Exactly solvable multichain supersymmetric t-J model

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We have constructed a Bethe ansatz solution for the multichain quantum supersymmetric frustrated t-J model. The system is in the antichiral state for both spin and charge degrees of freedom. Elementary spin and charge excitations carry nonzero chiralities (topological charges) but they are confined into pairs with opposite signs of their chiralities, so the state of the system remains antichiral. Topological terms of Wess-Zumino nature in the Hamiltonian cause such chiral behavior. We conjecture that a multichain supersymmetric frustrated t-J model without topological terms in the Hamiltonian has charge and spin excitations with gaps.

In recent years physicists have been greatly interested in the low-temperature properties of low-dimensional strongly correlated electron systems. The discovery of high-temperature superconductivity in metal oxides has focused much attention on electron systems with repulsive interaction. Understanding of the mechanism by which superconductivity occurs is strongly connected with the correct description of low-energy excitations of such systems. Following Anderson,[1] we believe that superconductivity is connected with the interaction between spin and charge excitations of strongly correlated systems: spinons and holons. One can construct the special two-dimensional (2D) theory of superconductivity using, e.g., the concept of anyons carrying intermediate statistics between bosons and fermions;[2] anyons are specific to the $(2+1)$ quantum theories. A way to describe anyons is in adding the Chern-Simons topological term (violating T and P symmetries) to the Lagrangian of the system, which changes the statistics of excitations.[2] One of the intriguing features of such "topological" superconductivity in the 2D systems[3–6] is the connection between the chiral spin ordering and the superconductivity.[7,6] It is known that frustrated spin lattices with antiferromagnetic Heisenberg interaction reveal the main features of metal oxide behavior.[8] Some examples of real cuprates exist in which so-called ladders (multichain subsystems) divided from each other by frustrated spin bonds play an essential role and the conductivity for such systems is connected strongly with these ladders:[9,10] e.g., superconducting copper oxide $Sr_{2n-2}Cu_{2n}O_{4n-2}$, $(VO)_2P_2O_7$, and $La_{4+4n}Cu_{8+2n}O_{4n-2}$ contain weakly coupled arrays of metal oxide–metal ladders with spin frustrating bonds betweed them. These strongly correlated electron systems are of great interest now because of their intermediate properties between the 1D and 2D, which implies the better understanding of some aspects of high-temperature superconductivity in cuprates.

It is important to solve the low-dimensional quantum many-body problem exactly in order to understand the features of the 2D quantum strongly correlated electron systems (because of peculiarities in the density of states in low-dimensional systems quantum fluctuations are enhanced there). Theorists have to answer one essential question in high-temperature superconductivity (see, e.g., Ref. 1): whether elementary spin and/or charge excitations of the 2D strongly correlated repulsive electron systems have gaps and the quantum Anderson's disodered ground state emerges, or they are gapless and the ground state is ordered. We believe that the exact solution of a multichain strongly correlated electron model with spin frustration could help in answering this question. This paper is a generalization of Ref. 11, in which the exact solution for the multichain spin-1/2 frustrated magnet with the T and P symmetry violation was obtained. We study exactly, using the quantum inverse scattering method,[12] the multichain strongly correlated electron system with T and P violation. The model reveals some properties both of the 1D and 2D systems. We show that the system is in the antichiral charge and spin state for any values of an external magnetic field and for various band filling (except of trivial ferromagnetic ground state). Elementary spin and charge excitations (gapless for non-half-filled band and nonferromagnetic state) carry nonzero chiralities, but only pairs of them with opposite signs of their chiralities contribute to the ground-state energy without changing the total antichirality. We conjecture that for the multichain frustrated supersymmetric t-J model without T and P symmetry violation elementary charge and spin excitations have gaps even for non-half-filled band. Zhang and Rice[13] proposed the 2D t-J model for the description of high-temperature superconductors. The 1D supersymmetric version of the t-J model permits the exact solution using the Bethe ansatz.[14] In this paper we construct the multichain realization of the 1D supersymmetric t-J model. This is an attempt to describe exactly a multichain strongly correlated electron system and to show specific 2D topological features of spin and charge excitations for it.

We write the transfer matrix $\mathscr{T}(\lambda) = T(\lambda-\theta_1)T(\lambda-\theta_2)\cdots T(\lambda-\theta_L)$, and the Hamiltonian for the L supersymmetric t-J chains (L is even) with the inter- and intrachain interactions (the Hamiltonian is the logarithmic derivative of the transfer matrix as usual), see, also Ref. 11, which has the form

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$$\mathcal{H}=\sum_{r=1}^{L}\sum_{n}\left[\Pi_{X_{r,n}X_{r+1,n}}+\left(\prod_{i,k}\theta_{i,k}\right)\Pi_{X_{r,n}X_{r,n+1}}+\sum_{p<q}\theta_{p,q}^{-1}\left(\prod_{i,k}\theta_{i,k}\right)[\Pi_{X_{p,n}X_{q,n}},(\Pi_{X_{p,n}X_{p,n+1}}+\Pi_{X_{q,n}X_{q,n+1}})]+\cdots\right]+\text{const}, \tag{1}$$

where $T(\lambda)$ is the standard transfer matrix of a single supersymmetric t-J chain;[15] λ is the spectral parameter (this construction is the generalization of the multichain spin-1/2 magnetic frustrated model[11] for the system with internal degrees of freedom), $\theta_{p,q}$ are the interchain coupling parameters, $\theta_{i,k}=\theta_i-\theta_k$, $\theta_{L+k}=\theta_k$, Π_{XX} is the graded permutation operator for the supersymmetric t-J model[15] ($X_{r,n}$ is the Hubbard operator, see later), and square brackets denote commutator. In the first term we must replace $\Pi_{X_{L,n}X_{1,n}}$ with $\Pi_{X_{L,n}X_{1,n+1}}$, which means toroidal winding boundary conditions (see, also, Refs. 11 and 16), and we have omitted in Eq. (1) higher order in permutations terms. The third term and omitted (nonlocal) terms do not change classical equations of motion for spin and charge (they are total derivatives)—they are higher-order topological charge and spin lattice numbers of our multichain strongly correlated electron system. On the other hand, the structure of the third term and omitted terms of the Hamiltonian (1) is similar to the structure of higher moments for the supersymmetric t-J model, see Ref. 15. Let N_a be the number of sites on each chain, N the total number of electrons, M the number of electrons with down spins, and μ_j and λ_α the rapidities for the unbound electrons and spinons, respectively, which are determined from the Bethe ansatz equations in FFB (fermion-fermion-boson) grade. The ground state is formed by the bound pairs,[14] for which $\mu=\lambda\pm(i/2)$ and unbound electrons (we have M pairs and $N-2M$ unbound electrons). Thus the set of rapidities μ_j consists of $(N-2M)$ real values and M pairs of complex conjugated values (spin-paired electrons) $\mu_\alpha^{\pm}=\lambda_\alpha\pm(i/2)$ for the ground state. Taking it into account, the energy of the state is equal to

$$E=C+\sum_{r=1}^{L}\left[2\sum_{k=1}^{M}[(\lambda_k+\theta_{r,1})^2+1]^{-1}+\sum_{j=1}^{N-2M}[(\mu_j+\theta_{r,1})^2+(1/4)]^{-1}\right], \tag{2}$$

where C is the shift of the energy which does not depend on the interaction, and λ_k and μ_j are the solutions to the sets of equations

$$\prod_{r=1}^{L}[f(2(\mu_j-\theta_{r,1}))]^{N_a}=\prod_{r=1}^{L}\prod_{\beta=1}^{M}f(2(\mu_j-\lambda_\beta-\theta_{r,1})), \tag{3}$$

$$\prod_{r=1}^{L}[f(\lambda_\alpha-\theta_{r,1})]^{N_a}=\prod_{r=1}^{L}\prod_{k=1}^{N-2M}f(2(\lambda_\alpha-\mu_k-\theta_{r,1}))\prod_{\beta=1}^{M}f(\lambda_\alpha-\lambda_\beta), \tag{4}$$

where $f(x)=(x+i)/(x-i)$; $j=1,2,\ldots,N-2M$; $\alpha\neq\beta$; $\alpha=1,2,\ldots,M$.

It can be seen from Eqs. (3) and (4) that in the limit $\theta_{i,k}\to 0$ one has the 1D supersymmetric t-J chain with LN_a sites, and if $\theta_{i,k}\to\infty$ we describe L noninteracting supersymmetric t-J chains with N_a sites. We can solve Eqs. (3) and (4) in the limit $N,M,N_a\to\infty$, M/N_a, N/N_a being fixed; for the nonmagnetic case and for the band filling $N=LN_a$ the ground-state energy is minimal and equal to

$$E_0=C-N_a\sum_{r=2}^{L}\int_{-\infty}^{\infty}\frac{|1+\exp(i\omega\theta_{r,1})|^2}{1+\exp(-|\omega|)}d\omega. \tag{5}$$

To understand better chiral topological properties of the multichain model Eq. (1), let us consider the structure of the Hamiltonian for the simplest case of two supersymmetric t-J chains, $L=2$, which reveals some properties of the 2D (a number of papers are devoted to the quantum description of the strongly correlated multichain case[17] and especially to the two-chain problem,[18] most of them used Abelian bosonization technique[19]):

$$\begin{aligned}\mathcal{H}=-\sum_{n}\big(&8(H_{1,n,2,n}+H_{1,n,2,n+1})+4\theta^2(H_{1,n,1,n+1}+H_{2,n,2,n+1})\\&+8i\theta\{[X_{1,n}^{\tau\sigma}-X_{2,n+1}^{\tau\sigma}+\delta_{\tau,\sigma}(X_{1,n}^{00}-X_{2,n+1}^{00})]X_{1,n+1}^{\sigma 0}X_{2,n}^{0\tau}\\&+(X_{1,n}^{\sigma 0}-X_{2,n+1}^{\sigma 0})(X_{1,n+1}^{0\tau}X_{2,n}^{\tau\sigma}+X_{1,n+1}^{0\sigma}X_{2,n}^{00}-X_{1,n+1}^{\tau\sigma}X_{2,n}^{0\tau}-X_{1,n+1}^{00}X_{2,n}^{0\sigma})\\&+(X_{1,n}^{\sigma\tau}-X_{2,n+1}^{\sigma\tau})X_{1,n+1}^{\tau'\sigma}X_{2,n}^{\tau\tau'}-\text{H.c.}\}\big)+\text{const},\end{aligned} \tag{6}$$

where $H_{i,n,j,m}=(X^{\sigma 0}_{i,n}X^{0\sigma}_{j,m}+\mathrm{H.c.})-X^{\sigma\tau}_{i,n}X^{\tau\sigma}_{j,m}+X^{00}_{i,n}X^{00}_{j,m}$, $\sigma,\sigma',\tau,\tau'=1,-1$, $X^{a,b}_{1,2,n}$ are the Hubbard operators [we used Hubbard operators in terms of which the supersymmetric (permutation) properties could be seen obviously] $X^{ab}=|a\rangle\langle b|$, of the site electron $(a,b=0,1,-1)$ on the first or second chain in site n, and θ is the interchain coupling parameter. A classical 3D analog of the quantum two-chain strongly correlated electron system could be the double-layer fractional quantum Hall system, studied recently in Ref. 20, where excitations have specific (2D) topological properties like in our case. The third (topological) term in Eq. (6) could be originated from spin-orbit coupling for low enough temperatures, for which orbital degrees of freedom of electrons were frozen.[11] It breaks both T and P symmetries separately, the TP symmetry being conserved. The effect of this term for spin degrees of freedom of electrons is equivalent (for the nonmagnetic phase) to the topological magnetic charge (or Noether charge) for the chiral (magnetic) classical field. This term is equivalent to the time component of conserved topological current; the Lagrangian of the system's spin part has the usual form of the nonlinear σ-model Lagrangian (or CP^1 model) with the Wess-Zumino term. For a classical field the term, linear in θ, is an integer number[21,22] (Pontryagin index). On the other hand this term is connected with the (statistical) current known as the Chern-Simons term,[21] which is specific for the 2D systems. Note that in $D=1$ or 3 we cannot construct a scalar (or pseudoscalar) with the properties of the topological charge, e.g., for 3D we have the vector instead of this term, and can construct the Hopf invariant, which has integer values only.[21] Our pseudoscalar (topological charge) is the homotopy class characteristic $\pi_2(S^2)=\mathbf{Z}$;[22] it is the total time derivative, so it does not change classical equations of motion. The spin-charge and charge parts of the chiral term in the Hamiltonian (6) are total time derivatives too, so they do not change classical equations of motion. They are charge and spin-charge lattice quantum analogs of a spin topological charge, describing (charge) chiralities of a strongly correlated electron system. Another explanation of those terms nature may be a generalization of the well-known Shraiman-Siggia model;[23] see also, Ref. 24. Note that for all sites occupied with electrons the Hamiltonian of our model is equal, naturally, to one of the multichain spin-1/2 frustrated antiferromagnet.[11]

Equation (5) is an even function of the chirality parameter $\theta_{r,1}$. In previous papers[11,25] we named such states as "antichiral." We can see that the supersymmetric multichain t-J model reveals antichirality too in the ground nonmagnetic half–filled state. For the "ferromagnetic" case (with the nominal value of the magnetization of the system) the ground-state energy, naturally, is an even function of $\theta_{r,1}$ and spin chirality is equal to zero. But this state has trivial ferromagnetic ordering. If we apply a small magnetic field the magnetization of the system behaves like a spin-1/2 frustrated multichain system,[11] and 1D spin-1/2 antiferromagnet.[26] The ground-state energy remains an even function of the chirality parameter $\theta_{r,1}$, so we can conclude that the weak magnetic field does not change the antichirality of the system. Thus a small magnetic field does not change the number of spin topological excitations (instantons) which form the (spin and charge) Dirac seas of the system. For the nonmagnetic situation, if the band is almost half-filled, following Schlottmann,[14] we can see that at non-half-filling the model reveals the antichiral properties too.

There exist two types of low-energy excitations for the model. A charge excitation carries no spin, and spin excitation (spinon) is pure spin only at half-filling, like the 1D case.[27] Charge excitation has a gap at half-filling. For a charge hole, non-half-filled and nonmagnetic case, the energy is not an even function of θ, so a hole carries nonzero chirality (nonzero topological charge) $E_h=E_0+(\pi/2)\Sigma_{r=1}^{L}\mathrm{sech}[\pi(\lambda_0+\theta_{r,1})/2]$, where λ_0 is the hole rapidity (from this point of view, hole or particle carries topological chargelike spin instanton, similar to the quasiparticle in the fractional quantum Hall effect). But, analogously to the 1D case, the elementary charge excitation of the system is the particle-hole excitation: a particle carries topological charge of the opposite sign as to the hole's, so total chirality (or topological winding number) of a particle-hole charge excitation remains zero. For a simple spin doublet excitation (which is the quantum analog of a classical instanton solution) we have, following Refs. 28 and 27, $E=E_0+\pi\Sigma_{r=1}^{L}\mathrm{sech}[\pi(\mu_0+\theta_{r,1})]$, where μ_0 is the spinon rapidity. Two doublet excitations (spinons) carrying different chiralities (winding numbers with opposite signs) with both spin $S=\frac{1}{2}$ confines into singlet and triplet states, see Refs. 28 and 27, or the Dirac seas for holons and spinons follow excitations carrying nonzero charge and spin chiralities (or nonzero topological charges) in such a way that the total multichain correlated electron system remains antichiral. It is connected with the periodic (toroidal) boundary conditions in our case. Note that away from half-filling spin excitations carry nonzero charge. At half-filling, we can see that a hole is the pairing of a holon with a charge e and zero spin and a spinon with zero charge and spin 1/2, like the 1D case, see Ref. 27, carrying nonzero topological charge.

We have shown that the exact elementary excitations (doublet, singlet, triplet spin excitations, and particle-hole charge ones) of the Hamiltonian (1) are gapless. But the Hamiltonian (1) contains the θ-vacuum (topological) terms. The situation is analogous to Haldane's picture of a 1D antiferromagnetic spin chain (see, e.g., Refs. 29 and 30); Haldane argued that an integer spin chain has a gap, and a half-integer one has not due to the θ-vacuum term for the last case. Using reasoning analogous to Haldane's (see, also Ref. 11), we may suppose that the existence of the θ term in the Hamiltonian (1) implies the gapless behavior of our system, similar to the multichain spin-1/2 model.[11] The same difference takes place in the 2D classical Wess-Zumino model and the 2D classical nonlinear σ model.[31–33] Thus for the frustrated supersymmetric multichain t-J model without topological terms in the Hamiltonian we can conjecture that the spin and charge excitations do have gaps. In our case hole, particle, and doublet spin excitations change chiral properties of the system, because their energies are not even functions of $\theta_{r,1}$. In Ref. 11 we suggested that such excitations (from the classical point of view they are similar to instantons[34]) could form antichiral spin liquid. But the temperature does not change topological properties, so our strongly correlated electron system is in the antichiral state for any temperatures, like the multichain spin-1/2 frustrated magnet.[11] To change antichirality into chiral state one has to change the periodic boundary conditions into, say, free ones for which meronlike topological excitations could exist. The generalization of the results of this paper for the case of the multichain strongly correlated electron exactly solvable model with correlated pair hopping[35] with spin and charge chiralities (or T and P symmetry violation) will be reported elsewhere.

To summarize, we have studied the Bethe ansatz exact solution of the multichain t-J supersymmetric model. We have shown that the ground state of the model reveals antichiral spin and charge properties for any values of the external magnetic field and band filling (except for the trivial "ferromagnetic" case and the half-filled point for which the charge Fermi velocity is zero). Elementary excitations of the strongly correlated electron system are spinons and holons carrying nonzero spin and charge chiralities (or nontrivial spin and charge topology). Only pairs of such excitations with opposite spin and charge chiral numbers form the ground state of the system and contribute into the free energy. We cannot divide "charge" and "spin topological charges" (or chiralities) for our model, that is, undoubtedly, the lack of the considered model, but it reveals topological properties of the 2D, and has spin-charge separation (for the ground state and excitations). Elementary excitations of the model are gapless, but for the multichain t-J supersymmetric frustrated system without topological terms in the Hamiltonian (breaking T and P symmetries separately) we can conjecture that the elementary excitations (holons and spinons) have gaps.

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