confinement, in contrast to the SE system where the electrons are free to move in the whole plane.

The methods for creating Q1D electron systems, by using the properties of liquid helium, were first conceived theoretically.[1–3] Recently, physical arrangements of the system were realized experimentally as pointed out by Kovdrya and Nikolaenko[4] and by Kirichek *et al.*[5] In Ref. 4 a strong anisotropy of the electron mobilities, along and across a series of parallel channels created by grooves filled of helium, was observed. In Ref. 5 experimental data were obtained for the electron conductivity in a single Q1D channel on the helium surface strongly distorted by capillary forces due to a wedge geometry. Sokolov and Kirichek[6] found the dispersion relation of the plasma excitations in the Q1D electron system confined by a parabolic potential and Sokolov and Studart[7] determined the electronic structure in the presence of a magnetic field in order to establish connection with the spectroscopic transitions between the confined states.

The study of the kinetic properties is the most useful tool to get information about the peculiarities of the electron systems in one more reduced dimensionality, as it was demonstrated in the case of SE. In a previous paper,[8] hereafter referred to as I, the authors made a theoretical analysis of the electron mobility in the Q1D system in the linear regime of small driving fields when the electrons are in thermal equilibrium with the liquid helium. The role of electron-electron correlations was investigated by solving the Boltzmann equation in the two opposite limiting cases of electron correlations, the one-electron approximation and the complete control regime, where the electron-electron collision frequency is much larger than that typical frequencies of electron-ripplon and electron-gas collisions. It is well known that for SE's over a bulk helium surface, the gaseous, liquid and crystal phases of the two-dimensional (2D) system are realized in terms of the fundamental plasma parameter of the 2D electron plasma $\Gamma = e^2(\pi n_s)^{1/2}$, where n_s is the electron density. The electron density can be easily varied by changing the holding field $E_\perp$ in the case of SE's. For $\Gamma < 1$, the one-electron approximation describes well the gas state. For $\Gamma \geqslant 137$ the system undergoes a transition to a Wigner crystal. At intermediate values, $1 < \Gamma < 137$, the strong electron-electron interactions lead to the liquid state. It was shown in Ref. 9 that for low holding fields and, hence, low electron densities (small Γ), the experimental data are coincident with the theoretical mobility calculated by neglecting the interactions between electrons. But in the liquid state (intermediate Γ) the so-called complete control regime, where the average drift velocity of electrons and the effective electron temperature can be introduced, was realized in a definite range of electron densities up to 4×10^8 cm^{-2}, where $\Gamma = 47$ and the single-particle approach is no longer applicable. The model was found to describe the kinetic properties of the SE's system over bulk helium.[9] The basic question should be understanding how a model of independent electrons can describe the main features of the transport properties of the system in the correlated regime. Nevertheless, it has been demonstrated that such a classical model based on quasielastic scattering of independent electrons gives results

in good agreement with the experimental results for zero magnetic field.[9–11] At low-field the results are still close to zero-field theoretical values,[11,10,12–16] but at higher fields, the magnetoconductivity depends on the electron density indicating that many-electron effects are important.[11,14] The data have been interpreted with excellent agreement in terms of the theory of Dykman[10,17] where the effect of the electron-electron interaction upon the electron scattering is due to the presence of a density dependent fluctuating electric field. The influence of many-electron effects on the high-field magnetoresistivity and the limit of applicability of single-electron approximation has been discussed in Refs. 10 and 18. At extremely strong magnetic field, the ultraquantum limit, the extended self-consistent Born approximation was demonstrated to describe the magnetotransport data perfectly.[19] Unfortunately, the conditions necessary to study many-electron effects in the Q1D system are not so well established at the present time because the anisotropic character of the electron motion and a complicated dependence of the electron density on the holding field. However, we hope that, as in the zero-field case for SE's, the complete control approximation should describe the main features of the electron transport in the Q1D system.

It was shown in I that in view of the weakness of the electron-ripplon and electron-vapor gas interactions, which are the main mechanisms of scattering, the mobility depends strongly on the temperature and the clamping electric field. So, we expect that such a weakness of the electron-scatter interactions can lead to substantial nonlinear effects with a strong driving-field dependence of the mobility. The role of nonlinear effects has been demonstrated since the earliest times of the investigations of SE transport.[20–31] In the present work we calculate the high-field mobility in the Q1D electron channel. We found in our calculation that the nonlinear effects in the electron mobility take place at driving electric fields $E_{\parallel}$ near 10^{-2} V/cm. We also show that the contribution of the two-ripplon process to the energy dissipation rate is the dominant one as compared with the one-ripplon process. As in I, we also consider in the nonlinear regime two opposite limiting cases of electron correlations, the partial control regime where the momentum relaxation frequency ν are much larger than the frequency of the electron-electron collisions ν_{ee} (corresponding to the one-electron approximation with an effective electron temperature) and the complete control regime, where ν_{ee} is much larger than that typical frequencies of electron-ripplon and electron-gas collisions. In both regimes ν is much bigger than the energy relaxation frequency ν^*. We show that electron correlations influence significantly the structure of the electron distribution function and lead to different dependences of the mobility on the driving field. The theoretical approach and the calculations of the energy loss rate functions and the electron mobilities in both approximations are presented in Sec. II. Section III is devoted to the discussion of the results obtained. Our conclusions are summarized in Sec. IV.

II. THEORETICAL FORMULATION

As in I, we consider a single Q1D channel formed by two dielectric plates meeting at a sharp angle (the wedge geometry) filled with liquid helium whose surface is strongly distorted due to the capillarity forces from the substrate with a curvature radius R about 10^{-4}–10^{-3} cm.[3–5] The channel is along the x axis. Under the action of a clamping electric field along the z axis (normal to the helium surface), the electrons move to the bottom of the channel. At $E_{\perp} > 10^2$ V/cm, the energy $eE_{\perp}R$ is much larger than the binding energy of SE on the helium film covering the top of the dielectric plates (near 100 K) and the electrons are favored energetically to stay at the bottom of the channel.[3] If the electron is displaced by a distance $y \ll R$ from the bottom across the channel, it is subjected to a parabolic potential given by

$$U(y) = \frac{m\omega_0^2 y^2}{2}, \tag{1}$$

which laterally confines the electron in the y direction with $\omega_0^2 = eE_{\perp}/mR$, where m and e are the mass and the charge of the electron, respectively. Under this potential the energy spectrum of the electron turns out to be

$$E_{n,l}(k_x) = \frac{\hbar^2 k_x^2}{2m} + (n+1/2)\hbar\omega_0 + \Delta_l, \tag{2}$$

where $n = 0,1,2,\ldots$ and k_x is the one-dimensional wave vector. The label $l = 1,2,3,\ldots$ numerates the quantized surface states along the z axis. Because the condition $\langle z\rangle_l \ll R$ is satisfied for not very large l, where $\langle z\rangle_l$ is the mean electron distance from the surface, the energy spectrum Δ_l can be taken as in the case of a flat helium surface. As it was shown in I, Eq. (2) is valid for wide range of clamping fields and for large enough values of n. According to our estimates in I, the energy gap $\hbar\omega_0$ between the states with different n (we shall call them transverse states) is near 0.3 K at $E_{\perp} = 450$ V/cm and 0.8 K at $E_{\perp} = 3000$ V/cm. For this reason at temperatures $T \sim 1$ K one should take into account the finite population of not only the ground ($n = 0$) but also of the excited transverse levels ($n \geq 1$). One should note that the experiments are carried out at $T = 1.5$ K.[5] By the contrary, the energy gap between the first two vertical levels ($l = 0$ and $l = 1$) is near 14 K for $E_{\perp} = 1000$ V/cm and 30 K for $E_{\perp} = 3000$ V/cm.[23] So, for high clamping fields, we can consider only the subband $l = 1$. In this case the electron subbands corresponding to the energy spectrum, given by Eq. (1) can be written as

$$\Psi_n(k_x) = \frac{1}{\sqrt{L_x}} \exp(i\ k_x x)\varphi_n(y)\chi_1(z)\ , \tag{3}$$

where L_x is the size of the system in the x direction

$$\varphi_n(y) = \frac{1}{\pi^{1/4} y_0^{1/2}} \frac{1}{\sqrt{2^n n!}} \exp\left(-\frac{y^2}{2y_0^2}\right) H_n\left(\frac{y}{y_0}\right), \tag{4}$$

$$\chi_1(z) = 2\gamma^{3/2} z e^{-\gamma z}\ , \tag{5}$$

where $y_0 = (\hbar/m\omega_0)^{1/2} \ll R$ is the localization length in the y direction and the parameter γ is related to the mean distance of the electron from the surface, $\langle z\rangle_1 = 3/2\gamma^{-1}$. In general, γ is dependent on $E_{\perp}$.[23] $H_n(t)$ is the Hermite polynomial.

The above description of the subband structure is used to calculate in the Born approximation the electron probabilities for scattering by helium atoms in the vapor phase which dominates at $T>1$ K and ripplons at lower temperatures. As it was shown earlier for SE on bulk helium, because the weakness of the electron-ripplon and electron-atom interactions, the temperature of the system can become quite different from the helium bath temperature, even at low enough driving electric fields.[22] As a consequence, the electron mobility may depend drastically on the driving field along the plane.[23,24] In the Q1D system, the warming up of electrons by a driving electric field along the x axis will change the occupation of the transverse levels even though the vertical level ($l=1$) is not altered. For this reason, a consistent theoretical analysis, taking into consideration the intra- as well intertransverse subband collisions is necessary in order to determine the dependence of the mobility on the driving field. This analysis should be given by using the explicit dependence of the electron distribution function on the driving field. However, previous studies demonstrated that the mathematical treatment is quite cumbersome and this method does not lead to much better results when compared with those obtained by using the approximation of effective electron temperature.[29,30] Such an approach has been successfully employed in theoretical calculations of the nonlinear mobility $\mu(E_\parallel)$ of SE.[22–24] Then we choose to employ the effective-electron-temperature approximation to derive $\mu(E_\parallel)$ in the case of the Q1D system. In this case, the electron distribution at the subband n depends explicitly on the effective temperature T_e and can be approximated by

$$f_{n0}(T_e)=\sqrt{\frac{2\pi\hbar^2}{mT_eL_x^2}}\frac{1}{Z_n}\exp\left(-\frac{n\hbar\omega_0}{T_e}-\frac{\hbar^2k_x^2}{2mT_e}\right), \quad (6)$$

where $Z_n=1/2[1+\coth(\hbar\omega_0/2T_e)]$. The procedure for calculating the electron mobility as a function of the effective temperature T_e is different in the partial-control and the complete-control regimes, but is the same as that used in semiconductor physics.[32,33]

A. The partial-control regime

In this case, the relation $\nu^* \ll \nu_{ee} \ll \nu$ is fulfilled. Due to the fact that the electron-ripplon and the electron-gas scattering are almost elastic the condition $\nu^* \ll \nu$ is easily satisfied. As described in I, the distribution function of the nth subband can be written as

$$f_n=f_n^{(0)}+\frac{k_x}{|k_x|}f_n^{(1)}. \quad (7)$$

If we substitute Eq. (7) into the Boltzmann transport equation for the distribution function f_n, one obtains in a straightforward way the following expressions:

$$f_n^{(0)}=f_{n0}(T_e) \quad \text{and} \quad f_n^{(1)}=-\frac{eE_\parallel}{\hbar(\nu_{\rm er}^{(n)}+\nu_{\rm eg}^{(n)})}\frac{\partial f_n^{(0)}}{\partial|k_x|}. \quad (8)$$

The frequencies of momentum relaxation, due to the electron-ripplon scattering $\nu_{\rm er}^{(n)}$ and to the electron-gas scattering $\nu_{\rm eg}^{(n)}$ are derived in detail in I and their expressions are

$$\nu_{\rm er}^{(n)}(x)=\frac{4}{\sqrt{\pi}}\frac{e^2E_\perp^2T}{\alpha\hbar^2\omega_0\sqrt{x}}B_r^{(n)}(x) \quad (9)$$

and

$$\nu_{\rm eg}^{(n)}(x)=\frac{4}{\sqrt{\pi}}\frac{\hbar n_g(T)A\gamma}{m\sqrt{x}}B_g^{(n)}(x), \quad (10)$$

where $x=\hbar^2k_x^2/(2m\hbar\omega_0)$, α is the surface tension coefficient of the liquid helium, $n_g(T)$ is the volume concentration of helium atoms in the gas phase, and $A=4.676\times10^{-16}$ cm^2 is the effective cross section for the electron-helium atom scattering. The functions $B_r^{(n)}(x)$ and $B_g^{(n)}(x)$ were given in I by Eqs. (24) and (27), respectively. Following the same steps as in I, we arrive at the electron mobility

$$\mu(\Theta)=\frac{2e}{\sqrt{\pi}mZ_n(\theta)}\left(\frac{\hbar\omega_0}{\theta}\right)^{3/2}\times\sum_{n=0}^{\infty}\exp\left(-\frac{n\hbar\omega_0}{\theta}\right)\int_0^\infty\frac{\sqrt{x}e^{-\frac{\hbar\omega_0}{\theta}x}dx}{\nu_{\rm er}^{(n)}(x)+\nu_{\rm eg}^{(n)}(x)} \quad (11)$$

[Eq. (22) in I] which leads to the following expression of the mobility in the partial control regime:

$$\mu(T_e)=\frac{\mu_\perp(\hbar\omega_0/T_e)^{3/2}}{1+\coth(\hbar\omega_0/2T_e)}\sum_{n=0}^{\infty}\exp\left(-\frac{n\hbar\omega_0}{T_e}\right)\times\int_0^\infty\frac{xe^{-(\hbar\omega_0x/T_e)}dx}{(T/\hbar\omega_0)B_r^{(n)}(x)+(\mu_\perp\hbar n_gA\gamma/e)B_g^{(n)}(x)}, \quad (12)$$

where $\mu_\perp=\alpha\hbar/meE_\perp^2$. We remark that Eq. (12) has a similar expression as that of Eq. (31) in I, which gives the mobility obtained in the equilibrium case in the one-electron approximation at $T_e=T$.

B. The complete-control regime

In this case, the frequencies must satisfy the condition $\nu^* \ll \nu \ll \nu_{ee}$. Now the distribution function should be represented by

$$f_n=f_{n0}(T_e)\exp\left(\frac{\hbar uk_x}{T}\right), \quad (13)$$

where u is the drift velocity of the electron system. To find the mobility $\mu=u/E_\parallel$, we multiply the Boltzmann equation by k_x and sum it over the possible electron states. After some manipulations we arrive at the following expression of the electron mobility $\tilde{\mu}$ in the complete-control regime:

$$\tilde{\mu}(T_e)=\mu_\perp\frac{T_e}{T}[1+\coth(\hbar\omega_0/2T_e)]\left[G_r(T_e)+\frac{\mu_\perp\hbar n_g(T)A\gamma T_e}{eT}\left(\frac{\hbar\omega_0}{T_e}\right)^{1/2}G_g(T_e)\right]^{-1}, \quad (14)$$

where

$$G_{r(g)}(T_e)=\sum_{n,n'}\exp\left(-\frac{\hbar\omega_0(n+n')}{2T_e}\right)D_{nn'}^{r(g)}(T_e)\ . \quad (15)$$

The functions $D_{nn'}^{(r)(g)}(T_e)$ are expressed in the same form as in I [see Eqs. (42), (43), (45), and (46) of I] with the replacement of T by T_e.

In order to obtain the dependence of the electron mobility on the driving field $E_\parallel$ in both regimes, by using Eqs. (12) and (14), we need to find out the dependence of the effective temperature on $E_\parallel$. This relation can be determined from the solution of the energy balance equation, which is obtained from the Boltzmann equation, by multiplying it by the energy $E_n(k_x)=\hbar^2k_x^2/2m+(n+1/2)\hbar\omega_0$ and sum over n and k_x. In the stationary case, the energy balance equation can be written as

$$e\mu E_\parallel^2=W_r(T_e)\ , \quad (16)$$

where $W_r(T_e)$ describes the electron energy transmitted to the scatter system due to the electron-ripplon interaction and μ stands for the mobility in both partial- and complete-control regimes. We have neglected the electron-gas scattering in the energy dissipation because the scattering by helium atoms is negligible for $T\leq 1$ K at moderate T_e even in the limit $E_\parallel\rightarrow 0$.[24] As was shown in Ref. 31, the function $W_r(T_e)$ can be split out into two terms

$$W_r(T_e)=W_{1r}(T_e)+W_{2r}(T_e)\ . \quad (17)$$

The function $W_{1r}(T_e)$ represents the scattering process involving one long-wavelength ripplon. In Ref. 31, it was demonstrated that the main contribution to $W_r(T_e)$, under the condition $T_e\gg T$, arises from the processes with two short-wavelength ripplons created in opposite directions, which we are representing in Eq. (17) by $W_{2r}(T_e)$. Even though these processes give negligible contribution in momentum relaxation, they can dominate in energy relaxation and are 2 orders of magnitude more intense than the one-ripplon process for $E_\perp\leq 1000$ V/cm.[23,24,28,34]

The electron energy loss can be written as

$$W_{1(2)r}(T_e)=\left(1-\frac{T}{T_e}\right)J(T_e)\ , \quad (18)$$

where $J(T_e)$ is the energy loss rate which is given by

$$J(T_e)=\sum_{n,k_x}P_n^{(1),(2)}(k_x)f_n(k_x,T_e)\ . \quad (19)$$

The functions $P_n^{(1),(2)}(k_x)$ determine the rate of energy losses due to the electron-ripplon interaction in one- and two-ripplon processes. The general expressions for these functions were derived in Ref. 31 for the SE system. Following the same steps, one obtains straightforwardly

$$P_n^{(1)}(k_x)=\frac{2\pi}{\hbar S}\sum_{n,\vec{q}'}|\langle n'|e^{iq_yy}|n\rangle|^2\times Q_{\vec{q}}^2\,|\langle 1|V_q(z)|1\rangle|^2\hbar\omega_r(\vec{q})\times\delta\left(\hbar\omega_0(n-n')+\frac{\hbar^2}{2m}(q_x^2+2k_xq_x)\right) \quad (20)$$

where the matrix elements must be evaluated with the subband wave functions given by Eqs. (3)–(5). In Eq. (20) $\vec{q}$ is the two-dimensional wave vector of the ripplon with a dispersion law $\omega_r(\vec{q})^2\simeq(\alpha/\rho)q^3$ (ρ is the liquid-helium density); $Q_{\vec{q}}=(\hbar q/2\rho\omega_r)^{1/2}$; S is the area of the liquid surface and $V_q(z)$ is the electron-ripplon interaction.[35] For the two-ripplon processes, we obtain in the same manner

$$P_n^{(2)}(k_x)=\frac{4\pi\kappa^2F_{11}^2}{\hbar^2S^2}\sum_{n,\vec{q},\vec{s}'}|\langle n'|e^{is_yy}|n\rangle|^2\times Q_{\vec{q}}^2\,Q_{\vec{q}-\vec{s}}^2(\hbar\omega_r(\vec{q})+\hbar\omega_r(\vec{q}-\vec{s}))\times(N_{\vec{q}}+1)(N_{\vec{q}-\vec{s}}+1)\,\delta\left(\hbar\omega_0(n-n')+\frac{\hbar^2}{2m}\times(s_x^2+2k_xs_x)+\hbar\omega_r(\vec{q})+\hbar\omega_r(\vec{q}-\vec{s})\right), \quad (21)$$

where $N_{\vec{q}}$ is the Bose distribution function for the ripplons, $\vec{s}$ is the wave vector of the pair of ripplons created (we assume that $s\ll q$), $\kappa^2=2mV_0/\hbar^2$, and $F_{11}=V_0[\Phi_1(0)]^2$. V_0 is the height of the potential barrier which prevents the electron to penetrate in the liquid-helium phase and $\Phi_1(0)$ is the value of the electron wave function at the liquid-vapor boundary which is located in $z=0$. Using the results of Ref. 36, we take $\Phi_1(0)=59.7\ \mathrm{cm}^{-1/2}$ for $V_0\simeq 1$ eV.

Substituting Eqs. (19) and (20) into Eq. (18), we obtain the following expression:

$$W_{1r}(T_e)=(T_e-T)\nu_{1r}^{(0)}(T_e)\left(1+\coth\frac{\hbar\omega_0}{2T_e}\right)^{-1}\times\sum_{n,n'}\exp\left(-\frac{\hbar\omega_0(n+n')}{2T_e}\right)C_{nn'}^{(1r)}(T_e), \quad (22)$$

where

$$\nu_{1r}^{(0)}(T_e)=\frac{2\sqrt{2}m^{3/2}e^2E_\perp^2}{\pi^{3/2}\rho\hbar^3\sqrt{T_e}},$$

and

$$C_{nn'}^{(1r)}(T_e)=\frac{\hbar\omega_0}{2T_e}\frac{[\min(n,n')]!}{[\max(n,n')]!}\int_0^\infty dy\ \ y^{|n-n'|-1/2}\exp(-y)\times[L_{\min(n,n')}^{|n-n'|}(y)]^2I_{n,n'}(y/4), \quad (23)$$

with

$$I_{n,n'}(y)=\left(1+\frac{16\Delta_0\hbar\omega_0}{9\Delta_\perp^2}y\right)\int_0^\infty\frac{\sqrt{x+y}}{x}\exp\left[-(x+\beta_{nn'}^2/x)\frac{\hbar\omega_0}{T_e}\right]dx+\frac{16}{3}\frac{(\Delta_0\hbar\omega_0)^{1/2}}{\Delta_\perp}\left[|\beta_{nn'}|\,K_1\left(2|\beta_{nn'}|\frac{\hbar\omega_0}{T_e}\right)\right.$$
$$\left.+y\,K_0\left(2|\beta_{nn'}|\frac{\hbar\omega_0}{T_e}\right)\right]+\frac{16}{9}\frac{\Delta_0\hbar\omega_0}{\Delta_\perp^2}\int_0^\infty\sqrt{x+y}\exp\left[-(x+\beta_{nn'}^2/x)\frac{\hbar\omega_0}{T_e}\right]dx\ . \tag{24}$$

In the equation above $\beta_{nn'}=(n-n')/4$, $\Delta_\perp=eE_\perp/\gamma$, and $\Delta_0=m\Lambda_0/2\hbar^2$ is the deepness of the surface subband $l=1$, with $\Lambda_0=e^2(\varepsilon_{\rm He}-1)/4(\varepsilon_{\rm He}+1)$, $\varepsilon_{\rm He}$ being the dielectric constant of helium. $K_j(x)$ are the modified Bessel functions. For $n'=n$, the integrand has a singularity at $x=0$. To avoid the divergence in the integral, we cut off the lower limit to a small chosen value x_0 ($<10^{-4}$), which does not influence the results within the accuracy of the numerical calculations.

For the calculation of $W_{2r}(T_e)$ we must choose the model of the ripplon dispersion relation $\omega_r(\vec{q})$ in the short-wavelength limit. The exact dispersion law in this range is unknown at present. From data of helium surface tension, a model of the short-wave spectrum was proposed with a surface roton minimum for $q>3\times10^7$ cm^{-1}.[37] However the comparison of calculations[24] with experimental results[27] did not give support to this conjecture. Similar conclusion was presented in Ref. 38 and more recently in Refs. 39 and 40, where the authors concluded that there is no ground to suppose a large deviation of the ripplon dispersion in the short-wave region from its long-wavelength asymptotic. For these reasons, we use in Eq. (21) the same dispersion law $\omega_r(\vec{q})\simeq(\alpha/\rho)^{1/2}q^{3/2}$ as in Eq. (20). The final expression for $W_{1r}(T_e)$ can be written as

$$W_{2r}(T_e)=(T_e-T)\nu_{2r}^{(0)}(T_e)\left(1+\coth\frac{\hbar\omega_0}{2T_e}\right)^{-1}\times\sum_{n,n'}\exp\left(-\frac{n\hbar\omega_0}{T_e}\right)C_{nn'}^{(2r)}(T_e), \tag{25}$$

where

$$\nu_{2r}^{(0)}(T_e)=\frac{2^{7/3}\Gamma(\frac{2}{3})}{7\pi^2\Gamma(\frac{1}{6})}\frac{\kappa^2F_{11}^2m\omega_0^{1/2}T_e^{1/6}}{\alpha^{4/3}\rho^{2/3}\hbar^{7/6}},$$

and

$$C_{nn'}^{(2r)}(T_e)=\Theta(n'-n)\exp\left(-4\,\beta_{nn'}\frac{\hbar\omega_0}{T_e}\right)a_{n'n}\times\int_0^\infty dx\frac{x^{7/6}e^{-x}}{\sqrt{x+4\,\beta_{n'n}\hbar\omega_0/T_e}}+\Theta(n-n')\,a_{nn'}\times\int_0^\infty dx\frac{(x+4\,\beta_{nn'}\hbar\omega_0/T_e)^{7/6}}{\sqrt{x}}e^{-x}, \tag{26}$$

where $\Theta(n)$ is the step function and

$$a_{nn'}=\frac{(n')!}{n!}\int_0^\infty dy\ \ y^{n-n'-1/2}e^{-y}[L_{n'}^{n-n'}(y)]^2.$$

Even though the functions $a_{nn'}$ can be expressed in terms of the Legendre functions, in the numerical calculations of Eq. (25), we found it more convenient to use directly the integral representation for these coefficients.

It is interesting to remark that the similar expression of the two-ripplon contribution to the dissipation of the electron energy W_{2r} in the SE problem, given in Ref. 31, does not depend on the clamping field $E_\perp$ if the electrons are in the lowest subband $l=1$. However in the Q1D system, W_{2r} depends on ω_0 which in turn is determined by $E_\perp$. This result reflects the nature of the Q1D confinement in which the matrix elements of the scattering potential, appearing in Eq. (21), depend on the clamping field through the wave function given by Eqs. (3)–(5).

Once W_{1r} and W_{2r} are obtained, the relationship between the driving electric field $E_\parallel$ and the effective temperature T_e is given by Eq. (16), i.e., $E_\parallel=[W_r(T_e)/e\mu]^{1/2}$, where $W_r(T_e)$ is the sum of the two contributions coming from the two processes for creating ripplons. This relation also allows us to obtain the driving-field dependence of the mobility $\mu(E_\parallel)$ in both partial- and complete-control regimes given by Eqs. (12) and (14), respectively.

III. NUMERICAL RESULTS

The results of the numerical computation of the functions $W_{1r}(T_e)$ and $W_{2r}(T_e)$, given by Eqs. (22) and (25), are depicted in Figs. 1 and 2, respectively, for the clamping fields $E_\perp=1000$, 2000, and 3000 V/cm and temperatures T_e lower than 10 K. The helium bath temperature was chosen as $T=0.6$ K. As one can see the contribution of $W_{1r}(T_e)$ to the

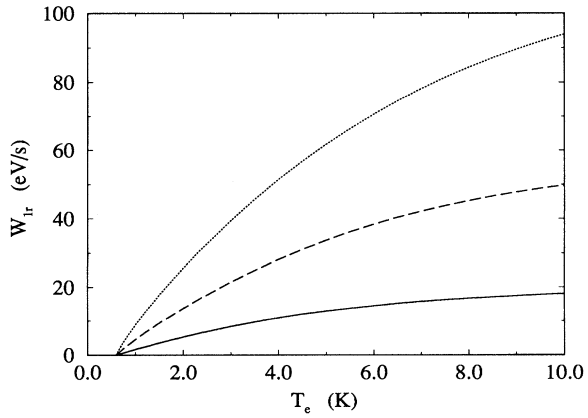

FIG. 1. The function $W_{1r}(T_e)$ at $E_\perp=$ 1000 V/cm (solid curve), 2000 V/cm (dashed curve), and 3000 V/cm (dotted curve). $T=0.6$ K.

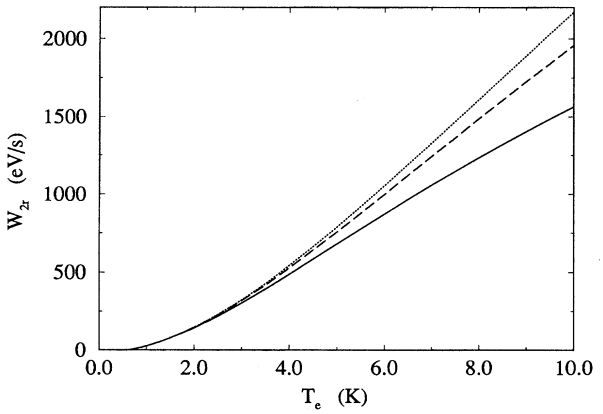

FIG. 2. Same as Fig. 1 but now for function $W_{2r}(T_e)$.

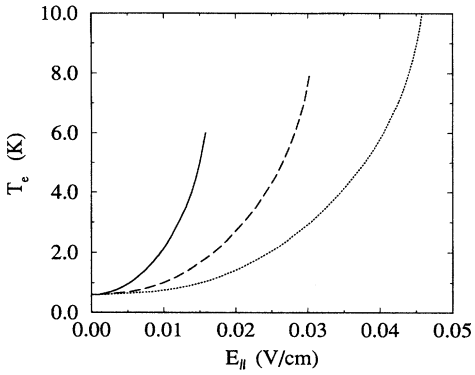

FIG. 4. Same as Fig. 3 but within the complete-control approximation.

total energy loss rate is small in comparison with the two-ripplon contribution $W_{2r}(T_e)$ in the range of clamping field investigated. The dependence of W_{2r} on the clamping field $E_{\|}$ for $T_e > 3$ K, shown in Fig. 2, is a consequence of the explicit dependence of W_{2r} on the frequency ω_0, as discussed of the end of Sec. II. Once $W_{1r}(T_e)$, $W_{2r}(T_e)$, and $\mu(T_e)$ [given by Eqs. (12) and (14) in the two regimes] are calculated, the energy balance equation, Eq. (16), is solved and the driving-field dependence of the effective temperature $T_e(E_{\|})$ is obtained. The results are presented in Fig. 3 for the partial-control approximation, where the mobility was calculated using Eq. (12), and, in Fig. 4, for the complete-control approximation whose mobility was given in Eq. (14). The curves exhibit qualitatively the same behavior. However, for a given driving field, the values of T_e in the partial control regime are larger than those in the complete-control regime. It does mean that the effective temperature T_e, calculated in the complete-control approximation, goes up to higher driving fields as compared with that calculated in the partial-control approximation. From the knowledge of the function $T_e(E_{\|})$, the electron mobilities, given by Eqs. (12) and (14) in the two opposite regimes, can be obtained as a function of the driving field. One should point out that the contribution coming from the electron-gas scattering is negligible in comparison with the contribution due to the electron-ripplon scattering for all effective temperature and clamping fields considered in this work. The results of $\mu(E_{\|})$ in the partial-control approximation are shown in Fig. 5 and those of $\tilde{\mu}(E_{\|})$ in the complete-control approximation are depicted in Fig. 6. The figures show that the mobilities are almost constant for $E_{\|} < 10^{-2}$ V/cm, but increase fast for higher driving fields. However, one should remark that the energy dissipation rate for the two-ripplon process, given by Eqs. (21) and (25), is valid only if the effective temperature is significantly higher that the helium bath temperature. For this reason the effective temperature dependence of the mobilities is only qualitative for intermediate T_e. But at $E_{\|} > 10^{-2}$ V/cm, which corresponds to $T_e > 1.5-2.0$ K, the curves obtained give a reliable description of the behavior of $\mu(T_e)$ and $\tilde{\mu}(T_e)$.

A comparison between $\mu(E_{\|})$ and $\tilde{\mu}(E_{\|})$ for two values of the clamping field is depicted in Fig. 7. As one can see again, the mobility in both partial- and complete-control re-

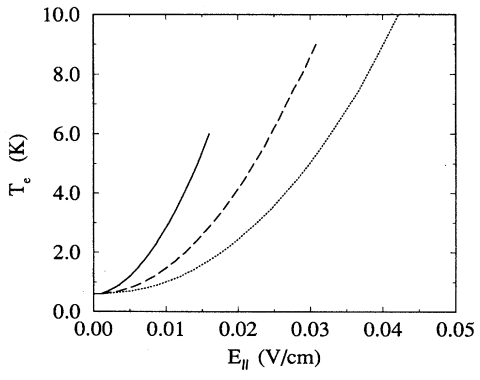

FIG. 3. The effective temperature as a function of the driving field within the partial control approximation. The solid, dashed, and dotted curves present the results for $E_{\perp} = 1000$, 2000, and 3000 V/cm, respectively.

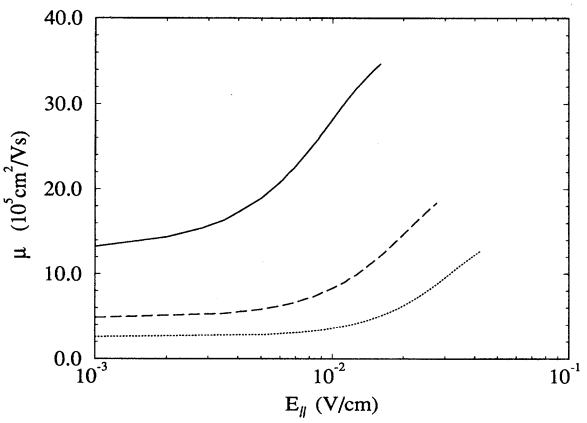

FIG. 5. The dependence of the mobility $\mu(E_{\|})$ for the partial-control regime at $T = 0.6$ K for $E_{\perp} =$ 1000 V/cm (solid curve), 2000 V/cm (dashed curve), and 3000 V/cm (dotted curve).

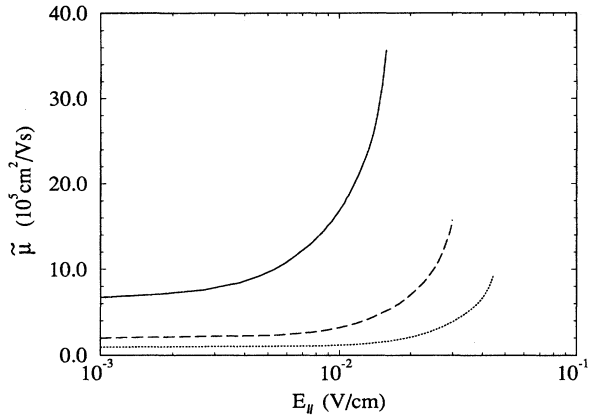

FIG. 6. The dependence of the mobility $\tilde{\mu}(E_{\|})$ for the complete-control regime at $T=0.6$ K for $E_{\perp}=$ 1000 V/cm (solid curve), 2000 V/cm (dashed curve), and 3000 V/cm (dotted curve).

gimes start to increase near 10^{-2} V/cm. However the values of the mobility in the complete-control case are 2.5−3 times smaller than those in the partial-control regime. In addition, $\tilde{\mu}(E_{\|})$ increases more strongly than $\mu(E_{\|})$ for driving fields above the critical value of 10^{-2} V/cm. As one can observe, from Figs. 6 and 7, the curves of $\tilde{\mu}(E_{\|})$ become steeper with decreasing the clamping field $E_{\perp}$. As a consequence, one can guess a possible negative differential $d[\tilde{\mu}(E_{\|})]/dE_{\|}$ at $E_{\perp}<1000$ V/cm which would lead to an instability in the electron current. However we must emphasize that for $E_{\perp}<1000$ V/cm, the electron transitions to excited subbands with $l>1$ become essential at $T_e<10$ K and the theory that we have developed in this paper cannot be applied to calculate the mobilities below $E_{\perp}=1000$ V/cm.

Finally, our results are very similar to those obtained for the dependence of the mobility on the driving electric field for electrons in SE over bulk helium, as calculated by Monarkha in Ref. 23 using the partial-control approximation

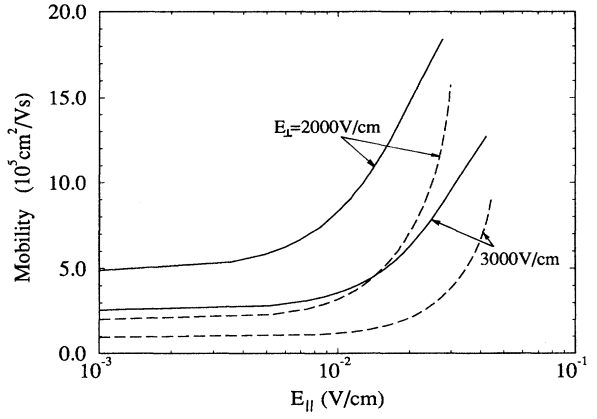

FIG. 7. A comparison between the mobility $\mu(E_{\|})$ within the partial-control approximation (solid curves) and $\tilde{\mu}(E_{\|})$ within the complete-control approximation (dashed curves) for $E_{\perp}=2000$ and 3000 V/cm.

and clamping field larger than 3000 V/cm. In both cases, the results show a monotonic increase of the mobility with the driving field. Such a behavior demonstrates that the confinement of the electron motion in the y direction does not influence significantly the nonlinear characteristics of the electron transport at high fields, which is determined by the electron-ripplon scattering. On the other hand, as shown in I, in the equilibrium case $T_e=T$, the restricted character of the electron motion along the Q1D channel leads to a nonmonotonic temperature dependence of the mobility, which is not observed in SE over bulk helium.

IV. CONCLUDING REMARKS

We have calculated the dependence of the mobility of electrons localized on Q1D channels over the helium surface on the driving electric field. The bath temperature was $T=0.6$ K and we have considered the electron-ripplon scattering as the dominant scattering mechanism. The calculations were carried out in the effective temperature approximation in the two opposite regimes of the relationship between the frequency of the electron-electron collisions ν_{ee} and the frequencies of the relaxation of electron momentum and energy. The widely used effective temperature approximation is valid only if ν_{ee} is large enough compared with other characteristic frequencies of the system. For SE on bulk helium, where the electron densities are easily changed by varying the clamping field, the frequency ν_{ee} can be changed in a very wide range, and one can fulfill the condition $\nu_{ee}\gg\nu$, as it was demonstrated in Ref. 9. In the Q1D system, the dependence of the electron density on the clamping field is more complicated and, as far as we know, was not carried out in the general form up to now. Furthermore, the specific nature of the electron motion in the channel is different of the 2D SE and we cannot say that we have the conditions of validity of the effective temperature approximation. However, we believe that this approximation has described at least the main features of the nonlinear effects of the electron mobility in the Q1D system as in the 2D case. We have obtained a strong dependence of the mobility on the driving field for $E_{\|}>10^{-2}$ V/cm. We have also observed large differences in the mobilities calculated in the two different regimes of electron-electron correlations. Unfortunately up to the present, there is no systematic experimental study of the kinetic properties of electrons in a single Q1D system, except the results presented in Ref. 5, where the dependence of the mobility on the clamping field was investigated. We hope that the different behaviors of the driving-field dependence of the mobility can lead to checking the possibility to achieve the complete-control regime in Q1D, as in the 2D case, and in addition, to different temperature dependence of the mobility in the partial- and complete-control regimes, as predicted in I. The theory developed here is not appropriate for clamping fields such that the effective temperature, which is related to the driving field, are higher than 10 K, because the electron transitions to higher subbands ($l>1$) become significant and can influence the mobility drastically. This question is however out of frame of the present work.

ACKNOWLEDGMENTS

This work was partially sponsored by the Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) and the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq). One of us (S.S.S.) is grateful to FAPESP for financial support and to B. I. Verkin Institute for Low Temperature Physics and Engineering, Kharkov, for a leave of absence. The other one (G.Q.H.) thanks the CNPq for financial support.

[1] Vl. L. Ginzburg and Yu. P. Monarkha, Fiz. Nizk. Temp. **4**, 1236 (1978) [Sov. J. Low Temp. Phys. **4**, 580 (1978)].

[2] A. V. Chaplik, Pis'ma Zh. Éksp. Teor. Fiz. **31**, 275 (1980) [JETP Lett. **31**, 252 (1980)].

[3] Yu. Z. Kovdrya and Yu. P. Monarkha, Fiz. Nizk. Temp. **12**, 1011 (1986) [Sov. J. Low Temp. Phys. **12**, 571 (1986)].

[4] Yu. Z. Kovdrya and V. A. Nikolaenko, Fiz. Nizk. Temp. **18** , 1278 (1992) [Sov. J. Low Temp. Phys. **18**, 894 (1992)].

[5] O. I. Kirichek, Yu. P. Monarkha, Yu. Z. Kovdrya, and V. N. Grigor 'ev, Fiz. Nizk. Temp. **19**, 458 (1993) [Low Temp. Phys. **19**, 323 (1993)].

[6] S. S. Sokolov and O. I. Kirichek, Fiz. Nizk. Temp. **20**, 764 (1994) [Low Temp. Phys. **20**, 599 (1994)].

[7] S. S. Sokolov and N. Studart, Phys. Rev. B **51**, 2640 (1995).

[8] S. S. Sokolov, G. Q. Hai, and N. Studart, Phys. Rev. B **51**, 5977 (1995).

[9] V. A. Buntar, V. N. Grigoriev, O. I. Kirichek, Yu. Z. Kovdrya, Yu. P. Monarkha, and S. S. Sokolov, J. Low Temp. Phys. **79**, 323 (1990).

[10] M. I. Dykman, M. J. Lea, P. Fozooni, and J. Frost, Phys. Rev. Lett. **70**, 3975 (1993).

[11] M. J. Lea, P. Fozooni, P. J. Richardson, and A. Blackburn, Phys. Rev. Lett. **73**, 1142 (1994).

[12] Yu. P. Monarkha, Fiz. Nizk. Temp. **19**, 737 (1993) [J. Low Temp. Phys. **19**, 530 (1993)].

[13] R. W. van der Heijden, M. C. M. van de Sanden, J. H. G. Surewaard, A. T. A. M. de Waele, H. M. Gijsman, and F. M. Peeters, Europhys. Lett. **6**, 75 (1988).

[14] O. Tress, P. Scheuzger, H. Bluyssen, and P. Wyder, Physica B **211**, 481 (1995).

[15] Yu. Z. Kovdrya, V. A. Nikolaenko, O. I. Kirichek, S. S. Sokolov, and V. N. Grigoriev, J. Low Temp. Phys. **91**, 371 (1993).

[16] R. W. van der Heijden, H. M. Gijsman, and F. M. Peeters, J. Phys. C **21**, L1165 (1988).

[17] M. I. Dykman and L. S. Khazan, Zh. Eksp. Teor. Fiz. **77**, 1488 (1979) [Sov. Phys. JETP **50**, 747 (1979)]; M. I. Dykman, J. Phys. C **15**, 7397 (1982).

[18] M. J. Lea and M. I. Dykman, Philos. Mag. B **69**, 1059 (1994).

[19] P. J. M. Peters, P. Scheuzger, M. J. Lea, Yu. P. Monarkha, P. K. H. Sommerfeld, and R. W. van der Heijden, Phys. Rev. B **50**, 11 570 (1994).

[20] R. S. Crandall, Phys. Rev. A **6**, 790 (1972).

[21] R. S. Crandall, Phys. Rev. B **12**, 119 (1975).

[22] Yu. P. Monarkha and S. S. Sokolov, Fiz. Nizk. Temp. **4**, 685 (1978) [Sov. J. Low Temp. Phys. **4**, 327 (1978)].

[23] Yu. P. Monarkha, Fiz. Nizk. Temp. **5**, 994 (1979) [Sov. J. Low Temp. Phys. **5**, 470 (1979)].

[24] Yu. P. Monarkha and S. S. Sokolov, Fiz. Nizk. Temp. **5**, 1283 (1979) [Sov. J. Low Temp. Phys. **5**, 605 (1979)].

[25] V. S. Edel'man, Pis'ma Zh. Eksp. Teor. Fiz. **25**, 422 (1977) [JETP Lett. **25**, 394 (1977)].

[26] C. C. Grimes, Surf. Sci. **73**, 379 (1978).

[27] B. N. Esel'son, A. S. Rybalko, and Yu. Z. Kovdrya, Fiz. Nizk. Temp. **5**, 1354 (1979) [Sov. J. Low Temp. Phys. **5**, 641 (1979)].

[28] A. S. Rybalko, Fiz. Nizk. Temp. **6**, 141 (1980) [Sov. J. Low Temp. Phys. **6**, 67 (1980)].

[29] M. Saitoh and T. Aoki, J. Phys. Soc. Jpn. **44**, 71 (1978).

[30] T. Aoki and M. Saitoh, J. Phys. Soc. Jpn. **46**, 423 (1979).

[31] Yu. P. Monarkha, Fiz. Nizk. Temp. **4**, 1093 (1978) [Sov. J. Low Temp. Phys. **4**, 515 (1978)].

[32] F. G. Bass and Yu. G. Gurevich, *Hot Electrons and Strong Electromagnetic Waves in Semiconductor and Gas-Discharge Plasma* (Nauka, Moscow, 1975).

[33] E. M. Conwell, *High Field Transport in Semiconductors* (Academic Press, New York, 1967).

[34] Yu. P. Monarkha, S. S. Sokolov, and V. B. Shikin, Solid State Commun. **38**, 611 (1980).

[35] V. B. Shikin and Yu. P. Monarkha, J. Low Temp. Phys. **16**, 193 (1974).

[36] O. Hipólito, J. R. D. de Felício, and G. A. Farias, Solid State Commum. **28**, 365 (1978).

[37] L. S. Reut and I. Z. Fisher, Zh. Eksp. Teor. Fiz. **60**, 1814 (1971) [Sov. Phys. JETP **33**, 981 (1971)].

[38] D. O. Edwards and W. F. Saam, in *Progress in Low Temperature Physics VII A*, edited by D. F. Brewer (North-Holland, Amsterdam, 1978), p. 302.

[39] K. A. Gernoth, J. W. Clark, G. Senger, and M. L. Ristig, Phys. Rev. B **49**, 15 836 (1994).

[40] A. Lastri, F. Dalfovo, L. Pitaevskii, and S. Stringari, J. Low Temp. Phys. **98**, 227 (1995).