Phonon-induced electron-electron interaction in disordered superconductors

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We have measured the resistivities as a function of temperature of crystalline disordered bulk $Ti_{1-x}Al_{x-y}(Co,Cr,Au)_y$ ($x \approx 0.072$) alloys below 25 K. With a total doping level of 7.2 at. % for Ti, our samples are disordered enough to manifest electron-electron interaction effects. As the temperature T is reduced, a resistivity increase $\Delta\rho(T)/\rho(10\text{ K}) = [\rho(T)-\rho(10\text{ K})]/\rho(10\text{ K})$ on the order of a tenth of a percent is observed in all alloys before they eventually undergo superconducting transitions at sufficiently low T (depending on y). Both the functional forms and magnitudes of the observed $\Delta\rho(T)/\rho(10\text{ K})$ are interpreted in terms of electron-electron interaction effects in the presence of disorder. Particularly, the values of the screened Coulomb interaction parameter $\tilde{F}$ defined in electron-electron interaction theory are extracted. In the Co- and Cr-doped alloys where the superconducting transition temperatures are raised from that of $Ti_{1-x}Al_x$, we find that the electron-phonon coupling, including exchange of virtual phonons, is of crucial importance in determining the value of $\tilde{F}$. However, the theory for $\tilde{F}$ in its current form fails to account for our experimental results. In the Au-doped alloys, the spin-orbit scattering introduced by the heavy Au atoms causes a small decrease in the value of $\tilde{F}$, i.e., $\tilde{F}$ becomes slightly more negative for higher spin-orbit scattering.

I. INTRODUCTION

Over the past 15 years, it has been realized that both the electron-electron (e-e) interaction and also the weak localization (WL) effects play essential roles in understanding the electron conduction in disordered metals.[1,2] Intensive theoretical and experimental work has been performed under various circumstances using various material systems, and it is now accepted that the two effects are basically understood. At low temperatures, and in zero magnetic field, both effects may cause a noticeable temperature dependent correction, $\Delta\rho(T)$, to the constant residual resistivity of a given sample. In two dimensions (e.g., thin metal films), the two effects are (could be) about equally important in producing $\Delta\rho(T)$. In three dimensions (i.e., bulk samples), only the e-e interaction effects are responsible for $\Delta\rho(T)$, while WL effects are comparatively small and can be ignored.[1,2] It is thus possible to extract the value of $\tilde{F}$ from high-precision $\Delta\rho(T)$ measurements in bulk samples. Here $\tilde{F}$ is a screening parameter characterizing the strength of Coulomb interaction between electrons; it is one of the most important, yet not very well-understood parameters defined in the e-e interaction theory.[1–3] Owing to the enormously complicated interaction effects between electrons, the magnitude of $\tilde{F}$ in a real disordered material is extremely difficult to calculate theoretically. The (only) theoretical prediction for $\tilde{F}$ available to date is evaluated within the Thomas-Fermi approximation.[3] It is thus not surprising that in literature one frequently finds discrepancies between the experimental and theoretical values of $\tilde{F}$. For the case of superconducting materials the situation can be even more complex and it was proposed that electron-phonon (e-ph) interactions, including exchange of virtual phonons, should be incorporated into the expression for $\tilde{F}$.[2,4–6] As a consequence, if the e-ph coupling is sufficiently strong, then $\tilde{F}$ can change from a positive (a repulsive case) to a negative (an attractive case) value. To our knowledge, this issue has never been experimentally examined in a systematic manner, though the effects of e-ph coupling on $\tilde{F}$ had occasionally been conjectured in some previous experiments.

In this work, our objective is to experimentally investigate the values of $\tilde{F}$ in three-dimensional (3D) crystalline disordered $Ti_{1-x}Al_{x-y}(Co,Cr)_y$ ($x \approx 0.072$) alloys whose superconducting transition temperatures T_c can be systematically changed by varying the concentration of Co or Cr.[7] The total doping level of 7.2 at. % for Ti is chosen such that the samples are disordered enough to manifest e-e interaction effects, and $\Delta\rho(T)$ is then sufficiently large to be accurately measured. As discussed below, a quantitative analysis of our results of $\Delta\rho(T)$ measurements suggest that the e-ph coupling, which is a measure of T_c, is of crucial importance in determining the value of $\tilde{F}$. Nevertheless, our observation cannot be explained by the theory of $\tilde{F}$ in its current form. For a consistency check of our experimental method, we also determine the values of $\tilde{F}$ in several $Ti_{1-x}Al_{x-y}Au_y$ (again, $x \approx 0.072$) alloys where the effects of spin-orbit scattering introduced by the heavy Au atoms are studied. In this latter case, we observe that spin-orbit scattering causes a small decrease in the value of $\tilde{F}$, i.e., $\tilde{F}$ becomes slightly more negative for higher spin-orbit scattering.

The paper is organized as follows: In Sec. II we discuss corrections to residual resistivity due to e-e interaction effects in a disordered superconductor. These effects are required to interpret the temperature dependence of our measured $\Delta\rho(T)$. The current theoretical expression of the screening parameter $\tilde{F}$ serves as a starting point for comparison with our experimental results as discussed in Sec. IV. In Sec. III, we describe sample fabrication and resistivity measurements methodology. Section IV contains our experimental results and discussion. We first compare our measured $\Delta\rho(T)$ with the prediction of e-e interaction theory, extract then the values of $\tilde{F}$ and finally determine the variation of

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$\tilde{F}$ with T_c which is a measure of the strength of the e-ph coupling. This section also contains a short survey of the current experimental status of $\tilde{F}$ in various material systems, as well as our experimental results for the effects of spin-orbit scattering on $\tilde{F}$. Our conclusions are given in Sec. V.

II. THEORY

The effects of e-e interaction in 3D disordered superconductors cause a resistivity change at low temperatures (i.e., in the residual resistivity regime) given by[2,4–6]

$$\frac{\Delta\rho(T)}{\rho} = -\frac{0.919e^2}{4\pi^2\hbar}\left(\frac{4}{3}-\frac{3}{2}\tilde{F}-\frac{2}{\ln(T_c/T)}\right) \times\rho\left[\frac{k_BT}{\hbar D}\right]^{1/2}, \quad (1)$$

where e is the electronic charge, k_B is the Boltzmann constant, $\hbar$ is the Planck constant divided by 2π, D is the diffusion constant, and $\tilde{F}$ is the electron screening parameter given by[2]

$$\tilde{F}=\frac{32}{3F}\left[\left(1+\frac{F}{2}\right)^{3/2}-1-\frac{3}{4}F\right], \quad (2)$$

where F is the dimensionless screened Coulomb potential between electrons averaged over the Fermi surface. The first two terms, $\frac{4}{3}-\frac{3}{2}\tilde{F}$, in the large parentheses of Eq. (1) stem from the e-e interaction effects in the diffusion channel (small momentum and frequency transfers), while the third term, $-2/\ln(T_c/T)$, stems from the interaction effects in the Cooper channel (small total momentum and small frequency difference). The common origin of all these three correction terms is a suppression of the single-electron density of states at the Fermi level caused by enhanced e-e interactions as a result of electronic diffusive motion in a random potential.[8] The Cooper channel contribution is due to the influence of the fluctuating interaction on the density of states and is believed to foreshadow the superconducting gap. On the other hand, both the Maki-Thompson and the Aslamazov-Larkin corrections to $\Delta\rho/\rho(10\text{ K})$ are unimportant in 3D,[4,9] if one is not concerned with the immediate vicinity of the transition to superconducting state where $k_B(T-T_c) \ll \hbar/\tau_\phi$, with τ_ϕ being the electronic phase-breaking time defined in the WL theory, which can be reliably inferred from magnetoresistance measurements.[1,2] Because we have previously derived $\hbar/(k_B\tau_\phi) \approx 0.01-0.1$ K for our Ti-Al alloys for Al composition in the range 4–15 at. %,[10] and since we shall concentrate on the temperature region at $T \gtrsim 1.5\,T_c$, i.e., $k_B(T-T_c) \gg \hbar/\tau_\phi$, in this work, we will ignore both the Maki-Thompson and the Aslamazov-Larkin terms in the following discussion.

In the case of normal conductivity, the parameter F equals the usual Coulomb repulsion constant $2\mu = (1/a)[\ln(1+a)]$ within the static Thomas-Fermi approximation.[3,11] Here is $a=(2k_F/\kappa_s)^2$, with k_F being the Fermi wave number, and κ_s^{-1} the Thomas-Fermi screening length. $F \to 1$ in the limit of small a (complete screening), while $F \to 0$ in the limit of large a (no screening).[3] If the exchange of virtual phonons or phonon mediated attraction, i.e., superconductivity, is taken into account, then F is proposed to carry the opposite sign and be given by[2,4–6]

$$F=2(\mu^*-\lambda_{\rm ph}), \quad (3)$$

where $\mu^*=\mu/[1+\mu\ln(T_F/\theta_D)]$ is the renormalized or reduced Coulomb pseudopotential between electrons known from the theory of superconductivity[12] (T_F is the Fermi and θ_D the Debye temperature, respectively), and $\lambda_{\rm ph}=2\int[d\omega G(\omega)/\omega]$ is the e-ph coupling constant [$G(\omega)$ is the Eliashberg function at phonon frequency ω]. In practice, the detailed nature of the Coulomb interaction between electrons in either clean or dirty metals is not very well known. In Eq. (3), we have considered the difference in the energy scales of interelectron repulsion and attraction, T_F and θ_D, and have replaced 2μ by $2\mu^*$. (Fortunately, although there are various definitions of μ^*, the magnitude of μ^* does not vary much among various superconductors.) The value of $\lambda_{\rm ph}$ for a particular superconductor can be estimated from McMillan's expression for T_c,[12] provided that the magnitudes of T_c and μ^* are known.

Equation (2) indicates that $\tilde{F} \approx F$ in the limit $F \ll 1$. If F is not small, it can readily be shown that $\tilde{F}$ equals F to within $\approx 10\%$ for F in the range $-1 \lesssim F \lesssim 1$. Thus, it should not be critical to distinguish between F and $\tilde{F}$ in most experiments, including the present one discussed below.

III. EXPERIMENTAL METHOD

Crystalline disordered $Ti_{1-x}Al_{x-y}(Co,Cr)_y$ (hereafter referred to as Ti-Al-Co or Ti-Al-Cr) alloys were prepared by a standard arc-melting method as reported previously.[10] The Al atoms were introduced into a Ti host to serve as static disorder, while Co or Cr atoms were introduced to enhance the superconducting properties of the system. In this work we concentrate on alloys with $x \approx 0.072$. (All the compositions quoted in this work are *nominal*.) This total doping level of 7.2 at. % for Ti in each alloy had been chosen such that every alloy had a residual resistivity $\rho(10\text{ K}) \approx 82\pm15$ and $\approx 86 \pm 10\ \mu\Omega$ cm for Ti-Al-Co and Ti-Al-Cr alloys, respectively. Experimentally, the resistivity correction $\Delta\rho(T)$ in samples with $\rho(10\text{ K})$ of this order can be accurately measured. This means that the alloys are disordered enough to manifest e-e interaction effects, while their structure remains essentially single-phased (the hexagonal α phase of Ti).[10] Furthermore, residual resistivity $\rho(10\text{ K})$ of this order in a Ti-Al-(Co,Cr) alloy implies the estimate $k_F\ell \approx 10$ (ℓ being the elastic mean free path),[10] justifying the application of Eq. (1) valid in the weakly disordered regime.

Previously it has been found that the addition of a small amount of Co or Cr for Ti results in a remarkable increase in T_c from that of pure Ti. For instance, Matthias *et al.*[7] found that T_c monotonically increases from ≈ 0.4 K for pure Ti to ≈ 2.8 K for Ti-Co with a doping level of 3–6 at. % of Co, while T_c shows a maximum of ≈ 3.7 K for Ti-Cr with a doping level of 3 at. % of Cr. Our results reveal trends of variation of T_c with y in both Ti-Al-Co (11 alloys with $0.005 \leq y \leq 0.06$) and Ti-Al-Cr (seven alloys with $0.01 \leq y \leq 0.05$) alloys similar to those reported by Matthias *et al.*, but our T_c value for a given level of Co or Cr doping is comparatively somewhat higher. Physically, the enhanced

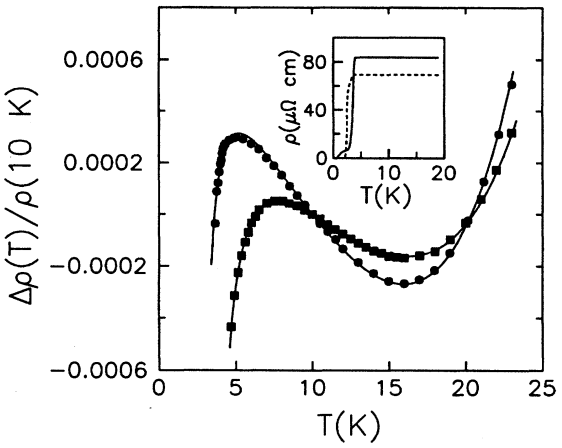

FIG. 1. Variations of the resistivity increase $\Delta\rho(T) = \rho(T) - \rho(10\ \mathrm{K})$, normalized to $\rho(10\ \mathrm{K})$, with T for $Ti_{0.928}Al_{0.032}Co_{0.040}$ (closed circles) and $Ti_{0.928}Al_{0.042}Cr_{0.030}$ (closed squares) alloys, respectively. The solid curves are predictions of the theory (1) including the usual Boltzmann AT^5 term normalized to $\rho(10\ \mathrm{K})$ as described in the text. The inset shows the full superconducting transition curves for these two alloys (dashed curve for the former and solid curve for the latter).

superconductivity is ascribed to an increase in the e-ph coupling constant λ_{ph} as a result of Co or Cr doping.[7] Therefore, it is quite appropriate to utilize Ti-Al-Co and Ti-Al-Cr alloys to investigate the effects of e-ph coupling on $\tilde{F}$, since λ_{ph}, which is a measure of T_c, can readily be "tuned" by the amounts of Co or Cr doped.

Eleven $Ti_{1-x}Al_{x-y}Au_y$ ($x \approx 0.072$ and $0.005 \leq y \leq 0.04$, hereafter referred to as Ti-Al-Au) alloys were also prepared by the same arc-melting method. These samples were used for a consistency check of our experimental method, as well as for a study of the effects of spin-orbit scattering on the value of $\tilde{F}$. For this series of alloys, $\rho(10\ \mathrm{K}) \approx 100 \pm 30\ \mu\Omega$ cm and $T_c \approx 0.52 \pm 0.03$ K, i.e., the superconducting transition temperatures remained essentially constant.

Our samples, typically of $0.2 \times 0.2 \times 10\ \mathrm{mm}^3$, were 3D with regard to e-e interaction effects. The measurements of ac resistivities were carried out by a standard four-probe technique on a ^{3}He fridge and temperature was monitored with a calibrated RuO_2 and a calibrated carbon-glass thermometer.

IV. RESULTS AND DISCUSSION

A. Results

Figure 1 shows the resistivity change $\Delta\rho(T) = \rho(T) - \rho(10\ \mathrm{K})$ normalized to $\rho(10\ \mathrm{K})$ as a function of temperature for $Ti_{0.928}Al_{0.032}Co_{0.040}$ (closed circles) and $Ti_{0.928}Al_{0.042}Cr_{0.030}$ (closed squares) alloys, respectively. The figure shows that for both alloys the resistivities rise with reducing temperature as T is lowered from 25 to below about 16−17 K. Below about 4 K (depending on alloys) the alloys undergo superconducting transitions. The magnitudes of the normalized resistivity rises $\Delta\rho(T)/\rho(10\ \mathrm{K})$ before the superconducting transitions take place are of order a tenth of a percent (while the increase in the resistivity at $T \gtrsim 16 - 17$ K is associated with the T dependence of the electron-phonon scattering.) For comparison, the inset of Fig. 1 shows the full superconducting transition curves for these two alloys (dashed curve for the former and solid curve for the latter). Behavior of $\Delta\rho(T)/\rho(10\ \mathrm{K})$ similar to that displayed in Fig. 1 has been observed in 11 Ti-Al-Co and seven Ti-Al-Cr alloys, but with the value of T_c depending strongly on the composition of Co or Cr, as mentioned in Sec. II.

According to current understanding the resistivity correction $\Delta\rho(T)/\rho(10\ \mathrm{K})$ in a disordered metal or superconductor originates from two effects, namely the WL and e-e interaction effects. (We ignore superconducting fluctuations in this work, as specified above.) However, in 3D the correction due to WL effects is negligibly small, compared with that due to e-e interaction effects.[2] Indeed, from magnetoresistance measurements in low magnetic fields we have previously extracted the inelastic scattering and spin-orbit scattering times in Ti-Al and Ti-Al-Co alloys.[10] With those times having been determined, we deduced that the correction to $\Delta\rho(T)/\rho(10\ \mathrm{K})$ due to WL effects in our measuring T range is *no* more than 5% of the measured resistivity change in each alloy. Therefore, in the following analysis, it should be safe to compare our observed $\Delta\rho(T)/\rho(10\ \mathrm{K})$ with the e-e interaction effects alone.

To compare our experimental results with Eq. (1), we use the value of $D = 78.0/[\rho(10\ \mathrm{K})]\ \mathrm{cm}^2/\mathrm{s}$ previously established from specific heat measurements in our Ti-Al alloys,[10] where $\rho(10\ \mathrm{K})$ is in $\mu\Omega$ cm. Then, the only free parameters left in Eq. (1) are $\tilde{F}$ and the directly measured value of T_c. In performing least-squares fits of our experimental results to the prediction of (1), we focus on temperatures $T \gtrsim 1.5\ T_c$ to avoid any appreciable complication which might result from fluctuational superconductivity induced paraconductivities.[9] [The upper bound of T for our least-squares fits is 24 or 25 K for every alloy. To include data points up to this temperature, a usual Boltzmann AT^5 term normalized to $\rho(10\ \mathrm{K})$ needs to be added to Eq. (1) to fully account for the measured $\Delta\rho(T)/\rho(10\ \mathrm{K})$, with A being a constant.] Our fitted values of the relevant parameters are summarized in Fig. 2 where we plot $-\tilde{F}$ as a function of T_c. We notice in passing that the fitted value of T_c in the majority of cases is very close to the zero-resistivity point in the measured ρ-T curve. For this reason, the T_c values plotted in Fig. 2 are actually the fitted values.

Figure 1 clearly demonstrates that the prediction of (1) plus a usual Boltzmann AT^5 term normalized to $\rho(10\ \mathrm{K})$ (the solid curves) can well describe our experimental results. [The values for the coefficient A lie in the range $(0.8-1.8) \times 10^{-8}$ and $(1.2-2.0) \times 10^{-8}\ \mu\Omega\ \mathrm{cm/K}^5$ for Ti-Al-Co and Ti-Al-Cr alloys, respectively.] This observation strongly suggests that the correction term originating from the Cooper channel is required for a quantitative description of $\Delta\rho(T)/\rho(10\ \mathrm{K})$ in a superconducting material at $T > T_c$. Without this term, the correction terms from the diffusion channel alone would have caused $\Delta\rho(T)/\rho(10\ \mathrm{K})$ to vary strictly with the square root of T as the temperature is decreased (assuming that $\tilde{F}$ is T independent). On the other hand, the figure clearly indicates that neither the Maki-Thompson term nor the Aslamazov-Larkin term are impor-

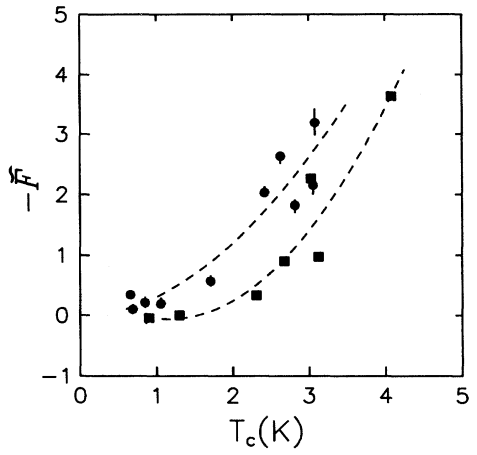

FIG. 2. Variation of the screened electron-electron interaction parameter $-\tilde{F}$ with superconducting transition temperature T_c for Ti-Al-Co (closed circles) and Ti-Al-Cr (closed squares) alloys, respectively. The dashed curves are guides to the eye. Note that $-\tilde{F}$ monotonically increases with increasing T_c in each set of alloys.

tant in the observed temperature range which is far above the immediate vicinity of T_c. Therefore, we conclude that the *e-e* interaction theory (1) alone can already well describe the functional dependence of $\Delta\rho(T)/\rho(10\text{ K})$ with T in 3D disordered superconductors outside the critical transition region. Moreover, the prediction of (1) actually describes $\Delta\rho(T)/\rho(10\text{ K})$ over a wider T range than expected. While our least-squares fits are performed on data points at $T \gtrsim 1.5\ T_c$ as mentioned, the theory (1) with the fitted parameters actually well describes the data down to much lower temperatures, down to at least $\approx 1.3\ T_c$ for the two samples shown in Fig. 1.

Figure 2 shows the variation of the screening parameter $-\tilde{F}$ with T_c for Ti-Al-Co (closed circles) and Ti-Al-Cr (closed squares) alloys, respectively. The dashed curves are guides to the eye. In plotting this figure, we use $-\tilde{F}$ instead of $\tilde{F}$ to emphasize the negativeness of $\tilde{F}$ in our superconducting alloys. This figure clearly indicates that the magnitude of $-\tilde{F}$ monotonically increases from ≈ 0.1 (0) to ≈ 3.2 (3.6) as T_c is increased from ≈ 0.6 (0.9) to ≈ 3 (4) K in Ti-Al-Co (Ti-Al-Cr) alloys. For a given T_c, the magnitude of $-\tilde{F}$ in Ti-Al-Co is slightly higher than that in Ti-Al-Cr alloys. For the time being, we have no explanation for this difference. Following the standard BCS theory of superconductivity, T_c is known to increase for larger values of λ_{ph} (with other material parameters being kept unchanged). Thus, according to Eq. (3), $-\tilde{F}$ should increase with increasing T_c, as is indeed the case in Fig. 2. Our experimental results therefore support the theoretical idea that the effects of *e*-ph coupling should be taken into account in evaluating $\tilde{F}$ in superconducting materials in which λ_{ph} could dominate over μ^*. Quantitatively, the large increase in $-\tilde{F}$ shown in Fig. 2 renders an interpretation of $\tilde{F}$ using the expression (3) impossible. (As has been mentioned in Sec. III, we focus on the samples with the hexagonal α phase of Ti in this work, and we study only the variation of $-\tilde{F}$ with T_c up to $T_c \approx 4$ K, the maximum superconducting transition temperature achievable in this phase by Co or Cr doping.[7])

An attempt to perform a quantitative comparison of our data with the prediction (3) has failed. For our alloys, the magnitude of $-\tilde{F}$ varies roughly from 0.0 to 3.6, and this variation implies a large increase in λ_{ph} according to Eq. (3). However, first, according to recent experiments the magnitude of μ^* is virtually unaffected by disorder in a given superconductor material system.[13] If one takes $\mu^* = 0.17$ (the pseudopotential for pure Ti, Ref. 12) and applies Eq. (3) to our alloys, then λ_{ph} would have to increase from about 0.2 to 2.0 to account for our observation in Fig. 2. Although λ_{ph} is known to generally increase with disorder, such a large increase by a factor ~ 10 in a given material system is hardly realistic. Secondly, even with these values of μ^* and λ_{ph}, one will still not be able to obtain theoretical T_c values reasonably close to our experimental values using, say, McMillan's expression for the superconducting transition temperature.[12] Therefore, although it is quite clear from Fig. 2 that the *e*-ph coupling is of crucial importance in determining the magnitude of $\tilde{F}$, the theoretical expression (3) needs to be refined to fully account for the role of λ_{ph}.

B. Discussion of $\tilde{F}$

An accurate evaluation of the value of F ($\tilde{F}$) for a given material is extremely difficult, both theoretically and experimentally. (We will not always explicitly distinguish between F and $\tilde{F}$ in this and the following subsections, because the two values are approximately equal, as discussed in Sec. II.) Physically, the detailed nature of the Coulomb interaction between electrons in either clean or dirty metals is not very well known. Theoretically, it is difficult to calculate F ($\tilde{F}$) beyond the Thomas-Fermi approximation. It would be even more difficult to attempt to go beyond the first-order Hartree-Fock terms in *e-e* interactions and to include high order terms.[14] Experimentally, the values of F ($\tilde{F}$) deduced from $\Delta\rho(T)$ measurements and those deduced from high-magnetic-field magnetoresistance measurements are often not in agreement with one another in various materials, including amorphous Ni-Ti,[5] Ca-(Mg,Al),[15] and Cu-Ti alloys,[16] and icosahedral Al-Cu-Fe alloys.[17] Even worse, the sign of F ($\tilde{F}$) is not always clear, especially when one is concerned with superconducting materials, and also with materials with strong spin-orbit scattering. Several groups have reported observation of *positive* values of F ($\tilde{F}$) in amorphous superconducting Cu-Ti and Au-doped Ti-Cu alloys.[18] Kokanović *et al.*[19] have observed positive values of F which increase with increasing T_c in hydrogen-doped Zr-Ni and Zr-Cu metallic glasses. The trend of the variation of F with T_c reported by Kokanović *et al.* is entirely opposite to that displayed in Fig. 2 where we obtain negative values of $\tilde{F}$ which become more negative with increasing T_c. On the other hand, *negative* values of F ($\tilde{F}$) have been observed in granular aluminum,[20] amorphous $Cu_{72}Y_{28}$,[21] $Lu_{75}Pd_{25}$, and $Lu_{75}Ni_{25}$ alloys.[22] Initially, the observation of negative F ($\tilde{F}$) has caused much confusion in literature, as it was originally taken for granted that F ($\tilde{F}$) could only take positive values, according to the first theoretical work of Al'tshuler *et al.*[3] Recently, based on magnetoresistance and $\Delta\rho(T)$

measurements in both nonsuperconducting and superconducting Ca-Al metallic glasses, Steep *et al.*[6] have argued that $\tilde{F}$ is most likely to assume positive values (i.e., a repulsive case) in normal metals, while it is most likely to assume negative values (i.e., an attractive case) in superconductors. Our results in Fig. 2 are in line with this recent opinion of Steep *et al.*, namely, our work supports the idea that the e-ph coupling plays essential roles in determining the value of $\tilde{F}$ in superconducting materials.

A puzzling, large negative value of $F \approx -2.7$ has recently been found in superconducting PtSi thin films by Ishida *et al.*[23] Large positive or negative values of F have also been reported for the two-dimensional electrons and/or holes in semiconductor devices. Bishop *et al.*[24] reported in Si MOSFET's a positive F which increased to 3.5 as $k_F\ell$ is decreased to ~ 2. Lin *et al.*[25] reported in GaAs-$Al_\delta Ga_{1-\delta}As$ heterostructures an F of -2.2 in the regime $k_F\ell \gg 1$ where the theory is expected to be valid. Washburn *et al.*[26] observed in GaSb-InAs-GaSb double heterostructures an F as large as ~ -60. These large values of F are totally unexpected from the theory which predicts $0 \leq F \leq 1$. Those experiments, and also our observation of Fig. 2, are some examples that explicitly point out a failure of the theory of F in its current form. [Notice, however, that in the case of semiconductor devices intervalley scattering either by impurities or by interactions could be of significance, which might in turn noticeably affect the value of F ($\tilde{F}$).]

C. Spin-orbit scattering effect on $\tilde{F}$

Another relevant issue of interest is the effect of spin-orbit scattering on the value of F ($\tilde{F}$). Sahnoune *et al.*[15] and others[22,27] have reported that the (positive) value of $\tilde{F}$ decreased for higher spin-orbit scattering rate τ_{so}^{-1}, and that (in the case of nonsuperconducting materials) $\tilde{F}$ might even become negative in materials with sufficiently large τ_{so}^{-1}. They found that $\tilde{F}$ changed sign from positive to negative at $\tau_{so} \lesssim 10^{-12}$ s in several metallic glasses. This systematic change of $\tilde{F}$ with τ_{so}^{-1} cannot be satisfactorily explained by the current theory for $\tilde{F}$.[15] For example, the theory of Al'tshuler *et al.*[28] only predicts that the $\frac{3}{2}\tilde{F}$ term in Eq. (1) should be replaced by $\frac{3}{8}\tilde{F}$ in the limit of strong spin-orbit scattering. (The triplet part of the e-e interaction in the diffusion channel is predicted to be suppressed.) On the contrary, Bergmann[29] found a barely noticeable change in F ($\approx 0.1-0.2$) between quench-condensed Mg films and Mg films covered by a small fraction of an atomic layer of Au (the films were in the strong spin-orbit scattering limit when covered by this amount of Au). Hickey *et al.*[30] reported a constant F (≈ 0.4) in amorphous Cu-Ti alloys with different levels of Au doping up to 6 at. %. As for our samples, since spin-orbit scattering is moderate in Ti-Al-(Co,Cr) alloys, our observation of negative values of $\tilde{F}$ must mainly arise from the large values of λ_{ph} rather than from strong spin-orbit scattering. For comparison, we have previously determined a $\tau_{so} \approx 8\times 10^{-12}$ s in Ti-Al and Ti-Al-Co alloys for Al composition in the range 4–15 at. %.[10]

To investigate the effects of spin-orbit scattering on the magnitude of F in a quantitative manner, we have followed the standard method by doping $Ti_{1-x}Al_x$ alloys with minor amounts of Au to form $Ti_{1-x}Al_{x-y}Au_y$ alloys. As usual, we focus on $x \approx 0.072$ and $0.005 \lesssim y \lesssim 0.04$. With a few at. % of Au doping, the samples should be already close to the strong spin-orbit scattering limit (i.e., the weak antilocalization regime), while their electronic structure is expected to be barely changed from that of $Ti_{1-x}Al_x$. This is (in part) confirmed by our observation of an essentially constant $T_c \approx 0.52\pm 0.03$ K in 11 Au-doped alloys. This result of a constant T_c is also in accordance with the theory of Fukuyama *et al.*,[31] who predicts that T_c should be unaffected by strong spin-orbit scattering in a disordered superconductor. The change in $\tilde{F}$, if any, can in this case be ascribed to a variation in τ_{so}^{-1} (and λ_{ph} remains basically constant). In Fig. 3, we plot the variation of $-\tilde{F}$ (symbols) versus the amount of Au-doped y. The dashed line is a guide to the eye. This figure indicates that $\tilde{F}$ *becomes more negative for larger values of* y, or equivalently, for larger values of τ_{so}^{-1}. As y is increased from 0.005 to 0.04, $\tilde{F}$ varies correspondingly from $\approx 0.01\pm 0.07$ to $\approx -0.27\pm 0.07$. Importantly, despite the value of $\tilde{F}$ being negative in Ti-Al-Au alloys, both the trend and the amount of change revealed in Fig. 3 are in accordance with the just discussed observation of Sahnoune *et al.*[15] (They observed $\tilde{F}$ decreasing from ≈ 0.55 to ≈ 0.0 as τ_{so}^{-1} was increased from $\approx 10^{11}$ to $\approx 10^{13}$ s^{-1}.) Note that this amount of change in $\tilde{F}$ is about *an order of magnitude smaller* than that which originates from the change in T_c or λ_{ph}, Fig. 2.

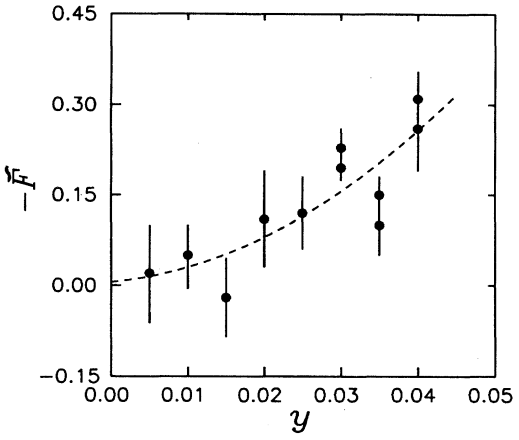

FIG. 3. Variation of the screened electron-electron interaction parameter $-\tilde{F}$ with the composition of Au in $Ti_{0.928}Al_{0.072-y}Au_y$ alloys. The dashed curve is a guide to the eye. Note that $\tilde{F}$ becomes more negative for larger values of y.

The agreement between our results and those of Sahnoune *et al.* provides a convincing consistency check of our experimental method for sample fabrication and data analysis. This in turn supports the assertion that our observation of the large variation of $\tilde{F}$ with T_c (λ_{ph}) in Fig. 2 is an intrinsic behavior of $\tilde{F}$. This is one of the key issues which the theory for F ($\tilde{F}$) should address.

V. CONCLUSION

We have shown that the temperature dependence of resistivity in superconducting Ti-Al-Co and Ti-Al-Cr alloys

above T_c can be accounted for in terms of electron-electron interaction effects in the presence of disorder. The values of the screened electron-electron interaction parameter $\tilde{F}$ are extracted. We find that electron-phonon coupling, including exchange of virtual phonons, is of crucial importance in determining the value of $\tilde{F}$. Our results, however, lead to the conclusion that, even though Eq. (1) predicts the correct functional dependence on temperature, the theory of $\tilde{F}$ in its current form, given by Eq. (3), needs to be refined. We also find in Ti-Al-Au alloys that strong spin-orbit scattering causes $\tilde{F}$ to assume slightly more negative value.

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